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ESTIMATION OF SALINITY INTRUSION IN TIDAL ESTUARIES

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SYNOPSIS

The salinity distribution in estuaries has generally been calculated on the basis of a one-dimensional advection-dispersion equation, and expressed as a continuous function of longitudinal position x and time t. By means of transforming the independent variable from x into storage volume V, we found that the salinity concentration C observed in tidal estuaries under constant fresh water inflow is a function of V only. Consequently, the unsteady dispersion equation expressed in terms of the ordinary x - t coordinate system reduces to the quasi-steady dispersion equation in terms of V. Using this equation, the longitudinal distribution of dispersion coefficients is evaluated with ease from the observed C - V relationship and fresh water discharge.

Further, the dispersion mechanism in a homogeneous oscillating flow accompanied with steady inflow is investigated theoretically. The dispersion coefficient in tidal estuaries is formulated, introducing the empirical correction factor to include buoyancy effects into the theoretical result. We also found that both the correction factor for the dispersion coefficient and non-dimensional storage volume v_* (the storage volume where the salinity concentration is equal to that of sea water imposed on the quasi-steady transformed dispersion equation) are correlated well with the overall Richadson number.

INTRODUCTION

Salinity intrusion in estuaries is controlled mainly by geometry of estuary, tidal motion, inflow of fresh water and the density differential between fresh and sea water. Analytical and experimental studies on the basis of one-dimensional advection-dispersion equation have been carried out by various investigators. Among them, Ippen and Harleman(7) have conducted large-scale flume experiments on salinity intrusion, and introduced the analytical formulation known as the "slack-tide" approximation. Harleman and Thatcher(5) have proposed the one-dimensional numerical model on the basis of a set of equations of flow continuity, momentum and salt balance. The salinity intrusion expressed in terms of the ordinary coordinate system x - t is essentially time-varying due to periodic tidal motion.

The authors(8) have proposed a numerical method to calculate unsteady salinity intrusion in tidal estuaries by means of transforming the independent variable from the longitudinal position x into the storage volume V. The significant feature of this approach is that the position for a certain value of V shifts continuously with tide, and the variations of cross-sectional parameters for a fixed V in the avdection-dispersion equation are much smaller than those for a fixed x within a tidal cycle.

Another important problem is the evaluation of a dispersion coefficient. The dispersion coefficient for estuaries is naturally determined by a number of complex factor,i.e. tidal oscillation, fresh water inflow, vertical motion and transverse motion. Recently, Fischer(2), Smith(9) and others have demonstrated that the dominant mechanisms of dispersion in stratified estuaries are not the boundary shear and internal gravitational circulation in vertical direction, but the ones in transverse direction. The characteristic length scale controlling the dispersion mechanisms is channel width. Since the duration of a tidal cycle is generally shorter than the time scale of transverse diffusion of contaminant in real estuaries, it is also of significance to estimate the effects of oscillating flow accompanied with fresh water inflow on the dispersion.

In this paper, the analysis using the independent variable V is re-examined and extended, utilizing the data of salinity and tidal elevation in the Chikugo and the Sendai Rivers. It is found that salinity concentration expressed in terms of V is practically independent of tidal motion within a tidal cycle, provided that fresh water inflow is constant. The dispersion coefficient is obtained from the quasi-steady dispersion equation using the observed salinity distributions and the fresh water inflows. Furthermore, the form of dispersion coefficient is proposed in a semi-theoretical manner, considering the effects of tidal oscillating flow with steady river inflow and the effects of buoyancy in stratified flow. The simple method is proposed in order to treat the salinity intrusion in partially- and well-mixed estuaries.

GOVERNING EQUATIONS

Salinity intrusion in a tidal estuary of partially- or well-mixed type is in general described by one-dimensional advection-dispersion equation

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \frac{1}{A}\frac{\partial}{\partial x}\left(AD\frac{\partial C}{\partial x}\right) \qquad (1)$$

where C = the cross-sectional average of salinity concentration; U = the cross-sectional average of velocity; A = the cross-sectional area; D = the longitudinal dispersion coefficient. The distribution of salinity concentration in a tidal estuary is customarily predicted by solving numerically Eq.1. All of the quantities in Eq.1 must be functions of longitudinal position x and time t. The authors (8) have introduced the new coordinate system moving with tide in order to circumvent this parametric complexity.

The definitions of variables and the coordinate system are shown in Fig.1. We take the origin of x-axis (positive in downstream direction) at the upstream end of an estuary, where the water level is not affected by tidal motion. The stored water volume V from the origin to any section is defined as

$$V = \int_0^x A(x,t)\,dx \qquad (2)$$

Integrating the continuity equation, U is given by

$$U = \frac{1}{A}\left(Q_F - \frac{\partial V}{\partial t}\right) \qquad (3)$$

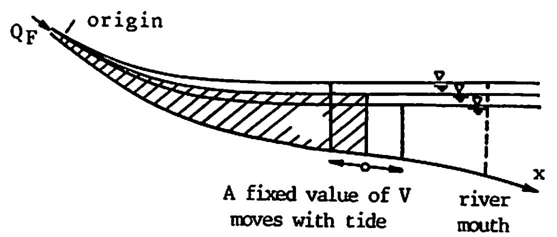


Fig.1 Definition sketch of a tidal estuary

where Q_F = the fresh water inflow. Transformation of the independent variable from (x,t) to (V,t) gives the following relationships.

$$\left(\frac{\partial}{\partial x}\right)_t = A\frac{\partial}{\partial V} \quad ; \quad \left(\frac{\partial}{\partial t}\right)_x = (Q_F - UA)\frac{\partial}{\partial V} + \frac{\partial}{\partial t} \qquad (4)$$

Eq.1 together with Eqs.3 and 4 yields

$$\frac{\partial C}{\partial t} + Q_F\frac{\partial C}{\partial V} = \frac{\partial}{\partial V}\left(A^2 D\frac{\partial C}{\partial V}\right) \qquad (5)$$

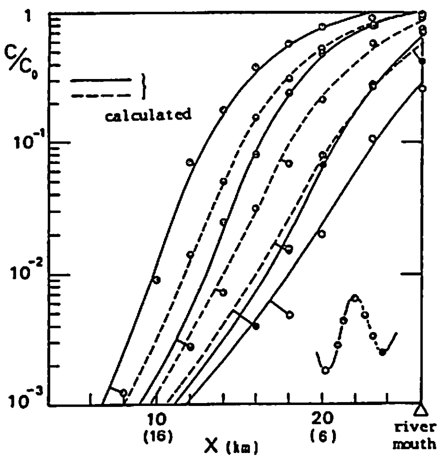
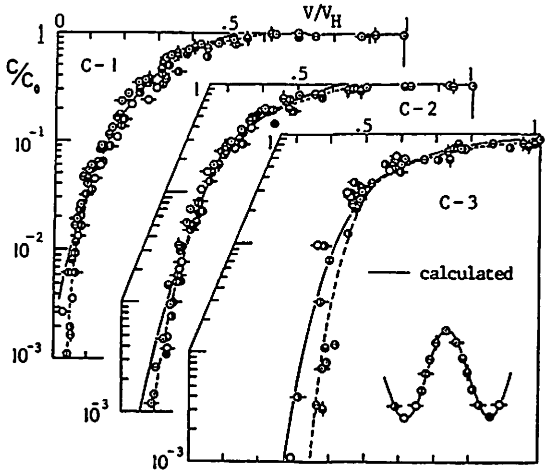
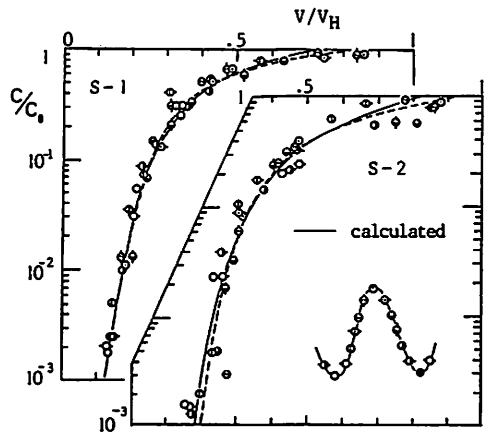


Fig.2 An example of salinity intrusion in the Chikugo River (C-2). The numbers in bracket denote the distance from river mouth.



(a) The Chikugo River



(b) The Sendai River

Fig.3 Salinity concentration versus non-dimensional storage volume V/V_H.

The position of a certain value of V shifts upstream and downstream corresponding to the flood and ebb tide respectively. Therefore, it may be expected that salinity concentration C and the cross-sectional parameters are practically independent of time t, and thus are functions of V only provided that Q_F is constant.

The salinity distribution observed within a tidal cycle in the Chikugo River is plotted against x in Fig.2. The observed relationships between $c = C/C_0$ and $v = V/V_H$ in the Chikugo and the Sendai Rivers are shown in Figs.3(a) and 3(b) respectively, where C_0 = the salinity concentration of sea water; v = the non-dimensional storage volume; V_H = the storage volume upstream of the river mouth at a high water slack. Upon comparing Fig.2 with Fig.3, it is apparent that the variations of salinity distributions resulting from tidal action in v − t plane are much smaller than those in x − t plane. This leads to the further simplification of Eq.5. The quasi-steady equation of $c = C/C_0$, negrecting the term $\partial C/\partial t$ in Eq.5, is thus given by

$$Q_F\, c = \frac{A^2 D}{V_H} \frac{dc}{dv} \qquad (6)$$

The specification of functional relation for dispersion coefficient in Eq.6 is of the most importance.

LONGITUDINAL DISPERSION COEFFICIENT IN ESTUARIES

Brief Review of Previous Investigations

A dispersion coefficient for homogeneous steady flows in cross-section with variable depth has been proposed by Fischer et al. (3) as

$$D = 0.011 \, (W^2U^2)/(du_*) \qquad (7)$$

where W = the channel width; d = the mean flow depth; u_* = the shear velocity.

The dispersion processes in estuaries are remarkably influenced by the buoyancy and the tidal oscillation. Smith (9) and others have argued that there are two effects of buoyancy on the dispersion mechanism of steady stratified shear flows. First, the longitudinal density gradient promotes the vertical velocity gradient and consequently increases the dispersion. Secondly, the transverse circulation caused by the lateral variation of density reduces the dispersion. Analytical investigation by Smith (9) shows that the dispersion coefficient in stratified estuaries tends to decrease with increase of the buoyancy parameter. However, no reliable functional relations have been developed for the effects of density stratification.

On the other hand, the dispersion process in oscillatory flows has been investigated analytically by Holley et al. (6) and Furumoto and Shimada (4). The dispersion coefficient D depends strongly on the ratio of the oscillatory period (T) to the Eulerian time scale (T_E) for cross-sectional mixing. The dispersion coefficient is proportional to the square of (T/T_E) when (T/T_E) is small. Since the values of (T/T_E) are very small in real estuaries, fresh water inflow is expected to have an important role on the formation of transverse concentration distribution and consequently on the dispersion coefficient. The dispersion process of a homogeneous oscillatory flow with steady infow in the channel with variable depth should be examined theoretically, taking into account the periodic variation of the turbulent mixing within a tidal cycle.

Dispersion in Oscillating Flow with Steady Inflow

Consider the dispersion in a homogeneous turbulent shear flow in a channel with a very large channel aspect ratio (=B/H). B refers to flow width (the half width if the cross-section is symmetrical about the centerline), and H to the representative flow depth. Since the variation of flow depth in the lateral direction is gradual in most estuaries, the vertical average velocity u and the discharge q per unit width at a position y respectively, taking y-axis along transverse direction, are assumed to be given by Manning's formula,

$$u = \frac{1}{n} h^{2/3} I^{1/2} \quad ; \quad q = \frac{1}{n} h^{5/3} I^{1/2}$$

where h = the flow depth at a position y; n = Manning's roughness; I = the water surface slope. The cross-sectional average velocity U is given by

$$U = \frac{1}{n} I^{1/2} \int_0^B h^{5/3} \, dy \; / \int_0^B h \, dy \qquad (8)$$

Introducing new variables $\eta = y/B$ and $\zeta = h/H$, the equations for u and q become respectively

$$u = k_q \, U \, \zeta^{2/3} \quad ; \quad q = k_q \, H \, U \, \zeta^{5/3} \qquad (9)$$

where

$$k_q = \int_0^1 \zeta \, d\eta \; / \int_0^1 \zeta^{5/3} \, d\eta \qquad (10)$$

k_q is a proportional constant dependent on the geometry of a channel cross-section.

Denoting the vertical mean concentration of dissolved substances by C, the equation of conservation of mass is

$$h \frac{\partial C}{\partial t} + q \frac{\partial C}{\partial x} = \frac{\partial}{\partial y} \left(h \, e_t \frac{\partial C}{\partial y} \right) \qquad (11)$$

where x = the longitudinal coordinate; e_t = the transverse eddy diffusivity. Assuming that the transverse eddy diffusivity e_t is proportional to the cross-product of shear velocity u_* and flow depth h, e_t is given by

$$e_t = au_*h = k_e|U|H\,\zeta^{3/2} \tag{12}$$

where

$$k_e = ak_q n g^{1/2} H^{-1/6} \tag{13}$$

We introduce the moving coordinate x_1 and the non-dimensional time ϑ as

$$x_1 = x - \int_0^t U\,dt$$
$$\vartheta = (k_e H/B^2)\int_0^t |U|\,dt \tag{14}$$

and considering the case that the concentration gradient is constant in the longitudinal direction, we put

$$C = \frac{\partial C}{\partial x_1}\left(x_1 + \frac{B^2}{k_e H}\hat{c} \right) + \text{Const.} \tag{15}$$

where $\hat{c}$ shows the deviation of concentration from the cross-sectional mean. Eq.11 becomes

$$\zeta\frac{\partial \hat{c}}{\partial \vartheta} + \frac{U}{|U|}\left(k_q\zeta^{5/3} - \zeta \right) = \frac{\partial}{\partial \eta}\left(\zeta^{5/2}\frac{\partial \hat{c}}{\partial \eta}\right) \tag{16}$$

The solutions of Eq.16 for $U \geqq 0$ and for $U \leqq 0$ can be obtained separately. The solution for each region is required to match at the time when the sign of U changes.

The solution of Eq.16 for $U \geqq 0$ is assumed to be the sum of the steady solution $\hat{c}_s$ and the transient solution $\hat{c}_i$ ($i = 1,2,3,\cdots$). The steady solution $\hat{c}_s$ is given by

$$\hat{c}_s = \int_0^\eta \zeta^{-5/2}\int_0^\eta (k_q\,\zeta^{5/3} - \zeta)\,d\eta\,d\eta \tag{17}$$

The transient solution $\hat{c}_i$ satisfies the following equation

$$\zeta\frac{\partial \hat{c}_i}{\partial \vartheta} = \frac{\partial}{\partial \eta}\left(\zeta^{5/2}\frac{\partial \hat{c}_i}{\partial \eta}\right) \tag{18}$$

The solution of Eq.18 is thus given by

$$\hat{c}_i = A_i Y_i(\eta)\exp(-\sigma_i\vartheta) \tag{19}$$

where $Y_i(\eta)$ is the solution of the ordinary differential equation

$$\frac{d}{d\eta}\left(\zeta^{5/2}\frac{dY_i}{d\eta}\right) + \sigma_i\zeta\,Y_i = 0 \tag{20}$$

σ_i denotes a constant to be determined as an eigenvalue for the boundary condition, transverse flux = 0 at $\eta = 0,\ 1$. The eigenfunction $Y_i(\eta)$ for σ_i is obtained numerically from Eq.20. The general solutions of Eq.16 for $U \geqq 0$ and for $U \leqq 0$ are thus given respectively as follows;

$$\hat{c} = \hat{c}_s + \sum_{i=1}^{\infty} A_i Y_i \exp(-\sigma_i\vartheta) \qquad (U \geqq 0) \tag{21}$$

$$\hat{c} = -\hat{c}_s + \sum_{i=1}^{\infty} B_i Y_i \exp(-\sigma_i\vartheta) \qquad (U \leqq 0) \tag{22}$$

The coefficients (A_i and B_i) must be determined so as to satisfy the condition that $\hat{c}$ is equal at a matching point.

One can determine the coefficients, assuming the mean velocity as the sum of a steady flow component and oscillating flow component,

$$U = U_F + U_T\cos(2\pi t/T) \tag{23}$$

where T = the oscillatory period. Both U_F and U_T are defined as positive values in Eq.23. It is obvious that when $U_T < U_F$ the solution of Eq.16 tends to be independent of ϑ , because U is always positive.

$$\hat{c} = \hat{c}_s \tag{24}$$

While, U may change its sign when $U_T > U_F$. Denoting the time when the sign of U changes from negative to positive by t_{2m} and the time from positive to negative by t_{2m+1}, t_{2m} and t_{2m+1} are given by

$$t_{2m} = \{ m - \frac{1}{2} + \frac{1}{2\pi} \cos^{-1}(U_F/U_T)\} T \ ; \ t_{2m+1} = \{ m + \frac{1}{2} - \frac{1}{2\pi} \cos^{-1}(U_F/U_T)\} T \tag{25}$$

We define ϑ_+ and ϑ_- as the increments of ϑ for the period while U is kept positive and negative respectively. They are given by

$$\begin{aligned} \vartheta_+ &= \frac{k_e H}{B^2} \int_{t_{2m}}^{t_{2m+1}} (U_F + U_T \cos\frac{2\pi t}{T})\, dt = \Theta\,(\Psi + U_F/2U_T) \\ \vartheta_- &= \Theta\,(\Psi - U_F/2U_T) \end{aligned} \tag{26}$$

where

$$\Theta = T \,/ \left(\frac{B^2}{k_e H U_T}\right) \quad ; \quad \Psi = \frac{1}{\pi}\left\{\sqrt{1-\left(\frac{U_F}{U_T}\right)^2} + \frac{U_F}{U_T}\sin^{-1}\left(\frac{U_F}{U_T}\right)\right\} \tag{27}$$

We take the origin of non-dimensional time ϑ in Eqs.21 and 22 respectively at the beginning time for $U \geq 0$ and $U \leq 0$. Matching the tail of an interval for $U \leq 0$ with the head of a following interval for $U \geq 0$ and the tail for $U \geq 0$ with the head for $U \leq 0$ respectively, we have for $0 < \eta < 1$

$$\begin{aligned} \hat{c}_s + \sum_{i=1}^{\infty} A_i Y_i &= -\hat{c}_s + \sum_{i=1}^{\infty} B_i Y_i \exp(-\sigma_i \vartheta_-) \\ -\hat{c}_s + \sum_{i=1}^{\infty} B_i Y_i &= \hat{c}_s + \sum_{i=1}^{\infty} A_i Y_i \exp(-\sigma_i \vartheta_+) \end{aligned} \tag{28}$$

Since $Y_i(\eta)$ is determined by Eq.20, the orthogonal condition exists.

$$\int_0^1 Y_i Y_j \zeta \, d\eta = 0 \qquad [i \neq j] \tag{29}$$

Using this condition and the periodicity of $\hat{c}$, after some calculations A_i and B_i become

$$A_i = B_i \exp(-\sigma_i \vartheta_-) - 2F_i \quad ; \quad B_i = A_i \exp(-\sigma_i \vartheta_+) + 2F_i \tag{30}$$

where

$$F_i = \int_0^1 \hat{c}_s Y_i \zeta \, d\eta \Big/ \int_0^1 Y_i^2 \zeta \, d\eta \tag{31}$$

Solving Eq.30 for A_i and B_i,

$$A_i = -2 \frac{1 - \exp(-\sigma_i \vartheta_-)}{1 - \exp[-\sigma_i(\vartheta_+ + \vartheta_-)]} F_i \quad ; \quad B_i = 2 \frac{1 - \exp(-\sigma_i \vartheta_+)}{1 - \exp[-\sigma_i(\vartheta_+ + \vartheta_-)]} F_i \tag{32}$$

The dispersion coefficient D averaged over an oscillatory period is given by

$$D = -\frac{1}{T} \int_0^T \int_0^B C(u-U)h \, dy \, dt \Big/ \left(\frac{\partial C}{\partial x_1} \int_0^B h \, dy \right) \tag{33}$$

When $U_T \leq U_F$, Eq.33 becomes

$$D = \phi_0 (B^2 U_F)/(k_e H) \tag{34}$$

where

$$\phi_0 = -\int_0^1 (k_q \zeta^{5/3} - \zeta)\, \hat{c}_s \, d\eta \Big/ \int_0^1 \zeta \, d\eta \tag{35}$$

Eq.34 suggests that the component of an oscillatory flow has no effect on the dispersion process when $U_T \leq U_F$.

When $U_T > U_F$, Eq.33 becomes

$$D = \frac{B^2U_T}{k_eH}\left[2\Psi\phi_0 - 4\sum_{i=1}^{\infty} \frac{[1-\exp\{-\sigma_i\Theta(\Psi+U_F/2U_T)\}][1-\exp\{-\sigma_i\Theta(\Psi-U_F/2U_T)\}]}{\sigma_i\Theta\{1-\exp(-2\sigma_i\Theta\Psi)\}}\phi_i \right] \qquad (36)$$

where

$$\phi_i = -F_i\int_0^1 (K_q\zeta^{5/3}-\zeta)Y_i d\eta \Big/ \int_0^1 \zeta d\eta \qquad (37)$$

ϕ_i , similar to ϕ_0 , is the quantity to be determined solely by geometry of cross-section. Accordingly, the non-dimensional dispersion coefficient is given as a function of non-dimensional period Θ and the velocity ratio U_F/U_T. Θ is the ratio of the oscillating period T to the reference time scale (B^2/k_eHU_T) required for complete lateral mixing due to diffusion mechanisms.

Eq.36 yields the dispersion coefficient D_{OT} in the case of $U_F = 0$ and $\Theta \to \infty$ as

$$D_{OT} = 2\Psi\phi_0\left(\frac{B^2U_T}{k_eH}\right) = \frac{2}{\pi}\phi_0\left(\frac{B^2U_T}{k_eH}\right) \qquad (38)$$

Eqs.34 and 36 are rewritten respectively in the non-dimensional forms as

$$D/D_{OT} = (\pi/2)(U_F/U_T) \qquad [\,U_T \leq U_F\,] \qquad (39)$$

$$D/D_{OT} = \pi\Psi - \frac{2\pi}{\phi_0}\sum_{i=1}^{\infty} \frac{[1-\exp\{-\sigma_i\Theta(\Psi+U_F/2U_T)\}][1-\exp\{-\sigma_i\Theta(\Psi-U_F/2U_T)\}]}{\sigma_i\Theta\{1-\exp(-2\sigma_i\Theta\Psi)\}}\phi_i \qquad (40)$$

$$[\,U_T > U_F\,]$$

The relationships between D/D_{OT}, U_F/U_T and the parameter Θ are shown in Fig.4. They are calculated from Eqs.39 and 40 for a triangular and a parabolic cross-section. The values of factor associated with both the cross-sections are shown in Table 1.

Here, the characteristics of Eq.40 in some limiting cases are examined.

When $U_F = 0$, Ψ reduces to $1/\pi$. We have

$$\frac{D}{D_{OT}} = 1 - \frac{2\pi}{\phi_0}\sum_{i=1}^{\infty}\frac{\phi_i}{\sigma_i\Theta}\tanh\frac{\sigma_i\Theta}{2\pi} \qquad (41)$$

Eq.41 is further simplified when $\Theta \to 0$ into

$$\frac{D}{D_{OT}} \doteqdot \frac{\Theta^2}{12\pi^2\phi_0}\sum_{i=1}^{\infty}\sigma_i^2\phi_i \qquad (42)$$

Eqs.41 and 42 resemble the result of Holley et al. (6), though the turbulent mixing coefficient is assumed to be constant in their analysis.

Using $\phi_0 = \Sigma\,\phi_i$ derived from Eqs.31,35 and 37 and letting $\Theta \to 0$, we have

$$D/D_{OT} \doteqdot (\pi/4\Psi)(U_F/U_T)^2 \qquad (43)$$

Using D_{OT} given by Eq.38 and the mean absolute velocity $\overline{U}$ given by

$$\overline{U} = \frac{1}{T}\int_0^T \left| U_F + U_T\cos\left(\frac{2\pi t}{T}\right)\right| dt = 2\Psi U_T$$

Table 1 The factors of cross-sections

cross-section	d/H	k_q	ϕ_0	$\sum_{i=1}^{\infty}\sigma_i^2\phi_i$
Triangle	1/2	1.33	.00732	.228
Parabola	2/3	1.17	.00341	.201

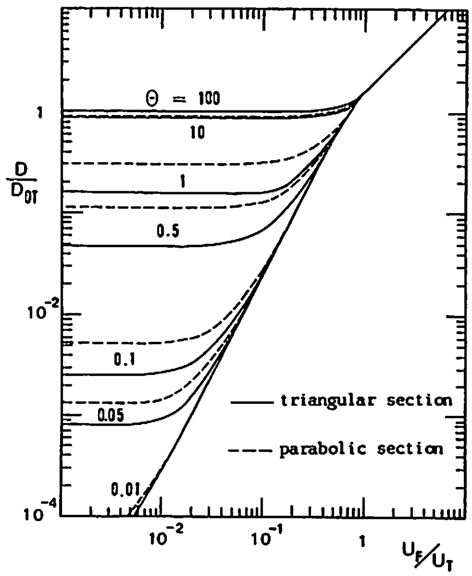


Fig.4 Relationship among D/D_{OT}, U_F/U_T and Θ

Eq.43 is rewritten as

$$D/(B^2\overline{U}/H) = (\ \phi_0/k_e\ )(\ U_F/\overline{U}\ )^2 \qquad (44)$$

D is proportional to $(B^2/H)(U_F{}^2/\overline{U})$. This means that U_F contributes to the formation of concentration distribution due to advection and $\overline{U}$ contributes to the transverse mixing.

Estimation of Dispersion Coefficient for Estuaries

(a) Description of prototype estuaries

We apply the model to the Chikugo and the Sendai Rivers to establish the functional form of the dispersion coefficient accounting for the buoyancy effects. The Chikugo River is in a well-mixed condition at spring tide and a partially-mixed one at neap tide. While, the Sendai River is a partially-mixed type. In both rivers the field surveys on tidal currents and the water qualities have been conducted by Chikugo River Construction Office and Sendai River Construction Office, Ministry of Construction, respectively. These field data are used in the following analysis.

Items of the observations are summarized in Table 2, where ΔH = the tidal range at river mouth; $\Delta\rho$ = the maximum excess density observed at the river mouth; U' = the r.m.s.current velocity within a tidal cycle. The subscript e denotes the quantities near the river mouth. Ri_v, K and v_* will be described later.

Table 2 Items of observation in the Chikugo and the Sendai Rivers.

River, No. (date)	Q_F m^3/s	ΔH m	$\Delta\rho/\rho$	A_e m^2	W_e m	U'_e m/s	V_H $\times 10^7 m^3$	Ri_v	K	v_*	Symbols in Figs.9,10,11
Chikugo,C-1 (1966.9.23)	222.4	1.75	.0233	3020	990	.17	3.38	16.8	.027	1.05	♀
Chikugo,C-2 (1967.9.1)	42.1	3.53	.0234	3230	970	.38	4.36	0.35	.59	.62	◑
Chikugo,C-3 (1967.10.5)	24.2	4.78	.0257	3540	1010	.54	5.54	0.085	1.85	.51	⊖
Sendai, S-1 (1973.9.26)	75.7	2.73	.0255	1650	570	.31	1.72	2.46	.15	.74	△
Sendai, S-2 (1980.10.29)	93.7	2.00	.0301	1620	620	.24	1.64	6.91	.075	.83	▲

(b) Transformation of variables

The geometric and hydraulic quantities (A,W,U') are expressed originally in terms of x and t. They must be transformed into functions of V in order to compute salinity distributions on the basis of Eq.6. The procedure is as follows;

(1) Fig.5 shows an example of the time variations of the tidal elevation H observed in the Chikugo River. The longitudinal variations of A and W at mean water level are shown in Fig.6. Using the diagrams H - A and H - W together with H - t curces, the value of the quantities of $A(x_i,t_j)$, $W(x_i,t_j)$, $U(x_i,t_j)$ and $V(x_i,t_j)$ at the longitudinal coordinate x_i and time t_j are obtained.

(2) V_m is defined as $V_m = m\,\Delta V$ ($m = 0, 1, 2,\ldots$). ΔV is the volume element.

(3) The location $x(V_m,t_j)$ corresponding to a fixed value of V_m at each time t_j is interpolated from $V(x_i,t_j)$. A calculated example on the trajectory of V_m within a tidal cycle is shown in Fig.7. $A(V_m,t_j)$, $W(V_m,t_j)$ and $U(V_m,t_j)$ are obtained in a similar manner.

(4) In order to take into account the sea region connected with the river, we assume that the river channel is extended seaward after Ippen and Harleman (7). The hypothetical channel has the area A and the width W, holding the tendency of exponential expansion of the actual one in the river region.

(5) The flow area A and the flow width W in Eq.6 are considered to be the average value $A(V_m)$ and $W(V_m)$,which are obtained by averaging $A(V_m,t_j)$ and $W(V_m,t_j)$ over a tidal cycle respectively. The calculated relation between the quantities

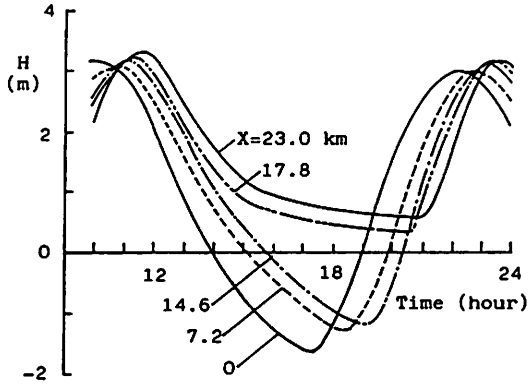


Fig.5 Tidal elevations within a tidal cycle in the Chikugo River (C-1). (Data from Ref.1, X=distance from river mouth)

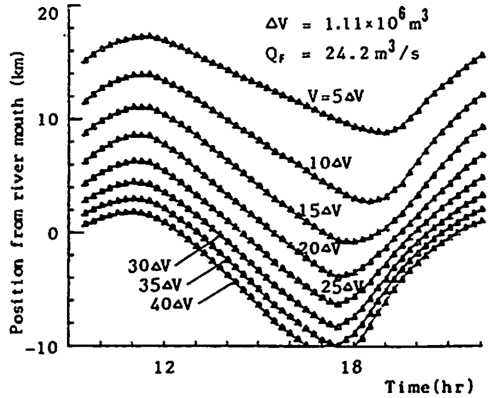


Fig.7 Trajectories of fixed values of V within a tidal cycle (C-3).

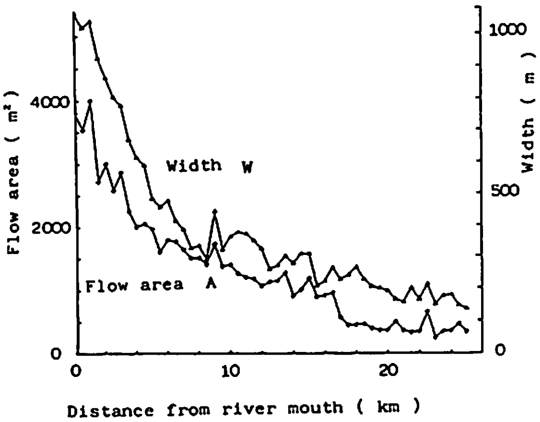


Fig.6 Longitudinal variations of flow area A and width W at mean water level in the Chikugo River.

(A,W) and V in the Chikugo and Sendai Rivers are shown in Fig.8(a) and 8(b) respectively. It may be seen that A and W are a function of V and almost independent of the fresh water discharge as well as tidal condition.
The r.m.s.velocity $U'(V_m)$ obtained from $U'(V_m,t_j)$ are also shown in the figures.

(c) Estimation of dispersion coefficient

The normalized dispersion coefficient (D/D_{OT}) for oscillating flows with the steady flow depends upon the non-dimensional period Θ($= T/(B^2/k_eHU_T)$) and the velocity ratio (U_F/U_T) as indicated by Eq.40.

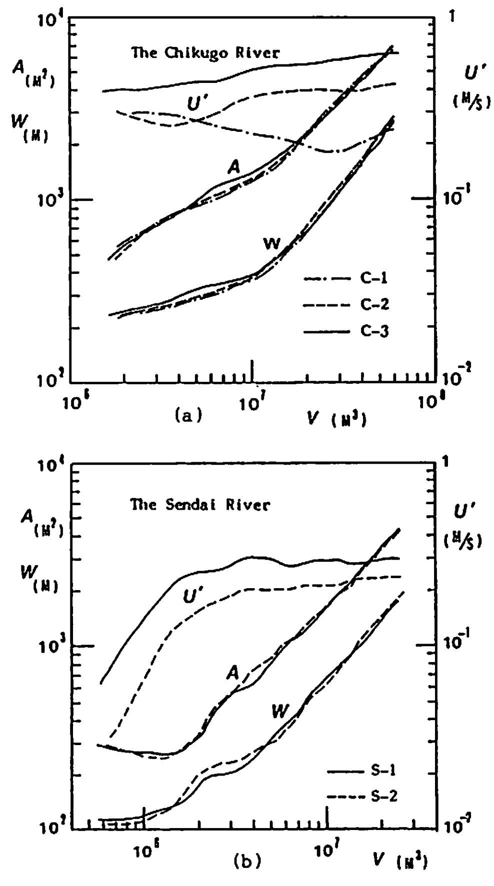


Fig.8 The relationship between flow area A, width W, r.m.s.current velocity U and storage volume V.

We estimate the values of Θ at the location near the upstream end of salinity intrusion in the Chikugo and the Sendai Rivers. Assuming the cross-section of the streams to be parabolic and using the values shown in Table 3, the calculated results are $\Theta = 0.046$ for the Chikugo River and $\Theta = 0.060$ for the Sendai River respectively. Fig.4 indicates that the influence of Θ on D/D_{OT} in Eq.40 may be considerably small, so that Eq.44 may be applicable for both rivers if the streams are assumed to be neutrally buoyant.

We attempt to estimate longitudinal distributions of the dispersion coefficients in both rivers from Eq.6, and to introduce the buoyancy effects into Eq.44. The relationships between A and V and between the observed salinity c and $v = V/V_H$ are presented in Fig.8 and Fig.3 respectively. The mean curves of the c - v relationship are also shown with the broken lines in Fig.3. Using the c - v curves observed under the given values of Q_F and V_H together with the use of the A - V relationships, we obtain the values of D at the position of v for c = 0.015, 0.05, 0.11, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8 from Eq.6. The values of A, W, U' ($\fallingdotseq U_T/\sqrt{2}$), U_F ($=Q_F/A$) and d ($=A/W$) at those positions of v are also obtained from Fig.8.

$D/(W^2U'/d)$ and (U_F/U') in Eq.44 are calculated by using the above values and $\overline{U} \fallingdotseq 0.9U'$, and are plotted in Fig.9. It is apparent that $D/(W^2U'/d)$ is proportional to $(U_F/U')^2$, and that individual lines corresponding to each observation shift in parallel. D is thus of the form,

$$D = K\frac{W^2U_F^2}{d\,U'} \qquad (45)$$

The values of K are shown in Table 2. The variation of K for each observation is not caused solely by the cross-sectional geometry, but significantly by the buoyancy effects due to tidal stratified flow.

The similar form of the parameter representing the effects of buoyancy in estuaries have been proposed by Ippen and Harleman (7), Fischer (2), Harleman and Thatcher (5) and others. We introduce the overall Richardson number Ri_v for estuaries expressed in terms of the storage volume V

$$Ri_v = \frac{Q_F T(\Delta\rho/\rho)gd_e}{(V_H-V_L)\,U_e'^2} \qquad (46)$$

where T = the tidal period; V_H and V_L = the storage volume V at the river mouth in high

Table 3 The values used to estimate Θ

River	$H(=\frac{3}{2}d)$ m	$B(=\frac{W}{2})$ m	U_T m/s
Chikugo	3.75	150	0.7
Sendai	3.75	100	0.4

$a\ (=e_t/du_*) = 0.15$
n (Manning's roughness)=0.02
T = 12.5x3600 s
$k_q = 1.17$
$\phi_0 = 0.00341$

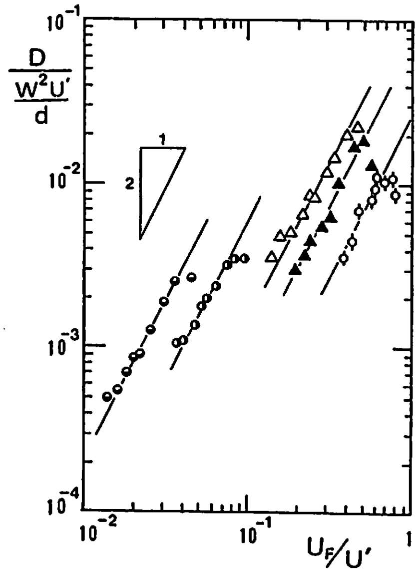


Fig.9 The relation between $D/(W^2U'/d)$ and U_F/U'

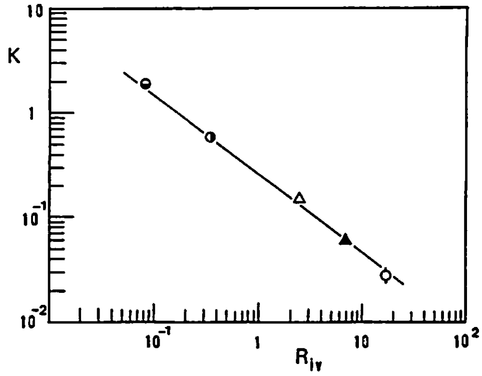


Fig.10 K-dependence on Ri_v

and low water slack respectively. The proportional coefficient K in Eq.45 correlated with Ri_v as shown in Fig.10. K decreases with the increase of Ri_v. K is thus given by

$$K = 0.28\, Ri_v^{-3/4} \qquad [\, 0.08 < Ri_v < 17 \,] \qquad (47)$$

The relationship between K and Ri_v in the channel with variable depth coincides qualitatively with one given by Smith (9), which was stated previously. We obtain the dispersion coefficient of the form,

$$D = K \frac{W^2 U_F{}^2}{d\, U'} = 0.28\, Ri_v^{-3/4} \frac{W^2 U_F{}^2}{d\, U'} \qquad (48)$$

BOUNDARY CONDITION AND ANALYSIS OF SALINITY CONCENTRATION

Substituting Eq.48 into Eq.6, the steady dispersion equation to predict salinity distribution in tidal estuaries is given by

$$\frac{dc}{dv} = \frac{V_H}{K\, Q_F} \left(\frac{AU'}{W^3} \right) c \qquad (49)$$

The determination of the boundary condition has still remained as a very difficult problem. The salinity distribution near the ocean entrance is very complex and can not be properly described by Eq.49. Because the flow field changes drastically from the river to the ocean.

We thus impose conveniently the boundary condition as

$$c = 1 \quad \text{at} \quad v = v_* = V_*/V_H \qquad (50)$$

v_* denotes the non-dimensional storage volume v to be determined as the intrusion position of the sea water. v_* for each observations was obtained as follows; The initial point of (c,v) to begin the computation of Eq.49 was chosen at the point, where c is about 0.3 on the broken line in Fig.3. Therein, the computation was proceeded in the upstream and downstream flow direction. v_* was determined as the v-value corresponding to the intersection between the straight line of c = 1 and the computed curve which is shown with solid line in Fig.3. The results are shown in Table 2. v_* correlates reasonably well with Ri_v as shown in Fig.11, and increases with the increase of Ri_v.

We can discuss the salinity concentration in terms of v, based on the quasi-steady equation (Eq.6) with the use of K and v_*.

A diagram of the c-v relationship can be converted back into that of the c - x relationship (the longitudinal salinity distribution in the physical plane within a tidal cycle), using the trajectories of V on x - t plane as shown in Fig.7. One example of calculated results is presented in Fig.2. This figure shows good agreement with the observed salinity distribution.

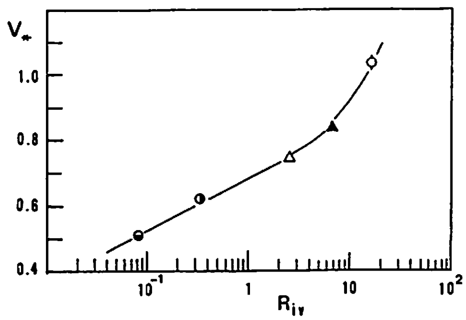


Fig.11 v_* -dependence on Ri_v

CONCLUSIONS

Salinity intrusion in tidal estuaries is investigated theoretically and experimentally by using the model based on storage volume V. The data obtained under the constant fresh water inflow from the Chikugo and the Sendai Rivers, which are typical examples of well- and partially-mixed estuaries, is used to verify the model.

The results obtained in this study are as follows;

(1) The variations of salinity concentration C in terms of independent variable V are comparably very small within a tidal cycle, and therefore can be regarded as a function of V only. Consequently, the one-dimensional dispersion equation in x - t plane reduces to the quasi-steady dispersion equation (Eq.6) expressed in terms of V.

(2) The longitudinal distribution of the dispersion coefficient in an estuary is obtained with ease, based on the quasi-steady transformed dispersion equation with aid of the observed C - V relationship and fresh water inflow.

(3) The dispersion process in a homogeneous oscillating flow with steady inflow in the channel with variable depth has been investigated theoretically. The dispersion coefficient reduces to Eq.44 under the condition that the ratio of the tidal period to the time scale of transverse diffusion is sufficiently small. The functional form of the dispersion coefficient agrees well with the one estimated from longitudinal salinity distributions, and the coefficient K in Eq.45 depends upon the overall Richardson number (Ri_v). The value of K decreases with increase of Ri_v (Fig.10). It implies that the transverse circulation is the dominant mechanism for the dispersion in the channel with variable depth.

(4) v_*, which is introduced to impose the boundary condition on governing equation, means the non-dimensional storage volume v corresponding to the intrusion position of ocean salinity. We found experimentally that v_* correlated well with Ri_v, and that v_* increased with the increase of Ri_v (Fig.11). The dispersion process can be analized properly, based on the quasi-steady dispersion equation with the use of the dispersion coefficient D and the boundary condition v_*.

(5) A diagram of the c - v relationship can be converted back into that of the physical plane, using the trajectories of V on x - t plane (Fig.7).

Detailed field studies in many estuaries should be required for the further elaboration of the model.

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APPENDIX - NOTATION

The following symbols are used in this paper:

- A = cross-sectional flow area;
- a = constant, Eq. 12;
- B = flow width;
- C, C_0 = salinity concentration and that of sea water;
- c = non-dimensional salinity concentration as defined by C/C_0;
- $\hat{c}$ = non-dimensional concentration associated with the deviation from cross-sectional mean as defined by Eq. 15;
- $\hat{c}_s$, $\hat{c}_i$ = steady solution of Eq. 16 and transient solution of Eq. 18;
- D = longitudinal dispersion coefficient;
- D_{0T} = longitudinal dispersion coefficient in the case of $U_F=0$ and $\Theta \to 0$
- d = mean flow depth;
- e_t = transverse eddy diffusivity;
- F_i = parameter as defined by Eq. 31;
- g = acceleration of gravity;
- H = representative flow depth or tidal elevation;
- ΔH = tidal range at river mouth;
- h = flow depth at a position y;
- I = surface slope;
- K = coefficient defined by $D=K(W^2U_F{}^2/dU')$, Eq45;
- k_q, k_e = parameters given by Eqs. 10 and 13;
- n = Manning's roughness coefficient;
- Q_F = fresh water discharge;
- q = discharge per unit width at a position y;
- Ri_v = modified estuarine Richardson number, Eq. 46;
- t = time;
- T = oscillatory period;
- T_E = Eulerian time scale for cross-sectional mixing;
- U = cross-sectional mean velocity;
- U_F = fresh water velocity;
- U_T = ampritude of oscillating flow velocity;
- U' = r.m.s. current velocity;
- $\overline{U}$ = mean absolute velocity over an oscillating cycle;
- u = vertical mean velocity at a position y;
- u_* = friction velocity;
- V = storage volume as defined by $\int_0^x A(x,t)dx$;
- V_H, V_L = storage volume V at the river mouth in high and low water slack;
- V_* = V corresponding to c = 1;
- v, v_* = non-dimensional storage volume as defined by V/V_H and V_*/V_H;

ΔV = volume element by which the estuary is divided;

V_m = $m\,\Delta V$ (m = 1,2,...);

W = flow width;

x, y = coordinates in longitudinal and transverse direction;

x_1 = coordinate moving with cross-sectional mean velocity;

Y_i = eigenfunction for Eq.18;

η = y/B;

ζ = h/H;

σ_i = eigenvalue for Eq.18;

ϑ = non-dimensional time as defined by $(k_e H/B^2)\int_0^t |U|\,dt$;

ϑ_+, ϑ_- = increments of ϑ for the teriod of $U \geqq 0$ and $U \leqq 0$;

Θ = non-dimensional oscillatory period as defined by Eq.27;

ρ, $\Delta\rho$ = density and density differential between fresh and sea water;

ϕ_0, ϕ_i = parameters as defined by Eqs.35 and 37; and

Ψ = parameter as defined by Eq.27.