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A NEW ATTEMPT TO EVALUATE RAINFALL-RUNOFF MODELS FROM THE VIEWPOINT OF STOCHASTIC TRANSFORMATION

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SYNOPSIS

At present we have many rainfall-runoff models available and their evaluation has become an important problem because we must find an adequate model which fits a specific hydrological problem. In this paper, a new method of evaluating rainfall-runoff models is presented and applied to a simple overland flow process.

First, the basic principle of model evaluation is proposed. Second, the properties of stochastic input-output transformation of a slope runoff system are studied by using the kinematic wave model and a Monte Carlo simulation technique. Subsequently, the so-called storage function models, which are regarded as those obtained by simplifying the kinematic wave model, are evaluated considering whether the models preserve the stochastic transformation properties of the system or not. The method presented here is appropriate and useful because rainfall-runoff phenomenon is essentially a physical and stochastic one.

INTRODUCTION

Various kinds of rainfall-runoff models have been developed since the early 1930's. However, there are so many models that we cannot clearly identify which model should be used for a specific hydrological problem. One of the reasons for the 'flood of models' is that it is difficult to describe the system in which the rainfall is transformed into discharge. Rainfall-runoff phenomenon is large scale and quite complex; in practice, even a small catchment has a complicated structure. Another important reason is that there is no efficient criterion to evaluate those models. In the past, some attempts were made to evaluate rainfall-runoff models by using the 'goodness of fit' criteria like the sum of the square errors between observed and computed runoff discharges and so on (e.g., Nash and Sutcliffe (8), Pilgrim (9) or WMO (14)). However, these attempts could not attain their desired end because most models can only reproduce the actual hydrographs given for model validation to a certain degree. Therefore, such criteria are not suitable for model evaluation.

As stated in the following chapter, rainfall-runoff phenomenon is a physical and stochastic one. However, the stochasticity has rarely been taken into account in conventional model evaluation. In this paper, we propose a new method of evaluating rainfall-runoff models from the viewpoint of stochastic transformation and show a typical application of the method.

A BASIC PRINCIPLE OF MODEL EVALUATION

A catchment is regarded as a system tranforming rainfall into discharge and can be described by the following equation.

$$\{Q\} = F(\{R\}, H_0, A) \tag{1}$$

where $\{Q\}$ = the sequence of system outputs (runoff discharges); $\{R\}$ = the sequence of system inputs (rainfall); H_0 = the initial condition of the catchment at the beginning of the series of rainfall; and A = the condition of the catchment. Eq. 1 does not necessarily mean a deterministic transformation because {Q}, {R}, H_0 and A change with time and space; and moreover, our recognition (or observation techniques) cannot determine those values quantitatively.

Eq. 1 should essentially represent the physical and stochastic transformation of rainfall-runoff. By using $\{\bar{Q}\}$, $\{\bar{R}\}$, $\bar{H}_0$ and $\bar{A}$, average quantities (or idealized quantities), the average behavior of the system is often expressed as

$$\{\bar{Q}\} = f(\{\bar{R}\}, \bar{H}_0, \bar{A}) \tag{2}$$

In the conventional runoff analysis, researchers have been looking for an optimal function f of $\{\bar{R}\}$, $\bar{H}_0$ and $\bar{A}$; and in most cases, regarding the sequence of observed discharges as {Q} in Eq. 1, they have tried to make $\{\bar{Q}\}$ conform to {Q}. Let {ε} denote 'variation components' caused by replacing {Q}, {R}, H_0 and A with $\{\bar{Q}\}$, $\{\bar{R}\}$, $\bar{H}_0$ and $\bar{A}$, respectively. We denote the system output obtained by considering the variation components {ε} in the model expressed by Eq. 2 as follows:

$$\{Q\} = f(\{\bar{R}\}, \bar{H}_0, \bar{A}, \{\varepsilon\}) \tag{3}$$

If the system description is appropriate, {Q} in Eq. 3 should agree with {Q} in Eq. 1.

Deviations between the system outputs obtained by Eqs. 2 and 3 are caused by {ε}. Let {δ} denote the deviations as

$$\{\delta\} = \{Q\} - \{\bar{Q}\} \tag{4}$$

Let us now consider the simplification of the model f. In the same manner as Eq. 2,

$$\{\bar{Q}^M\} = g(\{\bar{R}\}, \bar{H}_0, \bar{A}) \tag{5}$$

where $\{\bar{Q}^M\}$ = the sequence of outputs of a simplified model g. If the simplification is appropriate, $\{\bar{Q}\}$ and $\{\bar{Q}^M\}$ must agree well. Considering the variation components {ε} in the same manner as Eq. 3, we obtain

$$\{Q^M\} = g(\{\bar{R}\}, \bar{H}_0, \bar{A}, \{\varepsilon\}) \tag{6}$$

The deviation between $\{Q^M\}$ and $\{\bar{Q}\}$ is given by

$$\{\delta^M\} = \{Q^M\} - \{\bar{Q}\} \tag{7}$$

It is natural to regard $\{\bar{Q}\}$ as the reference output as in Eqs. 4 and 7 because the model output must agree with $\{\bar{Q}\}$ if the variation components {ε} do not exist in the system. In the conventional method of model evaluation, researchers paid more attention to the magnitude of the deviation between {Q} and $\{\bar{Q}^M\}$ or between $\{\bar{Q}\}$ and $\{\bar{Q}^M\}$, without considering the variation components {ε} as in Eq. 3 or Eq. 6. From the viewpoint that rainfall-runoff phenomenon is both physical and stochastic, we should take into consideration not only the magnitude of {δ} or $\{\delta^M\}$ but also their stochastic properties. The model in which the stochastic properties of $\{\delta^M\}$ differ much from those of {δ} is apparently inadequate. Therefore, the properties of {δ} and $\{\delta^M\}$ can be good criteria for model evaluation.

On this basis, we try to evaluate rainfall-runoff models in this paper according to the following procedure:

[1] Consider an ideal model, obtained by idealizing the rainfall-runoff system, as a fundamental model and evaluate some models which are obtained by simplifying the ideal model.

[2] Examine the properties of the stochastic transformation of the ideal model in the case where $\{\bar{R}\}$, $\bar{H}_0$, $\bar{A}$ and {ε} are given.

[3] Check the stochastic properties of $\{\delta^M\}$ obtained by using the simplified model with $\{\bar{R}\}$, $\bar{H}_0$, $\bar{A}$ and {ε}; then show that the model in which $\{\delta^M\}$ preserves the stochastic properties of {δ} is a valuable one.

IDEAL AND SIMPLIFIED MODELS FOR OVERLAND FLOW PROCESS

Regarding the kinematic wave model as an ideal one representing overland flow process on a slope system, we try to evaluate three storage function models which are obtained by simplifying the kinematic wave model.

Kinematic Wave Model

Runoff process is essentially the movement of rainwater in the catchment and should be described by a physically-based model. Ishihara and Takasao (4) initiated the research on the hydraulic mechanism of runoff process using the kinematic wave model with a time-variant input given by an arbitrary function of time. After their analytical research which gave fruitful results to the hydrologists and engineers, the kinematic wave model is now often used for rainfall-runoff analysis.

The kinematic wave model for overland flow is given by:

$$\frac{\partial h}{\partial t} + \frac{\partial w}{\partial x} = r \qquad (0 \le t \ ; \ 0 \le x \le \ell) \tag{8}$$

$$w = \alpha h^m \tag{9}$$

$$h(x,0) = 0 \ ; \ h(0,t) = 0 \tag{10}$$

where t = time; x = distance from the upper edge of the slope; h = depth of flow; w = discharge rate per unit width; r = effective rainfall (system input); ℓ = slope length; and α and m = the parameters defining the flow characteristics. Eq. 10 gives the initial and boundary conditions.

For simplicity, by using the following relationships

$$t = t_* T \ ; \ x = x_* X \ ; \ h = h_* H \ ; \ w = w_* W \ ; \ r = r_* R$$

Eqs. 8, 9 and 10 are made nondimensional as:

$$\frac{\partial H}{\partial T} + \frac{\partial W}{\partial X} = R \tag{11}$$

$$W = H^m \tag{12}$$

$$H(X,0) = 0 \ ; \ H(0,T) = 0 \tag{13}$$

where the normalizing operators used are:

$$t_* = (\ell r^{1-m}/\alpha)^{1/m} \qquad \text{(the time of concentration)} \ ;$$

$$x_* = \ell \ ; \ h_* = r t_* \ ; \ w_* = \alpha h_*^m = \ell r \ ;$$

$$r_* = r \qquad \text{(the average effective rainfall intensity)}$$

Storage Function Models

Kimura (6) first proposed the storage function model as:

$$\frac{ds}{dt} = r(t-T_L) - q \tag{14}$$

$$s = K q^P \tag{15}$$

where s = water storage height; r = effective rainfall intensity; q = runoff height; and K, P and T_L = model parameters. The parameter T_L, which represents 'lag-time', is introduced into the model so that s and q satisfy the one-valued relationship given by Eq. 15. This model is often used for the flood runoff calculation in a basin with an area of less than five hundred square kilometers in Japan.

In this paper, the following models are considered as the ones obtained by simplifying (or lumping) the nondimensional kinematic wave model (Eqs. 11 and 12):

$$\text{Model-F} \qquad S = K_1 Q^{P_1} \tag{16}$$

$$\text{Model-P} \qquad S = K_1 Q^{P_1} + K_{2P} \frac{dQ}{dT} \tag{17}$$

$$\text{Model-H} \qquad S = K_1 Q^{P_1} + K_{2H} \frac{dQ}{dT}^{P_2} \tag{18}$$

where S = water storage; Q = discharge; and K_1, K_{2P}, K_{2H}, P_1 and P_2 = model parameters. Eqs. 16, 17 and 18 are called storage equations. For all the models, the continuity equation is given by

$$\frac{dS}{dT} = R - Q \tag{19}$$

From the theoretical solution of the kinematic wave model with constant rainfall intensity (rectangular rainfall), K_1 and P_1 in Eq. 16 are given by

$$K_1 = m/(m+1) \quad ; \qquad P_1 = 1/m \tag{20}$$

Fujita (1) showed that Eq. 20 can be used not only for rectangular rainfall but also for triangular rainfall. Model-F is a special case of Kimura's model (Eqs. 14 and 15) with $T_L = 0$.

Prasad (10) tried to represent the two-valued relationship between storage and discharge by incorporating the term dQ/dT as in Eq. 17. On the basis of the storage equation derived from the kinematic wave model with rectangular rainfall, Hoshi and Yamaoka (3) showed that the parameter K_{2P} in Eq. 17 depends on the discharge Q and incorporated the new parameter P_2. According to Hoshi and Yamaoka, for the rectangular rainfall case,

$$K_{2H} = 0.1\,m^{0.2} \quad ; \qquad P_2 = m^{-1.5} \tag{21}$$

and for the triangular rainfall case, K_{2H} and P_2 are given as functions of m and T_a/T_r, where T_a = the time when peak rainfall occurs and T_r = the rainfall duration.

In the case of Prasad's model (Model-P), K_{2P} is not given quantitatively; therefore, in this paper, we determine it by numerical experiments under the assumption that K_{2P} is given as a function of m. First, changing m with a 0.1 increment in the range of 1.0-2.0, we simulated the overland flow by using the kinematic wave model (Eqs. 11-13); second, we optimized K_{2P} by the COMPLEX method (a direct search method of maximum value of multi-variables and nonlinear function under non-equal constraints; the objective function used was the sum of the square errors between the dischrges simulated at time intervals $\Delta T = 0.05$ by the kinematic wave model and those by Prasad's model). Four patterns of temporal distribution of rainfall are considered, namely, one rectangular and three triangular (T_a/T_r = 0.2, 0.5 and 0.8). The average rainfall intensity was 1.0 and the rainfall and flood durations were set to 2.0 and 4.0, respectively. Fig. 1 shows the result of the optimization. According to Fig. 1, we assumed the following relation between K_{2P} and m

$$K_{2P} = \exp(\,am + b\,) \tag{22}$$

and searched for the optimum values of a and b by the least squares method. Table 1 shows the optimum values of a and b for all the cases.

After this, we assume that the overland flow obeys Manning's law, i.e., $m=5/3$. Then the values of the parameters in Eqs. 16-18 are determined by Eqs. 20-22 as shown in Table 2. Fig. 2 shows the hydrographs obtained by using these parameter values. Computation was carried out by the Lax-Wendroff scheme for the kinematic wave model and by the Runge-Kutta-Gill method for storage function models.

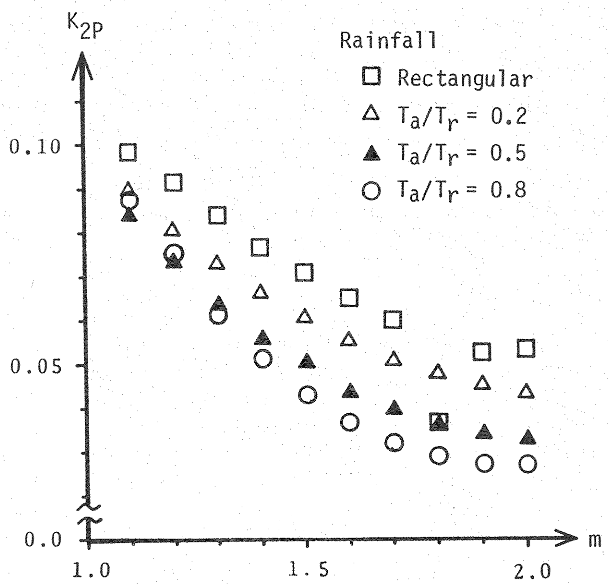

Fig. 1 Relation between the parameters K_{2P} and m in Prasad's model (Eq. 17)

Table 1 Fitted values a and b in Eq. 20

Rainfall (T_a/T_r)	a	b
Rectangular	-0.7790	-1.5235
Triangular (0.2)	-0.9195	-1.3830
Triangular (0.5)	-1.2564	-1.0462
Triangular (0.8)	-1.5148	-0.7878

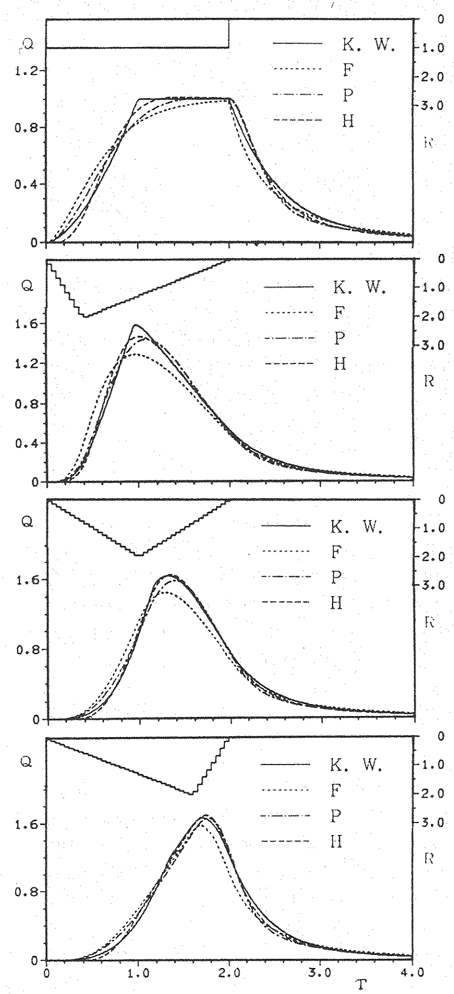

Fig. 2 Hydrographs by the kinematic wave model and the storage function models

Table 2 Parameter values of the storage function models

Rainfall (T_a/T_r)	K_1	P_1	K_{2P}	K_{2H}	P_2
Rectangular	0.625	0.6	0.05950	0.11076	0.4648
Triangular (0.2)	0.625	0.6	0.05418	0.07572	0.5537
Triangular (0.5)	0.625	0.6	0.04327	0.09608	0.4509
Triangular (0.8)	0.625	0.6	0.03643	0.10441	0.3586

STOCHASTIC RAINFALL-RUNOFF SIMULATION

Suppose that the condition of the catchment does not change with time or space, that is, the catchment slope has a constant gradient and uniform roughness. The initial condition is that there is no rainwater on the slope. Therefore, these conditions are given as:

$A = \bar{A}$; $H_0 = \bar{H}_0$

where

$\bar{A}$: $m = 5/3$

$\bar{H}_0$: $H(X,0) = 0$; $W(X,0) = 0$ for the kinematic wave model
$S(0) = 0$; $Q(0) = 0$ for the storage function models

In addition to the slope conditions, we suppose that the rain comes down over the slope uniformly. Consequently, only temporal variation of rainfall exists in the assumed runoff system.

The rainfall intensity changes with time at intervals of ΔT and takes a constant value within the interval; and so $\{\varepsilon\}$ is assumed to be a sequence changing at intervals of ΔT. Let the system input be given by

$$\{R\} = \{\bar{R}\} + \{\varepsilon\}$$

In this study, the four types of rainfall shown in Fig. 2 are considered as $\{\bar{R}\}$. We consider various rainfall patterns because the result of the evaluation may depend on the rainfall pattern. The sequence $\{\varepsilon\}$ is assumed to obey $NID(0,\sigma_\varepsilon^2)$, and σ_ε is supposed to be 0.1, 0.5 or 1.0, while the average effective rainfall $\bar{R} = 1.0$. The rainfall sequence $\{R\}$ is generated by adding random numbers obeying $NID(0,\sigma_\varepsilon^2)$ to $\{\bar{R}\}$ which represents the average behavior of rainfall. When $R(T) < 0$, $R(T)$ is assumed to be zero.

STOCHASTIC TRANSFORMATION PROPERTY OF THE KINEMATIC WAVE MODEL

The output $\{\bar{Q}\}$ of the kinematic wave model with the average input $\{\bar{R}\}$ is shown in Fig. 2. When $\{\varepsilon\}$ is added to $\{\bar{R}\}$, the output $\{Q\}$ fluctuates owing to $\{\varepsilon\}$. In general, when stochastic components exist in the system, the system output must fluctuate. In this chapter, we investigate the properties of stochastic transformation of the kinematic wave model by numerical experiments.

In the same manner as in Fig. 2, we set the rainfall duration $T_r = 2.0$, the flood duration $T_f = 4.0$, and the unit time interval $\Delta T = 0.05$. Using the rainfall added with the random numbers, we simulate runoff and obtain a sequence $\{\delta\}$ at intervals of ΔT. Since $\Delta T = 0.05$ for $T_f = 4.0$, eighty δ's are obtained for each hydrograph; the last thirty δ's are discarded because they are not significantly influenced by the rainfall. The simulation is repeated a hundred times and we obtain five thousand δ's. To analyze their stochastic properties, the correlogram and histogram of δ's are drawn.

Table 3 shows the mean $\bar{\delta}$ and standard deviation σ_δ of δ's. In all cases where $\sigma_\varepsilon = 0.1$, 0.5 and 1.0, σ_δ is about 20% of σ_ε.

The fluctuation of the output of the kinematic wave model with a stochastic input was investigated by Takasao and Shiiba (11) and Fujita *et al.* (2). Let us compare the results obtained here with their results.

Rewrite Eq. 8 as follows:

$$\frac{\partial h}{\partial t} + \frac{\partial w}{\partial x} = r(t) + v(t) \qquad (t \geq 0 \; ; \; 0 \leq x \leq 1) \tag{23}$$

where $r(t)$ = mean rainfall intensity given as a deterministic value; $v(t)$ = the stochastic component of rainfall; and the stochastic properties of $v(t)$, which is independent of the initial water depth $h(x,0)$, are given as:

$$E[\,v(t)\,] = 0$$
$$E[\,v(t)v(\tau)\,] = V\,\delta(t-\tau) \;;\; V > 0 \;;\; t,\tau > 0 \tag{24}$$

where $\delta(t)$ = the Dirac delta function; and $E[\cdot]$ denotes expectation. Note that

Table 3 Statistics of the output residuals

σ_ε	Model	Rectangular Rainfall		Triangular (T_a/T_r=0.5)	
		$\bar{\delta}$	σ_δ	$\bar{\delta}$	σ_δ
0.1	K.W.	0.000282	0.02153	-0.000007	0.02185
	F	0.015608	0.09177	0.024161	0.12684
	P	-0.005325	0.05952	0.005862	0.06501
	H	-0.005964	0.04477	0.001983	0.03567
0.5	K.W.	-0.004090	0.10319	-0.018760	0.10090
	F	0.011119	0.14387	0.004688	0.16226
	P	-0.009703	0.12194	-0.012802	0.11804
	H	-0.010188	0.11602	-0.017197	0.10774
1.0	K.W.	-0.068590	0.19134	-0.092123	0.20157
	F	-0.051834	0.21686	-0.069251	0.24445
	P	-0.075397	0.20197	-0.088143	0.21112
	H	-0.074789	0.20629	-0.091630	0.21208

the variables h, w and v in Eq. 23 are random variables; therefore, Eq. 23 is a stochastic differential equation. In this case, the transition of the probability distribution of water depth $h(x,t)$ is obtained analytically by linearizing Eq. 23 in an appropriate manner under the condition that

$$h(x,0) = 0 \; ; \qquad r(t) = \bar{r} = \text{constant}$$

The variance σ_h^2 of the water depth at the lower end of the slope is given as follows (see Takasao and Shiiba (7)):

$$\sigma_h^2 = (\bar{r}/\alpha)^{1/m} V / \{\bar{r}\,(2m-1)\} \tag{25}$$

In the nondimensional form as Eqs. 11-13, setting $\bar{r} = 1$ and $\alpha = 1$ in Eq. 25, the variance σ_H^2 of the water depth at the lower end of the slope $H(1,t)$ is given by

$$\sigma_H^2 = V / (2m - 1) \tag{26}$$

Though $v(t)$ in Eq. 23 is defined in continuous time, the variation components $\{\varepsilon\}$ is a discrete sequence at intervals of ΔT. Then the variance V of $v(t)$ and the variance σ_ε^2 of ε have the following relationship

$$V = \sigma_\varepsilon^2 \Delta T \tag{27}$$

By linearizing Eq. 12 about $\bar{H} = 1$ and using Eqs. 26 and 27, the variance σ_W^2 of the discharge rate at the lower end of the slope is given as

$$\sigma_W^2 = m^2 \sigma_H^2 = \frac{m^2}{2m-1} \sigma_\varepsilon^2 \Delta T \tag{28}$$

Since $m = 5/3$ and $\Delta T = 0.05$,

$$\sigma_W = m \sqrt{\Delta T / (2m-1)}\, \sigma_\varepsilon = 0.244\, \sigma_\varepsilon \tag{29}$$

As σ_δ mentioned above corresponds to σ_W, the result agrees with the relationship given by Eq. 29. Fig. 3, in which the correlogram of $\{\delta\}$ is shown, indicates that the sequence $\{\delta\}$ is highly autocorrelated. The results in Table 3 and Fig. 3 show superficial results of the stochastic properties of $\{\delta\}$. Let us see the characteristics of $\{\delta\}$ along the time horizon. Table 4 shows the mean and standard deviation of $\{\delta\}$ at $T = 0.5$, 1.0, 1.5, 2.0 and 2.5. The variance (standard deviation) of $\{\delta\}$ at $T = 1.0$, which corresponds to the time of concentration, is greater than that of the other cases regardless of the rainfall pattern and the magnitude of σ_ε. This result is the same as that shown by Fujita *et al.* (2). Therefore, we must pay attention to such temporal variation of δ's.

EVALUATION OF THE STORAGE FUNCTION MODELS

Three storage function models are evaluated considering whether they preserve the stochastic transformation properties of the ideal model or not.

In the same manner as in the case of the kinematic wave model, we obtained the output residual sequences $\{\delta^F\}$, $\{\delta^P\}$ and $\{\delta^H\}$ of the storage function models Model-F, Model-P and Model-H, respectively. Fig. 3 shows their correlograms as well as that of the kinematic wave model. Except for the case of $\sigma_\varepsilon = 0.1$ the correlograms are similar; therefore, it is impossible to discuss their merits and demerits on the basis of this result.

Tables 3 and 4 show the statistics (the mean and standard deviation) of the residual sequences of the storage function models as well as those of the kinematic wave model. Fig. 4 shows the histograms of the sequences. From these results, in all cases, the stochastic transformation properties of Model-H are most similar to those of the kinematic wave model; so that Model-H can be said to be better than the others from the viewpoint of stochastic trnsformation.

A few hydrographs in the case where the stochastic variation $\{\varepsilon\}$ is considered are shown in Fig. 5, which supports the evaluation made above according to the results in Table 3 and Fig. 4 because Model-H reproduces the behavior of the kinematic wave model better than the others.

Table 4 Statistics of the output residuals at some points of the time horizon

σ_ε	Time T		K.W. Model	Storage Function Models		
				F	P	H
0.5	0.5	Mean	-0.0099	-0.1705	-0.0864	-0.0057
		S.D.	0.0770	0.1028	0.0848	0.0895
	1.0	Mean	0.0131	0.1538	0.1318	0.0593
		S.D.	0.1498	0.1289	0.1274	0.1373
	1.5	Mean	-0.0267	0.0145	-0.0244	-0.4258
		S.D.	0.1229	0.1299	0.1293	0.1287
	2.0	Mean	-0.0136	0.0087	-0.0232	-0.0197
		S.D.	0.1330	0.1453	0.1354	0.1356
	2.5	Mean	0.0020	0.0738	0.0639	0.0329
		S.D.	0.0477	0.0327	0.0468	0.0446
1.0	0.5	Mean	-0.0409	-0.1986	-0.1275	-0.0506
		S.D.	0.1285	0.1718	0.1361	0.1445
	1.0	Mean	-0.0778	0.0671	0.0394	-0.0398
		S.D.	0.2700	0.2603	0.2330	0.2503
	1.5	Mean	-0.0916	-0.0411	-0.1016	-0.1100
		S.D.	0.2459	0.2410	0.2713	0.2661
	2.0	Mean	-0.0782	-0.0661	-0.0846	-0.0820
		S.D.	0.2319	0.2426	0.2429	0.2467
	2.5	Mean	-0.0147	0.0591	0.0489	0.0180
		S.D.	0.0749	0.0524	0.0753	0.0697

CONCLUSIONS

Given that rainfall-runoff phenomenon is a physical and stochastic one, we have proposed a new method of evaluating rainfall-runoff models. As a typical application of the method, we dealt with overland flow process on a simple slope system. The stochastic transformation properties of the system, in which only temporal variation of rainfall exists as variation component, were clarified by using the kinematic wave model and a Monte Carlo simulation technique. Three storage function models were considered as simplified models of the kinematic wave model and were evaluated by this method. Consequently, Model-H was evaluated as the best one. When the kinematic wave model should be replaced with a storage function model, the result obtained can be used effectively.

This approach is directly applicable to a natural system if the catchment slope is not covered with a highly permeable surface stratum through which the penetrated water flows downstream. In such a system, surface runoff (or overland flow) is the main component and the fundamental (ideal) model used is the kinematic wave model. However, when the slope is covered with a highly permeable surface stratum, not only surface runoff but also sub-surface runoff (or interflow) must be considered. In this case, the kinematic wave model containing a sub-surface component proposed by Ishihara and Takasao (5) or other models (see e.g., Kirkby (7)) are the fundamental models that could be used.

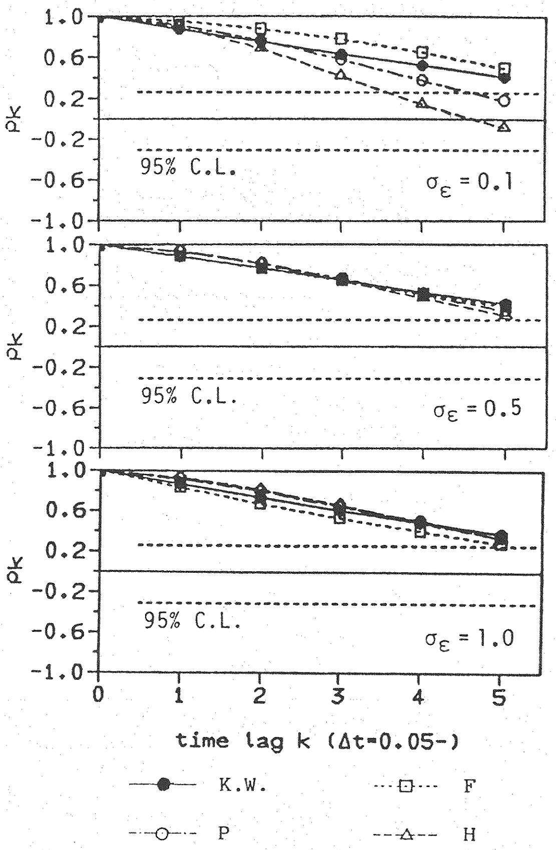

Fig. 3 Correlograms of the output residuals

This kind of study represent a new approach and we will have to try to construct better models by solving the following important problems:
[1] appropriate temporal and spatial scales of the rainfall-runoff field;
[2] stochastic properties of the field, the input and the initial conditions;
[3] quantities of the various stochastic components contained in actual input-output data.

Even considering the limitation of our model, we believe that the methodology presented here is plausible and can be used for specific hydrological problems. Furthermore, in the case where an ideal model must be replaced with a simplified one, the framework as presented here will play a significant role in evaluating models of a certain stochastic system.

This paper is the revised edition of the authors' previous paper (12) and the application of this approach to a slope-channel system is presented in another paper (13).

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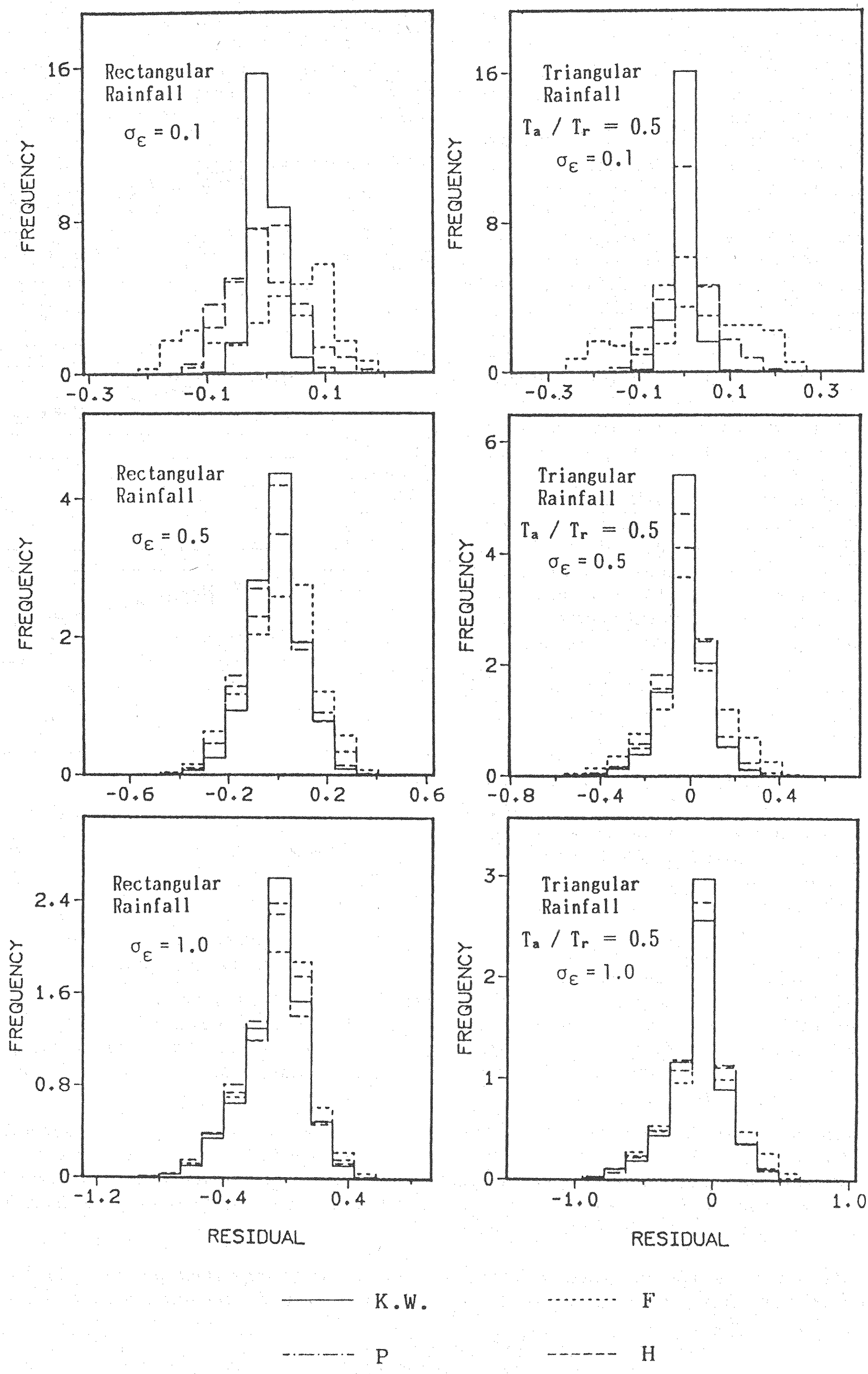

Fig. 4 Histograms of the output residuals

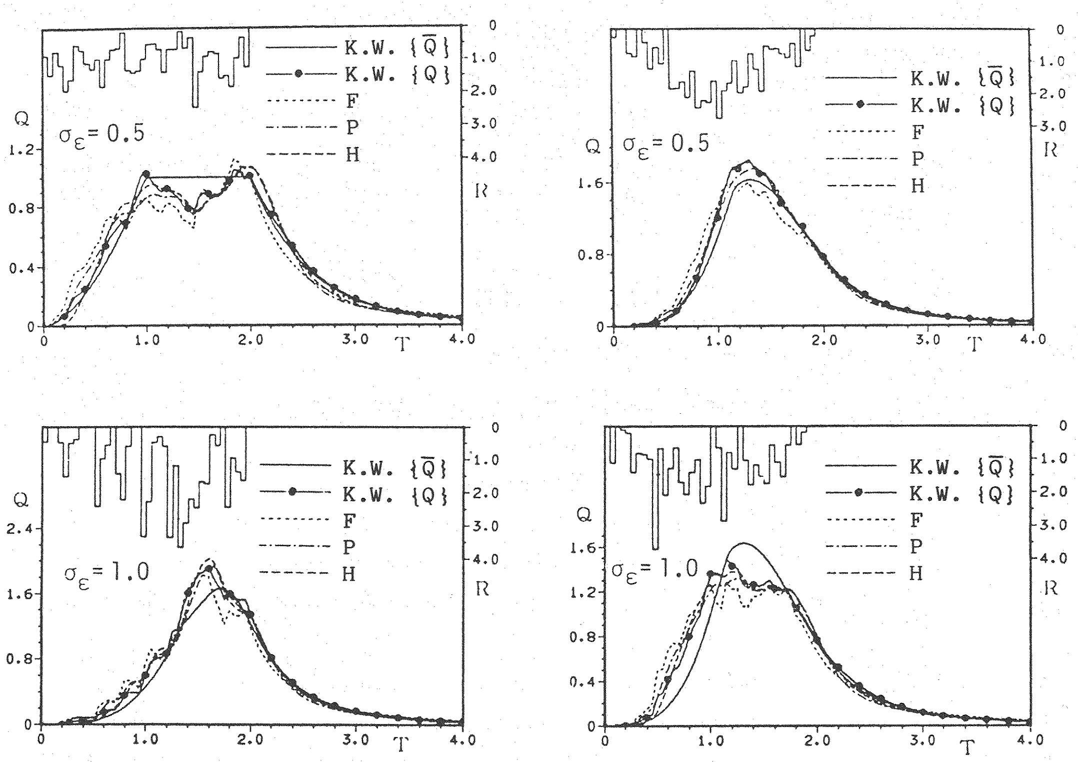

Fig. 5 Examples of the hydrographs with the stochastic input

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APPENDIX - NOTATION

The following symbols are used in this paper:

A, $\overline{A}$	= catchment condition and its ideal form;
$E[\cdot]$	= expectation operator;
f, g	= functions representing ideal and simplified models;
h, H	= flow depth and its nondimensional form;
H_0, $\overline{H}_0$	= initial catchment condition and its ideal form;
K, K_1, K_{2P}, K_{2H}	= parameters in the storage function models (see Eqs. 15-18);
ℓ	= slope length;
m	= parameter defining the flow characteristics;
$NID(\mu,\sigma^2)$	= normal independent distribution with mean μ and variance σ^2;
P, P_1, P_2	= parameters in the storage function models (see Eqs. 15-18);
q, Q	= runoff height and its nondimensional form (see Eqs. 14-19);
$\{Q\}$, $\{\overline{Q}\}$	= sequences of outputs of an ideal model (see Eqs. 2-3);
$\{Q^M\}$, $\{\overline{Q}^M\}$	= sequences of outputs of a simplified model (see Eqs. 5-6);
r, R	= effective rainfall intensity and its nondimensional form;
$\{R\}$, $\{\overline{R}\}$	= sequences of rainfall and its average (or ideal) quantity;
t, T	= time and its nondimensional form;
T_a	= time when peak rainfall occurs;
T_r, T_f	= rainfall duration and flood duration;
v, V	= stochastic component of rainfall and its statistic (see Eqs. 23-24);
w, W	= discharge rate per unit width and its nondimensional form;
x, X	= distance along the slope axis, positive in downstream direction and its nondimensional form;
α	= parameter defining the flow characteristics;
$\{\delta\}$, $\{\delta^M\}$	= sequences of output residuals obtained by ideal and simplified models (see Eqs. 4 and 7);
ΔT	= unit computation time interval in nondimensional domain;
$\{\varepsilon\}$	= sequence of the stochastic components existing in the system;
σ_h, σ_H, σ_W	= standard deviations of h, H and W; and
σ_δ, σ_ε	= standard deviations of δ's and ε's, respectively.