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MODELINGS OF DEPOSITION OF VOLCANIC ASH AND RUNOFF OF DEBRIS FLOW IN SAKURAJIMA VOLCANO

By

Muneo Hirano, Toshiyuki Moriyama

Department of Civil Engineering Hydraulics,
Kyushu University, Fukuoka 812, Japan

and

Makoto Hikida

Department of Civil Engineering,
Kagoshima National College of Technology, Kagoshima 899-51, Japan

SYNOPSIS

Sakurajima Volcano is the most active one in Japan, and has been erupting large amount of volcanic ash around the mountain. The deposited volcanic ash on the mountainside results in debris flow in the rivers caused by heavy rainfall.

Assuming that a particle erupted from the crater into the air falls at its terminal velocity, a theoretical equation which gives the depositing rate of volcanic ash and the distribution of grain size on the ground was obtained.

A parametric model to predict the hydrograph of debris flow was offered, in which the distributions of slope length and the depth of the deposited volcanic ash were introduced. The parameters of the model are the mean and standard deviation of the depth of deposits, the hydraulic conductivity, and the lag time in the stream, which are identified by use of the Simplex method. The applicability of this model was checked by the data from two rivers in Sakurajima Volcano.

INTRODUCTION

Sakurajima Volcano is located in the southwest part of Japan as shown in Fig.1. It has been in violent volcanic activity since 1955 and erupting large amount of volcanic ash and stones from the crater. The eruption frequency of the volcano is shown in Fig. 2. The annual amount of erupted volcanic ash is estimated to be about ten million tons in these last ten years. Falling volcanic ash has denuded the mountainside of vegetation and supplied the material for debris flow running down the streams. In 1974, debris flows claimed the lives of 8 persons as well as destroying some public facilities, and in 1985, debris flow occurred about a hundred times in six different rivers on Sakurajima Volcano.

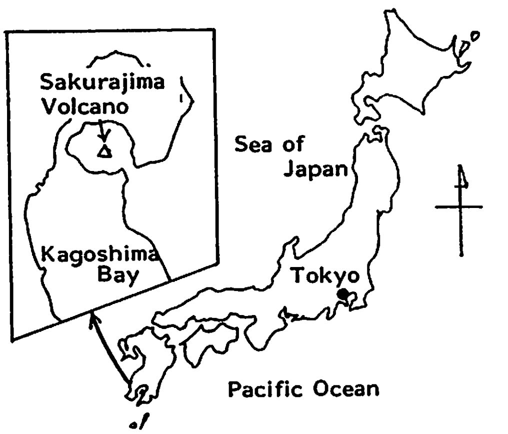

Fig. 1 Location of Sakurajima Volcano

Therefore, studies on the depositing rate of volcanic ash and the intensity of debris flow are required to plan countermeasures for disasters in this area. For this purpose, several observatories of debris flow were set up

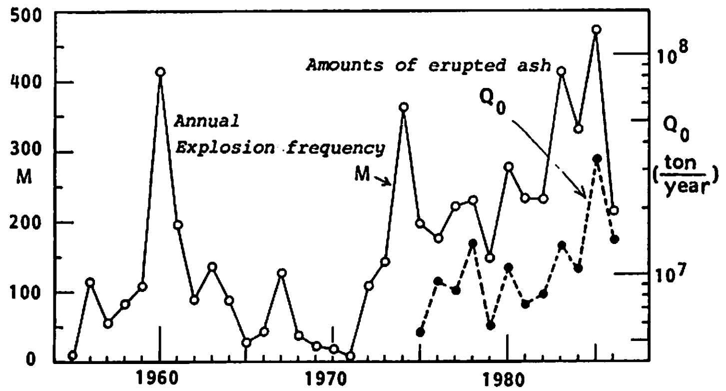

Fig. 2 Variation of annual numbers of explosions of Sakurajima Volcano and amounts of erupted volcanic ash

in some streams on Sakurajima Volcano, and field data has been collected by use of a VTR system. Through these observations, the nature of debris flow has been revealed to a certain extent by, for instance, Tahara (4) and Hirano, Hikida and Moriyama (1). But, the depositing rate of volcanic ash on the mountainside and runoff intensity of debris flow still remain unclear. In this paper, the mathematical models for estimation of the erupted volcanic ash and prediction of debris flow are studied.

DEPOSITING RATE AND SIZE DISTRIBUTION OF VOLCANIC ASH

Assuming that a particle erupted from the crater falls in the air at its terminal velocity as shown in Fig. 3, one obtains

$$\frac{W_0}{V} = \frac{D}{x} \qquad (1)$$

where W_0=terminal settling velocity of the particle; V=wind velocity; D=height of eruption column; and x=horizontal distance from the crater.

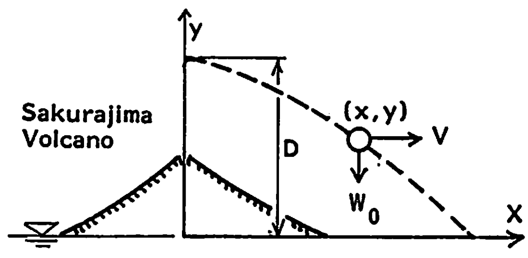

Fig. 3 Motion of a grain of volcanic ash from the crater

The amount of volcanic ash which is deposited within x from the crater on the ground, Q(x), is given by

$$Q(x) = Q_0\int_{DV/x}^{\infty} f(W_0)dW_0 \qquad (2)$$

where Q_0=amount of volcanic ash erupted from the crater; $f(W_0)$=a probability density function of settling velocity of erupted volcanic ash. From Eq. 2, the depositing rate of volcanic ash per unit area, q(x), is given by

$$q(x) = \frac{Q(x)-Q(x+\Delta x)}{b(x)\Delta x} = \frac{-\partial Q(x)}{b(x)\partial x}$$

$$\simeq \frac{Q_0}{b(x)x} f(W_0)W_0 \qquad (3)$$

where b(x)=width of deposition.

Now, the wind velocity may vary with time, the fluctuation of this value should be considered. From Eq. 1, one obtains

$$\Delta V = \Delta W_0 \frac{x}{D} \qquad (4)$$

Then the depositing rate of a particle having a settling velocity of W_0 is, expressed as

$$q(x,W_0)\Delta W_0 = \frac{Q_0}{b(x)x} f(W_0)W_0 g(V)\Delta V \qquad (5)$$

where g(V)=probability density functions of V.

In addition, the eruption column height may also vary with time. By putting Eq. 4 into Eq. 5, as

$$q(x,W_0)\Delta W_0 = \frac{Q_0}{b(x)x} f(W_0)\Delta W_0 \int_0^\infty g(\frac{xW_0}{D})\frac{xW_0}{D}h(D)dD \qquad (6)$$

where h(D)=probability density functions of D.

The distribution function of settling velocity of the deposited volcanic ash is defined by next equation:

$$F(x,W_0) = \int_0^{W_0} q(x,W_0)dW_0 / \int_0^\infty q(x,W_0)dW_0 \qquad (7)$$

where $q(x,W_0)dW_0$ is calculated by substituting Eq. 6 into Eq. 7.

Since the erupted volcanic ash is carried leeward by wind, the trailing direction of volcanic ash should also be considered. Therefore the depositing rate of volcanic ash is expressed as

$$P(x,\theta) = \int_{\theta-\Delta\theta/2}^{\theta+\Delta\theta/2}\int_0^\infty q(x,W_0)dW_0\phi(\theta)d\theta \simeq \int_0^\infty q(x,W_0)dW_0\phi(\theta)\Delta\theta \qquad (8)$$

where $\phi(\theta)$=frequency of the trailing direction; $\int_0^\pi \phi(\theta)d\theta=1$; and $b(x)=x\Delta\theta$.

It has been known that the wind velocity and eruption column height observed by the Meteorological Observatory distribute log-normally. The distribution of the settling velocity of volcanic ash from the crater, $f(W_0)$, may also be assumed to be log-normal. The assumption of log-normal distributions for V, D and W_0 yields

$$F(x,W_0) = \int_{-\infty}^{\log(W_0/W_m)} \frac{1}{\sqrt{2\pi}S_1}\exp\left(-\frac{(\zeta-\bar{\zeta})}{2S_1{}^2}\right)d\zeta \qquad (9)$$

and

$$\frac{P(x,\theta)}{W_d(\theta+\pi)} = \frac{Q_0}{(2\pi)^{1\cdot5}x^2c\ S}\exp\left(-\frac{1}{2S^2}(\log\frac{x\ W_m}{D_mV_m})^2\right) \qquad (10)$$

where c=ln10; $S_1=S_W\sqrt{S_V{}^2+S_D{}^2}/S$=standard deviation of $\log W_0$ for deposited volcanic ash; $\zeta=\log(W_0/W_m)$; $\bar{\zeta}=-(S_W/S)^2\log(xW_m/D_mV_m)$; $S^2=S_W{}^2+S_V{}^2+S_D{}^2$; S_W, S_W, S_D=standard deviations of $\log W_0$ for erupted volcanic ash at the crater, logV and logD, respectively; $\log W_m$, $\log V_m$, $\log D_m$=means of $\log W_0$ for erupted volcanic ash at the crater, logV and logD, respectively; and $W_d(\theta+\pi)=2\pi\phi(\theta)$=weight of frequency of wind direction.

Equation 9 shows that the settling velocity of deposited volcanic ash has also log-normal distribution with the standard deviation of S_1, and the mean settling velocity W_{50} is expressed by

$$\frac{W_{50}}{W_m} = \left(\frac{x W_m}{D_m V_m}\right)^{-(S_W/S)^2} \tag{11}$$

To check the applicability of Eq. 9 and 11, we collected the deposited volcanic ash from Jan. to Dec. in 1986 at 7 points and measured the size distribution of the volcanic ash. The settling velocity was calculated from the diameter of collected volcanic ash by using Fair's equation, and plotted in Fig. 4. As is expected, the distributions are log-normal, and have similar values of standard deviations. The average value of S_1 is 0.367. The mean values of settling velocity, W_{50}, are also plotted in Fig. 5, showing the validity of Eq. 11. By the least squared method, the value of $(S_W/S)^2$ is estimated to be 0.637. The grain size distributions were calculated by applying these values to Eq. 9 and compared with observed ones in Fig. 6. Good agreement is obtained.

The data of wind velocity and eruption column height observed by the Kagoshima Local Meteorological Observatory are available to estimate V_m, S_V, D_m and S_D. The weight of wind direction frequency is also obtainable from the data. In Eq. 10, therefore, Q_0 and W_m are the unknown parameters to be identified.

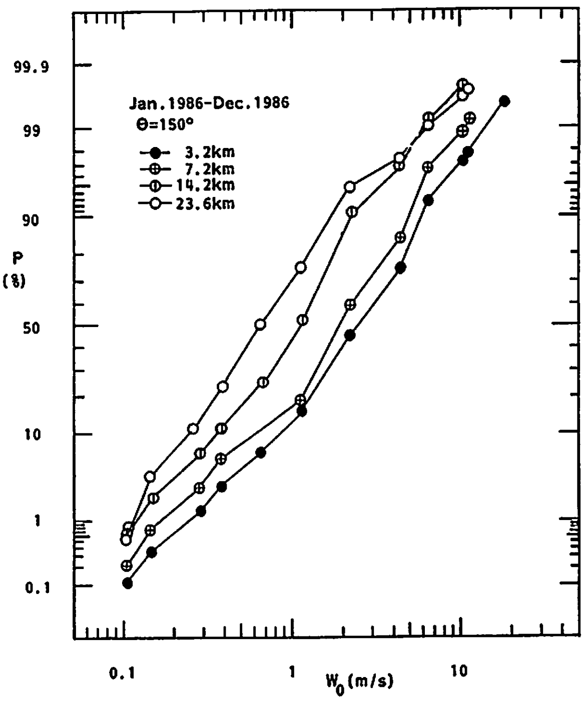

Fig. 4 Cumulative frequency distribution of settling velocity of the deposited volcanic ash

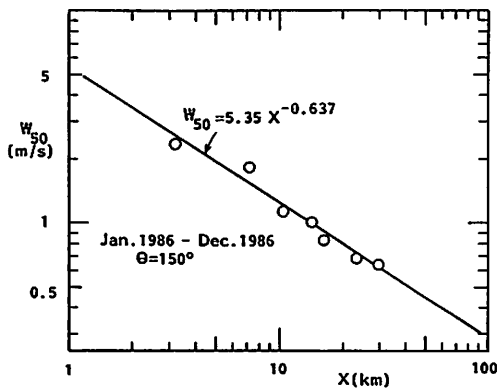

Fig. 5 Relation between mean settling velocity of the deposited volcanic ash and the distance from the crater

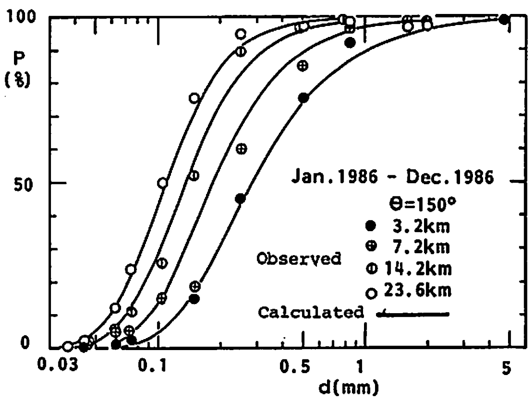

Fig. 6 Comparison between calculated and observed grain size distributions

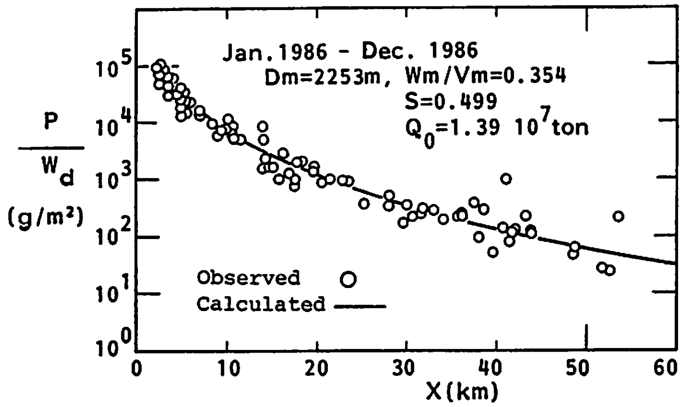

Fig. 7 Relation between depositing rate of volcanic ash and the distance from the crater

Using the data of deposited volcanic ash taken at 93 collection points around Sakurajima Volcano, the parameters were identified by the least squared method. The frequency of wind direction at 1500 m was also used to obtain $W_d(\theta+\pi)$. In Fig. 2, the identified values of Q_0 from 1975 to 1986 are shown with the frequency of eruption of Sakurajima Volcano. The annual amount of erupted volcanic ash was estimated to be thirteen millions tons. The depositing rate $P(x,\theta)/W_d(\theta+\pi)$ was computed and compared with the observed ones in Fig. 7. From the good agreement of the computed rates and observed ones, Eq. 10 is found to be acceptable for estimation of the depositing rate of volcanic ash around the volcano and the amount of erupted volcanic ash.

RUNOFF ANALYSIS OF DEBRIS FLOW

On Sakurajima Volcano, debris flow originates on the slopes and flows into a stream as illustrated in Fig. 8.

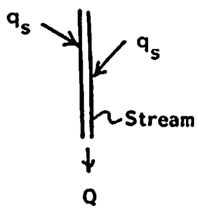

Fig. 8 Schematic sketch of debris flow in a stream

Equation of continuity in a stream is

$$\frac{\partial A_0}{\partial t} + \frac{\partial Q}{\partial x} = q_s \qquad (12)$$

where A_0=cross sectional area of the stream; Q=discharge of debris flow in the stream; x=coordinate taken in the downstream direction; t=time; q_s=rate of inflow from the slope. Assuming Q to be a function of A_0, the above equation is solved by the characteristic method as follows:

$$\frac{dx}{dt} = \frac{dQ}{dA_0} \quad \text{and} \quad Q(t) = \int_0^L q_s(t-\tau_\ell)dx \qquad (13)$$

where L=length of the stream; and τ_ℓ=time of concentration in the stream.

Volume of debris flow from a slope, V, is given by

$$V = q_s\Delta t = V_s+V_w+ \ell r\cos\theta\Delta t \qquad (14)$$

where V_s=net volume of the volcanic deposits; V_w=volume of water contained in the deposits; r=rainfall intensity; θ=angle of the slope; ℓ=slope length; and Δt=time from the occurrence to the end of the debris flow on the slope. The deposits are saturated by water at the moment of occurrence of debris flow, i.e., $V_w=\lambda V_s/(1-\lambda)$, where λ=porosity of the deposit. The concentration of debris flow, c, is defined by $c=V_s/V$. Consequently, the rate of debris flow from a slope per unit width, q_s, is expressed as

$$q_s = C\ell r\cos\theta \; ; \quad C = \frac{1-\lambda}{1-\lambda-c} \qquad (15)$$

The validity of this equation has been confirmed by experimentally by Iwamoto and Hirano (3).

Substituting, Eq. 15 into Eq. 13, one obtains

$$Q(t) = \int_0^L Cr(t+\tau_\ell)\ell\cos\theta dx \qquad (16)$$

Since the flow velocity in a stream is much higher than that on the slope, the deformation of the hydrograph in the stream is considered to be much smaller than that on slope. Then, Eq. 16 can be rewritten as

$$Q(t+\tau_\ell) \simeq \int_0^L Cr\ell\cos\theta dx = ACr(t)F(t); \quad F(t) = \int_0^L \ell\cos\theta dx/A \qquad (17)$$

where, A=catchment area. F(t) represents the ratio of area where debris flow is in existence, and is obtained as follows:

According to the experimental results of debris flow by Iwamoto and Hirano (3), debris flow occurs when surface flow appears on a slope due to heavy rainfall. Then, the occurrence criteria of surface flow on a slope are given by

$$\ell \geqq \frac{kT\sin\theta}{\lambda} \quad \text{and} \quad \int_0^T r\cos\theta dt \geqq \lambda D \qquad (18)$$

where k=hydraulic conductivity; T=time of concentration on the slope; and D=depth of the deposits.

The debris flow will occur on the slope longer than kTsinθ/λ when the accumulated rainfall becomes equal to λD/cosθ. The ratio of the area which satisfies these conditions, F_1, is given by

$$F_1(t) = f(\eta_0)\Delta\eta_0\int_{kt\sin\theta/\lambda}^{\infty} g(\ell)d\ell \qquad (19)$$

where $\eta_0=\lambda D=\int_0^t r\cos\theta dt$; $f(\eta_0)$, $g(\ell)$=probability density functions of $\eta_0=\lambda D$, ℓ, respectively.

While, on a slope whose length is in the range of $k(t-t_0)\sin\theta/\lambda$ and $kt\sin\theta/\lambda$, debris flow will occur on the slope when the condition in the following equation is met:

$$\eta = \lambda D = \int_{t_0}^{t} r\cos\theta dt \qquad (20)$$

where t=time after the beginning of the rainfall; t_0=arbitrary time between 0 to t. The ratio of occurrence area in these conditions, F_2, is

$$F_2(t) = \sum_{t_0} f(\eta)\Delta\eta\, g(\ell)\Delta\ell \tag{21}$$

Therefore, the ratio of occurrence area F(t) is given by as the sum of F_1 and F_2:

$$F(t) = f(\eta_0)\Delta\eta_0 \int_{kt\sin\theta/\lambda}^{\infty} g(\ell)d\ell + \sum_{t_0} f(\eta)\Delta\eta\, g(\ell)\Delta\ell \tag{22}$$

Assuming that all deposited sediments on a slope outflow in a short period of Δt, one obtains

$$\int_t^{t+\Delta t} q_s dt = C\int_t^{t+\Delta t} r\ell\cos\theta dt = D\ell + \int_t^{t+\Delta t} r\ell\cos\theta dt \tag{23}$$

$$\therefore \quad \Delta t \simeq \frac{D}{(C-1)r\cos\theta} \tag{24}$$

Therefore, considering that $\Delta\eta_0=\Delta\eta=r\cos\theta\Delta t$ and $\Delta\ell=-k\Delta t_0\sin\theta/\lambda$, and substituting Eq. 22 to Eq. 17, one obtains

$$Q(t+\tau_\ell) = Ar(t)\frac{C}{\lambda(C-1)}\left[f(\eta_0)\frac{\eta_0}{\lambda}\int_{kt\sin\theta/\lambda}^{\infty} g(\ell)d\ell + \frac{k\sin\theta}{\lambda}\int_0^t \eta\, f(\eta)g(\ell)dt_0\right] \tag{25}$$

This equation can be solved as a parametric model, when the forms of the probability density function, $f(\eta)$ and $g(\ell)$, are given. Distribution of slope length is obtainable by measurement on a topographical map. The measured slope lengths of the Hase and the Nojiri River on the map distributed log-normally. As for the distribution of depth of volcanic deposits, no data is available at present, but the assumption of log-normal function may be acceptable. Thus the parameters to be identified are four; i.e., the mean and standard deviation of λD, $k\sin\theta/\lambda$ and τ_ℓ.

This model was applied to the Nojiri River and the Hase River on Sakurajima. Debris flow has occurred about twenty times a year in each river. A Simplex method was used to find the optimum values of the parameters. Three examples of solutions are shown in Figs. 9-11. The computed hydrographs agree well with the observed ones, and the obtained values of the parameters are also considered to be reasonable. This indicates that the method presented here is accurate enough to predict the hydrograph of debris flow.

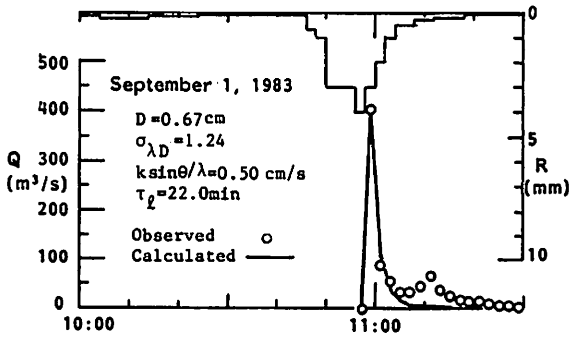

Fig. 9 Comparison between computed and observed hydrographs (the Nojiri River)

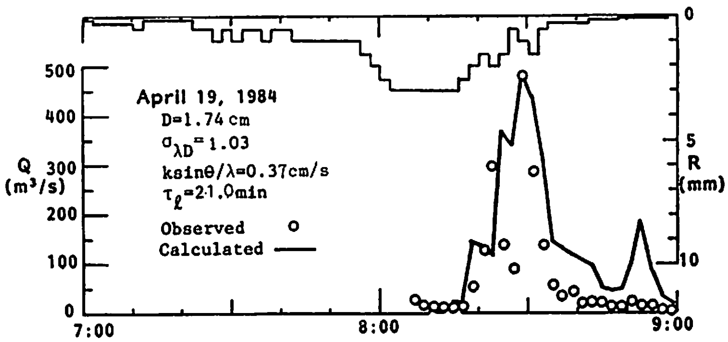

Fig.10 Comparison between computed and observed hydrographs (the Nojiri River)

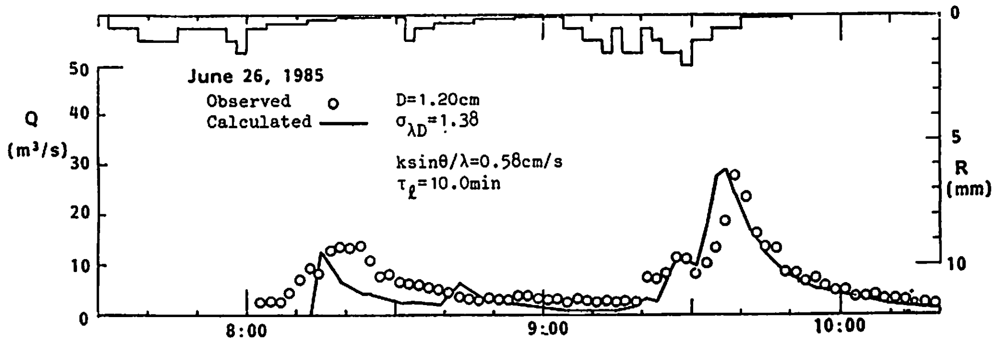

Fig.11 Comparison between computed and observed hydrographs (the Hase River)

CONCLUSION

Mathematical models for deposition of volcanic ash and debris flow on Sakurajima Volcano were established through this study. The results obtained are as follows:

Based on an assumption that a particle erupted from the crater into the air falls at its terminal velocity, theoretical equations which give the depositing rate of volcanic ash and the distribution of grain size on the ground were obtained. In derivation of this equation, the probability density functions of eruption column height, the terminal velocity of the erupted particle and the wind velocity were introduced. The computed values show fairly good agreement with the data taken from 93 volcanic ash collection points around the volcano. The annual amount of erupted volcanic ash from the crater was estimated to be about thirteen millions tons.

A method to predict the hydrograph of debris flows in the active volcano was offered, in which the distributions of slope length and the depth of the deposited volcanic ash were introduced parametrically. The parameters in this model are identified by use of the Simplex method. This method was checked by the data of hydrograph from 1975 to 1985 in two catchment areas, the Nojiri River and the Hase River in Sakurajima Volcano, and was confirmed to be useful for the prediction of hydrograph of debris flows in such an active volcanic area.

ACKNOWLEDGMENTS

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APPENDIX - NOTATION

The following symbols are used in this paper:

A	= catchment area;
A_0	= cross sectional area of the stream;
$b(x)$	= width of deposition of erupted volcanic ash at x;
c	= ln(10); or concentration of debris flow;
C	= defined in Eq. 15;
D	= height of eruption column; or depth of the deposits;
D_m	= mean of height of eruption column;
$f(W_0)$	= probability density function of W_0;
$f(\eta)$	= probability density function of $\eta=\lambda D$;
$F(t)$	= ratio of area where debris flow exists;
$F(x,W_0)$	= distribution function of settling velocity of the deposited volcanic ash at x of W_0;
$g(\ell)$	= probability density function of ℓ;
$g(V)$	= probability density function of wind velocity V;
$h(D)$	= probability density function of height of eruption column D;
k	= hydraulic conductivity;
ℓ	= slope length;
L	= length of the stream;
$P(x,\theta)$	= depositing rate of volcanic ash at x in the direction of θ;
$q(x)$	= depositing rate per unit area on the ground surface at x;
$q(x,W_0)\Delta W_0$	= depositing rate per unit area on the ground surface at x of W_0;
q_s	= rate of debris flow from a slope per unit width;
Q, $Q(t)$	= discharge of debris flow in the stream at t;
Q_0	= amount of volcanic ash erupted from the crater;
$Q(x)$	= amount of volcanic ash deposited within x from the crater;
r, $r(t)$	= rainfall intensity;
S	= $\sqrt{S_W{}^2+S_V{}^2+S_D{}^2}$ and defined in Eq. 10;
S_1	= defined in Eq. 9;

S_D, S_V, S_W	= standard deviations of logD, logV and $\log W_0$ for erupted volcanic ash at the crater, respectively;
t	= time after the beginning of the rainfall;
t_0	= arbitrary time between 0 to t;
T	= time of concentration on the slope;
V	= wind velocity; or volume of debris flow from a slope;
V_m	= mean velocity of wind;
V_s	= net volume of the volcanic deposits;
V_w	= volume of water contained in the volcanic deposits;
W_0	= settling velocity of a particle of erupted volcanic ash;
W_{50}	= mean of W_0 for deposited volcanic ash;
$W_d(\theta+\pi)$	= weight of frequency of wind direction in the direction of $\theta+\pi$;
W_m	= mean of W_0 for erupted volcanic ash at the crater;
x	= horizontal distance from the crater; or coordinate taken in the downstream direction;
ζ	= $\log(W_0/W_m)$ and defined in Eq. 9;
$\bar{\zeta}$	= defined in Eq. 9;
η	= defined in Eq. 22;
η_0	= defined in Eq. 22;
θ	= trailing direction of wind; or angle of slope;
λ	= porosity of the deposit;
τ_ℓ	= time of concentration in the stream; and
$\phi(\theta)$	= frequency of the trailing direction.