Journal of Hydroscience and Hydraulic Engineering
Vol. 5, No. 2, February, 1988, pp.53-64

A DAILY RAINFALL-RUNOFF MODEL FOR A MOUNTAINOUS BASIN

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SYNOPSIS

A new daily rainfall-runoff model for a mountainous basin is proposed in this paper. Its parameters are readily determined by hydrographic measurements. It is simple, relatively clear in its physical meaning, and includes all runoff components. In the model, a basin is considered to be composed of two parts: the saturated area and the infiltration area. Direct runoff parameters are determined from the relationship between storm rainfall and direct runoff. The groundwater runoff is expressed by the storage function and its parameter is determined from base-flow recession curves. In addition, moisture in excess of normal soil moisture is considered to become groundwater recharge. The applicability of the model is assessed by comparing it with observed data from an experimental basin in Japan.

INTRODUCTION

Daily rainfall-runoff models are considered to be important for water resources planning and management of river flow. Many daily rainfall-runoff models have been proposed and shown to be adaptable to real basins. However, almost all these models require trial-and-error process for parameter calibration, and consequently it takes a long time for the users to become familiar with these models and be able to use them.

In this paper, a new rainfall-runoff model without trial-and-error parameter calibration is proposed. The model is based on the hydrological model constructed by Ando, Musiake, and Takahasi (1). It is simple, relatively clear in its physical meaning, and includes all runoff components. And the suitability of the model is examined by applying the model to Uratukuba experimental basin (Yamaguti River basin) in Japan.

HYDROLOGICAL PROCESS OF NATURAL MOUNTAINOUS BASIN

The schematic diagram of hydrological processes is depicted in Fig.1, which is based on the study by Ando, Musiake, and Takahasi (1). The basin is considered to be composed of two parts. One part is a saturated area and the other is an infiltration area. The saturated area includes the channel surfaces of the stream system, lakes, swamps etc. The infiltration area consists of tops and slopes which usually have plants and soils. Then, if rainfall is little, it infiltrates into soils in the infiltration area. Precipitation onto the infiltration area (P_i) infiltrates into the soil except the part referred to as crowninterception (C), which is the portion caught by plants. This amount caught by plants finally evaporates. Rainfall infiltrating into the soil (I) is divided into three parts. The first part is the storaged water in the unsaturated zone (M_s), the second part is the direct runoff from the infiltration area (D_i), and the third part is the groundwater recharge (G). A portion of the rainfall stored in the unsaturated zone

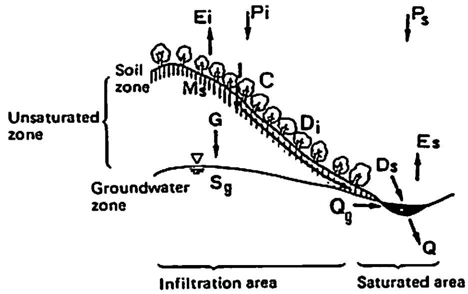

Fig. 1 Conceptual diagram of runoff process in a mountainous basin

contributes to the evapotranspiration from the infiltration area (E_i). The groundwater recharge (G) increases the groundwater storage (S_g), and the groundwater runoff (Q_g) results from the groundwater storage (S_g). On rainless days, the groundwater runoff becomes the only source of river flow. The rainfall onto the saturated area (P_s) becomes direct runoff (D_s). There is also evapotranspiration from the saturated area (E_s). Accordingly, the total river discharge (Q) can be given by the following expression.

$$Q=Q_g+D_i+D_s-E_s \tag{1}$$

DAILY RAINFALL-RUNOFF MODEL

A daily rainfall-runoff model for a mountainous basin is shown in Fig.2. In Fig.2, variables are expressed with capital letters and parameters are expressed with small letters.

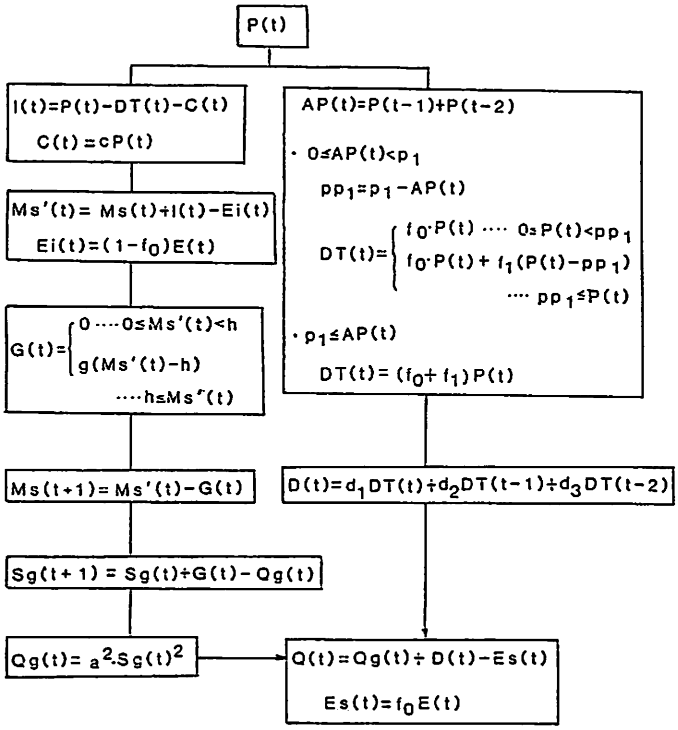

Fig. 2 Flow chart of the daily rainfall-runoff model

Flow Chart of Daily Rainfall-runoff Model

(1) Calculation of daily rainfall P(t)

If there are several daily rainfall data in a basin, daily rainfall P(t) may be obtained by Thiessen method.

(2) Estimation of coefficient of crown interception c

Annual potential evapotranspiration EH calculated by Hamon (4)'s equation vary little. However, according to observations, annual loss, E_Y, has a tendency to increase with annual precipitation, P_Y. Therefore, the author believes that a constant ratio of daily precipitation P(t) becomes crown interception, and the ratio is equal to coefficient of crown interception c. This "c" is equal to a slope of the line of P_Y-E_Y relation.

(3) Estimation of coefficient of evapotranspiration e and calculation of daily evapotranspiration E(t)

Hamon (4) pointed out that Thornthwaite's formula was too large in summer and too small in winter. Moreover, he proposed an Eq. 2 which can accommodate to the variation of evapotranspiration from real basins. Therefore, Hamon's formula is adopted in this paper.

$$E_{pi}=0.14D_{oi}{}^2P_{ti} \tag{2}$$

In which E_p represents the average potential evapotranspiration in mm per day; D_o is the possible hours of sunshine in units of 12 hr; P_t is the saturated water vapor density (absolute humidity) at the daily mean temperature in grams per cubic meter. Using this formula, monthly evapotranspiration E_{pi} and annual evapotranspiration EH are obtained. Since from annual precipitation P_Y, the portion of crown interception (cP_Y) evaporates besides usual evapotranspiration, annual loss E_Y can be expressed as follows.

$$E_Y=eEH+cP_Y \tag{3}$$

Where, "e" is the coefficient of evapotranspiration. When calculating E_Y, change of basin storage must be considered, but in the case of basins with little snowfall the change of basin storage may be negligible. If annual runoff is written as Q_Y, E_Y may be expressed as follows.

$$E_Y=P_Y-Q_Y \tag{4}$$

Therefore, values of E_Y, EH, c, P_Y are known, and so "e" can be calculated by Eq. 3.

To use the daily hydrological model, it is necessary to evaluate daily evapotranspiration. The products of the above e value and monthly potential evapotranspiration E_{pi} become monthly evapotranspiration. Moreover, daily evapotranspiration E(t) must be obtained. Ando and Takahasi (2) proposed that if daily evapotranspiration of a rainless day is expressed as 1, then daily evapotranspiration of rainy days can be written as follows:

$$r=\begin{cases} 1.0 & P=0.0 \\ 0.7 & 0<P<1.0 \\ 0.5 & 1.0\le P<5.0 \\ 0.4 & 5.0\le P \end{cases} \tag{5}$$

Where, P: daily rainfall. If the coefficient of a certain day is equal r(t), the daily evapotranspiration of the day is given by the next equation.

$$E(t)=eE_{pi}r(t)/\Sigma r(t) \tag{6}$$

Where, $\Sigma r(t)$ is equal to the sum of r(t) of the month.

(4) Estimation of the base runoff rate (f_0), the first additional runoff rate (f_1), and the first critical rainfall depth (p_1)

Ando and Takahasi (2) proposed that relationship between net storm rainfall and direct runoff is given by the following equations.

$$D_s=\begin{cases} f_0P_s & 0\le P_s<p_1 \\ f_0P_s+f_1(P_s-p_1) & p_1\le P_s<p_2 \\ \\ f_0P_s+f_1(P_s-p_1)+\dots+f_n(P_s-p_n) & p_n\le P_s \end{cases} \tag{7}$$

Where, P_s:net storm rainfall, D_s: direct runoff, f_0: the base runoff rate, f_1:the first additional runoff rate, f_n: n-th additional runoff rate, p_1: the first critical rainfall depth, p_n: n-th critical rainfall depth. And, according to Ando and Takahasi (2), in almost all cases it is enough to consider only up to n=1 term, and therefore in this paper n=1 is adopted from a practical point of view.

Values of the base runoff rate (f_0), the first additional runoff rate (f_1), and the first critical rainfall depth (p_1) can be obtained from correlation diagram of net storm rainfall and direct runoff. The physical meaning of these parameters are as follows. When the rainfall depth (P_s) is less than p_1, it is considered that rainfall onto the saturated area (channels, swamps, lakes etc) becomes direct runoff and that rainfall onto the infiltration area (tops and slopes) infiltrates into the soil. Here the author has termed f_0 as "the base runoff rate". On the other hand, in the case of the rainfall whose depth (P_s) is more than p_1, D_s is expressed as in the second Eq. 7. If P_s exceeds p_1, $f_1(P_s-p_1)$ joins f_0P_s as direct runoff. The physical meaning of $f_1(P_s-p_1)$ is both overland flow and interflow. The author names f_1 "the first additional runoff rate".

(5) Calculation of effective rainfall DT(t)

Effective rainfall in t-th day (DT(t)) is given by the following equation, considering antecedent rainfall (AP(t)). AP(t) is equal to antecedent two days' rainfall amount and is expressed by the next equation.

$$AP(t)=P(t-1)+P(t-2) \tag{8}$$

If AP(t) is less than p_1,

$$DT(t)=\begin{cases} f_0P(t) & 0\le P(t)<pp_1 \\ f_0P(t)+f_1(P(t)-pp_1) & pp_1\le P(t) \end{cases} \tag{9}$$

Where, $pp_1=p_1-AP(t)$.

If AP(t) is equal or greater than p_1,

$$DT(t)=(f_0+f_1)P(t) \tag{10}$$

(6) Calculation of direct runoff D(t)

Direct runoff in t-th day (D(t)) is expressed by the following equation, using the unit hydrograph method.

$$D(t)=d_1DT(t)+d_2DT(t-1)+d_3DT(t-2) \tag{11}$$

Where, d_1: distributing rate of unit hydrograph for the day of rainfall, d_2: distributing rate of unit hydrograph for the day after rainfall, d_3: distributing rate of unit hydrograph for the second day after rainfall.

(7) Calculation of daily infiltration I(t)

Daily infiltration I(t) is given as daily rainfall P(t) minus effective rainfall DT(t) and crown interception C(t).

$$I(t)=P(t)-DT(t)-C(t) \tag{12}$$

Crown interception C(t) is calculated by the next equation.

$$C(t)=cP(t) \tag{13}$$

Where, c: coefficient of crown interception.

(8) Calculation of soil moisture storage $M_s(t)$

Daily infiltration rate I(t) increases soil moisture storage $M_s(t)$ and evapotranspiration from infiltration area $E_i(t)$ decreases soil moisture storage $M_s(t)$.

$$M_s'(t)=M_s(t)+I(t)-E_i(t) \tag{14}$$

Where, evapotranspiration from infiltration area $E_i(t)$ is calculated by the following equation.

$$E_i(t)=(1-f_0)E(t) \tag{15}$$

(9) Calculation of groundwater recharge G(t)

Groundwater recharge is proportional to soil-moisture excess.

$$G(t)=\begin{cases} 0 & 0\le M_s'(t)<h \\ g(M_s'(t)-h) & h\le M_s'(t) \end{cases} \tag{16}$$

Where, h: normal soil moisture, g: constant of groundwater recharge. Values of h and g are 200mm and 1.0 respectively, using the same values obtained by Ando, Musiake, and Takahasi (1) in a natural hillslope basin covered with Kanto Loam formations. Then G(t) is subtracted from $M_s'(t)$.

$$M_s(t+1)=M_s'(t)-G(t) \tag{17}$$

Where, $M_s(t+1)$ represents the soil moisture on the (t+1) day.

(10) Calculation of groundwater runoff $Q_g(t)$ and groundwater storage $S_g(t)$ and estimation of fractional groundwater recession constant a

Groundwater runoff $Q_g(t)$ is proportional to the second power of groundwater storage $S_g(t)$.

$$Q_g(t)=a^2S_g(t)^2 \tag{18}$$

Where, a: fractional recession constant of groundwater runoff. "a" is estimated so that Eq. 19, which was introduced theoretically by Werner and Sundquist (8), Roche (6), and Takagi (7), fits the recession hydrograph without rain in winter.

$$Q_g=Q_0/(1+a\sqrt{Q_0}t)^2 \tag{19}$$

Because the performance of the fractional groundwater runoff recession equation was found to be better than that of the exponential one for mountainous basins in Japan by Ando, Takahasi, Ito, and Ito (3). Groundwater storage of the next day $S_g(t+1)$ is expressed as follows:

$$S_g(t+1)=S_g(t)+G(t)-Q_g(t) \tag{20}$$

(11) Calculation of total runoff Q(t)

Total runoff Q(t) can be calculated as groundwater runoff $Q_g(t)$ plus direct runoff D(t) minus evapotranspiration from saturated area $E_s(t)$.

$$Q(t)=Q_g(t)+D(t)-E_s(t) \tag{21}$$

Where, $E_s(t)$ is given by the following equation.

$$E_s(t)=f_0E(t) \tag{22}$$

Evaluation of Initial Value of $M_s(t)$ and $S_g(t)$

(1) $M_s(1)$ (the initial value of $M_s(t)$) is calculated by the next equation.

$$M_s(1)=h-M_d \tag{23}$$

Where, M_d equals the estimated soil-moisture deficiency.

(2) $S_g(1)$ (the initial value of the groundwater storage $S_g(t)$) is estimated from

the following formula.

$$S_g(1)=\sqrt{Q_g(1)}/a \tag{24}$$

Estimating Method of Model Applicability by Relative Error

Calculated hydrograph is compared with observed hydrograph using average of daily relative errors (ADRE) and yearly relative error of total runoff (YRE). The equations are expressed as follows.

$$ADRE=1/N\Sigma|Q_o(t)-Q_c(t)/Q_o(t)| \tag{25}$$

$$YRE=|\Sigma Q_o(t)-\Sigma Q_c(t)|/\Sigma Q_o(t) \tag{26}$$

Where, $Q_o(t)$: observed runoff in t-th day, $Q_c(t)$: calculated runoff in t-th day, N : number of days of one year.

RESULTS OF COMPARISON OF THE MODEL WITH URATUKUBA EXPERIMENTAL BASIN DATA AND SOME CONSIDERATIONS

Outline of Uratukuba Experimental Basin

Uratukuba experimental basin was equipped in Kanto Region by Public Works Research Institute, Japan Ministry of Construction. Uratukuba experimental basin includes two basins: Yamaguti River basin and Sofugamine basin. In this study, Yamaguti River basin's data are used. The hydrological data are listed by Public Works Research Institute (5). The catchment area of Yamaguti River basin is 3.12 square kilometers. An outline of the basin is shown in Fig.3. The basin geology is weathered granitic rocks, and is mainly covered with forest. The basin is equipped with 3 rain gauges and one discharge gauge.

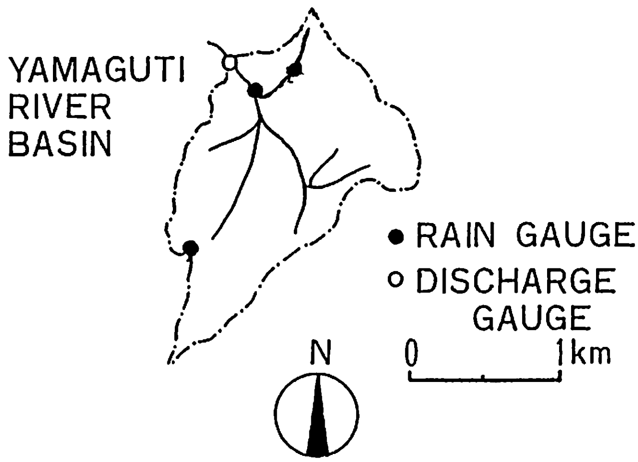

Fig. 3 Outline of Uratukuba experimental basin

Calibration Period and Test Period

There are 9 years data, from 1969 to 1977. In this paper, 6 years' data which are complete are used. Three years (1970, 1972, and 1973) are taken as calibration period, and three years (1974, 1975, and 1976) are taken as test period.

Estimation of Parameter Values

Parameter values are estimated with hydrological data of calibration period.
According to Fig. 4 which shows relationship between annual total precipitation (P_Y) and annual total loss (E_Y), coefficient of crown interception (c) and coefficient of evapotranspiration (e) are estimated.

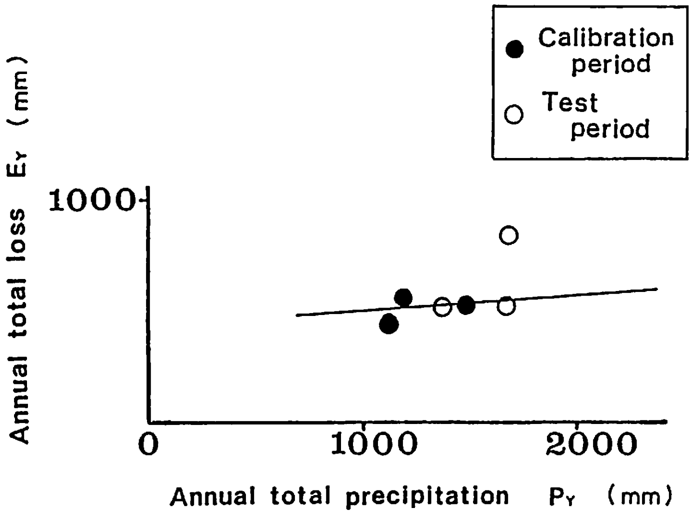

Fig. 4 Relationship between annual total precipitation and annual total loss

Direct runoff parameters (f_0, f_1, p_1) are estimated by the correlation between storm rainfalls and direct runoffs, as shown in Fig. 5.

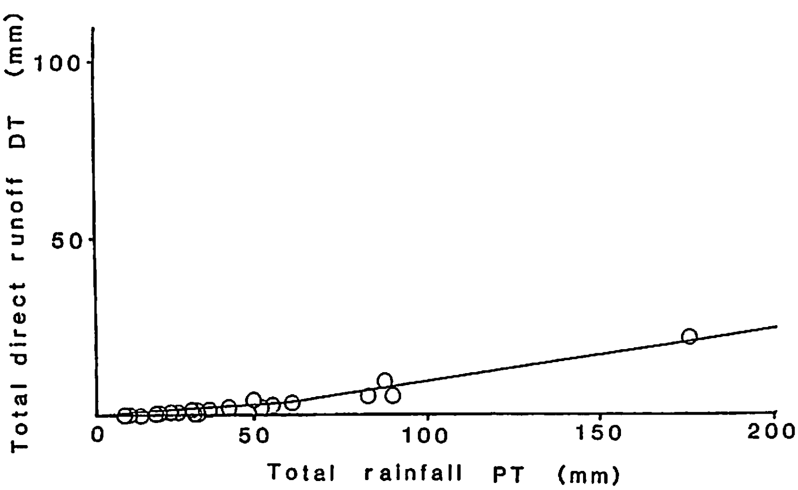

Fig. 5 Relationship between total rainfall and total direct runoff for each storm

Distribution rates of unit-hydrograph are estimated by distributing rates of solitary rainfalls as shown in Table 1.

Table 1 Distributing rates of unit hydrographs for solitary rainfall

Date	Rainfall (mm)	Direct runoff (mm)	Distributing rates d_1	d_2	d_3
May 20, 1970	32.8	1.9	0.76	0.18	0.06
Sep. 1, 1972	24.8	0.85	0.84	0.13	0.03
Oct. 6, 1972	10.0	0.35	0.80	0.14	0.06
Dec. 7, 1972	14.2	0.58	0.72	0.24	0.03
Apr. 4, 1973	21.0	0.51	0.75	0.18	0.08
May 2, 1973	27.0	1.71	0.75	0.18	0.08
Sep. 30, 1973	17.3	0.94	0.71	0.23	0.05
Oct. 13, 1973	72.8	5.78	0.79	0.14	0.07
		Average	0.77	0.17	0.06

Fractional recession constant (a) is estimated as winter value a=0.003, based on relationship between fractional recession constants and initial discharges, as shown in Fig. 6.

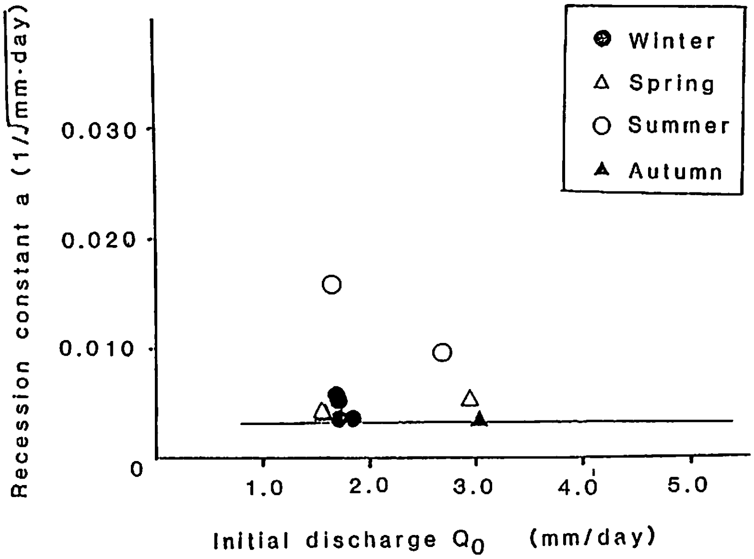

Fig. 6 Relationship between initial discharge and recession constant

In Table 2, values of model parameters of Yamaguti River basin are shown.

Table 2 Values of parameters of the model for Yamaguti River basin

Symbol of parameter	Value of parameter
a	0.003
c	0.07
d_1	0.77
d_2	0.17
d_3	0.06
e	0.70
f_0	0.06
f_1	0.09
g	1.0
h	200mm
p_1	60mm

Results of Applying the Model to the Basin

Using parameter values shown in Table 2, computation of hydrographs was done for both calibration period and test period. An example of the results during calibration period is shown in Fig. 7. The calculated hydrographs and the observed hydrographs are in good agreement. An example of the results during test period is shown in Fig. 8. The calculated hydrograph agrees well with the observed one.

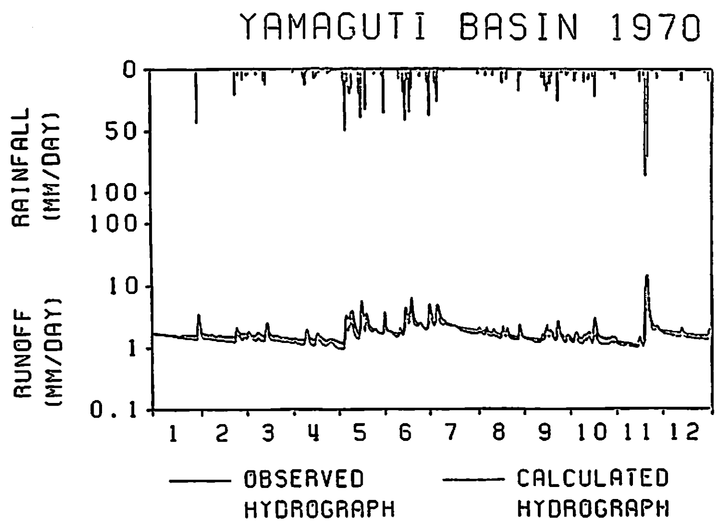

Fig. 7 Comparison between observed hydrograph and calculated hydrograph during calibration period

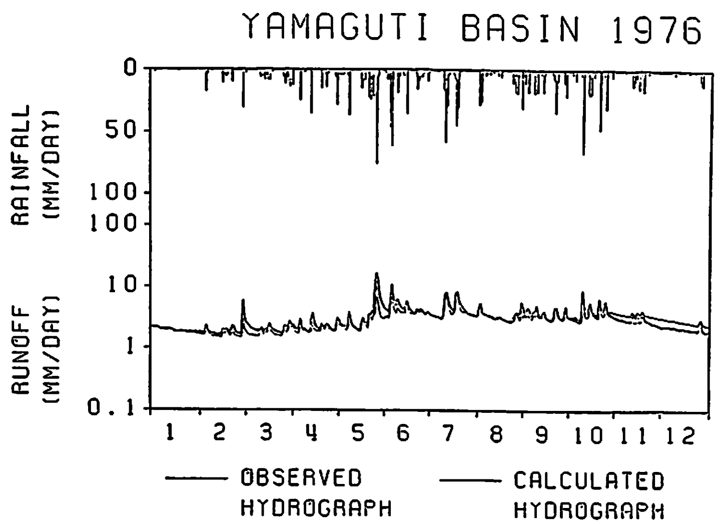

Fig. 8 Comparison between observed hydrograph and calculated hydrograph during test period

Moreover, Fig. 9 shows yearly relative errors of total runoff and averages of daily relative errors. Yearly relative errors of total runoff are from 0.04 to 0.15, and averages of daily relative errors are from 0.12 to 0.27. As these values are quite small, it can be concluded that the model has a high applicability to Yamaguti River experimental basin (Uratukuba experimental basin).

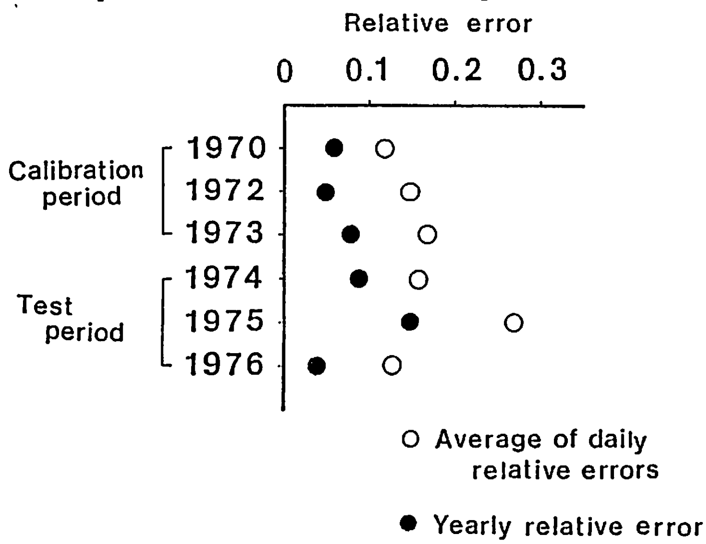

Fig. 9 Relative errors

CONCLUSION

A new daily rainfall-runoff model has been proposed. Its parameters are determined from the observed hydrograph. It has relatively well defined physical meaning, is simple, and includes all runoff components. The schematic hydrological process is shown in Fig. 1, and the daily rainfall-runoff model is given in Fig. 2. Furthermore, the applicability of the model is examined by simulations in an experimental basin.

ACKNOWLEDGEMENT

The author wishes to express his thanks to Professor Yutaka Takahasi (Shibaura Institute of Technology) for his useful advice and Mr. Koji Yamaguti and Mr. Sri Kantha Herath (University of Tokyo) for their assistance.

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APPENDIX - NOTATION

The following symbols are used in this paper:

ADRE	= average of daily relative errors;
AP	= antecedent two days' rainfall amount;
a	= fractional recession constant of groundwater runoff;
C	= crown interception;
c	= coefficient of crown interception;
D	= direct runoff
D_i	= direct runoff from the infiltration area;
D_o	= possible hours of sunshine in units of 12 hr;
D_s	= direct runoff from the saturated area;
DT	= effective rainfall;
d_1, d_2, d_3	= distributing rates of unit hydrograph;
E	= evapotranspiration;
E_i	= evapotranspiration from the infiltration area;
E_s	= evapotranspiration from the saturated area;
EH	= annual potential evapotranspiration;
E_p	= average potential evapotranspiration;
E_Y	= annual loss;
e	= coefficient of evapotranspiration;
f_0, f_1, f_n	= runoff rates;
G	= groundwater recharge;
g	= constant of groundwater recharge;
h	= normal soil moisture;
I	= infiltration;
M_s, M_s'	= soil moisture amount;
N	= number of days of one year;
P	= rainfall;
P_i	= rainfall onto infiltration area;
P_s	= rainfall onto saturated area;
P_t	= saturated water vapor density;
P_Y	= annual precipitation;
p_1, p_n	= critical rainfall depth;
Q	= total river discharge;
Q_c	= calculated discharge;
Q_g	= groundwater runoff;
Q_o	= observed discharge;

Q_Y	=	annual runoff;
Q_O	=	initial discharge;
r	=	coefficient of evapotranspiration for rainy days;
S_g	=	groundwater storage;
t	=	time; and
YRE	=	yearly relative error.