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FORMULATION OF THE JOINT RETURN PERIOD OF TWO HYDROLOGIC VARIATES ASSOCIATED WITH A POISSON PROCESS

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SYNOPSIS

This study aims to formulate the joint return period of two hydrologic variates, which are defined to be two random variables (marks) correlated with each other, associated with a single Poisson point process. Freund's bivariate exponential distribution is employed to describe the joint probability distribution of the partial-duration series of the two marks. The theory is applied to evaluate the joint return period of the peak rainfall intensity and the maximum storm surge caused by a typhoon in Osaka, where it might take place with quite high possibility that the flood following a heavy rainfall may move down the tidal river in the lowlying urban area while the storm surge may run up the river with a considerably short time lag.

INTRODUCTION

Rivers in lowlying urban basins generally consist of complicated channel networks with tidal reaches. It is quite possible in such a tidal river basin that the flood following a typhoon may travel downstream and the storm surge may run upstream after only a very short time lag. We have quantitatively evaluated this concurrent effect, and clarified the mechanism of interaction between flood flow and storm surge through close analyses of the running-up behavior of storm surge (Hashino and Kanda (3)). It is also important that the joint probability or joint return period of rainfall and storm surge is evaluated, for the planning and management of flood control system.

In this paper, the joint return period of two variates, the peak rainfall intensity and the maximum storm surge due to a typhoon, is formulated by using the theory of a marked point process, in which the two partial-duration series of peak rainfall intensity and maximum storm surge follow Freund's bivariate exponential distribution (Freund (2)). An application of this model to the data at osaka confirms its pertinence.

JOINT PROBABILITY OF TWO ANNUAL MAXIMUM VARIATES

Consider a single point process with an auxiliary variable, called a mark, such as the maximum storm surge at a shore due to a typhoon. It satisfies the following conditions: (i) The occurrences follow an inhomogeneous Poisson process; and (ii) $\{\eta_i\}$ is a sequence of mutually independent random variables whose distribution depends on their occurrence time t_i of the i-th exceedance of a fixed base level.

From these two conditions, the distribution of the annual maximum $\eta_{max}=\max \eta_i$ can be derived (North (4)) as

$$P[\eta_{max}\leq y] = \exp[-\int_0^{t_0}\{1-F_t(y)\}\lambda_p(t)dt] \tag{1}$$

where t_0 = 365 days, $\lambda_p(t)$ = time-dependent rate of occurrence, and $F_t(y)$ =

distribution function of exceedances η_i depending on occurrence time t_i. It is empirically known in this case that a gamma or an exponential distribution function may be appropriate for $F_t(y)$. In the light of simplicity and precision in fitting the distribution to the data, a so-called combined exponential distribution, composed of two or three exponential distributions, has been developed. For instance, a combined exponential distribution composed of three distributions is given as

$$F_t(y) = \begin{cases} 1 - \exp[-\lambda_1 y] & (0 \leqq y < y_1) \quad (2a) \\ 1 - \exp[-\lambda_2 (y-c_1)] & (y_1 \leqq y < y_2) \quad (2b) \\ 1 - \exp[-\lambda_3 (y-c_2)] & (y_2 \leqq y \quad) \quad (2c) \end{cases}$$

where y_i (i=1,2,3) = constants related to the range of y in Eqs. 2a - 2c, respectively, and c_1, c_2; λ_1, λ_2, λ_3 = parameters of the combined exponential distribution. Constants y_i (i=1,2,3) are decided from the visual inspection of the distribution of y, plotted on exponential probability paper (semilogarithmic paper) by Gringorten's formula:

$$P_i = (i-0.44) / (n+0.12) \quad (3)$$

in which i (i=1,2,...,n) = serial number of the i-th largest in the sample, and n = sample size.

The continuity conditions of F(y) for $y=y_1$, y_2 give the following equations for c_1 and c_2.

$$c_1 = \{1-(\lambda_1/\lambda_2)\}y_1; \quad c_2 = \{y_1(\lambda_2-\lambda_1)+y_2(\lambda_3-\lambda_2)\}/\lambda_3 \quad (4)$$

The use of the maximum likelihood method to estimate parameters λ_1, λ_2 and λ_3 leads to

$$1/\lambda_1 = \bar{y}_1+(\Sigma n_i/n_1-1)y_1; \quad 1/\lambda_2 = \bar{y}_2-y_1+n_3(y_2-y_1)/n_2$$

$$1/\lambda_3 = \bar{y}_3-y_2 \quad (5)$$

where n_i, $\bar{y}_i$ (i=1,2,3) = sample sizes and means of y for the ranges corresponding to Eqs. 2a - 2c, respectively.

In order to extend Eq. 1 to the case when two auxiliary variables (say, the maximum storm surge and the peak rainfall intensity due to a typhoon) are associated with a single point process, a third condition is added, namely, (iii) the occurrences related to the second auxiliary variable can be also represented by $\lambda_p(t)$ in Eq. 1 and magnitude ξ_i of the second mark is related to magnitude η_i of the first mark. Consequently, the joint probability of two maximum annual variates ξ_{max} and η_{max} can be given as

$$P[\xi_{max} \leqq x, \eta_{max} \leqq y] = \exp[-\int_0^{t_0}\{1-F_t(x,y)\}\lambda_p(t)dt] \quad (6)$$

where $F_t(x,y)$ = time-dependent joint probability distribution function of ξ_i and η_i.

FREUD'S BIVARIATE EXPONENTIAL DISTRIBUTION AND ITS COMBINATION

Consider a bivariate distribution, such as $F_t(x,y)$ in Eq. 6. The bivariate exponential distribution, which is a special case of the well-known bivariate gamma distribution, includes the zero-order modified Bessel function of the first kind, so that it is impossible without the help of supplementary numerical tables to estimate the value of X for given values of Y, and the conditional distribution function F(X|Y) of X given Y, although the value of F(X|Y) for given values of X and Y can be computed. On the other hand, Freund (1) proposed a quite different type of bivariate exponential distribution. It includes only exponential functions, so that the value of X for given values of $F(X|Y)$ and Y, as well as the value of $F(X|Y)$ for given X and Y, can be easily calculated.

We make use of Freund's distribution as the bivariate exponential distribution in essentials. If the combination of two or three Freund's distributions yields a better fit to data, then a combined Freund's distribution may be employed. A combined density function $f(X,Y)$ composed of two Freund's distributions is defined as (Hashino and Sugi (2))

$$f(X,Y) = \begin{cases} a_1 b_2 \exp[-b_2 Y-(a_1+b_1-b_2)X] & (0<X<Y, X<u) \quad (7a) \\ b_1 a_2 \exp[-a_2 X-(a_1+b_1-a_2)Y] & (0<Y<X, Y<u) \quad (7b) \\ \alpha_1 \beta_2 \exp[-\beta_2(Y-v)-(\alpha_1+\beta_1-\beta_2)(X-v)] & (u<X<Y) \quad (7c) \\ \beta_1 \alpha_2 \exp[-\alpha_2(X-v)-(\alpha_1+\beta_1-\alpha_2)(Y-v)] & (u<Y<X) \quad (7d) \end{cases}$$

where $a_1,b_1,a_2,b_2;\alpha_1,\beta_1,\alpha_2,\beta_2$ = parameters for the lower class functions of X and Y: Eqs. 7a and 7b, and for the upper classes: Eqs. 7c and 7d, v = constant, and u = critical value of X and Y that classifies the combined Freund's distribution into the two regions shown in Eqs. 7a; 7b and 7c; 7d. In the case of $u = \infty$, Eqs. 7a and 7b become the original Freund's equations. From continuity conditions of $f(Y)$, $F(X|Y)$, $F(X)$ and $F(Y)$ for X or Y = u, the following equations are derived.

$$b_1 = \beta_1;\; a_2 = \alpha_2;\; v = [1-\{(a_1+b_1)/(\alpha_1+\beta_1)\}]\, u \qquad (8)$$

The marginal density and distribution functions of Y, denoted by $f(Y)$ and $F(Y)$, the conditional distribution function $F(X|Y)$ of X given Y, and the joint distribution function $F(X,Y)$ can be given as

$$f(Y) = \begin{cases} f_1(Y) = \dfrac{a_1 b_2 \exp(-b_2 Y)}{(a_1+b_1-b_2)} + \dfrac{(a_1+b_1)(b_1-b_2)}{(a_1+b_1-b_2)} e^{-(a_1+b_1)Y} & (0<Y<u) \quad (9a) \\ f_2(Y) = \dfrac{a_1 b_2 \exp(-b_2 Y)}{(a_1+b_1-b_2)} \{1-e^{-(a_1+b_1-b_2)u}\} + \dfrac{\alpha_1 \beta_2}{(\alpha_1+\beta_1-\beta_2)} e^{-\{(\alpha_1+\beta_1)(u-v)+\beta_2(Y-u)\}} + \dfrac{(\alpha_1+\beta_1)(\beta_1-\beta_2)}{(\alpha_1+\beta_1-\beta_2)} e^{-(\alpha_1+\beta_1)(Y-v)} & (u<Y) \quad (9b) \end{cases}$$

$$F(Y) = \begin{cases} F_1(Y) = 1-\dfrac{a_1 \exp(-b_2 Y)}{(a_1+b_1-b_2)} - \dfrac{(b_1-b_2)}{(a_1+b_1-b_2)} e^{-(a_1+b_1)Y} & (0<Y<u) \quad (10a) \\ F_2(Y) = 1-\dfrac{a_1 \exp(-b_2 Y)}{(a_1+b_1-b_2)} \{1-e^{-(a_1+b_1-b_2)u}\} - \dfrac{\alpha_1}{(\alpha_1+\beta_1-\beta_2)} e^{-\{(\alpha_1+\beta_1)(u-v)+\beta_2(Y-u)\}} - \dfrac{(\beta_1-\beta_2)}{(\alpha_1+\beta_1-\beta_2)} e^{-(\alpha_1+\beta_1)(Y-v)} & (u<Y) \quad (10b) \end{cases}$$

$$F_1(X|Y) = \frac{a_1 b_2 \exp(-b_2 Y)}{(a_1+b_1-b_2) f_1(Y)} \{1-e^{-(a_1+b_1-b_2)X}\} \qquad (0<X<Y<u) \quad (11a)$$

$$F(X|Y) = \begin{cases} F_2(X|Y) = 1-\dfrac{b_1}{f_1(Y)}\, e^{-\{a_2X+(a_1+b_1-a_2)Y\}} & (0<Y<u, Y<X) \quad (11b) \\[2ex] F_3(X|Y) = F_1(X|Y)\cdot f_1(Y)/f_2(Y) & (0<X<u<Y) \quad (11c) \\[2ex] F_4(X|Y) = 1-\dfrac{\exp\{-\beta_2(Y-v)\}}{(\alpha_1+\beta_1-\beta_2)f_2(Y)}\{\alpha_1\beta_2 e^{-(\alpha_1+\beta_1-\beta_2)(X-v)} & \\ \qquad +(\alpha_1+\beta_1)(\beta_1-\beta_2)e^{-(\alpha_1+\beta_1-\beta_2)(Y-v)}\} & (u<X<Y) \quad (11d) \\[2ex] F_5(X|Y) = 1-\dfrac{\beta_1}{f_2(Y)}\, e^{-\{(\alpha_1+\beta_1)(Y-v)+\alpha_2(X-Y)\}} & (u<Y<X) \quad (11e) \end{cases}$$

$$F(X,Y) = \begin{cases} F_1(X,Y) = F_1(X)-F_1(X|Y)\cdot f_1(Y)/b_2 & (0<X<Y, X<u) \quad (12a) \\[2ex] F_2(X,Y) = F_1(Y)-\dfrac{b_1\exp(-a_2X)}{(a_1+b_1-a_2)}\{1-e^{-(a_1+b_1-a_2)Y}\} & (0<Y<X, Y<u) \quad (12b) \\[2ex] F_3(X,Y) = F_2(X)+F_2(Y)-1+\dfrac{(\beta_1-\beta_2)}{(\alpha_1+\beta_1-\beta_2)}\, e^{-(\alpha_1+\beta_1)(Y-v)} & \\ \qquad +\dfrac{\alpha_1}{(\alpha_1+\beta_1-\beta_2)}\, e^{-\{(\alpha_1+\beta_1)(X-v)+\beta_2(Y-X)\}} & (u<X<Y) \quad (12c) \\[2ex] F_4(X,Y) = F_2(X)+F_2(Y)-1+\dfrac{(\alpha_1-\alpha_2)}{(\alpha_1+\beta_1-\alpha_2)}\, e^{-(\alpha_1+\beta_1)(X-v)} & \\ \qquad +\dfrac{\beta_1}{(\alpha_1+\beta_1-\alpha_2)}\, e^{-\{(\alpha_1+\beta_1)(Y-v)+\alpha_2(X-Y)\}} & (u<Y<X) \quad (12d) \end{cases}$$

where $f_1(Y)$, $F_1(Y)$; $f_2(Y)$, $F_2(Y)$ = density and distribution functions of Y for $0<Y<u$ and $u<Y$, respectively, and $F_1(X)$, $F_2(X)$ = distribution functions of X for $0<X<u$ and $u<X$, respectively, which can be easily obtained by replacing Y,a,b,α and β with X,b,a,β and α, respectively. Equations 9 through 12 hold for the case of $a_1+b_1 \neq b_2$ and $\alpha_1+\beta_1 \neq \beta_2$. Equations for other cases are omitted herefrom. Under the continuity conditions shown in Eq. 8, independent variables of the combined Freund density function in Eq. 7 are the parameters: a_1, b_2, α_1, β_1, α_2 and β_2. These six parameters can be estimated by the maximum likelihood method as

$$\begin{aligned} 1/a_1 &= \{\sum_{j=1}^{N_{11}} X_j^{<11>} + \sum_{j=1}^{N_{12}} Y_j^{<12>} + N_2 u\}/N_{11} \\ 1/b_2 &= \sum_{j=1}^{N_{11}} (Y_j^{<11>} - X_j^{<11>})/N_{11} \\ 1/\alpha_1 &= \{\sum_{j=1}^{N_{21}} X_j^{<21>} + \sum_{j=1}^{N_{22}} Y_j^{<22>} - N_2 u\}/N_{21} \\ 1/\beta_1 &= 1/b_1 = N_{21}/(N_{22}\alpha_1) \\ 1/\alpha_2 &= 1/a_2 = \sum_{j=1}^{N_{22}} (X_j^{<22>} - Y_j^{<22>})/N_{22} \\ 1/\beta_2 &= \sum_{j=1}^{N_{21}} (Y_j^{<21>} - X_j^{<21>})/N_{21} \end{aligned} \qquad (13)$$

where $N_2 = N_{21}+N_{22}$, $(X_j^{<11>}, Y_j^{<11>})$, $(X_j^{<12>}, Y_j^{<12>})$, $(X_j^{<21>}, Y_j^{<21>})$, $(X_j^{<22>}, Y_j^{<22>})$ = samples of the bivariate (X,Y) satisfying the regions: $(0<X<Y, X<u)$, $(0<Y<X, Y<u)$, $(u<X<Y)$ and $(u<Y<X)$, respectively, and N_{11}, N_{12}, N_{21}, N_{22} = sample sizes for these four regions. The correlation coefficient ρ of X and Y for the upper class-density function shown in Eqs. 7c and 7d can be expressed in terms of the corresponding parameters: α_1, β_1, α_2 and β_2 as

$$\rho = (\alpha_2\beta_2-\alpha_1\beta_1) \;/\; \sqrt{(\alpha_2^2+2\alpha_1\beta_1+\beta_1^2)(\beta_2^2+2\alpha_1\beta_1+\alpha_1^2)} \tag{14}$$

It is easily found from Eq. 14 that the correlation coefficient varies within the domain: $-1/3 < \rho < 1$.

In practice, the original bivariate (x,y) should be transformed to the following nondimentional bivariate (X,Y):

$$X = x^m \;/\; \sigma_{xm} \quad ; \quad Y = y^m \;/\; \sigma_{ym} \tag{15}$$

in which σ_{xm}, σ_{ym} = standard deviations of the m-th power-transformed variates x^m and y^m of x and y, respectively.

JOINT RETURN PERIOD OF TWO ANNUAL MAXIMA

The joint exceedance probability $P[\xi_{max} \geq x, \eta_{max} \geq y]$ of the bivariate (ξ_{max}, η_{max}) can be expressed in terms of the joint non-exceedance probability $P[\xi_{max} \leq x, \eta_{max} \leq y]$ and the marginal probabilities $P[\xi_{max} \leq x]$ and $P[\eta_{max} \leq y]$ as

$$\begin{aligned} P[\xi_{max} \geq x, \eta_{max} \geq y] &= 1-P[\xi_{max} \leq x]-P[\eta_{max} \leq y] \\ &\quad +P[\xi_{max} \leq x, \eta_{max} \leq y] \end{aligned} \tag{16}$$

The inverses of exceedance probabilities $P[\xi_{max} \geq x]$, $P[\eta_{max} \geq y]$, $P[\xi_{max} \geq x, \eta_{max} \geq y]$ and $P[\xi_{max} \geq x | \eta_{max} \leq y]$ are defined as the return periods T_x, T_y, T_{xy}, and $T_{x|y<}$, namely,

$$T_x = 1/P[\xi_{max} \geq x] = 1/\{1-P[\xi_{max} \leq x]\} \tag{17a}$$

$$T_y = 1/P[\eta_{max} \geq y] = 1/\{1-P[\eta_{max} \leq y]\} \tag{17b}$$

$$T_{xy} = 1/P[\xi_{max} \geq x, \eta_{max} \geq y] \tag{17c}$$

$$\begin{aligned} T_{x|y<} &= 1/P[\xi_{max} \geq x | \eta_{max} \leq y] \\ &= 1/\{1-P[\xi_{max} \leq x | \eta_{max} \leq y]\} \end{aligned} \tag{17d}$$

Substitution of Eq. 17 into Eq. 16 gives

$$T_{xy}/(T_x \cdot T_y) = [(T_x/T_{x|y<})+T_y\{1-(T_x/T_{x|y<})\}]^{-1} \tag{18}$$

The right-hand side of Eq. 18 is designated by K, so that

$$T_{xy} = K \cdot T_x \cdot T_y \tag{19}$$

and

$$K = [(T_x/T_{x|y<})+T_y\{1-(T_x/T_{x|y<})\}]^{-1} \tag{20}$$

It follows from Eqs. 1 and 17b that

$$1-1/T_y = \exp[-\int_0^{t_0}\{1-F_t(y)\}\lambda_p(t)dt] \qquad (21)$$

in which replacement of y into x gives

$$1-1/T_x = \exp[-\int_0^{t_0}\{1-F_t(x)\}\lambda_p(t)dt] \qquad (22)$$

Combining Eqs. 1 and 5 with Eq. 17d gives

$$1-1/T_{x|y<} = P[\xi_{max} \leqq x, \eta_{max} \leqq y]/P[\eta_{max} \leqq y]$$

$$= \begin{cases} \exp[-\int_0^{t_0}\{F_t(y)-F_t(x,y)\}\lambda_p(t)dt] & (\rho \neq 0) \\ 1-1/T_x & (\rho = 0) \end{cases} \qquad (23)$$

Combining Eqs. 22 with 23 for the case of $\rho \neq 0$ yields

$$(1-1/T_x)/(1-1/T_{x|y<}) = \exp[-\int_0^{t_0}\{1-F_t(x)-F_t(y)+F_t(x,y)\}\lambda_p(t)dt]$$

$$= \exp[-\int_0^{t_0}\{\int_x^{\infty}\int_y^{\infty} f_t(x,y)dxdy\}\lambda_p(t)dt]$$

$$< 1 \qquad (24)$$

where $f_t(x,y)$ = bivariate joint density function of x and y. Assuming $1 << T_x$ and $T_{x|y<}$, Eq. 24 gives

$$T_x \leqq T_{x|y<} \qquad (25)$$

or

$$K = T_{xy}/(T_x T_y) \leqq 1 \qquad (26)$$

in which the sign of equality holds for the case of $\rho=0$. It should be noted from Eq. 26 that the joint return period T_{xy} for $|\rho|>0$ is always smaller than the product of the univariate return periods T_x and T_y, namely, T_{xy} for $\rho=0$.

In summary, the joint return period T_{xy} can be estimated as follows. K is first calculated using Eqs. 20 through 23, in which the marginal probability distributions of x and y derived from the bivariate probability distribution $F_t(x,y)$ of x and y are adopted as $F_t(x)$ and $F_t(y)$. Besides K, T_x and T_y are required to give T_{xy} in Eq. 19. We can calculate these values of T_x and T_y using probability distributions different from the marginal probability distributions $F_t(x)$ and $F_t(y)$ of x and y, that is, using univariate probability distributions of x and y, denoted by G(x) and H(y), by which $F_t(x)$ and $F_t(y)$ in Eqs. 21 and 22, respectively, are replaced and used for estimation of T_x and T_y. This method may result in better fitting of T_x and T_y to the data.

APPLICATION

The theory is developed to evaluate the joint return period of two concurrent variates: peak rainfall intensity ξ_i and maximum storm surge η_i, which occur almost simultaneously when a typhoon attacks the Osaka district. 117 typhoons brought storm surges with peak η_i over 26cm at Osaka Port for the 80-year period 1900-1980. All (seven) cases of typhoons bringing peak surges over 1.5m passed to the west of the apse line of Osaka Bay, and occurred in September. The fre-

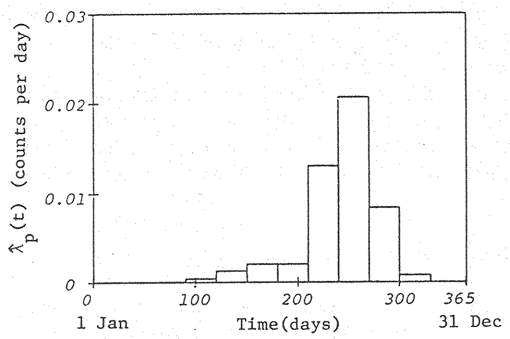

Fig. 1 Estimated rate of occurrence $\hat{\lambda}_p(t)$

quency of the range of occurrence time lag, $-2\sim2$ hours between η_i and peak rainfall intensity ξ_i, is about 40 percent, and time lags for almost all typhoons causing η_i over 1m are in this range. The upper observed limits of ξ_i against η_i are due to typhoons with the time lag $-1\sim1$ hour, for a wide range of η_i.

Figure 1 shows the monthly estimates of occurrence rate $\lambda_p(t)$ supposing an inhomogeneous Poisson process, of which the hypotheses were accepted by the χ^2 test at a signicance level of 5% for each month.

Univariate Distributions of the Exceedance and Maximum Exceedance

As mentioned above, the probability distributions of the two exceedances ξ_i and η_i seem to be time-dependent, and the spatial distribution of typhoon locations when η_i occurred depends on the typhoon course, east or west of Osaka. Taking account of these characteristics of ξ_i and η_i would be troublesome even if possible; for convenience, we may assume distributions G(x) and H(y) of ξ_i and η_i as univariates, respectively, to be time-independent and represented by combined exponential distributions, say, Eq. 2.

Figure 2 shows the fitting of G(x) and H(y) to the empirical distributions of peak rainfall intensity $x=\xi_i$(mm/hr) and peak storm surge $y=\eta_i-26$(cm) on semilogarithmic paper. In this case, the univariate distributions G(x) and H(y) are composed of two and three exponential distri-

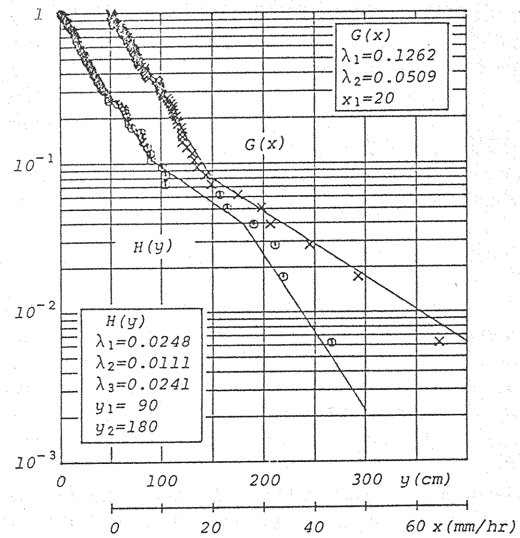

Fig. 2 Fitting of the combined exponential distribution functions G(x) and H(y) of univariates x and y, respectively, to their corresponding empirical distribution

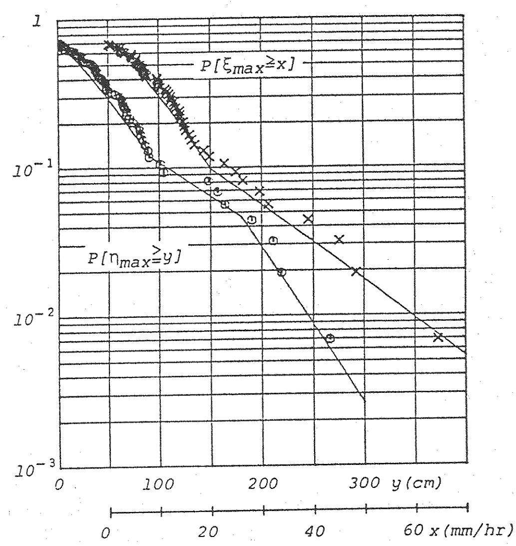

Fig. 3 Univariate distribution functions of two annual maxima

butions, respectively, because it is clearly seen in Fig. 2 that for each case of G(x) and H(y), single exponential distributions can not give good fits to the empirical distributions plotted by Gringorten's formula.

Figure 3 shows the univariate theoretical distributions of the annual maxima ξ_{max} and η_{max} using the above-mentioned distributions of G(x) and H(y), respectively, as $F_t(y)$ in Eq. 1. Good fits of these distributions in Fig. 3 confirm that the assumptions of the time-independence of G(x) and H(y) are acceptable in this case.

Combined Freund's Bivariate Distribution and Iso-Probability Curve

The use of Freund's bivariate exponential distribution requires the transformation of the original data of x and y to the nondimensional values X and Y, as shown in Eq. 15. As the values of the exponent m in Eq. 15, m=1/2, 3/4, 4/5, and 1 were set up, and combined Freund distributions shown in Eq. 7 were fitted to the transformed data X and Y. Comparing these four distribution functions, the best fit is by the combined Freund distribution in the case of m=3/5, of which the marginal distribution functions F(X) and F(Y) are shown in Fig. 4. These marginal distributions F(X) and F(Y) seem to have not as good fits as the univariate distributions G(x) and H(y) as shown in Fig. 2. The estimated values of parameters in the case of m=3/4 are also shown in Fig. 4. Substitution of these estimates into Eq. 14 gives the correlation coefficient ρ=−0.02 of X and Y.

Since we also have to consider the relationship between the two variates X and Y, we must examine the joint and the conditional probability distributions F(X,Y) and F(X|Y) for goodness of fit. Figure 5 shows the joint and the conditional iso-probability curves of F(X,Y) and F(X|Y), which are defined as contour lines connecting points of equal probability, F(X,Y) and F(X|Y). It is found in Fig. 5 that all data lies inside the iso-probability curves of F(X,Y)=0.90 and F(X|Y)=0.995. Therefore, we could confirm that the combined Freund distribution is appropriate to the bivariate which consists of two exceedances correlated each other.

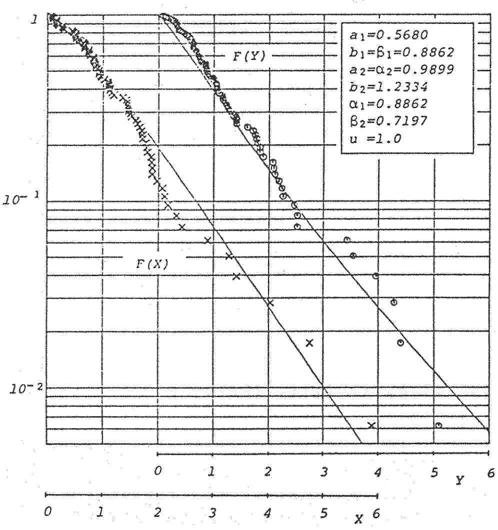

Fig. 4 Marginal distribution functions F(X) and F(Y) of the combined Freund bivariate distribution

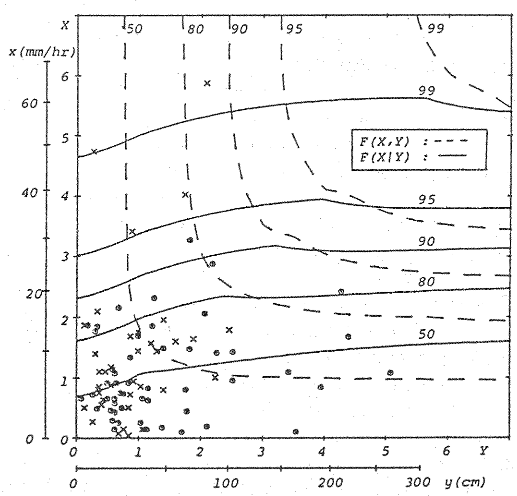

Fig. 5 Iso-probability curves of F(X,Y) and F(X|Y)

Joint Return Period of Two Annual Maxima

In Eq. 19, which gives the joint return period T_{xy}, K is calculated using F(Y), F(X), and F(X,Y) in place of $F_t(y)$, $F_t(x)$, and $F_t^y(x,y)$, respectively, in Eqs. 21, 22, and 23, while T_y and T_x in Eq. 19, and not in Eqs. 21 and 22, are estimated by replacing $F_t(y)$ and $F_t(x)$ with H(y) and G(x) in Eqs. 21 and 22, respectively.

Figure 6 shows contour lines connecting the points of equal joint return period of T_{xy} and $T_x \cdot T_y$ for ρ=-0.02 and ρ=0, respectively. The past three typhoons which give the largest values of T_{xy} and $T_x \cdot T_y$, are listed in Table 1.

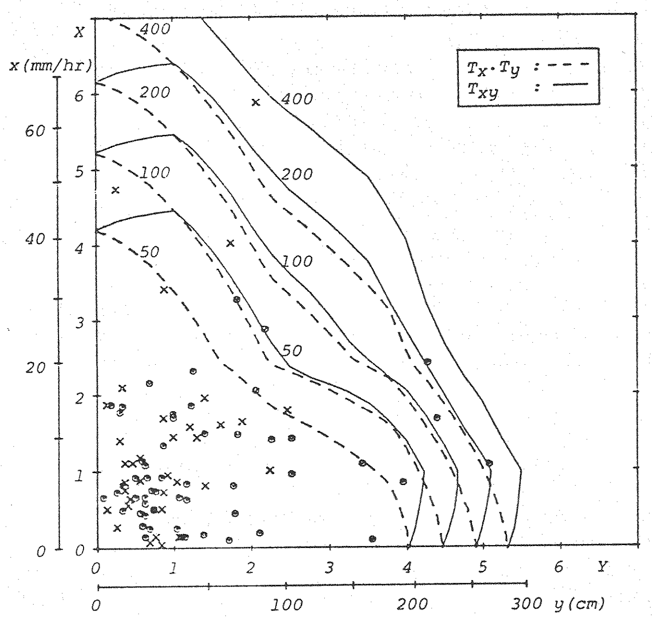

Fig. 6 Contour lines connecting the points of equal joint return period of T_{xy} and $T_x \cdot T_y$ for ρ=-0.02 and ρ=0, respectively (Symbols x and o denote typhoons passing east and west of Osaka, respectively.)

Table 1 Joint return periods of the three largest typhoons

Typhoon	Date	ξ_i=x (mm/hr)	η_i=y+26 (cm)	T_{xy} (yr)	$T_x T_y$ (yr)
Muroto	21/09/34	6.8	292	215	485
Jane	03/09/50	19.8	237	240	560
T 7916	30/09/79	64.5	107	338	785

It is clarified from Fig. 6 and Table 1 that taking the cross-correlation coefficient(=-0.02) into account somewhat reduces the joint return period; i.e., the assumption of an uncorrelation between two variates may lead to an overestimation of the joint return period. For instance, values of T_{xy} for the great typhoons Muroto and Jane, which attacked in 1934 and 1950 respcetively, are 215 and 240 years, while the values of $T_x \cdot T_y$ are 485 and 560 years, respectively. Typhoon T7916, which attacked in 1979, is, however, found to have the largest value of the joint return period: T_{xy}=338, with x=64.5mm/hr and y=81cm. Figure 6 can be used to estimate the joint probability for the combination of a given peak rainfall and a given peak storm surge that may occurr in the future, as well as evaluate the joint return period for a given past typhoon.

CONCLUDING REMARKS

It might take place with quite high possibility that the flood following heavy rainfall may flow down the tidal river in the lowlying urban area while the storm surge may run up the river after only a short time lag.

The joint probability of two variates, peak rainfall intensity and maximum storm surge, is formulated by using the theory of a marked point process, in which the two variates are supposed to follow a combined Freund distribution. The estimation technique of the joint return period is developed and demonstrated using the data at Osaka. It was found that the combined exponential and the combined Freund distributions are useful for univariate and bivariate probability distributions, respectively.

Taking the cross-correlation between two variates into accont by means of the combined Freund distribution somewhat reduces the joint return period, while the assumption of independence may lead to an overestimation of the joint return period. The three largest values of the joint return periods due to past typhoons at Osaka may be evaluated as T_{xy}=338, 240, and 215 years, due to typhoon T7916 in 1979, Jane in 1950, and Muroto in 1934, respectively.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a_1, b_1, a_2, b_2	= parameters of the lower class Freund distribution;
c_1, c_2	= parameters of combined exponential distribution;
$f(\cdot)$	= marginal probability density function of argument;
$f(A,B)$	= joint probability density function of A and B;
$F(\cdot)$	= marginal probability distribution function of argument;
$F(A,B)$	= joint probability distribution function of A and B;
$F(A\|B)$	= conditional probability distribution function A given B;

$f_1(Y)$, $F_1(Y)$; $f_2(Y)$, $F_2(Y)$	= density and distribution functions of Y for $0<Y<u$ and $u<Y$, respectively;
$F_t(x)$, $F_t(y)$	= distribution functions of exceedances ξ_i and η_i, respectively;
$f_t(x,y)$	= bivariate joint density function of x and y;
$G(x)$, $H(y)$	= univariate probability distribution functions of x and y, respectively;
K	= ratio of T_{xy} to $T_x \cdot T_y$;
m	= exponent in Eq. 15;
n	= sample size;
n_1, n_2, n_3	= sample sizes for the ranges of y corresponding to Eqs. 2a through 2c, respectively;
N_{11}, N_{12}, N_{21}, N_{22}	= sample sizes of bivariate (X,Y) satisfying regions : $(0<X<Y, X<u)$, $(o<Y<X, Y<u)$, $(u<X<Y)$ and $(u<Y<X)$, respectively;
N_2	= $N_{21}+N_{22}$ = sample size;
$P[\cdot]$	= probability of argument;
$P[A,B]$	= joint probability of A and B;
$P[A\|B]$	= conditional probability of A given B;
P_i	= empirical exceedance probability of i-th largest in the sample;
t_i	= occurrence time of i-th exceedance;
t_o	= 365 days = fundamental period;
T_x, T_y	= return periods of x and y;
T_{xy}	= joint return period of x and y;
$T_{x\|y<}$	= conditional return period of x given y;
x, y	= event magnitudes or variates equal to ξ_i and (η_i-26), respectively;
X, Y	= nondimentional event magnitudes of x and y;
y_1, y_2	= constants;
$\overline{y}_1$, $\overline{y}_2$, $\overline{y}_3$	= sample means of y for the ranges corresponding to Eqs. 2a through 2c, respectively;
$(X_j^{<11>}, Y_j^{<11>})$, $(X_j^{<12>}, Y_j^{<12>})$, $(X_j^{<21>}, Y_j^{<21>})$, $(X_j^{<22>}, Y_j^{<22>})$	= samples of bivariate (X,Y) satisfying the regions: $(0<X<Y, X<u)$, $(0<Y<X, Y<u)$, $(u<X<Y)$ and $(u<Y<X)$, respectively;
u	= critical value of X and Y that classifies the combined Freund distribution into Eqs. 7a, 7b and 7c, 7d;
v	= constant;
α_1, β_1, α_2, β_2	= parameters of the upper class Freund's distribution;

η_i, ξ_i	= variates or marks associated with single Poisson point process;
η_{max}, ξ_{max}	= annual maximum exceedances of η_i and ξ_i;
λ_1, λ_2, λ_3	= parameters of combined exponential distribution;
$\lambda_p(t)$	= time-dependent rate of occurrence;
ρ	= cross-correlation coefficient; and
σ_{xm}, σ_{ym}	= standard deviations of m-th power transformed variates x^m and y^m of x and y, respectively.