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TRANSFORMATION OF SAND WAVES DUE TO THE TIME CHANGE OF FLOW CONDITIONS

By

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SYNOPSIS

In this study the author devised some new measurement and regulation techniques for a flume experiment, and carried out two series of tests on the time response of sand waves to the change of flow conditions. The first is a test with periodic and continuous change of flow discharge and the second is that with continuous change of flume inclination.

From these experiments, it was found that bed form transfigures rapidly from dunes to antidunes with an increase in channel slope, whereas it takes a long time for the bed to recover the developed dune form after the decrease of the slope.

Based upon these experimental findings, the rate of change of mean wave length and mean wave height was formulated and some numerical simulations were carried out to estimate the bed configuration during floods of principal rivers in Japan.

INTRODUCTION

It is important to elucidate the time response of sand waves to the change of flow conditions in order to estimate the flow resistance, sediment discharge and river bed change during floods more precisely from the view point of sediment hydraulics. Many efforts have been made in pursuit of this subject (1), (3)-(11) and it has been found that because it takes much time for sand waves to change their form, they respond to any rapid change of flow conditions with a considerable time lag and loops appear in the stage-discharge relation during floods. Although some models have been proposed to formulate this process (3), (6), (7), (10), (11), they are mainly based on the features found through the experiments in which flow discharge changed discretely or continuously with a solitary wave hydrograph superimposed on a base flow. Therefore, it is doubtful whether these are applicable to the case of arbitrary change in flow conditions. In particular, many problems have remained for the case in which flow condition changes covering both the lower and upper regimes.

Some big reasons hindering this research are in the difficulties of the observation of unsteady phenomena of sediment laden flow on a movable bed and the difficulties of the regulation of experimental conditions such as flow rate, inclination and sediment supply. In this study the author devised some new measuring and regulating techniques for a flume experiment and carried out two series of tests on the time response of sand waves to the time change of flow conditions (12), (13).

The first is a test with periodic and continuous change of flow discharge and the second is with continuous change of flume inclination. The bed material used was almost uniform medium sand. In order to keep the flow condition as uniform as possible, the water surface level at the downstream end of the flume was frequently adjusted and the sediment transported from the flume was continuously circulated with water to the upstream end. The water surface shape and bed shape along the center line of the flume were frequently measured and the mean wave length, mean wave height and the mean flow depth were analyzed.

In the first series, sinusoidal and triangular patterns of a hydrograph with various periods were used within the dune regime. In the both patterns a remarkable time lag among the flow discharge, sand waves and flow depth was detected. Moreover not only the range and period of discharge variation but also the pattern of the hydrograph affected the response of sand waves.

In the second series, the hydraulic condition was chosen to cover both of the lower and upper regimes. Although there was a remarkable irregularity and regulation and sampling techniques were not good enough, it was found that the bed form transfigures rapidly from dunes to antidunes with the increase of flume inclination, but it takes a long time for the bed to recover the developed dunes after the decrease of the inclination.

Based upon these experimental findings, the rate of change of characteristic parameters such as mean wave length and mean wave height was formulated and some numerical simulation were carried out to ascertain its applicability.

Finally, some hydrological and hydraulic data of several principal rivers in Japan were rearranged and the bed configuration during floods were estimated by means of a numerical simulation according to the formulation.

EXPERIMENTAL EQUIPMENTS AND PROCEDURES

Fig. 1 shows the setup of the equipment. The flume is 50cm wide, 50cm deep, 21m long and its slope is adjustable from 0 to 1/30 at an arbitrary speed of less than 1/3000/min using motor driven jacks. Flow discharge can be changed continuously from 0 to 40ℓ/s within several seconds using an inverter. Sediment gathers in a downstream hopper and circulates with water through a sand pump. The uniform flow condition is kept by the frequent adjustment of the downstream gate, below which a slit exists so that the sediment may pass through. The carriage

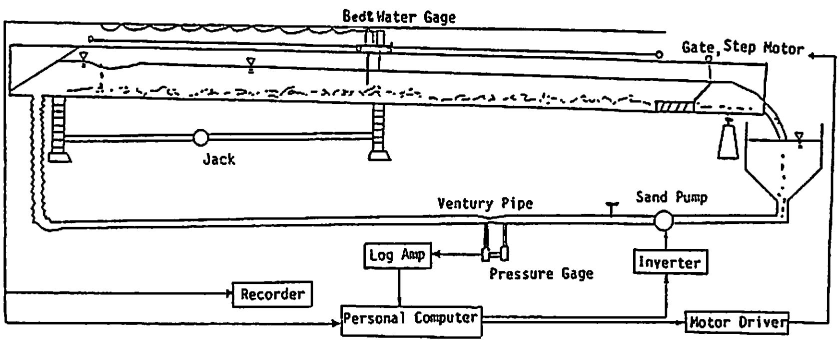

Fig. 1 Experimental equipment

moves at a speed of 50cm/s and the bed form and water surface form are measured along the channel center line.

Fig. 2 represents the size distribution of the material used. The median grain diameter is 0.77mm and the mean grain diameter is 0.8mm.

The experiments consist of two series. In the first series, the flume inclination was kept constant at 1/500, and the flow discharge was changed periodically between 20ℓ/s and 40ℓ/s with various periods. The patterns of hydrograph, sinusoidal and triangular waves were used. In each run, the experiment lasted for several periods until an equilibrium cycle appeared, and the data from the last period were used

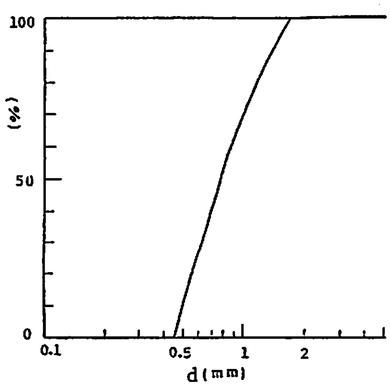

Fig. 2 Size distribution of bed material

for the analysis.

In the second series, the flow discharge was kept constant at 30ℓ/s, and the flume inclination was changed between 1/1000 and 1/150. Initially the flume inclination was kept constant at 1/1000 until the dunes developed fully, then the slope was increased from 1/1000 to 1/150 at a constant speed of 1/12000/min. After the slope became 1/150, it was decreased from 1/150 to 1/1000 with the same constant speed and kept constant at 1/1000 again for a long time.

In both series, in order to compare various experimental results with the results of fully developed sand waves under respective flow conditions, several runs were added under steady flow conditions.

The length of the measured reach was 9m for Series I, and 6m for Series II. In analyzing the bed forms, an unevenness of more than 2mm height difference was regarded as an independent wave unit. The individual wave length was defined as the distance between two adjacent troughs. The individual wave height was defined as the fall between neighboring crest and trough. The mean wave height and wave length were defined as their mean values over the measured reach. These definitions are somewhat subjective but convenient when considering the connection of the flow resistance to sand waves. Hereafter, only these mean scales will be referred to.

EXPERIMENTAL RESULTS

Series I

Fig. 3 shows the relation between the flow discharge (Q) and the wave height (Δ), wave length (Λ) and flow depth (h) under steady flow conditions in Series I. As a general trend, all of the three parameters seem to increase with the increase in the flow discharge, but a fairly irregular change is found in detail. For example, in spite of the smaller wave length and the larger wave height in the case of Q=22ℓ/s compared with the case of Q=20ℓ/s, the flow depth is somewhat smaller in the case of Q=22ℓ/s.

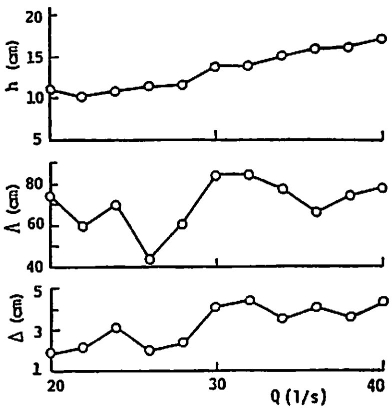

Fig. 3 Wave height, wave length and flow depth under steady flow conditions

Fig. 4 shows the time response of dune height, dune length and mean flow depth under the unsteady flow conditions in Series I. Fig. 4(a) corresponds to the case of triangular hydrographs, and Fig. 4(b) corresponds to the case of sinusoidal hydrographs. As a general tendency it was found that the response of the bed forms and flow depth to the change of flow discharge was delayed. But in the case of the period T=5040sec for the triangular hydrograph, the peak of the wave length and wave height preceded that of the discharge, which means the data scattered widely.

In Fig. 5, these processes were replotted taking the discharge as abscissa. From this figure, a remarkable loop was detected in some cases. In the case of the triangular hydrograph, the largest loop appeared for the period T=1260sec. But in the case of the sinusoidal hydrograph, it appeared for the period T=5280sec. As the general tendency (especially in the case of the triangular hydrograph), the following features were found. When the period is relatively large and the rate of the discharge change is relatively small, the loop is small and the ranges of the variation of wave length, wave height and flow depth are large. On the other hand, when the period is relatively short and the rate of the discharge change is relatively large, the loop becomes small again, but the ranges of the variation of these parameters also become small.

Not only the range but also the representative value of those parameters averaged over one period changed with the period and the pattern of the discharge variation. Although it is difficult to determine the phase lag definitely from

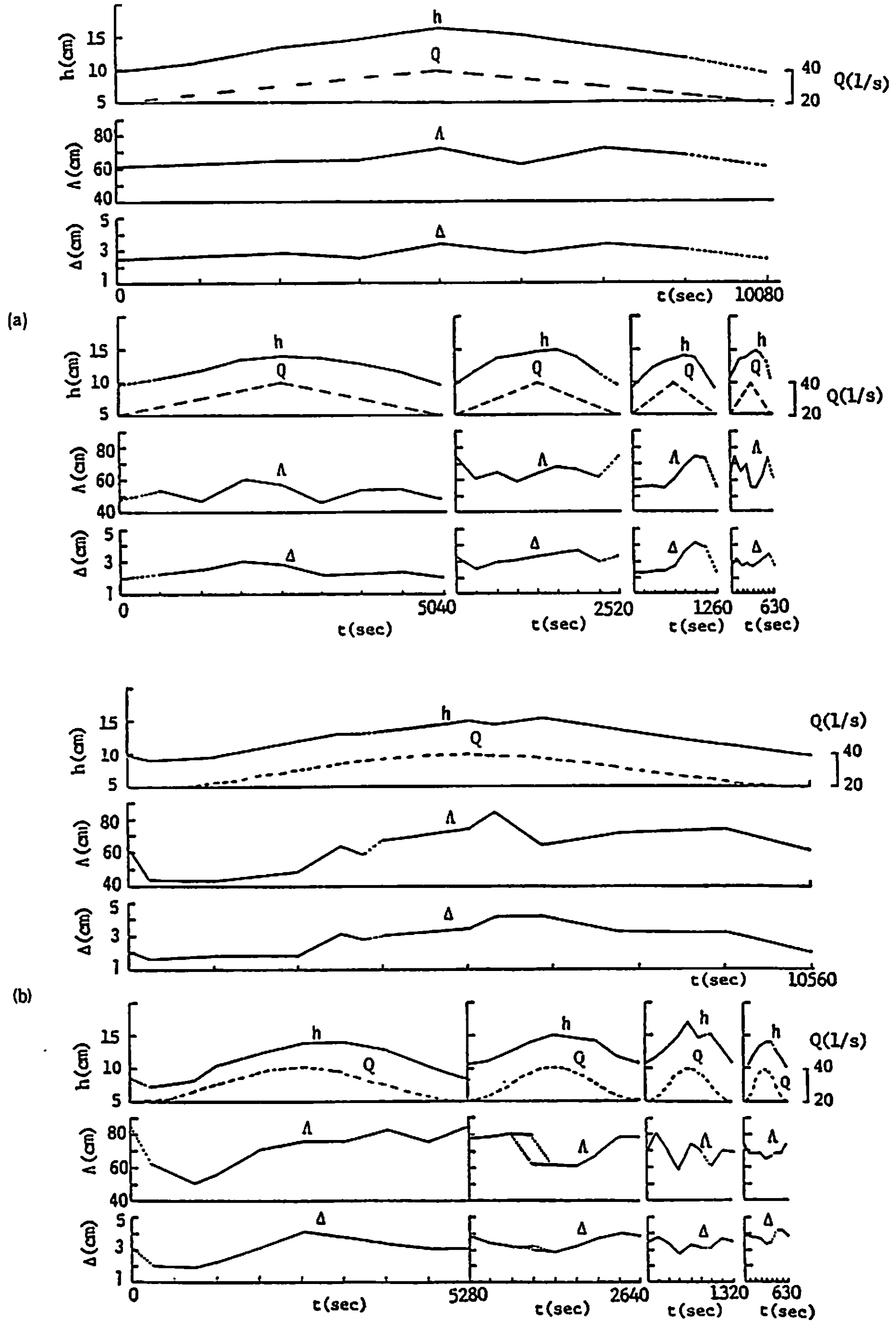

Fig. 4 Time response of dune height, dune length and mean flow depth
(a): under triangular hydrographs (b): under sinusoidal hydrographs

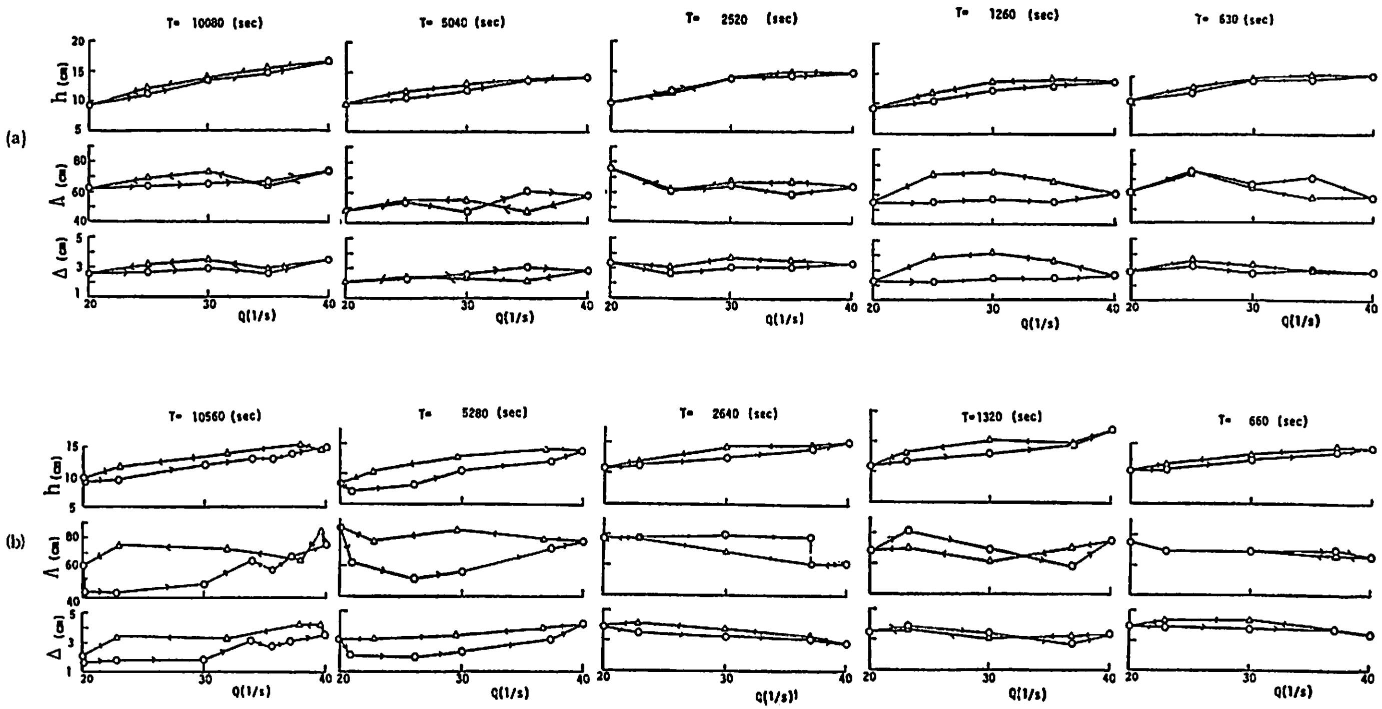

Fig. 5 Hystereses of dune height, dune length and mean flow depth
(a): under triangular hydrographs (b): under sinusoidal hydrographs

these data, as a general tendency, it seems to increase with the decrease of the period of the discharge variation.

Considering the effect of the pattern of each hydrograph, in the case of sinusoidal hydrograph, similar responses appeared for a much shorter period than in the case of triangular hydrograph. It means that not only the average but also the temporal rate of the discharge change considerably affects the response of sand waves.

Series II

Fig. 6 shows the relation between the flume inclination and the wave height, wave length and flow depth under steady conditions in Series II. As a general trend, all of the three parameters seem to decrease with the increase of the flume inclination. But a local maximum appeared at I=1/400 for the wave height and the flow depth, and a local minimum at I=1/200 for the wave height.

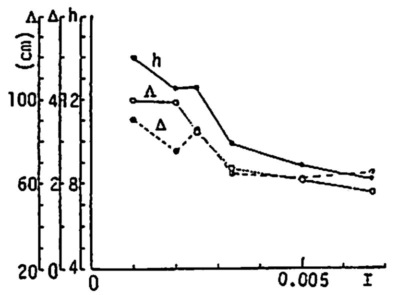

Fig. 6 Equilibrium wave length, wave height and flow depth

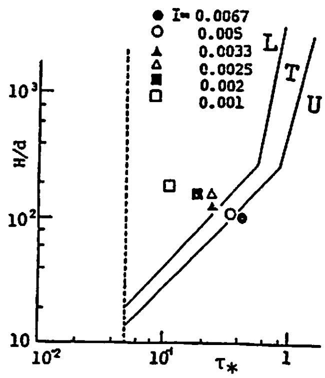

Fig. 7 Plots on the diagram of bed configuration

According to the bed configuration diagram (2) using dimensionless shear stress τ_* and relative flow depth h/d as shown in Fig. 7, flume inclination less than 1/400 corresponds to the lower regime, flume inclination I=1/150 corresponds to the upper regime and flume inclination between 1/300 and 1/200 corresponds to the transition under steady conditions.

Fig. 8 shows the time change of the bed profiles before, during and after the change of flume inclination in series II. Before the change of the slope, remarkable dunes were observed which propagated downstream with an almost constant speed maintaining similar forms. Once the flume inclination began to increase, dunes deformed to become highly irregular and when the slope became 1/200, almost all dunes had been replaced by antidunes of much smaller scale. It is difficult to pursue the propagation of antidunes from these data. Once the flume inclination began to decrease, antidunes disappeared gradually and dunes began to develop again. But it took a long time for dunes to fully develop. Two hours after the flume slope reached 1/1000 was not time enough.

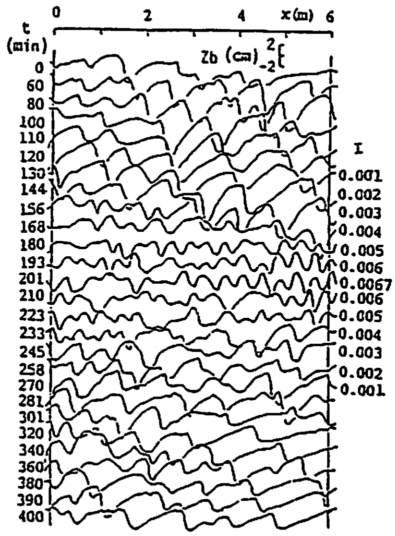

Fig. 8 The response of bed form to the time change of inclination

In Fig. 9, the time response of wave length, wave height and flow depth as well as the mean water surface level, mean bed level, standard deviation of the bed level, water surface inclination, bed inclination and flume inclination are plotted. A remarkable difference can be seen in the bed form and flow depth between the rising and falling stages of flume slope. The discrepancy among the three kinds of inclination means that the requirement of uniform flow condition was not attained completely. One of the reasons for the irregular change of bed inclination is the shortage of the length of the measured reach. Even only one wave unit of relatively large scale affects the mean bed inclination considerably.

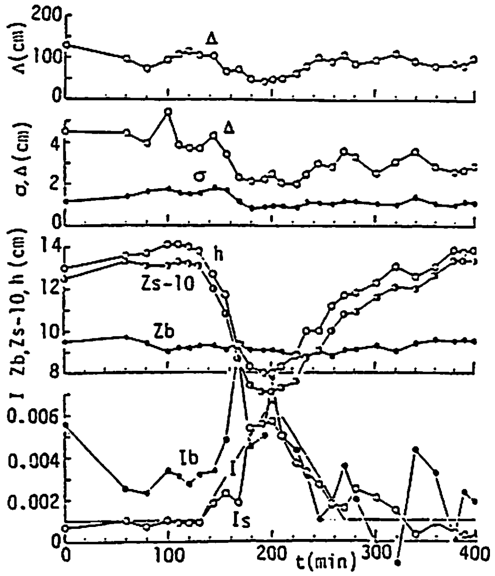

Fig. 9 Time change of some parameters averaged longitudinally

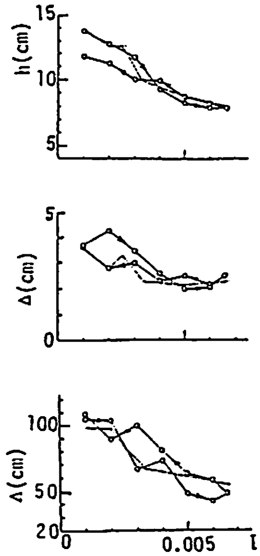

Fig. 10 Hystereses of wave length, wave height and flow depth

In Fig. 10, the response of the wave length, wave height and flow depth to the change of flume inclination is replotted taking the inclination as abscissa. For comparison, the responses for steady conditions are also replotted by the dotted lines. In the early stage of the slope increase, some time lag existed in the response of wave height, whereas wave length and flow depth almost coincided with those for the steady state. On the other hand, in the late stage of the slope increase, the wave height nearly coincided with that for the steady state, but the wave length was shorter than that for the steady state. In the decreasing stage of the flume inclination, the wave height almost coincided with that for the steady state but the wave length only approached that for the steady state after the length became larger than that for the steady state. Moreover, after the flume inclination became 1/1000 again, although the wave height and wave length coincided with those for the steady state, the recovery of the flow depth was considerably delayed.

As mentioned above, the response of sand waves including the regime transition is complicated. As a general tendency, in the lower regime the response seems to be slow and in the upper regime or transition the response seems to be quick. The data are not enough to discuss the effect of the rate of change of the inclination.

FORMULATION AND NUMERICAL SIMULATION OF THE TRANSFORMATION OF SAND WAVES

As the method to estimate the time response of sand waves to the change of flow conditions, it is considered to be effective to formulate and integrate the rate of change of characteristic parameters such as mean wave length and mean wave height according to the given time series of hydraulic parameters such as flow discharge or energy slope. It was found by Ashida et al. (4) that there is some linear relation between wave length and wave height for the respective hydraulic conditions and that in the right-down domain the splitting of waves dominates and in the left-upper domain burying dominates (see Fig. 11). In both cases, the shape of sand waves quickly approaches the line and asymptotes to an equilibrium state along the line exponentially with time.

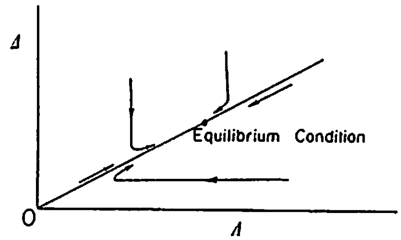

Fig. 11 Traces of sand wave transformation on Λ-Δ plane

Ashida et al. (3) derived the following equations and carried out some numerical simulations of the transformation process of dunes under steady flow conditions. It was found that these equations are applicable to the developing stage of dunes.

1) In the case of $\delta<\delta_e$

$$\frac{d\Lambda}{dt}=-A\frac{\Lambda^3}{l}(1-\beta\delta) \tag{1}$$

$$\frac{d\Delta}{dt}=-A\frac{\Lambda^2\Delta}{l}(1-\beta\delta) \tag{2}$$

2) In the case of $\delta=\delta_e$

$$\frac{d\Lambda}{dt}=-A\frac{\Lambda^3}{l}(1-\beta\delta)+f\frac{q_B}{1-\lambda}\frac{1}{\Delta} \tag{3}$$

$$\frac{d\Delta}{dt}=-A\frac{\Lambda^2\Delta}{l}(1-\beta\delta)+f\frac{q_B}{1-\lambda}\frac{1}{\Lambda} \tag{4}$$

3) In the case of $\delta>\delta_e$

$$\frac{d\Lambda}{dt}=0 \tag{5}$$

$$\frac{d\Delta}{dt}=-f_s\frac{q_B}{(1-\lambda)(1-\beta\delta)}\frac{1}{\Lambda} \tag{6}$$

where, δ = the steepness of sand waves ($=\Delta/\Lambda$); A= the rate of split of new waves during unit time and unit step length; l= the mean step length (=100d); β = the ratio of the length of separation zone to the wave height (=5); q_B= the sediment discharge in unit width; λ= the porosity (= 0.35); and f, fs= coefficients concerning the probabilistic distribution of sand waves (f=0.296, fs=2.55).

The first term of eqs. 3 and 4 represents the reduction of the wave length due to the splitting of sand waves, and the second term represents the increment of wave length due to the overtake of waves. As they balance in an equilibrium condition, the following is obtained.

$$A=f\frac{q_B}{1-\lambda}\frac{l}{\Delta_e\Lambda_e^3(1-\beta\delta_e)} \tag{7}$$

where, the subscript e denotes the equilibrium state.

In this model, the steepness can not change because $d\Delta/d\Lambda=\delta$ except for the case of $>\delta_e$. According to the experiments, however, only the wave length rapidly decreases keeping the wave height almost constant in the case of $\delta<\delta_e$. It is considered that this discrepancy comes from the simultaneous amplification of waves during splitting when $\delta<\delta_e$. In this study, therefore, eq. 2 is replaced by the following equation.

$$\frac{d\Delta}{dt}=0 \tag{8}$$

Fig. 12 shows the simulated results corresponding to the experiment under the condition of sinusoidal discharge hydrograph mentioned above. In this simulation, the following equations are used to estimate equilibrium wave length and wave height of sand waves, sediment discharge and flow resistance according to Ashida & Michiue (2) and Yalin (14), (15).

$$\Lambda_e = 2\pi h_e \tag{9}$$

$$\delta_e = 0.0127\left(\frac{\tau_*}{\tau_{*c}} - 1\right)\exp\left(-\left(\frac{\tau_*}{\tau_{*c}} - 1\right)/12.84\right) \tag{10}$$

$$\frac{q_B}{\sqrt{sgd^3}} = 17\tau_{*e}^{1.5}\left(1 - \frac{\tau_c}{\tau}\right)\left(1 - \sqrt{\frac{\tau_c}{\tau}}\right) \tag{11}$$

$$\frac{u}{u_{*e}} = 6.0 + 5.75 \log \frac{h}{d(1 + 2\tau_*)} \tag{12}$$

$$c_f = c_{f0}(1 - \bar{\delta} \cot \phi) + \frac{\Lambda}{h}\bar{\delta}^2 \tag{13}$$

$$c_{f0} = 2/\left(6.0 + 5.75 \log \frac{h}{d}\right)^2 \tag{14}$$

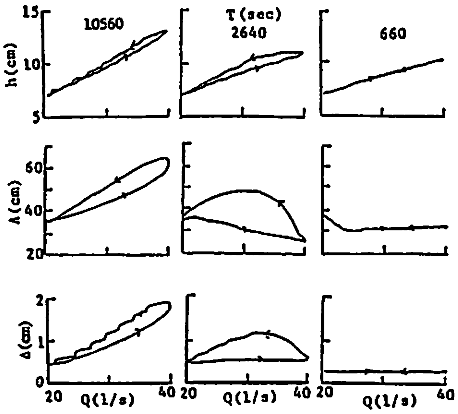

Fig. 12 Simulated hystereses of dune height, dune length and mean flow depth

where, τ_* is the dimensionless shear stress, τ_{*c} is the dimensionless critical shear stress, τ_{*e} is the dimensionless effective shear stress, Cf is the coefficient of flow resistance (=2(u/u$_*$)) and Cfo is that for the flat bed condition.

In the calculation, after the flow depth satisfying the resistance law was determined by iteration, the new bed form was calculated by the forward difference method of dt=10sec. Eqs. 3 and 4 were applied in case of $|\delta - \delta_e| < 0.001$. The initial condition was set as Λ=50cm and Δ=1cm. As the effect of the initial condition disappeared soon in all cases, only the steady cycle is shown in the figure.

In the case of the long period, wave length and wave height change almost in the same phase as discharge and the loop is small. But in the case of T=2640sec, wave length decreases in the rising stage of discharge and both of wave length and wave height increase in the early falling stage of discharge. As the period becomes much shorter, sand waves show less response and the range of the change of flow depth is also reduced.

Although these results partly coincide with the experimental one, the calculated value of wave length, wave height and flow depth are small on the whole. It is considered that this comes not only from the inaccuracy of the formulation of the rate of the change of the wave length and height, but also from the inaccuracy of the equilibrium wave length, height and flow resistance.

The above mentioned model can be applied only to the lower regime. For the upper regime, the time lag between sand waves and flow condition is negligible. In that case, the formulas for the equilibrium condition can be applied.

ESTIMATION OF THE VARIATION OF SAND WAVES DURING FLOODS IN SOME MAJOR RIVERS IN JAPAN

Hydrograph during Flood

There are various scales of floods and the river bed configuration is supposed to be dependent on the scale of the flood. As the typical flood which has a significant effect on the bed configuration, a flood with a return period of two years is considered. In Fig. 13, the relation between τ_* and R/d at the flood peak, whose return period is one year, from the middle to down stream reach of the major rivers in Japan are plotted according to the data by Yamamoto et al. (16). The lines represent the division of bed configuration. The data concentrates in the lower left and the upper right parts of the figure.

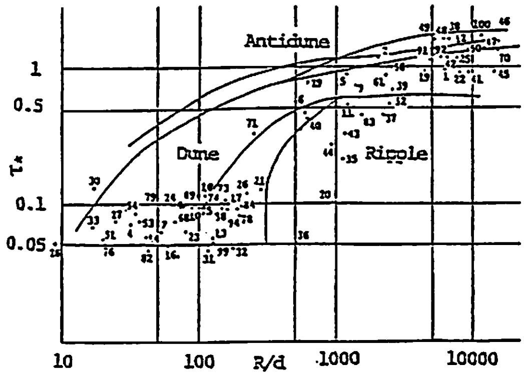

Fig. 13 τ_* and h/d at the mean annual maximum discharge

Eight points were chosen from the various domains in Fig. 13 and hydrographs for the maximum and the twentieth maximum peak discharge are plotted in Fig. 14. In the figure, the hydrographs with two-year return periods are interpolated and shown with dotted lines. For every river, flood duration is relatively short. The duration with a discharge higher than 50% of the maximum is about 10 hours, whereas for the discharge higher than 90% of the maximum it is only few hours.

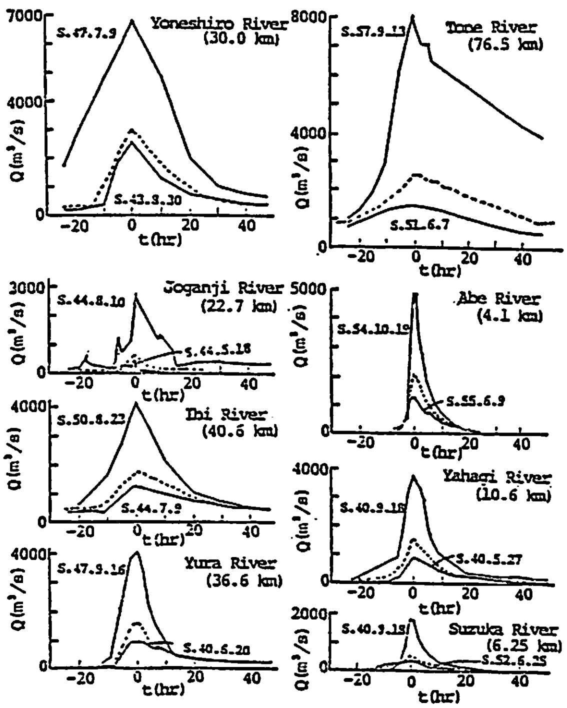

Fig. 14 Observed and estimated hydrographs for several floods

In Fig. 15, the change of hydraulic conditions during flood is shown in the plane of τ_* and h/d. The flow depth was estimated from stage-discharge diagrams at the respective observation points and the slope was substituted for by the surface slope of the design high water.

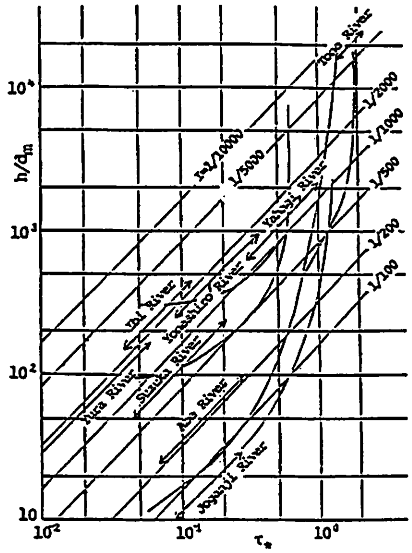

Fig. 15 The range of the variation of τ_* and h/d during floods

Estimation of Scales of Sand Waves

It is meaningful to estimate the time change of sand waves in actual rivers during floods using the knowledge obtained by experiments in laboratories, even if it is difficult. At such times, the author carried out a numerical simulation of sand waves for some of the aforementioned floods with two-year return periods. In principle, the same set of equations should be used as in the flume case. But, the accuracy of eqs. (1) to (8) is not so high. Therefore, the following equations derived by Nakagawa et al. (11) are used here.

$$\frac{d\Lambda}{dt} = 0.6\,\frac{q_B}{\Delta}\left(1-\frac{\Delta}{\Delta_e}\right) \qquad (15)$$

$$\frac{d\Lambda}{dt} = 0.4\,\frac{q_B}{\Delta}\left(1-\frac{\Lambda-100d}{\Lambda_e-100d}\right) \qquad (16)$$

These equations were derived mainly by taking into account the coalescence of adjacent sand waves with different propagating speed. Neverthless, it was ascertained to be applicable to wider conditions given some correction factors. Although Nakagawa et al. used different coefficients for increasing and decreasing stages, the same set of coefficients was used in this simulation. For equilibrium wave length and wave height, sediment discharge and flow resistance, the same set of equations was used as in the previous chapter.

As the initial conditions, the equilibrium scales were given at the 30th or 15th hour before the flood peak for the Tone River and other rivers, respectively.

The results are shown in Fig. 16, where solid lines correspond to the equilibrium state for the instantaneous hydraulic conditions, and broken lines correspond to the simulated data according to the model mentioned above.

These results show that a remarkable time lag exists in the response of sand waves during flood and the range of the variation of sand wave scales is considerably smaller than that estimated for the equilibrium state.

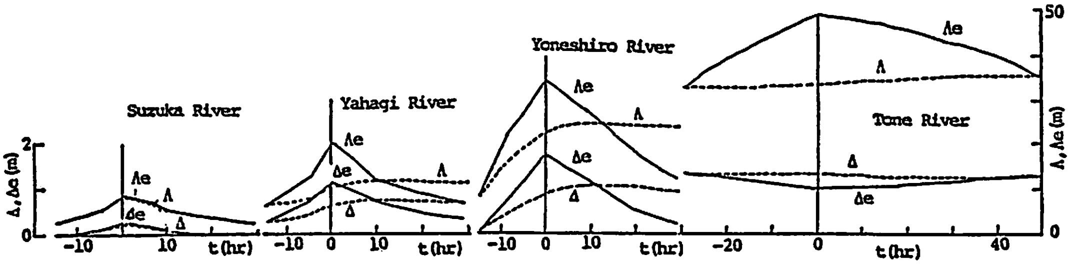

Fig. 16 Equilibrium and time integrated scales of sand waves during floods

CONCLUSIONS

In this study, the author devised some new measurement and regulation techniques for a flume experiment concerning the time response of sand waves to the change in flow conditions, and carried out two series of experiments. In the first series, the flow discharge was changed periodically keeping the flume inclination constant within any dune regime. On the other hand, in the second series, the flume inclination was changed with time keeping the discharge constant covering both the lower and upper regimes.

In the dune regime, there was a remarkable time lag in the response of the wave height, wave length and flow depth to the change of flow discharge. Consequently, a large loop appeared in the relation between these parameters and flow discharge when the rate of change of the latter was appropriate. The range of variation of these parameters decreased with the decrease of the period of the change of flow discharge. Not only the range but also the representative value of these parameters averaged over one period changed with the period and the pattern of the discharge variation. In the case of sinusoidal hydrograph, a similar response appeared for a much shorter period than in the case of triangular hydrograph.

The response of sand waves including the regime transition is complicated. As a general tendency, in the lower regime the response seems to be slow but in the upper regime or transition, the response seems to be so quick that the bed form almost coincides with that for the steady state.

In order to estimate the time response of sand waves during flood, the rate of change of characteristic parameters such as mean wave length and mean wave height was formulated based upon these experimental findings. Its applicability was ascertained through some numerical simulations corresponding to the experiments.

Finally, some hydrological and hydraulic data of several principal rivers in Japan were arranged and the bed configuration during floods was estimated by means of a numerical simulation according to the formulation. In some cases, the bed configuration was estimated to be dunes and the wave length and/or height would differ considerably from those for the equilibrium state.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A = rate of split of new waves during unit time and unit step length;
Cf = coefficient of energy loss;
Cf_o = coefficient of energy loss due to grain roughness;
d = particle diameter;
dm = mean particle diameter;
f, fs = coefficients concerning the probabilistic distribution of waves;
g = acceleration due to gravity;
h = flow depth;
he = flow depth in equilibrium state;
I = flume inclination;
Ib = bed inclination;
Is = water surface inclination;
q_B = bed load discharge in unit width;
Q = flow discharge;
R = hydraulic radius;
s = submerged specific weight of sand;
t = time;
T = period of hydrograph;
u = mean flow velocity;
u_* = shear velocity;
u_{*e} = effective shear velocity;
x = longitudinal distance;
zb = bed surface level;
zs = water surface level;
β = ratio of the length of separation zone to the wave height;
δ = steepness of sand waves;
δ_e = steepness of sand waves in equilibrium state;
Δ = wave height;
Δ_e = wave height in equilibrium state;
Λ = wave length;
Λ_e = wave length in equilibrium state;
σ = standard deviation of bed level;
τ_* = non-dimensional tractive force;
τ_{*e} = non-dimensional tractive force in equilibrium state; and
ϕ = angle of repose of bed material in water.