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STORM SURGES RUNNING UP THE RIVER OF UNIFORM FLOW

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SYNOPSIS

The present study gives a theoretical treatment to the storm surges running up a river of uniform flow. Sinusoidal waves with very long period are given at the downstream end of the river as substitute for storm surges. The analytical solutions for river water level and flow velocity are obtained by use of a perturbation method. From these solutions, an approximate formula for the highest water levels along a river channel is derived and their relations with the amplitude and the period of waves and with the river discharge are investigated. Basic characteristics of propagation of storm surges in a river are also revealed based on the obtained analytical solution.

INTRODUCTION

Typhoons as well as hurricanes and cyclones drive occasionally storm surges on the coasts and in the bays. To protect the coastal regions from inundation damages caused by storm surges, the seadikes have been constructed extensively along coastlines. It is apparent that such measures have been much effective for diminution in the inundation damages. However, there is the serious vulnerable point left in the tidal reach where storm surges invade through river mouths.

Typhoons cause not only the storm surges frequently, but also the rainfalls in river basins. Simultaneous occurrence of the flood flowing down and the storm surge running up a river, should make river water levels considerably high, compared with the case that the storm surge only runs up the river of a certain constant discharge (Hashino and Kanda (2)). In order to evaluate quantitatively this concurrent effect of the storm surge and the flood flow on river water levels, the mechanism of interaction between storm surge and flood flow has to be clarified.

The experimental studies on storm surges running up a river were reported in the past, for example, by Hayami et al.(3) and the theoretical ones by Ichiye (4) and by Yano (9). In recent years, numerical simulations have prevailed, especially for the problems with complex boundary conditions (Kanda (5)). The theoretical investigations on travelling-up of astronomical tides were performed for the river of uniform flow by Okamoto (7) and for the river of non-uniform flow by Unoki (8).

The present study treats the storm surges running up a river of uniform flow. At the downstream end of the river, is given the tidal wave with very long period whose lowest water level coincides with the water surface of uniform flow, and the analytical solutions are obtained. Based on these solutions, basic characteristics of propagation of storm surges are revealed.

BASIC EQUATIONS AND BOUNDARY CONDITIONS

The river channel with a wide rectangular cross section and a constant bed slope is considered. The river flow before invasion of a storm surge is assumed to be uniform throughout the river channel. Beginning with this initial condition, water level at a river mouth rises gradually and the river flow becomes unsteady.

If we neglect the effect of a density current caused by sea water intrusion, the one-dimensional analysis using the following basic equations will be allowed:

$$\frac{1}{g}\frac{\partial v}{\partial t} + \frac{v}{g}\frac{\partial v}{\partial x} + \frac{\partial h}{\partial x} - S + S_f = 0 \qquad (1)$$

$$\frac{\partial h}{\partial t} + \frac{\partial (hv)}{\partial x} = 0 \qquad (2)$$

where h = water depth; v = mean velocity in x-direction; S = river-bed slope; S_f = friction slope, $S_f = n^2 v|v|/h^{4/3}$; n = Manning's roughness coefficient; and x-axis has the origin (x=0) at river mouth and is positive in the upstream direction.

Neglecting the inertia terms in Eq. 1, then it becomes

$$v = \pm \frac{1}{n} h^{2/3}\left(\pm S \mp \frac{\partial h}{\partial x}\right)^{1/2} \qquad \text{for } v \gtrless 0 \qquad (3)$$

For uniform flows in the steady state,

$$v_0 = -\frac{1}{n} h_0{}^{2/3}(-S)^{1/2} \qquad (4)$$

where v_0 = velocity of uniform flow; and h_0 = uniform water depth.

From Eqs. 2 and 3, the basic equation in terms of water depth is obtained as

$$\frac{\partial h}{\partial t} \pm \frac{5h^{2/3}}{3n}\left(\pm S \mp \frac{\partial h}{\partial x}\right)^{1/2}\frac{\partial h}{\partial x} - \frac{h^{5/3}}{2n}\left(\pm S \mp \frac{\partial h}{\partial x}\right)^{-1/2}\frac{\partial^2 h}{\partial x^2} = 0 \qquad \text{for } v \gtrless 0 \qquad (5)$$

Boundary conditions at the downstream end (x=0) and at the infinitely upstream location ($x \to \infty$) are

$$h(0,t) = \frac{A_0}{2}(1 - \cos\gamma t) + h_0 \qquad (6)$$

$$h(\infty,t) = h_0 \qquad (7)$$

where A_0 = amplitude of time-varying water level, or wave height of storm surge at river mouth; $\gamma = 2\pi/T$ = angular frequency; and T = period of wave.

As the present study treats the behavior of storm surges running up a uniform flow, the boundary condition at the downstream end is given so that the lowest water level coincides with water surface of uniform flow in the steady state, as expressed by Eq. 6. Wave shape of storm surges is not the periodic one; rather, they have the shape of a single hump. Nevertheless, the characteristics of propagation of periodic waves with very long period are presumed to be fundamentally close to those of storm surges. From this point of view, the above sinusoidal wave, which would be convenient for analytical treatment, is used as substitute for storm surge.

DERIVATION OF SOLUTIONS

To obtain the solution of Eq. 5 (v<0) under the boundary conditions 6 and 7, the method of perturbation is used.

Suppose that the solution for water depth h is expressed by the power series in the following form:

$$h = h_0(1 + \varepsilon y_1 + \varepsilon^2 y_2 + \cdots) \tag{8}$$

where $\varepsilon = A_0/(2h_0)$.

Now, we neglect the terms of higher order than the second order with respect to ε in Eq. 8, and substitute it for Eq. 5. Then, we have

$$h_0^{4/3}(1 + \varepsilon y_1 + \varepsilon^2 y_2)^{1/3}\{- S + h_0(\varepsilon\frac{\partial y_1}{\partial x} + \varepsilon^2\frac{\partial y_2}{\partial x})\}^{1/2}(\varepsilon\frac{\partial y_1}{\partial t} + \varepsilon^2\frac{\partial y_2}{\partial t})$$

$$- \frac{5}{3n}h_0^2(1 + \varepsilon y_1 + \varepsilon^2 y_2)\{- S + h_0(\varepsilon\frac{\partial y_1}{\partial x} + \varepsilon^2\frac{\partial y_2}{\partial x})\}(\varepsilon\frac{\partial y_1}{\partial x} + \varepsilon^2\frac{\partial y_2}{\partial x})$$

$$- \frac{1}{2n}h_0^3(1 + \varepsilon y_1 + \varepsilon^2 y_2)^2(\varepsilon\frac{\partial^2 y_1}{\partial x^2} + \varepsilon^2\frac{\partial^2 y_2}{\partial x^2}) = 0 \tag{9}$$

Rearranging Eq. 9 and putting each of the coefficients for ε and ε^2 to be equal to zero, the following set of equations with respect to y_1 and y_2 is obtained:

$$\frac{\partial y_1}{\partial t} + \omega\,\frac{\partial y_1}{\partial x} - \mu\,\frac{\partial^2 y_1}{\partial x^2} = 0 \tag{10}$$

$$\frac{\partial y_2}{\partial t} + \omega\,\frac{\partial y_2}{\partial x} - \mu\,\frac{\partial^2 y_2}{\partial x^2} = \Gamma(x,t) \tag{11}$$

$$\Gamma(x,t) \equiv \frac{5}{3}\mu\{(\frac{\partial y_1}{\partial x})^2 + y_1\,\frac{\partial^2 y_1}{\partial x^2}\} - \frac{2}{3}\omega y_1\,\frac{\partial y_1}{\partial x} + \frac{5\mu^2}{3\omega}\,\frac{\partial y_1}{\partial x}\,\frac{\partial^2 y_1}{\partial x^2} \tag{12}$$

where

$$\omega = (5/3)v_0 \tag{13}$$

$$\mu = (h_0 v_0)/(2S) \tag{14}$$

The boundary conditions 6 and 7 are rewritten as

$$\left.\begin{aligned} &y_1(0,t) = 1 - \cos\gamma t; \qquad y_2(0,t) = 0 \\ &y_1(\infty,t) = y_2(\infty,t) = 0 \end{aligned}\right\} \tag{15}$$

First Approximation

The solution of Eq. 10 is given by

$$y_1 = \exp(\frac{\omega}{\mu}x) - \exp\{(\frac{\omega}{2\mu} - p_1)x\}\cos(\gamma t - q_1 x) \tag{16}$$

where

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \left[\{\sqrt{(\frac{\omega}{2\mu})^4 + (\frac{\gamma}{\mu})^2} \pm (\frac{\omega}{2\mu})^2\}/2\right]^{1/2} \tag{17}$$

Second Approximation

Substituting Eq. 16 for Eq. 12, $\Gamma(x,t)$ is represented as

$$\Gamma(x,t) = \frac{13}{3}\,\frac{\omega^2}{\mu}\,\exp(\frac{2\omega}{\mu}x) - \frac{1}{3}\exp\{(\frac{3\omega}{2\mu} - p_1)x\}$$

$$\cdot\ \{\omega(\frac{29\omega}{2\mu} - 23p_1)\cos(\gamma t - q_1 x) + \mu q_1(\frac{23\omega}{\mu} - 20p_1)\sin(\gamma t - q_1 x)\}$$

$$+ \exp\{2(\tfrac{\omega}{2\mu} - p_1)x\}\{(\tfrac{\omega}{2\mu} - p_1)(\tfrac{\omega}{2} - \tfrac{5\mu}{2}p_1 + \tfrac{5\mu^2}{3\omega}p_1{}^2)$$

$$+ (\tfrac{\omega}{2\mu} - p_1)(\tfrac{13\omega}{6} - \tfrac{5\mu}{6}p_1 - \tfrac{5\mu^2}{3\omega}p_1{}^2)\cos(2\gamma t - 2q_1x)$$

$$+ q_1(\tfrac{13\omega}{6} - \tfrac{35\mu}{6}p_1 + \tfrac{5\mu^2}{3\omega}p_1{}^2)\sin(2\gamma t - 2q_1x)\} \qquad (18)$$

The right hand side of Eq. 18 consists of the non-oscillatory terms, the oscillatory terms with fundamental frequency $\gamma/2\pi$ and those with bi-frequency γ/π.

Firstly, let the general solution of the homogeneous equation corresponding to Eq. 11 be denoted by y_{20}, that is,

$$\frac{\partial y_{20}}{\partial t} + \omega\frac{\partial y_{20}}{\partial x} - \mu\frac{\partial^2 y_{20}}{\partial x^2} = 0 \qquad (19)$$

Then, y_{20} is given by

$$y_{20} = A_{20}\exp(\tfrac{\omega}{\mu}x) + B_{20}\exp\{(\tfrac{\omega}{2\mu} - p_1)x\}\cos(\gamma t - q_1x + \theta_1)$$

$$+ C_{20}\exp\{(\tfrac{\omega}{2\mu} - p_2)x\}\cos(2\gamma t - q_2x + \theta_2) \qquad (20)$$

where A_{20}, B_{20}, C_{20} = integral constants; θ_1, θ_2 = phase constants; and

$$\begin{pmatrix} p_2 \\ q_2 \end{pmatrix} = \left[\{\sqrt{(\tfrac{\omega}{2\mu})^4 + (\tfrac{2\gamma}{\mu})^2} \pm (\tfrac{\omega}{2\mu})^2\}/2\right]^{1/2} \qquad (21)$$

Secondly, the equation for the oscillatory terms with fundamental frequency is represented as

$$\frac{\partial y_{21}}{\partial t} + \omega\frac{\partial y_{21}}{\partial x} - \mu\frac{\partial^2 y_{21}}{\partial x^2} = \Gamma_1(x,t) \qquad (22)$$

$$\Gamma_1(x,t) \equiv -\tfrac{1}{3}\exp\{(\tfrac{3\omega}{2\mu} - p_1)x\}\{\omega(\tfrac{29\omega}{2\mu} - 23p_1)\cos(\gamma t - q_1x)$$

$$+ q_1(23\omega - 20\mu p_1)\sin(\gamma t - q_1x)\} \qquad (23)$$

The solution of this equation is given by

$$y_{21} = D_1\exp\{(\tfrac{3\omega}{2\mu} - p_1)x\}\cos(\gamma t - q_1x + \theta_1) \qquad (24)$$

where

$$D_1 = \sqrt{M_1{}^2 + N_1{}^2} \qquad (25)$$

$$\theta_1 = \tan^{-1}(-\tfrac{N_1}{M_1}) \qquad (26)$$

$$M_1 = -\frac{1}{12p_1}(\tfrac{3\omega}{\mu} - 36p_1 + \tfrac{20\mu}{\omega}p_1{}^2) \qquad (27)$$

$$N_1 = \frac{q_1}{12p_1}(\tfrac{3\omega}{\mu} + 10p_1 - \tfrac{20\mu}{\omega}p_1{}^2)/(\tfrac{\omega}{2\mu} - p_1) \qquad (28)$$

Thirdly, the equation for the oscillatory terms with bi-frequency is represented as

$$\frac{\partial y_{22}}{\partial t} + \omega \frac{\partial y_{22}}{\partial x} - \mu \frac{\partial^2 y_{22}}{\partial x^2} = \Gamma_2(x,t) \qquad (29)$$

$$\Gamma_2(x,t) = \exp\{2(\frac{\omega}{2\mu} - p_1)x\}\{(\frac{\omega}{2\mu} - p_1)(\frac{13\omega}{6} - \frac{5\mu}{6}p_1 - \frac{5\mu^2}{3\omega}p_1^2)$$

$$\cdot \cos(2\gamma t - 2q_1x) + q_1(\frac{13\omega}{6} - \frac{35\mu}{6}p_1 + \frac{5\mu^2}{3\omega}p_1^2)\sin(2\gamma t - 2q_1x)\} \qquad (30)$$

The solution of this equation is given by

$$y_{22} = - D_2\exp\{2(\frac{\omega}{2\mu} - p_1)x\}\cos(2\gamma t - 2q_1x + \theta_2) \qquad (31)$$

where

$$D_2 = \sqrt{M_2^2 + N_2^2} \qquad (32)$$

$$\theta_2 = \tan^{-1}(- \frac{N_2}{M_2}) \qquad (33)$$

$$M_2 = - \frac{1}{24p_1}(\frac{2\omega}{\mu} + 25p_1 - \frac{10\mu}{\omega}p_1^2) \qquad (34)$$

$$N_2 = \frac{q_1}{24p_1}(\frac{2\omega}{\mu} - 5p_1 + \frac{10\mu}{\omega}p_1^2)/(\frac{\omega}{2\mu} - p_1) \qquad (35)$$

Lastly, the equation for the non-oscillatory terms is represented as

$$\frac{\partial y_{23}}{\partial t} + \omega \frac{\partial y_{23}}{\partial x} - \mu \frac{\partial^2 y_{23}}{\partial x^2} = \Gamma_3(x,t) \qquad (36)$$

$$\Gamma_3(x,t) \equiv \frac{13\omega^2}{3\mu}\exp(\frac{2\omega}{\mu}x) + \exp\{2(\frac{\omega}{2\mu} - p_1)x\}$$

$$\cdot (\frac{\omega}{2\mu} - p_1)(\frac{\omega}{2} - \frac{5\mu}{2}p_1 + \frac{5\mu^2}{3\omega}p_1^2) \qquad (37)$$

The solution of this equation is given by

$$y_{23} = - \frac{13}{6}\exp(\frac{2\omega}{\mu}x) - D_0\exp\{2(\frac{\omega}{2\mu} - p_1)x\} \qquad (38)$$

where

$$D_0 = - \frac{1}{4p_1}(\frac{\omega}{2\mu} - \frac{5}{2}p_1 + \frac{5\mu}{3\omega}p_1^2) \qquad (39)$$

Consequently, from Eqs. 20, 24, 31, 38 and the boundary conditions 15, the second approximation y_2 is given as follows:

$$y_2 = \frac{13}{6}\{\exp(\frac{\omega}{\mu}x) - \exp(\frac{2\omega}{\mu}x)\} + D_0[\exp(\frac{\omega}{\mu}x) - \exp\{2(\frac{\omega}{2\mu} - p_1)x\}]$$

$$+ D_1[\exp\{(\frac{3\omega}{2\mu} - p_1)x\} - \exp\{(\frac{\omega}{2\mu} - p_1)x\}]\cos(\gamma t - q_1x + \theta_1)$$

$$+ D_2\Big[\exp\{(\frac{\omega}{2\mu} - p_2)x\}\cos(2\gamma t - q_2 x + \theta_2)$$

$$- \exp\{2(\frac{\omega}{2\mu} - p_1)x\}\cos(2\gamma t - 2q_1 x + \theta_2)\Big] \quad (40)$$

After all, by using y_1 and y_2 obtained above, the water depth h is given as

$$h = h_0 + \frac{A_0}{2}y_1 + \frac{A_0{}^2}{4h_0}y_2 = h_0 + \Delta H \quad (41)$$

$$\Delta H \equiv \frac{A_0}{2}y_1 + \frac{A_0{}^2}{4h_0}y_2 \quad (42)$$

where ΔH will be called the wave height, or the increase in water depth measured from water surface of uniform flow.

Solution for Flow Velocity

From Eq. 3, the solution for the flow velocity v is given as

$$v = -\frac{1}{n}\Big(h_0 + \frac{A_0}{2}y_1 + \frac{A_0{}^2}{4h_0}y_2\Big)^{2/3}\Big\{-S + \frac{A_0}{2}\frac{\partial y_1}{\partial x} + \frac{A_0{}^2}{4h_0}\frac{\partial y_2}{\partial x}\Big\}^{1/2} \quad (43)$$

where y_1 and y_2 are given by Eqs. 16 and 40, respectively.

ACCURACY OF SOLUTIONS OBTAINED

Accuracy of the analytical solution 41 is examined by comparing it with the numerical solution of Eq. 5 for river discharge per unit width $q_0 = 5(m^2/s)$, river-bed slope $S = -1/10000$, wave amplitude $A_0 = 0.5\sim2(m)$ and period $T = 1\sim8(hr)$. In addition, because Eq. 5 has been obtained as a result of omitting the inertia terms in Eq. 1, the numerical solution of Eqs. 1 and 2 is also compared with that of Eq. 5 to confirm the pertinence of this omitting. The exact solution of Eq. 5 and that of Eqs. 1 and 2 are never to be given analytically. Therefore, the numerical solutions by a finite difference method—Preissmann four-point implicit scheme (Cunge et al.(1); Kanda and Kitada (6))—are substituted for the exact solutions.

As examples of the computed results, Figs. 1 and 2 show the magnitude of each term in Eq. 1 at river mouth and the variations of water level and velocity with time, respectively, for $A_0 = 1(m)$, $T = 4(hr)$, and Figs. 3 and 4 for $A_0 = 1(m)$, $T = 8(hr)$. Analytical solutions 41 and 43 show good agreement with numerical solutions except those for the velocity near a river mouth. Inertia terms, $(1/g)(\partial v/\partial t)$ and $(v/g)(\partial v/\partial x)$ tend to have a significant magnitude with increasing wave amplitude and with decreasing wave period. Although the omission of inertia terms may yield a slight time-lag between both the wave phases, there is little difference in peak values of water level and velocity between the numerical solutions to Eqs. 1, 2 and Eq. 5.

After all, these results together with the results for the other conditions demonstrate that Eq. 41 is a fairly good approximation to the exact solution of Eqs. 1 and 2 for $A_0 \leq 1(m)$ and $T \geq 4(hr)$.

CHARACTERISTICS OF PROPAGATION OF LONG PERIOD WAVE

Structural Feature of Solutions

The first term of y_1 in Eq. 16 is the mean water level, the second term being the deviation from it. The former is the function of river discharge, and the latter the function of river discharge and wave period. The first and the second terms of y_2 in Eq. 40 denote the non-oscillatory variations, the third term the fundamental oscillation, and the fourth term the oscillation with bi-frequency.

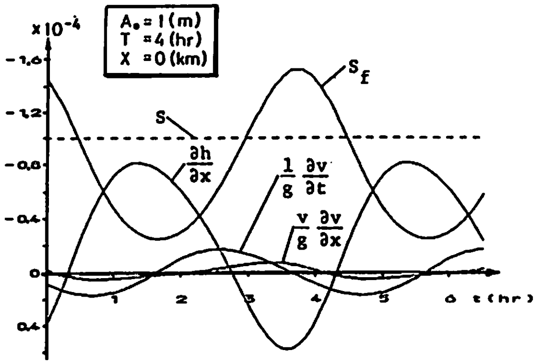

Fig. 1 Comparison of magnitude of the terms in Eq. 1 for wave period T=4 (hr).

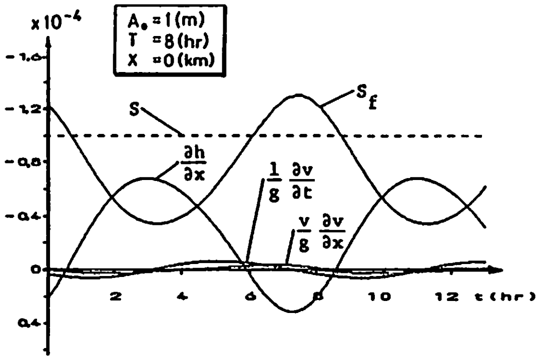

Fig. 3 Comparison of magnitude of the terms in Eq. 1 for wave period T=8 (hr).

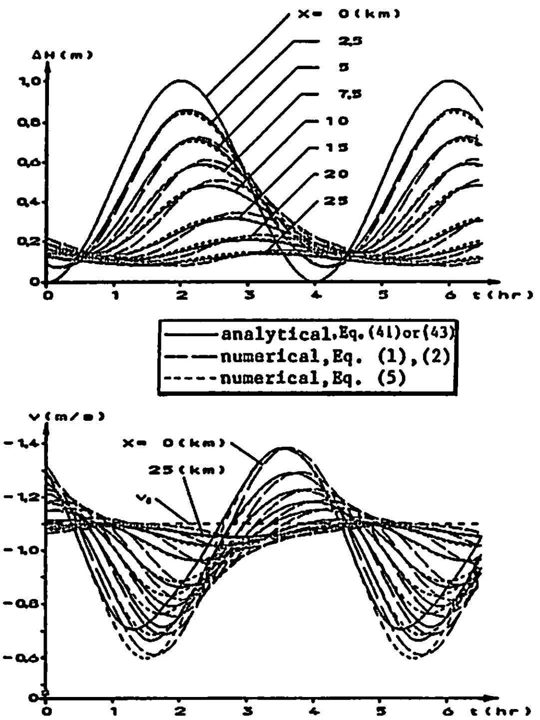

Fig. 2 Variations of water level and velocity with time for T=4 (hr).

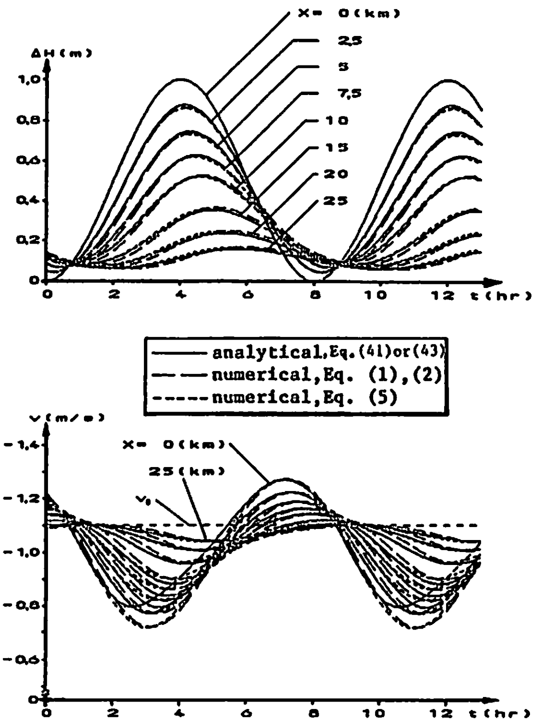

Fig. 4 Variations of water level and velocity with time for T=8 (hr).

In Fig. 5, the upper figure shows the magnitude proportion of the first approximation $(A_0/2)y_1$ and the second approximation $(A_0^2/4h_0)y_2$ for wave height ΔH. It can be seen that the wave of the latter term propagates with a larger speed than the wave of the former term, which brings about the change in wave shape, that is, makes the foreside of the wave steep and the backside mild. The lower figure is a comparison of the respective terms of the second approximation $(A_0^2/4h_0)y_2$. The third term refering to the fundamental oscillation is the most significant in magnitude, and the fourth term refering to the oscillation with bi-frequency has non-negligible magnitude. Figure 6 shows the phase lag δ for each term included in y_1 and y_2, where $\delta_0 = q_1x$, $\delta_1 = q_1x-\theta_1$, $\delta_2 = (q_2x-\theta_2)/2$, $\delta_3 = q_1x-\theta_2/2$. As is

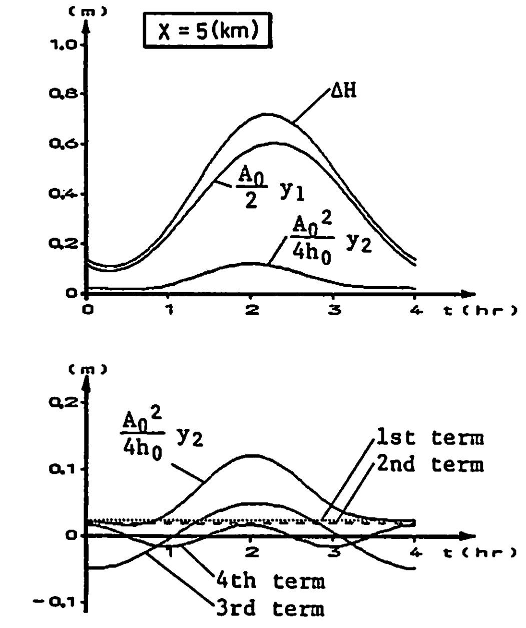

Fig. 5 Magnitude of the terms in Eq. 42 and those in Eq. 40.

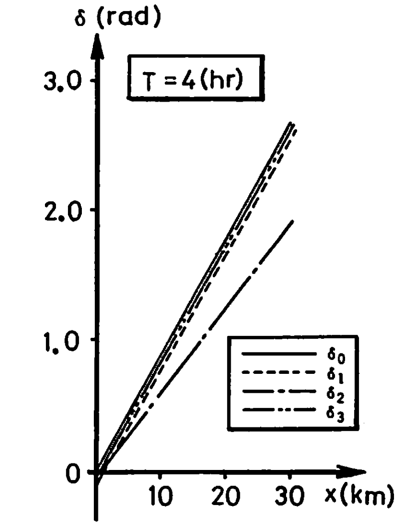

Fig. 6 Phase lags for the terms in y_1 and y_2.

obvious from the figure, only the phase lag δ_2 for the term with bi-frequency is considerably small compared with the other phase lags. This means that the corresponding wave propagates faster and accordingly results in deformation of the total wave, as described above.

Highest Water Level along River Channel

If we neglect the difference among phase lags of the elementary waves in Eqs. 16 and 40, then Eq. 42 yields the highest water level ΔH_{max}, namely, the maximum increase in water depth, as follows:

$$\frac{\Delta H_{max}}{A_0} = \frac{1}{2} y_{1.max} + \frac{A_0}{4h_0} y_{2.max} \tag{44}$$

$$y_{1.max} = \exp\left(\frac{\omega}{\mu}x\right) + \exp\left\{\left(\frac{\omega}{2\mu} - p_1\right)x\right\} \tag{45}$$

$$\begin{aligned} y_{2.max} = {} & \frac{13}{6}\left\{\exp\left(\frac{\omega}{\mu}x\right) - \exp\left(\frac{2\omega}{\mu}x\right)\right\} \\ & + D_0\left[\exp\left(\frac{\omega}{\mu}x\right) - \exp\left\{2\left(\frac{\omega}{2\mu} - p_1\right)x\right\}\right] \\ & + D_1\left[\exp\left\{\left(\frac{\omega}{2\mu} - p_1\right)x\right\} - \exp\left\{\left(\frac{3\omega}{2\mu} - p_1\right)x\right\}\right] \\ & + D_2\left[\exp\left\{\left(\frac{\omega}{2\mu} - p_2\right)x\right\} - \exp\left\{2\left(\frac{\omega}{2\mu} - p_1\right)x\right\}\right] \end{aligned} \tag{46}$$

In Fig. 7, Eq. 44 is shown together with the highest water level obtained from numerical solution of Eq. 5 and that from Eq. 42. This figure substantiates that Eq. 44 is the approximation with high accuracy to the highest water level.

In case that the river discharge q_0 and the wave period T are kept constant, both of $y_{1.max}$ and $y_{2.max}$ in Eq. 44 take constant values. Therefore, $\Delta H_{max}/A_0$ is linearly dependent on the relative wave height A_0/h_0.

Secondly, the relations of the highest water level with wave period and river discharge in case of the constant wave amplitude are given from Eq. 44, as follows. Figure 8 shows its relation with wave period. For the shorter period, the highest water level attenuates more largely as the wave runs upstream. If the period approaches infinity in Eq. 44, the following equation is obtained:

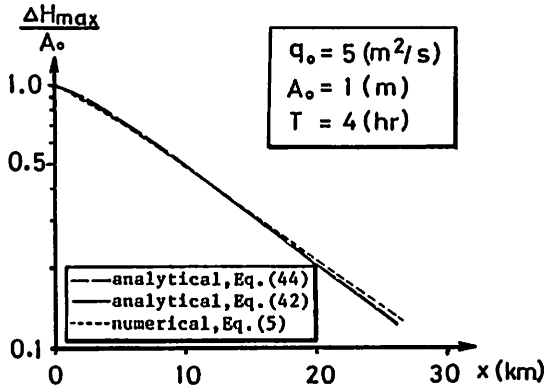

Fig. 7 Maximum increase in water depth due to storm surge.

$$\frac{\Delta H_{max}}{A_0} = \exp\left(\frac{\omega}{\mu}x\right) + \frac{13A_0}{6h_0}\left\{\exp\left(\frac{\omega}{\mu}x\right) - \exp\left(\frac{2\omega}{\mu}x\right)\right\} \tag{47}$$

The above equation would provide a fairly good approximation to surface profiles for the steady non-uniform flow, when we regard (h_0+A_0) and $(h_0+\Delta H_{max})$ as water depths at the downstream end and the arbitrary location along a river, respectively.

The relation of the highest water level with river discharge is shown in Fig. 9. As the discharge is smaller, the attenuation becomes larger. This is due to the fact that attenuation coefficients in Eqs. 45 and 46, $-\omega/\mu$, $-\omega/2\mu+p_1$, $-2\omega/\mu$, etc. are larger for the smaller discharge.

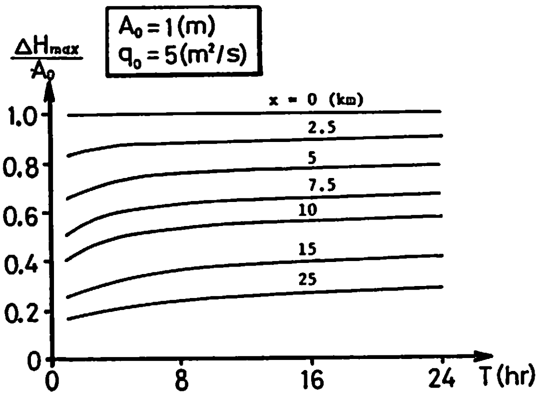

Fig. 8 Relation of ΔH_{max} with wave period T.

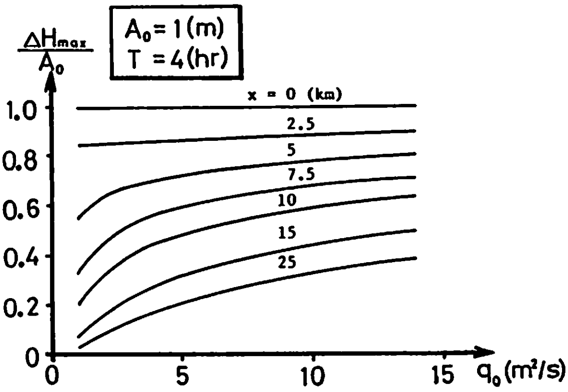

Fig. 9 Relation of ΔH_{max} with river discharge q_0.

Velocity of Propagation of Highest Water Level

Time-distance relations which trace the propagation of the highest water level are shown by the broken line for numerical solution of Eq. 5 and by the solid line for Eq. 41 in Fig. 10. Dot-dash-lines denote the reference waves whose propagation velocities are γ/q_1 and $2\gamma/q_2$, respectively. The solid line as well as the broken line becomes parallel to the upper dot-dash-line, as the wave runs upstream. This means that the elementary waves with bi-frequency attenuate and the wave with the fundamental frequency and the velocity γ/q_1 becomes dominant at the upstream region. The velocity γ/q_1 is the function of river discharge and wave period, and the relation between them is illustrated in Fig. 11.

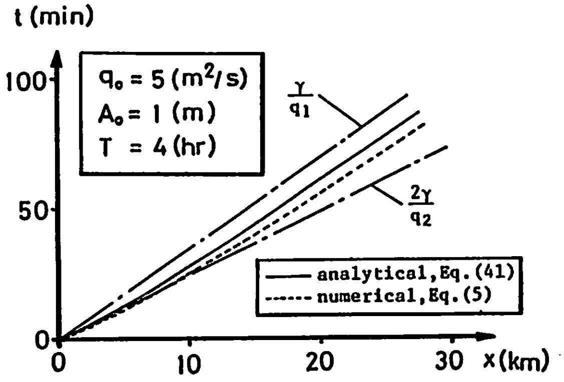

Fig. 10 Propagation of the highest water level.

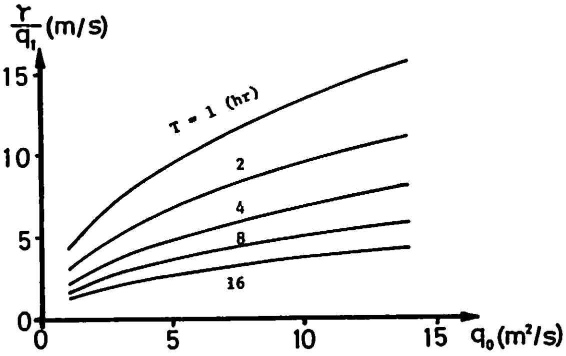

Fig. 11 Relation of γ/q_1 with river discharge and water period.

CONCLUDING REMARKS

Real wave shapes of storm surges are made up mainly of a single hump, although they are generally accompanied by the forerunner and small oscillatory waves at the front and the tail of a hump, respectively. On the contrary, analytical solutions in the present study have been obtained for the successive wave and they are the stationary solutions to which the transient waves converge after the lapse of time. In this respect, the applicability of these solutions to storm surges has been examined, as follows.

A sine wave of one cycle from a trough to the next trough has been used as the wave shape of storm surges at a river mouth, instead of the successive sine wave used in the above-described analyses. As a result of comparison between the wave shape in a river channel calculated for this boundary condition and that for the obtained analytical solution, both wave shapes have agreed precisely except the portion near troughs or front and tail of the wave. Consequently, the characteristics of propagation of the successive waves obtained in this study, especially those near the wave peak are regarded to be fundamentally identical with those of storm surges.

Equation 41 is a fairly satisfactory solution of Eq. 5 for the amplitude; $A_0/h_0 \leq 0.3$. However, this basic equation itself is not applicable to the overall boundary conditions. For the larger amplitude and the smaller period of waves at a river mouth, inertia terms in Eq. 1 cannot be neglected and accordingly Eq. 5 comes to lose the validity. Therefore, further investigations are necessary to clarify the running-up behavior of storm surges under such conditions.

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REFERENCES

1. Cunge, J.A., F.M. Holly and A. Verwey : Practical Aspects of Computational River Hydraulics, Pitman Publishing Limited, pp.53-131, 1980.
2. Hashino, M. and T. Kanda : Characteristics of concurrence of rainfall, flood and storm surge associated with typhoon, Jour. Hydroscience and Hydraulic Engineering, Vol.3, No.2, pp.31-47, 1985.
3. Hayami, S., K. Yano, S. Adachi and H. Kunishi : Experimental studies on meteorological tsunamis travelling up the rivers and canals in Osaka City, Disaster Prevention Research Institute, Kyoto Univ., Bull. No.9, pp.1-47, 1955.

4. Ichiye, T. : On the abnormal high waters in rivers, The Oceanographical Magazine, Vol.5, No.1, pp.45-60, 1953.
5. Kanda, T. : Concurrence of flood flow and invasion of storm surge in the tidal river, Proc. 21st Congress of IAHR, Vol.3, pp.15-20, 1985.
6. Kanda, T. and T. Kitada : An implicit method for unsteady flows with lateral inflows in urban rivers, Proc. 17th Congress of IAHR, Vol.2, pp.213-220, 1977.
7. Okamoto, G. : Tides in a river, Chikyu Butsuri, Vol.4, pp.62-80, 1940 (in Japanese).
8. Unoki, S. : Study on tides in a river (Part 2), Proc. 16th Japanese Conference of Coastal Engineering, pp.377-384, 1969 (in Japanese).
9. Yano, K. : Theoretical research on the surging phenomena of the high tide by the typhoon into the rivers and canals, Annuals of the Disaster Prevention Research Institute, Kyoto University, No.4, pp.194-197, 1969 (in Japanese).

APPENDIX - NOTATION

The following symbols are used in this paper:

A_0 = amplitude of time-varying water level or wave height of storm surge at river mouth;

$(A_0/2)y_1$ = first approximation to solution of river water depth;

$(A_0^2/4h_0)y_2$ = second approximation to solution of river water depth;

g = gravitational acceleration;

h = water depth at arbitrary location along river channel;

h_0 = uniform water depth;

ΔH = increase in water depth due to storm surge, measured from water surface of uniform flow;

ΔH_{max} = maximum value of ΔH;

n = Manning's roughness coefficient;

p_1, p_2, q_1, q_2 = function of ω, μ and γ;

q_0 = river discharge per unit width;

S = river-bed slope;

S_f = friction slope;

t = time;

T = wave period;

v = mean velocity at the arbitrary location along river channel;

v_0 = velocity of uniform flow;

x = coordinate which has the origin (x=0) at river mouth and is positive in the upstream direction;

$y_{1.max}$ = maximum value of y_1;

$y_{2.max}$ = maximum value of y_2;

γ = $2\pi/T$, angular frequency;

δ = phase lag for each term included in y_1 and y_2;

ε = $A_0/(2h_0)$;

θ_1, θ_2 = phase constant;

μ = $h_0v_0/(2S)$; and

ω = $(5/3)v_0$.