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EFFECT OF ARMORING ON LOCAL SCOUR AROUND A CIRCULAR CYLINDER

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SYNOPSIS

A model to describe scouring process around a circular cylinder has been derived in order to apply it especially to the scouring in a graded sand bed, where sediment sorting process is inevitably accompanied with scouring. A combination of the model for scouring process and that for sediment transport for each grain size of sand mixtures makes it possible to describe scouring process and armor coat formation simultaneously. Some calculations based on the present model and some experiments clarify that the scouring around a cylinder is appreciably suppressed by armor-coat formation. Such a fact might be applied to reducing scour depth. A method to mingle coarser materials and the original fine sand bed materials around a bridge pier is investigated, and it is clarified that it works effectively by the present model and laboratory experiments.

INTRODUCTION

Because of recent severe change in river environments (river bed degradation, river-improvement works, etc.) and the subsequent increase of disasters of hydraulic structures (including bridges) caused by local scour, it becomes more important to investigate the mechanism of local scour. A lot of research works have been done (these are reviewed in the literatures (4, 11, 14, 17)), but some of them are somehow empirical and some have limitations in their applications. Recent change of river environments make it difficult for them to describe essential characteristics of actual phenomena as causes of disasters. Hence, it is required to derive a model to describe the scouring process which inherits both physical soundness and simplicity for applications to several variations of river environments.

The authors believe that a model proposed by Tsujimoto & Nakagawa (19, 21) somehow satisfies the aforementioned requisites. It expresses the essential characteristics of scouring process and it is so simple to follow the changes of surrounding conditions around scouring. In fact, it has been applied to a prediction of fluctuation of scour depth due to dune migration which is significant to estimate the maximum scour depth (19), and to an estimation of the effect of protection works (20). One of the remaining important problems related to the local scour is an evaluation of the effect of the gradation of bed materials. In a graded sand bed, scouring process inevitably accompanies a sediment sorting process, and it can be easily imagined that armoring may suppress the scouring (see Photo. 1)

As for behaviors of graded bed materials, a model to describe armor coat formation was derived by Nakagawa-Tsujimoto-Hara (15). It is constituted by pick-up rate and step length defined for each grain size and it is still available in a scour hole where sediment transport is non-equilibrium. A combination of the aforementioned models for scouring and sediment transport of graded materials might promise us a success in quantitative evaluation of the effect of armoring on local scour around a pier.

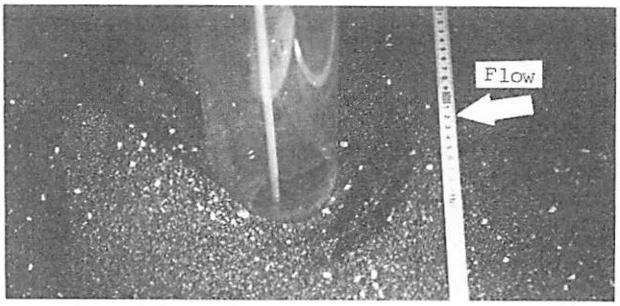

Photo 1 Sediment sorting around a pier

Previously, a few works were done to evaluate the effect of armoring on the local scour around a bridge pier. Nakagawa & Suzuki (12) applied the method to estimate the grain size distribution of armor coat derived by Gessler (7), but it could not essentially describe the temporal change of the process but the final stage. On the other hand, Nakagawa-Otsubo-Nakagawa (10) reasonably applied the model of armor-coat formation proposed by Nakagawa-Tsujimoto-Hara (15), but they studied the scour around a four-cornered pier, where no analytical model for scouring was developed.

In this paper, an analytical model to describe the scouring process is prepared specially for application to a graded sand bed, which is principally similar to that proposed by Tsujimoto-Nakagawa (19, 21) but somehow modified in order to be coupled with the model for the change of bed composition derived by Nakagawa-Tsujimoto-Hara (15). The characteristics of individual sediment motion for each grain size are evaluated by the method used in the model originally proposed by Nakagawa-Tsujimoto-Hara (15). Some calculated examples are shown based on the combined model to emphasize the effect of armoring on scouring process, and they suggest an idea to reduce scour depth around a pier by intermixing coarser materials to the original fine bed materials. Although the bed near the pier is sometimes covered by very rough materials for protection work, the present method uses comparatively small but coarse materials and is essentially different from covering protection. The effect of such a device is inspected both analytically and experimentally.

MODELLING OF SCOURING PROCESS

Several researchers succeeded in prediction of the equilibrium scour depth by macroscopic modelling of scouring (5, 9). Figure 1 shows a control volume for which they considered the continuity of sediment transport. In such a model, however, we have to ambiguously estimate the sediment transport rate because the local effect is predominant within a comparatively broad control volume. On the other hand, Nakagawa & Suzuki (11) and Tsujimoto & Nakagawa (19,

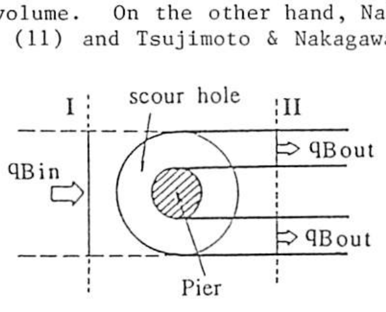

Fig.1 Macroscopic control volume for sediment balance

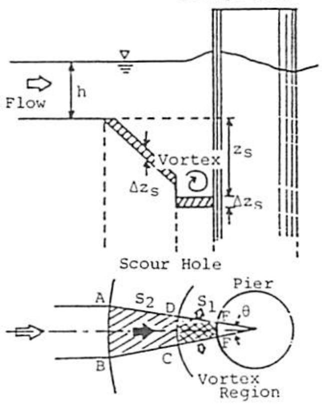

Fig.2 Microscopic control volume for sediment balance

21) proposed a model using more microscopic control volume to consider the continuity of sediment transport. Since the scour hole is kept geometrically similar through a development of scour depth as clarified experimentally (11), we can simply focuss our attention on the variation of the scour depth at the front foot of a pier instead of that of the whole volume of the scour hole. When the flow and the sediment transport in the stagnant plane (see Fig. 2) are focussed, they are clearer and easier to analyze than those considered in the former macroscopic control volume. In the models of Nakagawa & Suzuki (11) and Tsujimoto & Nakagawa (21), the region AEFB is considered. Furthermore, a more microscopic model is here adopted because only the so-called clear water scour is considered (the armoring effect is appreciable for the case of clear water scour) and even in the inside of the region ABFE the sediment sorting process is different between ABCD and CDEF (in only the region CDEF, the bed becomes armored). The difference between sediment amount supplied through the arc CD and that transported away through DE and CF (see Fig.2) is here considered and it determines the progress rate of scouring.

Since almost all the sediments picked-up from the region CDEF are transported away from this region immediately through DE and FC, the volume of sediment-out during the time interval $(t,t+\Delta t)$ is expressed as

$$\Delta V_1 \equiv \sum_{i=1}^{N} \Delta M_i(t) A_3 d_i^3 = (A_3/A_2) \sum_{i=1}^{N} p_{si}(t) p_i(t) d_i \cdot S_1 \Delta t \tag{1}$$

in which $\Delta M_i(t)$=the number of sand particles of the i-th fraction picked-up from the region CDEF during the time interval $(t,t+\Delta t)$; A_2, A_3=2- and 3-dimensional geometrical coefficients of sand; S_1=horizontally projected area of CDEF; p_i=the volumetric fraction of sand of which diameter is represented by d_i (p_i equals the areal fraction of sand of d_i at the surface (15)); N=the number of fractions of sand; and p_{si}=pick-up rate of sand of the i-th fraction from the region CDEF.

In the case of clear water scour, the sediment inflow through the arc AB is absent, but the sliding of sand along the slope S_2-S_1 like an "avalanche" to keep the angle of repose supplies sediments to the region CDEF through the arc CD. This amount of sediment supply during the time interval $(t,t+\Delta t)$ is expressed by

$$\Delta V_2 \equiv \sum_{i=1}^{N} \Delta Q_i(t) A_3 d_i^3 = (1-\rho_0)(S_2-S_1)\Delta z_s \tag{2}$$

in which $\Delta Q_i(t)$=the number of sand particles of the i-th fraction supplied to the region CDEF during $(t,t+\Delta t)$; Δz_s=the bed descending of the scour hole during $(t,t+\Delta t)$; ρ_0 =porosity of sand; and S_2=horizontally projected area of ABFE. When p_{i0}=the original (initial) fraction of sand of d_i, ΔQ_i can be expressed as follows:

$$\Delta Q_i = (1-\rho_0)(S_2-S_1) p_{i0} \Delta z_s / A_3 d_i^3 \tag{3}$$

Meanwhile, the continuity of sediment leads us the following equation.

$$\Delta V_1 - \Delta V_2 = (1-\rho_0) S_1 \Delta z_s \tag{4}$$

And thus,

$$dz_s/dt = \{A_3 S_1 / [A_2 S_2 (1-\rho_0)]\} \sum_{i=1}^{N} p_{si}(t) p_i(t) d_i \tag{5}$$

The temporal variation of scour depth can be described by the above equation. In the case of uniform sand, the above equation becomes

$$dz_s/dt = \{A_3 S_1 / [A_2 S_2 (1-\rho_0)]\} p_s(t) \cdot d \tag{6}$$

and this is quite identical to that deduced from the model where another control volume is considered (19).

Accompanying the progress of scouring, the sediment sorting occurs and $p_i(t)$ changes. Therefore, the variation of the composition of bed materials exposed at the surface should be simultaneously analyzed. When the number of the i-th fraction sand exposed at the surface of the region CDEF at the time t is represented by $n_i(t)$, the following equation can be obtained.

$$n_i(t+\Delta t) = n_i(t) - \Delta M_i(t) + \Delta Q_i(t) + \Delta R_i(t) \qquad (7)$$

in which $\Delta R_i(t)$=the number of the i-th fraction sand newly exposed at the surface as a result of sediment pick-up and supply during $(t, t+\Delta t)$, and it is expressed as follows:

$$\Delta R_i(t) = \sum_{j=1}^{N}\{[\Delta M_j(t) - \Delta Q_j(t)]d_j^2\}\cdot p_{i0}/d_i^2 \qquad (8)$$

This equation means that the newly exposed bed surface has the originally given gradation curve of bed materials.

$n_i(t)$ and $p_i(t)$ are related each other as

$$n_i(t) = S_1 p_i(t)/(A_2 d_i^2) \qquad (9)$$

$$p_i(t) = n_i(t)d_i^2/[\sum_{j=1}^{N} n_j(t)d_j^2] \qquad (10)$$

By Eqs. 5 and 7, the temporal variations of the scour depth and bed composition in the region CDEF can be simultaneously described.

SCOUR-HOLE GEOMETRY AND BED SHEAR STRESS IN VORTEX REGION

The flow structure around a bridge pier is determined by the geometry of the scour hole, and the scour hole formed in a bed composed of sand mixtures is almost similar to that formed in a uniform sand bed. In the other words, the flow structures with the same values of (h_0/k_s) and (h_0/D) (h_0= undisturbed flow depth; k_s=equivalent sand roughness of the bed; and D=pier diameter) are determined only by the dimensionless scour depth $\zeta=z_s/D$. Hence, the flow model developed for local scour in the case of uniform sand can be applied here without any modification. Furthermore, the vortex scale is determined by the pier diameter, and the angle of repose of sand is almost invariant irrespectively of grain size. Hence, S_1 and S_2 in the present model can be also given by the model for uniform sand.

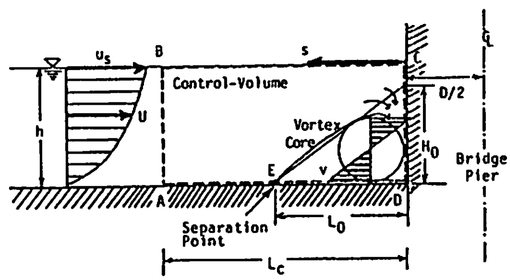

Fig.3 Control volume to estimate circulation of vortex

Shen-Schneider-Karaki (18) suggested that the difference between the circulation along ABCD in the stagnant plane (see Fig. 3) without pier and that with pier was redistributed as the circulation of the vortex formed at the front foot of a pier. Applying the potential flow theory to the horizontal flow field around a pier, the change of the circulation is expressed as

$$\Delta\Gamma = -u_{s0}D/2 \qquad (10)$$

in which u_{s0}=undisturbed surface flow velocity.

The scale of the separation zone represented by the triangular CED (see Fig. 3) has been clarified experimentally (10) and the radius of the vortex core, r_0, has been estimated as follows by identifying it with that of an inscribed circle of this triangular as shown in Fig. 3.

$$\Omega_0 \equiv r_0/D \simeq 0.183 \qquad (12)$$

If the change of the circulation is quitely redistributed to the vortex core, the bed shear stress due to the vortex before scouring, τ_{v0}, can be related to the bed shear stress in the undisturbed region, τ_0 , as follows:

$$\gamma_0 \equiv \tau_{v0}/\tau_0 = \{\phi_s/[4\pi\Omega_0\phi_v]\}^2 \tag{13}$$

in which $\phi_s = u_{s0}/u_{*0}$; $u_{*0} = \sqrt{\tau_0/\rho}$; $\phi_v = v/u_{*v}$; v=the edge-velocity of the vortex core; $u_{*v} = \sqrt{\tau_v/\rho}$; and ρ=mass density of fluid. Although ϕ_v in Eq. 13 cannot be exactly evaluated, the experimental data of direct measurements of bed shear stress around a pier (8, 11) suggested that $\gamma_0 = 1.5\sim2.0$.

The bed shear stress in the vortex region, τ_v , changes with the progress of scouring, but unfortunately we have few informations about it. Thus, the diffusion of the vortex with enlargement of the scour hole is here assumed to be simply expressed as

$$\Omega(\zeta) \equiv r(\zeta)/D = \Omega_0(1+k_\omega\zeta) \tag{14}$$

in which k_ω=empirical constant. Baker (2) assumed the above in his analytical model to predict the scour depth, too. Then, the change of bed shear stress due to vortex with the progress of scouring is written as

$$\psi_\tau(\zeta) \equiv \tau_v(\zeta)/\tau_{v0} = (1+k_\omega\zeta)^{-2} \tag{15}$$

An empirical constant k_ω is roughly determined as follows: The final scour depth z_{sf} can be practically obtained from the following relationship.

$$\tau_c/\tau_{v_0} = [1+k_\omega(z_{sf}/D)]^{-2} \tag{16}$$

in which τ_c=critical tractive force, and thus,

$$\zeta_f \equiv z_{sf}/D = (\sqrt{\gamma_0\eta} - 1)/k_\omega \tag{17}$$

in which $\eta \equiv \tau_0/\tau_c$. Considering the fact that the equilibrium depth of scour with continuous sediment motion is approximately 90% of the practically final scour depth at η=1 (17), and that the equilibrium scour depth is about 1.5 times pier diameter (3), k_ω can be roughly estimated to be 1/7 (19).

Figure 4 illustrates a general side view of scour hole around a cylindrical pier. Fig. 4(b) shows a typical geometry as aforementioned; while at early stage of scouring, the slope of the angle of repose of sand cannot appear and the scour hole looks like that shown in Fig. 4(a). The transitional scour depth from the type (a) to (b) is given as follows (19, 21):

$$\zeta_1 \equiv z_{s1}/D = \Omega_0/(1-k_\omega\Omega_0) \tag{18}$$

The characteristic scales of scour hole, L_1 and Λ_s (see Fig. 4), are easily obtained as follow (19, 21):

$$\lambda_1(\zeta) \equiv L_1/D = \Omega_0(1+k_\omega\zeta)(1+\sin\theta_c) \tag{19}$$

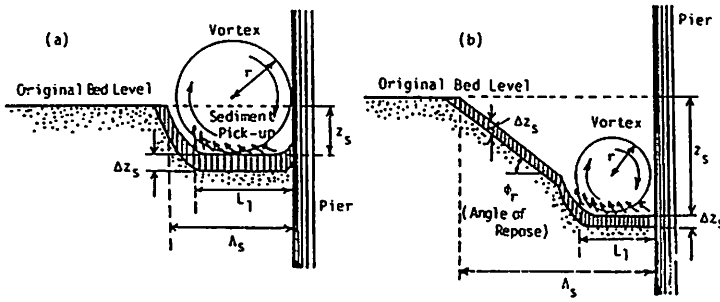

Fig.4 Geometrical properties of scour hole at front foot of a pier

$$\theta_c \equiv \begin{cases} \pi/2 & [\chi(\zeta) > 1] \\ \arcsin\chi(\zeta) - \phi_r & [\chi(\zeta) \leqq 1] \end{cases} \tag{20}$$

$$\chi(\zeta) \equiv (\gamma_0 \eta / k_3)(\sin\phi_r) \cdot \psi_\tau(\zeta) \tag{21}$$

$$\lambda_s(\zeta) \equiv \frac{\Lambda_s}{D} = \begin{cases} \Omega(\zeta)\{1+\sqrt{\zeta[2\Omega(\zeta)-\zeta]}/\Omega(\zeta)\} & (0 < \zeta < \zeta_1) \\ [\cot\phi_r + (2-\cot\phi_r)k_\omega \Omega_0]\zeta + (2-\cot\phi_r)\Omega_0 & (\zeta \geqq \zeta_1) \end{cases} \tag{22}$$

in which ϕ_r=angle of repose of sand, and $k_3 \fallingdotseq 1.0$. The quantities, S_1 and S_2, involved in Eqs. 1 and 2 are calculated as follows:

$$S_1 = (\lambda_1+1)\lambda_1 \theta D^2/2 \tag{23}$$

$$S_2 = (\lambda_s+1)\lambda_s \theta D^2/2 \tag{24}$$

in which θ=a small angle as shown in Fig. 2 but it is eliminated in analyses of equations.

SEDIMENT PICK-UP RATE FOR EACH GRAIN SIZE

The most contributive work as for sediment transport of graded bed materials was done by Egiazaroff (6), and he proposed the concept of the critical tractive force for each grain size. According to his idea,

$$\tau_{*ci}/\tau_{*c0} = [\ln 30.1(\bar{z}_0/k_{s0}) / \ln 30.1(\bar{z}_i/k_{sm})]^2 \tag{25}$$

in which τ_{*ci}=dimensionless critical tractive force for the i-th fraction of sand; τ_{*c0}=dimensionless critical tractive force of uniform sand; $\bar{z}$=representative height of sand particles on the bed; k_{sm}= equivalent sand roughness of a bed composed of graded materials; and the subscript 0 indicates the values for uniform sand. As easily seen from Eq. 25, the logarithmic law is applied for flow velocity profile. Even in the vortex region, it is assumed to be valid in the region very near the bed. Although k_{sm} cannot always be identified with the mean diameter d_m (16), it is somehow averaged value of d_i and here it is simply assumed that $k_{sm}=d_m$. Namely,

$$k_{sm}(t) = d_m(t) = \sum_{i=1}^{N} d_i p_i(t) \tag{26}$$

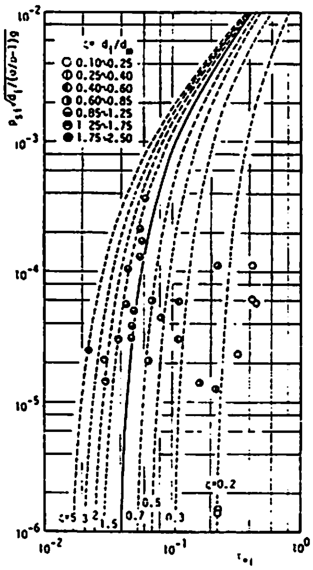

Fig.5 Pick-up rate for each grain size

On the other hand, the representative height is certainly well in proportion to the individual diameter as investigated by numerical simulation (16), and the coefficient of proportionality a_0 is about 0.63 as originally proposed by Egiazaroff (6). Moreover, Ashida & Michiue (4) modified Eq. 25 for finer part of sand mixtures ($d_i/d_m < 0.4$) mainly because of the difference of incipient motion of them. Concludingly, the following is here used instead of Eq. 25.

$$\tau_{*ci}/\tau_{*c0} = \begin{cases} [\ln 30.1a_0 / \ln(30.1a_0 d_i/d_m)]^2 & (d_i/d_m \geqq 0.4) \\ 0.85 d_m/d_i & (d_i/d_m < 0.4) \end{cases} \tag{27}$$

The concept of the critical tractive force for each grain size can be reasonably applied to evaluation of pick-up rate for each grain size of sediment mixtures, and the formula for pick-up rate of uniform sand established by Nakagawa & Tsujimoto (13) can be modified as follows:

$$p_{s*i} = 0.03\tau_{*i}[1-(0.7\tau_{*ci}/\tau_{*i})]^3 \quad (28)$$

in which $p_{s*i}=p_{si}\sqrt{d_i/(\sigma/\rho-1)g}$; σ=mass density of sand; $\tau_{*i}=u_*^2/[(\sigma/\rho-1)gd_i]$; u_*= shear velocity; and g=gravity acceleration.

As shown in Fig. 5, the above equation may make a somehow appreciable error for each grain size, but the essential characteristics of "selective pick-up" can be sufficiently well demonstrated by this equation and its applicability to description of armor-coat propagation for uniform flow have been confirmed (15).

EFFECT OF GRADATION CURVE OF BED MATERIALS ON SCOURING PROCESS

In order to inspect the effect of gradation of bed materials and armoring on scouring process around a circular cylinder, some calculations are shown based on the present model. The gradation curves of bed materials used for the present calculations are shown in Fig. 6. 4 kinds of bed materials have the same median diameter (d_{50}) but different values of $\sigma_d=\sqrt{d_{84}/d_{16}}$ (d_r=the size for which r% of the bed materials is finer).

In Fig. 7, some examples of the calculated temporal variation of the scour depth at the front foot of a cylinder are shown, where the scour depth and the elapsed time are dimensionlessed as z_s/D and $t\sqrt{(\sigma/\rho-1)/d_{50}}$, respectively. On the calculations, D/d_{50}=100, and σ_d and η (calculated for d_{50}) are changed as parameters. It is clearly seen from this figure that the progress of scouring is much retarded and the final scour depth is much suppressed with the increase of σ_d. These facts are caused by armor-coat formation at the front foot of a pier as easily confirmed by Fig. 8, where some examples of the calculated temporal change of gradation curve of bed materials exposed at the surface of the vortex region are shown. Moreover in Fig. 9, the decrease of the final scour depth against σ_d is demonstrated.

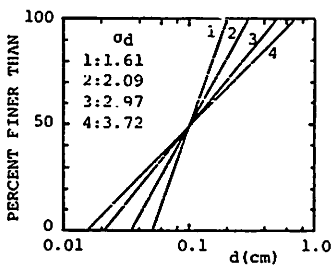

Fig.6 Several sand mixtures for numerical calculation

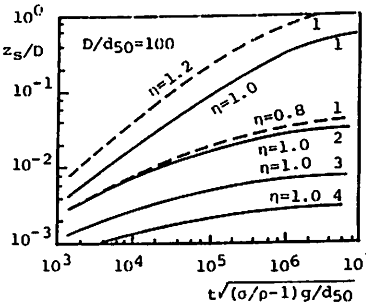

Fig.7 Calculated example of progress of scour depth

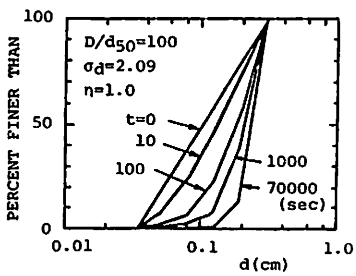

Fig.8 Calculated example of armor-coat formation

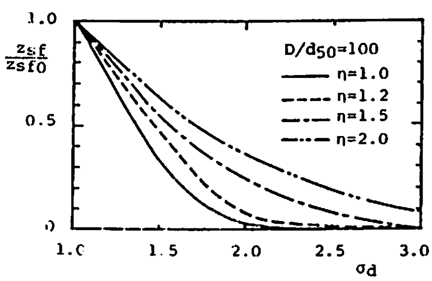

Fig.9 Relation between final scour depth and σ_d

REDUCING SCOUR DEPTH BY INTRUSION OF COARSER MATERIALS INTO ORIGINAL FINE BED MATERIALS NEAR A PIER

Local scour around a cylinder in the case of graded bed materials is considerably suppressed compared with that in the case of uniform sand, and this fact suggests us a reducing work of scour depth around a bridge pier by intermixing of coarser materials in the original fine bed materials near the pier. The region where a scour hole may develop without a scour reducing work should be replaced by the mixtures, and it might be possible to use such an occasion as this region is often dredged and refilled up for other protection works or maintenance of the pier.

The effect of the intermixing of coarser materials on scouring around a pier can be easily examined by the presently derived analytical model. Fig. 10 shows the calculated examples to demonstrate the effect of the intermixing of coarser materials, in which p_{21} is the volumetric ratio of coarser materials to the total materials. An intermixing of a few volumetric percent of coarser materials of which diameter is merely twice the original bed materials reduces the scour depth appreciably. The change of the final scour depth by intermixing of coarser materials (d_2) to the original bed materials (d_1) is depicted against the volumetric ratio of new materials to the total materials in Fig. 11, with (d_2/d_1), (D/d_1) and η (calculated for d_1) as parameters. This figure also demonstrates the intermixing effect.

In order to verify the aforementioned effect as well as the applicability of the present model for scour in graded bed materials, a laboratory experiment was conducted. The experiment was carried out in the 0.23m wide, 12m long flume. The model

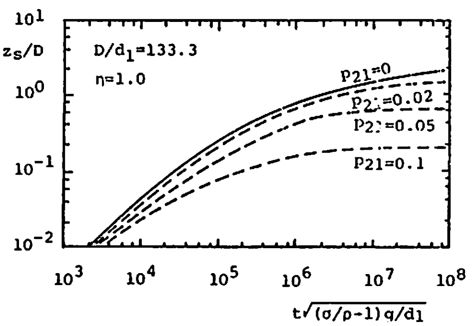

Fig.10 Effect of intermixing of coarser materials on scouring

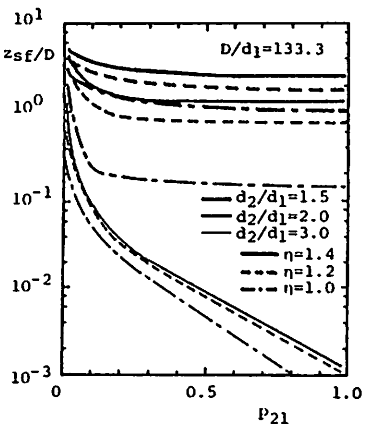

Fig.11 Decrease of final scour depth by intermixing

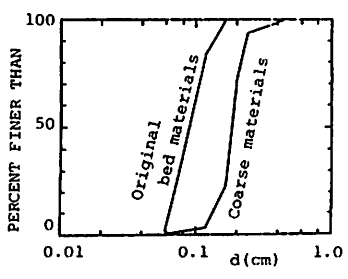

Fig.12 Gradation curves of original and mixing materials

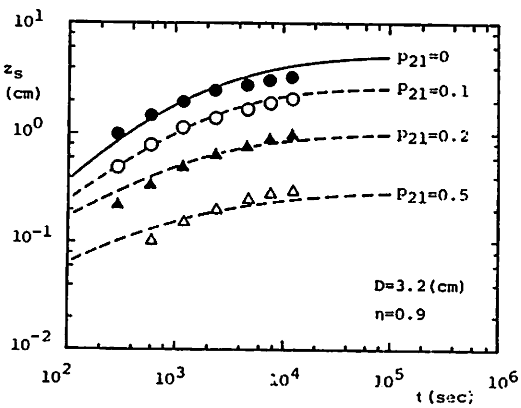

Fig.13 Temporal change of scour depth at front foot of a pier

of a pier was a circular cylinder of which diameter is 3.2cm. The original and intermixed materials have gradation curves shown in Fig. 12, and their median diameters are d_1=0.089cm and d_2=0.183cm, respectively. The bed slope was adjusted as i_b=0.002, and the uniform flow depth (h_0) was kept to be 3.30cm. The volumetric ratio of coarser materials, p_{21}, was changed as 0, 0.1, 0.2, 0.5 and 1.0. The cases p_{21}=0 and p_{21}=1.0 correspond to those for uniform sand of d=d_1 and d=d_2, respectively.

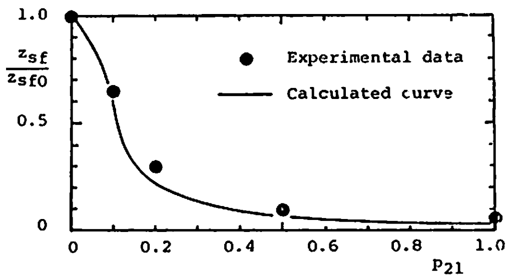

Fig.14 Suppressing effect of mixing materials on final scour depth

The temporal changes of scour depth at the front foot of the cylinder are shown in Fig. 13 with the calculated curves based on the present model. On the calculation, γ_0 was experimentally determined as 1.8 according to the experiments for uniform sand. Moreover, the experimental data of the change of the final scour depth with the intermixing ratio of coarser materials are compared with the analytically expected curve in Fig. 14. These show good agreements, and the effect of the proposed scour-depth reducing work and the applicability of the proposed analytical model for local scour in graded sand beds have been confirmed.

CONCLUSIONS

The results obtained in this study are summarized below:

(1) Taking notice of the sorting process and the applicability of the present model for local scour around a circular cylinder, a microscopic control volume for which the continuity of sediment for each grain size is considered has been prepared to make a model to describe scouring process and subsequent armoring process simultaneously. The requisite parameters involved in the present model have been determined by comparing them with those used in the researches of scouring in a uniform sand bed (19) and armor coat propagation for uniform flow (15).

(2) The calculated results based on the present analytical model, it has been somehow quantitatively estimated that scouring in a graded sand bed is retarded than that in a uniform sand bed and that the final scour depth is suppressed as a result of armoring effect.

(3) The facts mentioned in (2) suggest us a protection work to suppress the scour around a bridge pier by intermixing coarser materials into the original bed materials, and its effect can be evaluated by the presently derived analytical model for scouring.

(4) A laboratory experiment to simulate the above mentioned protection work has been carried out. The comparison of the experimental data and the calculated results based on the present model shows good agreements, and the availability of the proposed scour-depth reducing work and the applicability of the present analytical model of local scour in a graded sand bed have been satisfactorily confirmed.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A_2, A_3 = geometrical coefficients of sands;

D = pier diameter;

d_i = sand diameter of the i-th fraction of sand mixtures;

d_r = the size for which r% of sediment is finer;

g = gravitational acceleration;

h = flow depth in the undisturbed region;

k_3, k_ω = empirical constants;

k_s = equivalent sand roughness;

L_1 = horizontal scale of the vortex region at the front foot of a pier;

N = number of fractions of sediment mixtures;

n_i = number of particles of the i-th fraction exposed at the surface in the vortex region;

p_i = fraction of sand of which diameter is d_i exposed at the surface in the vortex region;

p_{i0} = fraction of sand of which diameter is d_i in the original bed materials;

p_{si} = sediment pick-up rate of the i-th fraction of sands;

p_{s*} = dimensionless pick-up rate $(=p_s\sqrt{d/(\sigma/\rho-1)g})$;

r, r_0 = the radius of vortex core formed at front foot of a pier and its value without scouring;

S_1 = horizontally projected area of CDEF in Fig. 2 (vortex region);

S_2 = horizontally projected area of ABFE in Fig. 2;

t = elapsed time;

u_{s0} = surface velocity of undisturbed flow;

u_{*0}, u_{*v} = shear velocities of undisturbed flow and the vortex;

v = flow velocity at the outer edge of the vortex core;

z_s = scour depth;

z_{s1}, z_{sf} = transitional and final scour depths;

ΔM_i = the number of sand particles of the i-th fraction picked-up from the bed in the vortex region during $(t,t+\Delta t)$;

ΔQ_i = the number of sand particles of the i-th fraction supplied into the vortex region during $(t,t+\Delta t)$;

ΔR_i = the number of sand particles of the i-th fraction newly exposed at the surface in the vortex region during $(t,t+\Delta t)$;

ΔV_1, ΔV_2 = volume of sediment out and that of sediment supply, from or into the vortex region during $(t,t+\Delta t)$;

$\Delta\Gamma$ = the change of flow circulation due to the presence of a cylinder;

γ_0 = the initial ratio of bed shear stress in the vortex region to that in the undisturbed region;

Λ_s = horizontal scale of scour hole;

λ_1, λ_2 = L_1/D and Λ_s/D;

ϕ_r = angle of repose of sand;

ϕ_{s0}, ϕ_v = u_{s0}/u_{*0} and v/u_{*v};

$\psi_\tau(\zeta)$ = the ratio of the bed shear stress in the vortex region with scouring to that without scouring;

ρ = mass density of fluid;

ρ_0 = porosity of sand;

σ = mass density of sand;

σ_d = $\sqrt{d_{84}/d_{16}}$;

τ_0, τ_v = bed shear stress in the undisturbed region and that in the vortex region;

τ_c = critical tractive force;

τ_* = $u_*^2/[(\sigma/\rho-1)gd]$=dimensionless bed shear stress;

τ_{*i} = $u_*^2/[(\sigma/\rho-1)gd_i]$;

τ_{*c0}, τ_{*ci} = dimensionless critical tractive force for uniform sand and that for each grain size;

η = τ_0/τ_c; and

ζ = z_s/D.