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# ANALYSIS OF TURBULENT STRUCTURE OF OPEN-CHANNEL FLOW WITH SUSPENDED SEDIMENTS

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## SYNOPSIS

It is known that the turbulent structure of sediment-laden flow is different from that of clear water. This variation of the turbulent structure of the flow with suspended sediments is represented by the reduction of von Karman's constant in the log law of mean velocity distribution. Thus it is observed that the velocity gradient at the near bed becomes steep, and the resistance of the flow is reduced. The main reason of this phenomenon is considered to be the suppression of turbulent energy production by density gradient of suspended sediments near the bed. It is important to take into consideration of the negative buoyancy effect by suspended sediments to explain the properties of the sediment laden flow. In the present study, the k-ε turbulence model, which is the simplest two-equation model, is adopted to analyze the open channel flow with suspended sediments. The values of some parameters in the model are determined by the experimental results of velocity and concentration distributions. Some interesting results on turbulent structure are obtained in the numerical calculations.

## INTRODUCTION

The study of suspended sediment-laden flow is one of the most important problems in river engineerings. It is difficult to predict their velocity and concentration distributions, because the development of turbulent energy is suppressed by the negative buoyancy effect of suspended sediments. It is a well-known fact that the velocity gradient near the bed increases and the resistance of the flow decreases.

Most studies on suspended sediment so far have used the classical mixing length theory (Shimura (5), Hino (2), Itakura and Kishi (3)). Recent studies in hydraulic engineering use more advanced turbulence model (Rodi (4), Devantier and Larock (1)). Two equation turbulence models can take into consideration of the buoyancy effects more naturally. The k-ε model is the simplest turbulence model among such models (Rodi (4)). It solves Reynolds stresses of mean flow, k-equation of turbulent energy, and ε-equation of energy dissipation rate. In the model, the mixing length and kinematic eddy viscosity are the variables unkown to be solved. However, the applicability of k-ε model to the suspended sediment laden flow is not tested yet.

The present study applied the k-ε model for two-dimensional uniform flow with suspended sediments. The values of some parameters in the model are determined by the experimental results of velocity and concentration distributions. Some interesting results on turbulent structure are obtained in the numerical calcuations.

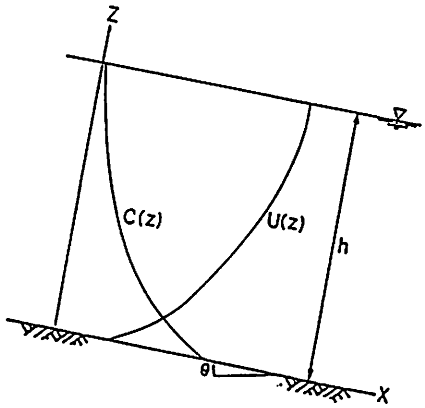


Fig. 1 Coordinates system and simbols

THE k-ε MODEL FOR SUSPENDED SEDIMENT-LADEN FLOW

Consider that sediments keep in suspension in open channel flow, of which the channel width is sufficiently large compared with the depth. The equation of mass conservation of suspended sediments is added to the standard k-ε model. Coordinate system and simbols used herein are shown in Fig. 1. It is not easy to solve the full set of the k-ε model. The main purpose of the present study is to examine how the turbulent structure of open channel flow with suspended sediments varies. The flow is considered to be two-dimensional and uniform in x-direction, and the turbulence is fully developed so that the viscous effect in the flow is ignored.

The set of fundamental equations of the flow with suspended sediment becomes (Rodi (4));

Momentum balance in the flow direction:

$$g \sin \theta \ (1 + Rc) + \frac{\partial}{\partial z}(\nu_t \frac{\partial u}{\partial z}) = 0 \qquad (1)$$

Mass balance of suspended sediments:

$$v_s \frac{\partial c}{\partial z} + \frac{\partial}{\partial z}(\frac{\nu_t}{\sigma_t} \frac{\partial c}{\partial z}) = 0 \qquad (2)$$

Turbulent kinematic energy:

$$\frac{\partial}{\partial z}(\frac{\nu_t}{\sigma_t} \frac{\partial k}{\partial z}) + \nu_t(\frac{\partial u}{\partial z})^2 + Rg \cos \theta \frac{\nu_t}{\sigma_t} \frac{\partial c}{\partial z} - \varepsilon = 0 \qquad (3)$$

Energy dissipation rate:

$$\frac{\partial}{\partial z}(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial z}) + c_{1\varepsilon} \frac{\varepsilon}{k} [\nu_t(\frac{\partial u}{\partial z})^2 + (1 - c_{3\varepsilon})Rg \cos \theta \frac{\nu_t}{\sigma_t} \frac{\partial c}{\partial z}] - c_{2\varepsilon}\frac{\varepsilon^2}{k} = 0 \qquad (4)$$

Kinematic eddy viscosity:

$$\nu_t = c_\mu \frac{k^2}{\varepsilon} \qquad (5)$$

where u =mean flow velocity in the x-direction; $v_s$ = fall velocity of sediment in still water; g = gravity acceleration; ρ, $\rho_o$ = densities in sediment-laden flow and clear water flow, respectively; c = mean sediment concentration; $\nu_t$ = the eddy viscosity; and $\sigma_t$ = the turbulent Schmidt number. Among the values of non-dimensional coefficients included in Eqs. 1-5, $c_\mu$, $\sigma_k$, $\sigma_\varepsilon$, $c_{1\varepsilon}$, and $c_{2\varepsilon}$ are shown in Table 1 (note that $\sigma_t$ and $c_{3\varepsilon}$ are not included).

Table 1 Coefficients in the k-$\varepsilon$ model

| $c_\mu$ | $c_{1\varepsilon}$ | $c_{2\varepsilon}$ | $\sigma_k$ | $\sigma_\varepsilon$ |
|---|---|---|---|---|
| 0.09 | 1.44 | 1.92 | 1.0 | 1.3 |

There is a relationship between concentration and density of suspended sediment as follow;

$$\rho = \rho_o (1 + Rc) \tag{6}$$

where R $(=(\rho_s - \rho_o)/\rho_o$ and $\rho_s$ are submerged specific gravity and density of sediment particles, respectively. The quantities, u, c, k, and $\varepsilon$, are unknown, but all of these are functions of only z.

## BOUNDARY CONDITIONS

Boundary conditions which should be satisfied are as follows. At the water surface, the shear stress and the normal flux of sediments are zero, and symmetric conditions are imposed on k and $\varepsilon$. At the bed, the wall function method which is often used in turbulence models is employed, because turbulence models cannot be applied to the region close to the bed, where the viscous effect becomes dominant.

At the water surface ( z = h );

$$\frac{\partial u}{\partial z} = 0 \qquad \frac{\partial k}{\partial z} = 0$$
$$v_s c + \frac{\nu_t}{\sigma_t}\frac{\partial c}{\partial z} = 0 \qquad \frac{\partial \varepsilon}{\partial z} = 0 \tag{7}$$

near the bed ( $z = z_o$ );

$$\frac{u}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{z_o}{k_s}\right) + A_r \qquad k = \frac{u_\tau^2}{\sqrt{c_\mu}}$$
$$c = c_b \qquad \varepsilon = \frac{u_\tau^3}{\kappa z_o} \tag{8}$$

where $z_o$ = the level where boundary condition will be imposed; $\kappa$ = von Karman's constant; $k_s$ = the height of bed roughness, $A_r$ ( = 8.5 ) = non-dimensional constant; and $u_\tau$ = the shear velocity near the bed.

The boundary conditions, Eq. 7, are the same as those of a pipe flow. The model used herein may predict the larger value of the eddy viscosity near the water surface than thode of an open channel flow. THe concentration of suspended sediment is, however, very small near the water surface. Thus the boudary condit-tions at the water surface is not so important as those at the bed. Therefore, Eq. 7 is adopted as a first approximation for an open channel flow.

Integrating Eq. 2 with respected to z and using the boundary condition Eq.7, the following equation is derived.

$$v_s c + \frac{\nu_t}{\sigma_t}\frac{\partial c}{\partial z} = 0 \tag{9}$$

## SET OF NON-DIMENSIONAL EQUATIONS

All variables in Eqs.1 to 9 are non-dimensionalized for numerical calculation. Representative length and representative velocity are the depth h and the shear velocity $u_{\tau o}$ of clear water flow, respectively. All quantities are expressed in the non-dimensional form as follows.

$$z = h\eta \qquad u = u_{\tau o} u_* \qquad c = c_b c_*$$
$$k = u_{\tau o}^{\,2}\, k_* \qquad \varepsilon = \left(\frac{u_{\tau o}^{\,3}}{h}\right) \varepsilon_* \qquad \nu_t = u_{\tau o} h\, \nu_{t*} \tag{10}$$

in which the subscript " * " indicates non-dimensional variables. The shear velocity of clear water flow $u_{\tau o}$ is defined as;

$$u_{\tau o} = (\, gh \sin\theta\,)^{0.5} \tag{11}$$

Eqs. 1, 9, 3, 4 become in the non-dimensional form as follows;

$$1 + \chi c + \frac{\partial}{\partial\eta}\left(\nu_t \frac{\partial c}{\partial\eta}\right) = 0 \tag{12}$$

$$\frac{v_s}{u_{\tau o}} c + \frac{\nu_t}{\sigma_t}\frac{\partial c}{\partial\eta} = 0 \tag{13}$$

$$\frac{\partial}{\partial\eta}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial\eta}\right) + \nu_t\left(\frac{\partial u}{\partial\eta}\right)^2 + R_i \frac{\nu_t}{\sigma_t}\frac{\partial c}{\partial\eta} - \varepsilon = 0 \tag{14}$$

$$\frac{\partial}{\partial\eta}\left(\frac{\nu_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial\eta}\right) + c_{1\varepsilon}\frac{\varepsilon}{k}\left[\nu_t\left(\frac{\partial u}{\partial\eta}\right)^2 + (1 - c_{3\varepsilon}) R_i \frac{\nu_t}{\sigma_t}\frac{\partial c}{\partial\eta}\right] - c_{2\varepsilon}\frac{\varepsilon^2}{k} = 0 \tag{15}$$

In the above equations, the subscript " * " is omitted for a simplicity. The non-dimensional parameters $\chi$ and $R_i$ are defined as follows:

$$\chi = R\, c_b \tag{16}$$

$$R_i = \frac{R c_b gh \cos\theta}{u_{\tau o}^{\,2}} = \frac{R c_b}{\tan\theta} \tag{17}$$

In a similar way, the boundary conditions Eqs. 7 and 8 are also non-dimensionalized.

Usually the bed slope of channel (inclined angle θ), water discharge of unit width $q$ and sediment discharge of unit width $q_s$ are given conditions in the experiment. On the other hand, depth $h$ and bed concentration of suspended sediments, $c_b$ , are unknown. For the case that values of $q$ and $q_s$ are given, the depth $h$ and the near bed concentration $c_b$ can be obtained as the following way. The definition of water and sediment discharges per unit width, $q$ and $q_s$ are

$$q = \int_{z_o}^{h} u\, dz \doteqdot u_{\tau o} h \int_0^1 u_*\, d\eta \tag{18}$$

$$q_s = \int_{z_o}^{h} uc\, dx \doteqdot c_b u_{\tau o} h \int_0^1 u_* c_*\, d\eta \tag{19}$$

From Eqs. 18 and 19, the near bed concentration $c_b$ can be obtained as;

$$c_b = \left(q_s \int_0^1 u_*\, d\eta\right) / \left(q \int_0^1 u_* c_*\, d\eta\right) \tag{20}$$

From Eq. 8, with the aid of Eq. 11, the depth $h$ can be obtained as follows;

$$h = \left[\, q / \left(\sqrt{g \sin\theta} \int_0^1 u_*\, d\eta\right)\right]^{2/3} \tag{21}$$

When calculated results are compared with results of experiments with narrow channel, the effect of resistance of side walls may not be negligibly small. Therefore, it is necessary to replace the depth $h$ in Eq. 11 to the hydraulic radius $h_R$ and the expression of Eq. 21 must be corrected.

## NUMERICAL ANALYSIS

The set of equations from Eq. 12 to Eq. 15 are highly non-linear. These differential equations are converted to the simultaneous equations by using relaxation method. Both of regular and irregular intervals of grids are tested in the analysis. It is found that solutions using the regular interval method have comparatively good results. The values of coefficients of k-ε model used in the analysis are set to the standard values shown in Table 1.

It is convenient in the numerical calculation that the integration is from $\eta = 0$ to $\eta = 1$ in the non-dimensional form in Eq. 10. It is noted that for most cases of interest the parameter $\chi$ is regared as $\chi = Rc_b << 1$, i.e. the sediment concentration is not so large. Thus the relation of $u_{\tau*} = 1$ is a good approximation.

## COMPARISON BETWEEN CALCULATIONS AND EXPERIMENTS

The same hydraulic conditions as Runs 1, 3, 5, 7 by Vanoni and Nomicos (7)'s experiments are adopted to the calculations. Value of $\sigma_t$ and $c_{3\varepsilon}$ are estimated as the calculated results fit best to measured velocity and concentration distributions, and depths. Parameters estimated and calculation conditions used are summarized in Table 2. Note that $q$ and $q_s$ are directly obtained from measured velocity and concentration distributions. In Fig. 2 to Fig. 5, numerical results of velocity and concentration distributions of each Run is compared with experimatal results. In Fig, 4a, the numerical result of velocity distribution of clear water flow with the same hydraulic condition is also plotted. It is found that the k-ε model, which is comparatively simple turbulence model, is able to simulate experiment data well by proper selection of values of the parameter in the model. Comparing the velocity distributions of sediment laden flow and clear water flow (Fig. 4a), it is found that velocity of suspended sediment-laden flow is larger than that of clear water flow.

The equivalent bed roughness $k_s$ in Table 2 obtained fitting data are of the order of diameter of sand particles in Runs 3, 5, and 7, in which no aparent sand waves were reported and bed conditions were flat or nearly flat. On the other hand, it is 0.94 cm in Run 1, in which generations of dunes were reported. Thus, the values of the equivalent bed roughness seem to be reasonable. The distributions of kinetic energy of turbulence $k$ and viscous dissipation rate $\varepsilon$ for each run is plotted in Fig. 6. The turbulent energy is slightly different for each run near the water surface. The turbulent energy tend to decrease as $c_b$ increses. As the near bed concentration $c_b$ increases, the viscous dissipation rate increases mostly near the bed. A comparison of the distributions of eddy viscosity for each run and that of clear water flow is made in Fig. 7. The largest eddy viscosity is appeared in the case of clear water flow. As the near bed concentration $c_b$

Table 2 Hydraulic conditions and calculation conditions

| Run. No | $q$ $cm^2/s$ | $q_s$ $cm^2/s$ | $h$ cm | $\theta$ | $\rho_s$ $g/cm^3$ | $D_s$ mm | $v_s$ cm/s | $k_s$ cm | $z_0/k_s$ | $\sigma_t$ | $c_{3\varepsilon}$ | $c_b$ | $R_i$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Run. 1 | 382 | .542 | 8.66 | .00250 | 2.65 | .105 | .945 | .941 | .4 | 1.20 | 1.0 | .007 | 4.6 |
| Run. 3 | 500 | .612 | 7.44 | .00200 | 2.65 | .105 | .945 | .018 | 16.4 | 1.70 | 3.0 | .014 | 11.6 |
| Run. 5 | 571 | .678 | 7.83 | .00206 | 2.65 | .105 | .945 | .020 | 16.0 | 1.80 | 6.0 | .018 | 14.4 |
| Run. 7 | 608 | .183 | 7.77 | .00258 | 2.65 | .161 | 1.89 | .016 | 19.5 | 1.30 | 1.0 | .006 | 3.7 |

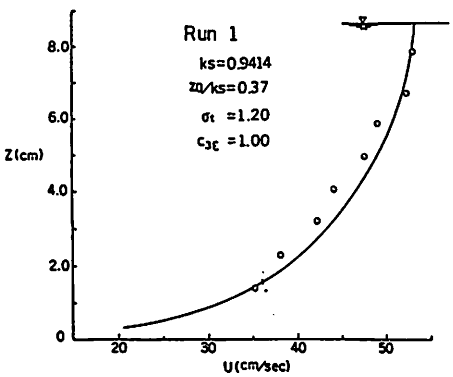


Fig. 2a Velocity distribution of Run 1

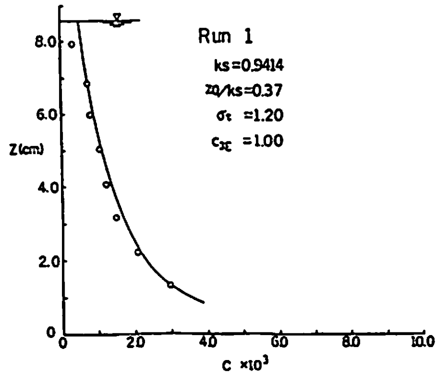


Fig. 2b Concentration distribution of Run 1

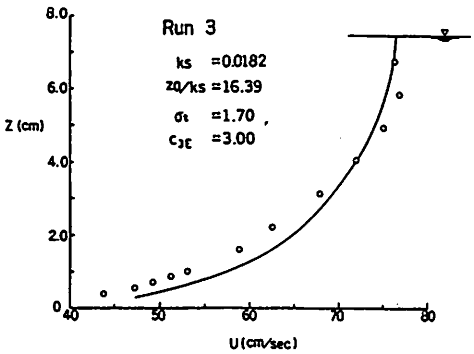


Fig. 3a Velocity distribution of Run 3

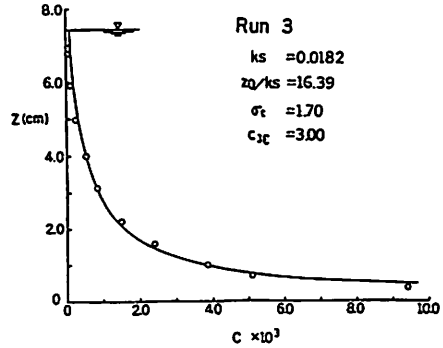


Fig. 3b Concentration distribution of Run 3

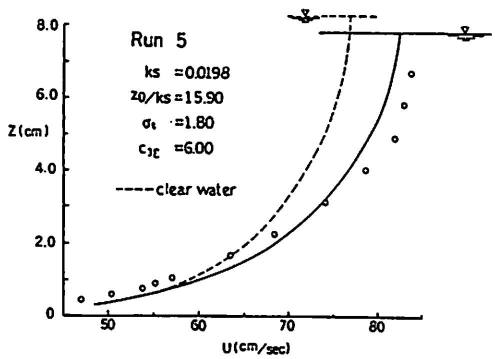


Fig. 4a Velocity distribution of Run 5

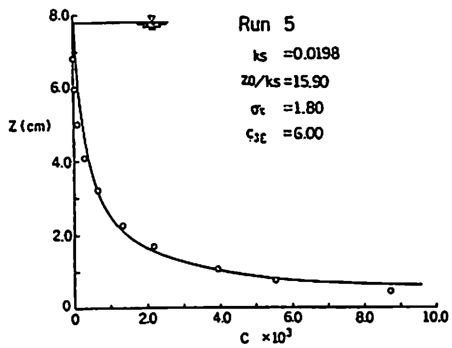


Fig. 4b Concentration distribution of Run 5

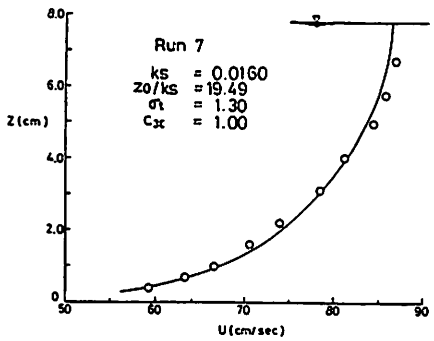


Fig. 5a Velocity distribution of Run 7

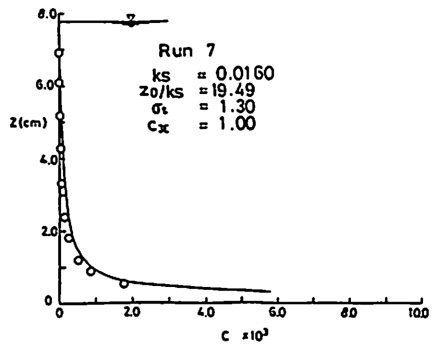


Fig. 5b Concentration distribution of Run 7

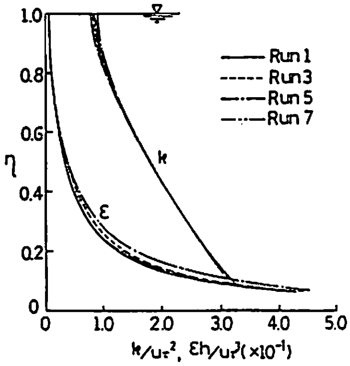


Fig. 6 Distributions of kinetic energy of turbulence and viscous dissipation rate

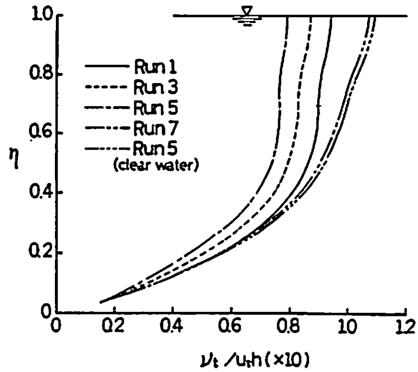


Fig. 7 Distribution of eddy viscosity

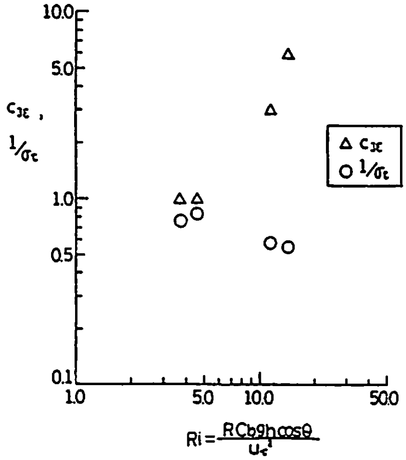


Fig. 8 $1/\sigma_t$ and $c_{3\varepsilon}$ versus $R_i$

becomes large, the eddy viscosity becomes smaller apparently for all flow regions. Thus turbulent properties such as kinetic energy of turbulence, the viscous dissipation rate and eddy viscosity vary significantly for the open channel flow with suspended sediments, comparing with those of clear water flow. The k-ε turbulence model simulates well these changes of turbulent properties.

Finally, the applicability of estimated values of coefficients $\sigma_t$ and $c_{3\varepsilon}$ is discussed. Flow properties seem to be dependent on the negative buoyancy effect caused by the concentration distribution of suspended sediments. From this point of view, $1/\sigma_t$ and $c_{3\varepsilon}$ are plotted against the overall Richardson number $R_i$ in Fig. 8. Ratio of the eddy diffusivity to the eddy viscosity, $1/\sigma_t$, decreases as the Richardson number increases; thus the stability of the flow increases. This tendency seems to be reasonable. The dependency of $c_{3\varepsilon}$ on the overall Richardson number is not clear. As the value of $c_{3\varepsilon}$ becomes large, the velocity gradient near the bed becomes steeper. For this reason, the value of $c_{3\varepsilon}$ of Run 5 becomes very large. Some researchers reported that the value of $c_{3\varepsilon}$ is 0.8-1.0, which is relatively small in comparison with the values obtained herein. Further discussions on the value of $c_{3\varepsilon}$ are necessary.

## CONCLUSIONS

Open-channel flow with suspended sediments are analyzed by the k-ε turbulence model. It is shown that the k-ε model explains well the change of the turbulent structure, mean velocity, and concentrations. Parameters included in the model, $1/\sigma_t$ and $c_{3\varepsilon}$, are functions of the overall Richardson number $R_i$.

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## APPENDIX - NOTATION

The following symbols are used in this paper:

| | |
|---|---|
| $A_r$ | = non-dimensional coefficient in the log low; |
| $c$ | = mean sediment concentration in volume; |
| $c_b$ | = near bed concentration; |
| $c_{1\varepsilon}$, $c_{2\varepsilon}$, $c_{3\varepsilon}$ | = non-dimensional coefficients in $\varepsilon$ equation; |
| $c_\mu$ | = non-dimensional coefficient in Eq. 5; |
| $g$ | = gravity acceleration; |
| $h$ | = depth of the flow; |
| $k$ | = kinematic energy of turbulence; |
| $q$ | = water discharge per unit width; |
| $q_s$ | = sediment discharge per unit width; |
| $R$ | = submerged specific gravity of sediment particles; |
| $R_i$ | = overall Richardson number; |
| $u$ | = mean velocity of x-direction component; |
| $u_\tau$ | = shear velocity of the bed; |
| $v_s$ | = fall velocity of sediment particle in still water; |
| $x$ | = downstream direction; |
| $z$ | = direction normal to x-direction; |
| $z_o$ | = height from the bed where boundary conditions are specified; |
| $\varepsilon$ | = viscous dissipation rate of turbulence; |
| $\eta$ | = non-dimensional coordinate in z-direction; |
| $\theta$ | = slope angle of channel; |
| $\kappa$ | = von Karman's constant; |
| $\nu_t$ | = eddy viscosity; |
| $\rho$, $\rho_o$, $\rho_s$ | = densities of water and sediment mixture, clear water, sediment particles, respectively; |
| $\sigma_k$ | = non-dimensional coefficient in k equation; |
| $\sigma_t$ | = turbulent Schmidt number; |
| $\sigma_\varepsilon$ | = non-dimensional coefficient in $\varepsilon$ equation and; |
| $\chi$ | = $Rc_b$ . |