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OBSERVATION OF INTERNAL SEICHE AND WAVE IN THERMALLY STRATIFIED LAKE

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SYNOPSIS

Based on field experiments performed in Lake Chuzenji, one of deep lakes in Japan, wind action upon thermal stratification seems effective only to break isotherm near surface and of little importance to erode thermocline. Variation of thickness of surface layer over the thermocline is rather reasonably explained by the model of hydrostatic pressure balance.

At the thermocline in July 1983 we can observe a uninodal internal seiche with approximately 12 hours period which well coincides with that calculated by using Holmboe's model. The oscillation continues for several days once internal seiche is generated, and its waveheight in initial stage reaches 5 meters. Besides, it proves through time series of water temperature that internal seiche attends both internal wave with modal structure and waves with short period.

INTRODUCTION

Temperature in deep lakes shows distinctive features throughout seasons. From view of seasonal variation of water temperature, Dake and Harleman (1) simulated skillfully the thermal structure with use of the one-dimensional heat transfer, without stir and convection due to wind. In the thermal structure during a day or week, however, many kinds of temperature variation are commonly contained and are closely associated with inner motion and mixing process in the water body. Especially when a lake is stratified in density, Mortimer (7) emphasized that returned current induced by wind action produced shifting location of thermocline slightly downward in the leeward of the lake. As to this phenomenon, Ura (13) studied experimentally.

Thermal stratification contains in general various types of internal waves in thermocline. Everywhere in the lake can be seen the internal wave with short period of minutes (Brunt-Väisälä wave), and the internal Kelvin wave with longer period may be expected in large lake such as Lake Biwa as suggested by Kanari's investigation (5). These internal waves play an important role in momentum transfer from surface layer to bottom layer. Overturning of them is thought to be one of principal mechanisms for vertical mixing in thermally stratified lake and ocean. Woods (14) observed growth of Kelvin-Helmholtz billow and its subsequent overturning in thin layer near thermocline, and has proposed that step-like structure in thermocline is the result of overturning of finite-amplitude billow. Experimentally fine features of Kelvin-Helmholtz instability were obtained by Thorpe (10,11). In recent works, Spiegel and Imberger (9) and Fischer et al. (2) suggest that hypolimnetic mixing in lakes of small or medium size may originate from the boundary mixing induced by the internal seiching motion on the bottom with rough shape. On the basis of this concept Ivey and Corcos (3) examined experimentally the relation between boundary mixing and a vertically oscillating grid in the stratified media. However, their result remains unrealistic because of the scarcity of evidence showing internal mixing driven by internal seiching motion.

Main subject in this paper is to study the influence of wind action upon thermal stratification and attendant internal seiche and waves. The questions we face as to internal seiche are what magnitude of waveheight and what lifetime the internal seiche has after its generation. The data we deal with here were gathered at Lake Chuzenji from 1981 to 1983, which is a representative deep lake in Japan. First, observation of vertical thermal profiles was carried out by the bathythermograph before and after wind to investigate the wind action in mixing process. Next, time series of temperature variation in the vicinity of thermocline were examined to understand the behavior of internal waves in deep lakes.

DESCRIPTION OF LAKE CHUZENJI AND FIELD EXPERIMENTS

Lake Chuzenji is located in the Nikko National Park which is about 100 km northern from Tokyo. Topographical image of Lake Chuzenji is shown in Fig. 1. The lake is 6.54 km long and 1.85 km wide, and has a mean depth of 94.7 m and the maximum of 163.0 m. The outline of seasonal thermal features is depicted in Fig. 2, where density interface is defined by the location indicating mean value of densities between those of surface layer (1 m depth) and bottom layer (100 m depth). The minimum and maximum temperatures observed in surface layer were 2.9°C in March 1983 and 21.8°C in August 1983. The water in surface layer warms from early spring with the increase of solar radiation, and thermal stratification is fully established in August every year. Thermocline in this season lies at the depth of 10 m. From autumn cooling from surface reduces the temperature of surface layer and three curves of temperature on Fig. 2 cross at 4°C in winter, as seen usually in deep lakes.

Along the longitudinal direction of lake, we set seven stations for observing vertical thermal profile, as shown in Fig. 1 and Table 1. Observation time from St-1 to St-7 can be kept approximately within an hour by using the bathythermograph (Tsurumi Seiki Co. Ltd. Micom-BT) which is equipped with a platinum resistance thermometer and a pressure gauge and can detect temperature and depth within ±0.05°C and ±0.1 m. We used another thermal detector (Hydrolab Co. Ltd. Model-2001) for long term record of temperature variation at fixed depth in thermocline. This detector has the accuracy within ±0.05°C and memorizes 3704 data with any sampling intervals from 1 to 60 minutes.

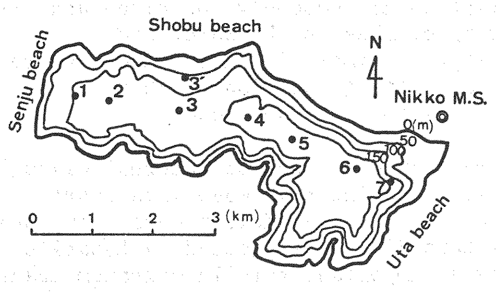

Fig. 1 Topographical feature of Lake Chuzenji and measurement stations.

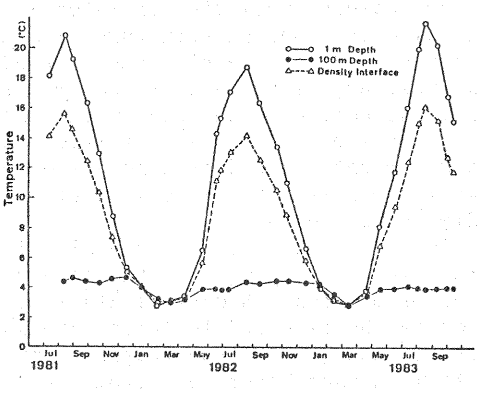

Fig. 2 Seasonal temperature variation in Lake Chuzenji.

Table 1 Depth at each station and distance between stations. Mean depth along the longitudinal line is 131.2 m.

Station	West end	1	2	3	4	5	6	7	East end	
Depth (m)	0	104	115	128	152	158	160	140	0	Mean depth 131.2 (m)
Distance (m)	546	534	1246	1074	762	1292	534	550		Total distance 6538 (m)

DEFORMATION OF THERMAL STRUCTURE DUE TO WIND STRESS

During the field survey in June 1982, the wind condition is illustrated in Fig. 3, referring to the record at the Nikko Meteorological Station from 23 to 26 June. It was calm on 23rd, but the east wind started to blow at the speed of 5-6 m s^{-1} from early morning on 24th and continued during 12 hours. In general, wind velocity on the water surface is 10-50 % greater than one on the land surface due to the difference of roughness elements of surface. In this case the wind on the lake is reasonably considered to have blown at the speed of 6-9 m s^{-1}. The following two days were moderate again. Under this wind condition four sets of isotherm data in the water body are displayed in Fig. 4 (a)-(d). Isotherms in Fig. 4 (a) appear approximately parallel to the water level because of no wind. Then, surface layer in Fig. 4 (b) , observed after 9 hours since the onset of wind on 24th, is clearly recognized to have developed toward the leeward area. In addition the isotherm of 16°C near surface has completely collapsed and disappears at that time. This result may suggest the lowering of thermocline level by stir and convection due to the wind stress. To confirm the deepening rate of thermocline, we calculated the location of density interface as drawn in the thick solid lines in Fig.4. Throughout the observation densities ρ_1 and ρ_2 of surface and bottom layers are 0.99908 g cm^{-3} and 1.000 g cm^{-3}, corresponding to 15.3°C and 4.0°C, respectively, and therefore, the density of interface becomes 0.99954 g cm^{-3} with respect to the temperature to be 11.9°C. Results show (1) the mean depth of surface layer h_1=10.2 m on 23rd, (2) h_1=13.4 m at St-1 and h_1=7.3 m at St-7 on 24th (see Fig. 4 (b)), and accordingly, density interface is inclined at the magnitude of 1.12×10^{-3} and (3) h_1=10.1 m in early morning on 25th when the wind stress has been entirely removed (see Fig. 4 (d)). Against our speculation of deformation of thermal structure, the locations of density interface scarcely alter before and after the

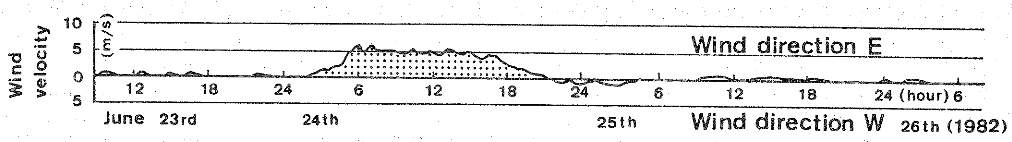

Fig. 3 Mean wind velocity and direction during 23-26 June 1982.

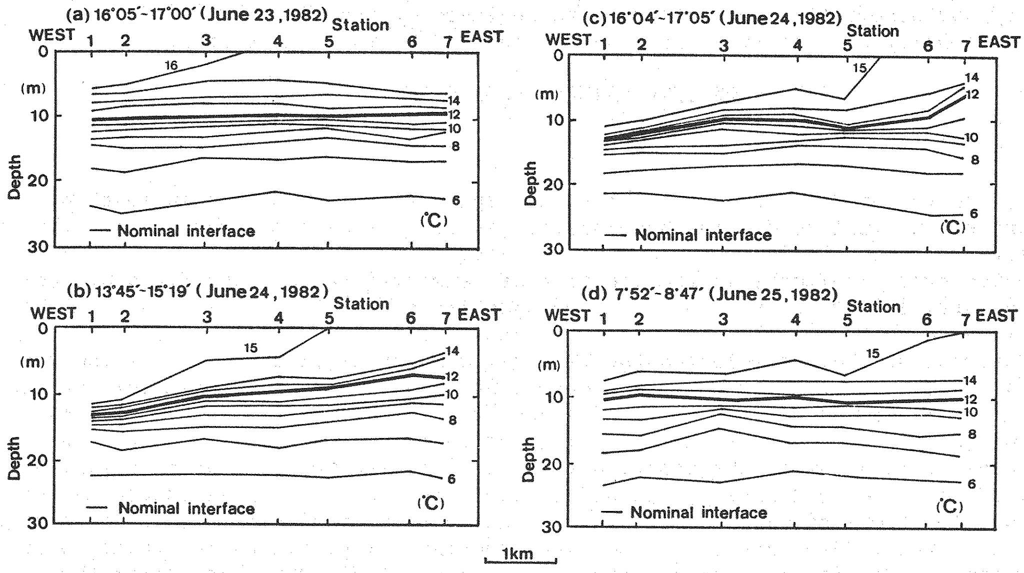

Fig. 4 Deformation of isotherms due to wind stress. (a) steady state under no wind on 23, (b) quasi-steady state during wind blow for about 9 hours on 24, (c) 2 hours later from (b) and (d) quasi-steady state after wind on 25, June 1982.

wind exertion of about 12 hours. It should be mentioned here that when surface layer develops to be 10 m, the wind action, except for a severe storm as studied by Kraus (6), is effective only to break the isotherms near surface and has little importance in upward entrainment of bottom water across the interface.

Physical interpretation for the mechanism of density interface inclination may be provided as follows:

1) Surface elevation induced by the wind set-up yields the increase of hydrostatic pressure in the leeward area, and inversely the decrease in the windward area.

2) Consequently, surface layer composed of lighter liquid thickens gradually toward the leeward area to adjust the increase of hydrostatic pressure.

On the basis of this idea, interface inclination will be computed. Now we assume for simplicity that the lake is two-dimensional rectangular basin and its density profile is separated into two layers remaining a slippery interface with little mixing across it. The density interface tends to reply to the wind forcing but the response is slow actually. At least, a quarter of the period Ti of uninodal internal seiche will be required for the density interface to reach the equilibrium slope. Density field in Fig.4 (b) has been obviously free from this restriction, because the isotherms are observed after 9 hours since initial wind blow that are enough longer than Ti/4 as further discussed in the following section. Under the condition of stationary wind stress and of steady state, surface elevation η measured from original surface is theoretically given by Kajiura (4) as

$$\eta = -\frac{a}{2} + \frac{1}{2}\{ a^2 + 4 (bx + A)\}^{1/2} , \qquad (1)$$

$$a = 2\varepsilon h_1 \quad , \quad b = 2\varepsilon\tau/\rho_1 g$$

where $\varepsilon = (\rho_2 - \rho_1)/\rho_1$; h_1=undisturbed mean depth of surface layer ; τ=wind stress ; g=gravitational acceleration ; x=distance from windward end and A=constant determined from mass conservation $\int_0^l \eta\, dx = 0$ in which l is the length of lake. Here the wind stress τ proves 0.94 dyne cm^{-2} , if the wind velocity is regarded as 8 m s^{-1} and a wind drag coefficient of 1.2×10^{-3} is used. Based on h_1=10.2 m and quantities mentioned above, Eq. 1 yields η_1=0.24 cm at St-1 and η_7=-0.28 cm at St-7. The depth $\bar{h}_1$ of surface layer is estimated from these elevations to be 12.8 m at St-1 and 7.2 m at St-7 through the relation $\bar{h}_1 = h_1 + \eta/\varepsilon$. The magnitude of density interface inclination is subsequently appreciated to be 1.03×10^{-3}. Theory and observation indicate that the balance of hydrostatic pressure can describe precisely interface inclination induced by the wind stress.

RESULTS OF TEMPERATURE RECORDS IN LAKE CHUZENJI

Internal Seiche

When the wind stops, natural tendency recovering thermocline in level position generates internal seiche in whole basin. Typical results observed by two thermal detectors at St-2 in July 1983 are illustrated in Fig. 5. Temperature variations detected every 5 minutes at the location of density interface and 5 m apart below the density interface are separated individually as the time series (a) and (b), and wind velocity and wind direction are also displayed there. In the analysis of internal seiche, we selected the vertical density profile at St-4 as the representative one for the whole basin, because St-4 was located approximately at the node of uninodal internal seiche. Besides, we can exclude observation errors in thermal profile due to internal seiching motion to some extents. One of density profiles at St-4 during the observed terms is demonstrated in Fig. 6, in which zero point on the ordinate is set at density interface and arrows denote the measurement locations of water temperature. The density profile has a linear stratification in the middle part of thermocline, and this property enables to calculate the vertical displacement of thermocline by means of dividing the temperature variation by the thermal gradient near thermocline. Vertical scale of the displacement was made for the time series (a) which was indicated in Fig. 5. The records are characterized as follows:

1) At time 06°00' on 12 July wind direction turned from west to east. According to the wind forcing, water temperatures increase in both time series (a) and (b). Temperature from 13°00' to 18°00' in time series (a) takes approximately constant value of 16.8°C, and this result indicates that the thermal detector still keeps its location in surface layer.

2) The time when wind velocity reaches the maximum is denoted by arrows in Fig. 5. Temperature at that time in time series (b) rises to the maximum simultaneously. Hereafter, temperature begins to fall with the decrease of wind velocity, and consequently internal seiche occurs as mentioned later. We can find out the periodical pattern of temperature driven by internal seiche for 4-5 days after its generation as numbered waves as (1) - (8) in Fig. 5. Each period and waveheight of internal seiche detected from 12th to 16th are arranged in Table 2.

Fig. 7 shows temperature spectra for time series (a) and (b) through waves (1) to (8). Dominant peak related to uninodal internal seiche is easily recognized at 12.2 hours and spectra also contain second and third harmonic components.

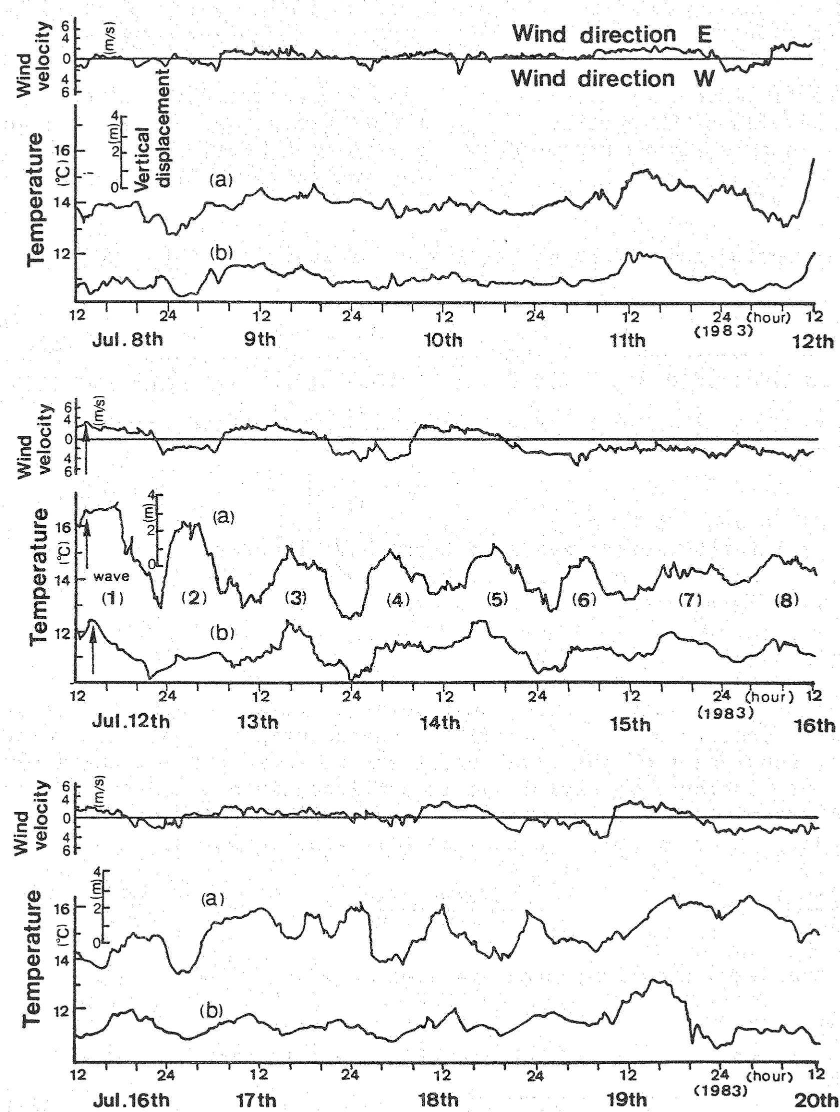

Fig. 5 Temperature time series at St-2 during 8-20 July 1983. (a) was observed at the location of density interface and (b) at the location 5 m downward from density interface.

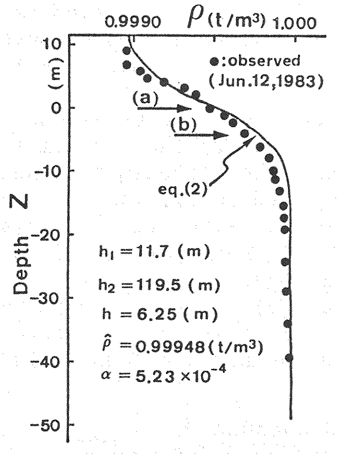

Fig. 6 Vertical density profile at St-4 on July 1983. Parametric constants in this figure are used in Eq. 2.

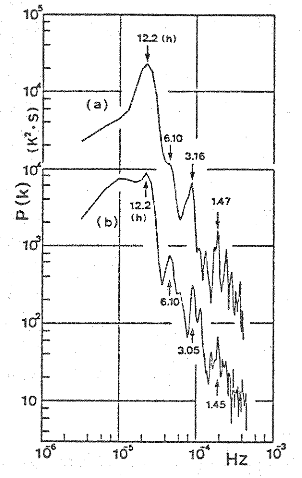

Fig. 7 Spectra with respect to temperature variation. Curves (a) and (b) are based on temperature time series (a) and (b) in Fig. 5, respectively.

Table 2 Period and waveheight of series of wave (1) to (8) in Fig. 5.

Wave	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Average value
Waveperiod (h)	13.0	13.7	12.7	12.7	12.0	12.8	12.4	12.4	12.7
Waveheight (m)		4.9	4.2	3.5	3.3	2.8	1.8	1.9	3.2

(a) Period of internal seiche

Period of internal seiche averaged throughout 8 waves is 12.7 hours. Let us evaluate the seiche period by using Holmboe's model developed by Tominaga (12). Density profile ρ_0 in Holmboe's model is expressed as

$$\rho_0 = \hat{\rho} \exp\{ -\alpha \tanh (z/h)\} \qquad (2)$$

where $\hat{\rho}=(\rho_1 \cdot \rho_2)^{1/2}$; α=dimensionless parameter of relative density difference determined from density profile ($\fallingdotseq \varepsilon/2$) ; h=characteristic length scale of thermocline (about half of the total thickness of thermocline) and z=vertical axis. Eq. 2 is displayed in Fig. 6 and is sufficiently applicable to represent the observed density profile. Linear solution concerning the dispersion relation is theoretically given by Tominaga (12) as following equation.

$$\frac{\Gamma(1+m)}{\Gamma(1-m)} \exp(2kh_1) = \frac{P_+ + Q_- \exp(-2kh_2)}{Q_+ + P_- \exp(-2kh_2)} \qquad (3)$$

where Γ(x) = the Gamma function; m=kh; k=wavenumber and

$$P_\pm = \frac{\Gamma(\pm m)}{\Gamma(1+n)\Gamma(-n)} , \qquad Q_\pm = \frac{\Gamma(\pm m)}{\Gamma(\pm m-n)\Gamma(1\pm m+n)} ,$$

$$n = (-1 + \sqrt{1 + 4gh\alpha/(\sigma/k)^2})/2 \qquad (4)$$

where σ is the frequency. To determine the phase velocity in general, eigenvalue n is iteratively computed for given conditions. Provided wavelength is enough longer than thermocline thickness and internal wave has the structure of 1st mode in the vertical direction, that is, $m \ll 1$ and $n \ll 1$ respectively, approximation of Eq. 3 and 4 is finally reduced to

$$C_h^2 = \frac{2\alpha g}{k} \cdot \frac{1}{(\coth kh_1 + \coth kh_2)\{1 + m(\coth kh_1 + \coth kh_2)/2\}} \tag{5}$$

where $C_h = \sigma/k$. Benefit by taking into account density profile in thermocline appears in second parentheses of the righthand side, and Eq. 5 equals simplified two-layered model when $m=0$. From Eq. 5, using parameters in Fig. 6, the velocity of internal long wave becomes $C_h = 0.29\ \mathrm{m\,s^{-1}}$, and immediately the period of uninodal internal seiche with a wavelength twice the lake length is evaluated to be 12.5 hours. Computed value shows fairly good agreement with observed one. For reference, waveperiod by the two-layered model is as small as 11.0 hours.

(b) Viscous damping

Once internal seiche is set into motion, frictional effect begins to work to diminish the amplitude of oscillation. Though the waveheight just after the occurrence is 5 m in practice, it falls to 1.9 m after 4 days. Time-dependent damping rate of the observed amplitude is $3.7 \times 10^{-6}\ \mathrm{s^{-1}}$. If the lake has two rectangular layers, damping rate of the amplitude can be estimated by the energy integral method in the laminar boundary layer. From the result of this method we get damping rate of $3.5 \times 10^{-7}\ \mathrm{s^{-1}}$ on the inteface of two layers , $4.8 \times 10^{-9}\ \mathrm{s^{-1}}$ on the side walls and $3.3 \times 10^{-9}\ \mathrm{s^{-1}}$ on the bottom, and the damping rates both on the end walls and of internal friction in the water body seem to be negligibly small (Muraoka and Hirata (8)). Sum of them becomes $3.6 \times 10^{-7}\ \mathrm{s^{-1}}$, but it is in one order below the observed one. Although the wind action which accelerates or dissipates the seiching motion is not considered in computation, the discrepancy between observed and theoretical results may suggest the mixing due to topographical undulation of bottom.

Internal Wave with Modal Structure

Fig. 4 (c) shows the spatial distribution of temperature observed about 2 hours later from Fig. 4 (b). As wind has decreased, the inclination of density interface in the horizontal direction is somewhat less than that of Fig. 4 (b). Especially the isotherms diverge and converge from time to time, and are very similar to isotherm pattern of the internal wave with three nodes in the horizontal direction and two nodes in the vertical one as shown in Fig. 8. This fact indicates the appearance of the internal wave with a modal structure of 3rd mode and 2nd mode in the horizontal and vertical directions, respectively. As to the 3rd mode in horizontal direction, wavelength of the internal wave is 4360 m. Substituting this wavelength and hydraulic parameters given in Fig. 9 into Eq. 3, the eigenvalue n in the Holmboe's model is numerically obtained as 0.247, 1.346

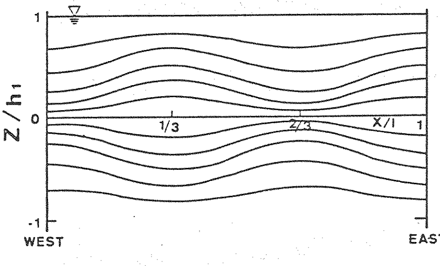

Fig. 8 Schematic of isotherms of internal wave with three nodes in horizontal direction and two nodes in vertical one. 1 in the figure indicates the lake length.

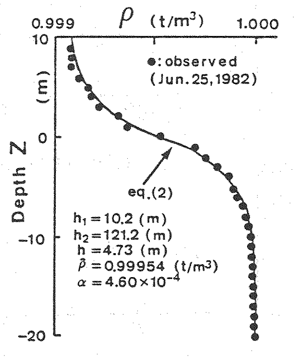

Fig. 9 Vertical density profile at St-4 on 25 June 1982.

and 2.422, correspondingly to 1st, 2nd and 3rd mode in the vertical direction, respectively. Therefore, the phase velocity of internal wave is presumed from Eq. 4 to be 0.082 m s^{-1} , to say about the waveperiod , to be 14.7 hours.

To confirm the evidence of internal wave mentioned above, temporal variations at various depths are displayed in Fig. 10, based on vertical thermal profiles observed approximately every 2 hours for 26 hours from 08°00' on 25 to 10°00' on 26 June 1982. They were mainly made from the vertical temperature records at St-3, except for the night time from 18°00' on 25th to 04°00' on 26th when the location was moved to St-3' about 500 m northern from St-3. Vertical displacement of isotherm due to internal wave having a structure of 2nd mode in the vertical direction vanishes theoretically at density interface. Actually periodical pattern of 12 hours induced by uninodal internal seiche can be seen in the temperature time series at 10 m depth which means approximately the location of density interface. However, the temperature records at 12-15 m depth contain temperature variation of 14-16 hours period, and this result leads to the evidence of internal wave with the modal structure.

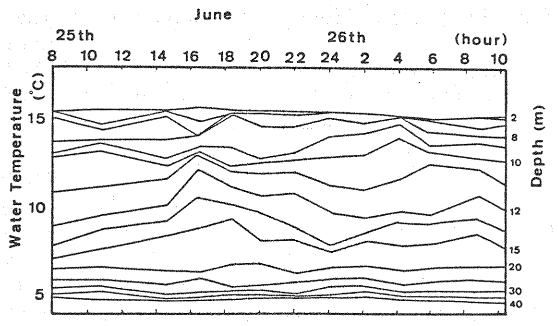

Fig. 10 Temperature variation at various depths observed at St-3 and 3' during 25-26 June 1982.

Internal Wave with Shorter Period

It is well known that short internal wave of which frequency nearly equals the Brunt-Väisälä's one tends to develop in thermocline. In order to obtain the further insights into the Brunt-Väisälä wave, temporal variations of temperature in thermocline were also examined in the middle September 1982. Two examples concerning shorter internal waves observed at density interface of St-2 are demonstrated in Fig. 11. Vertical density profile at St-2 during the observed terms

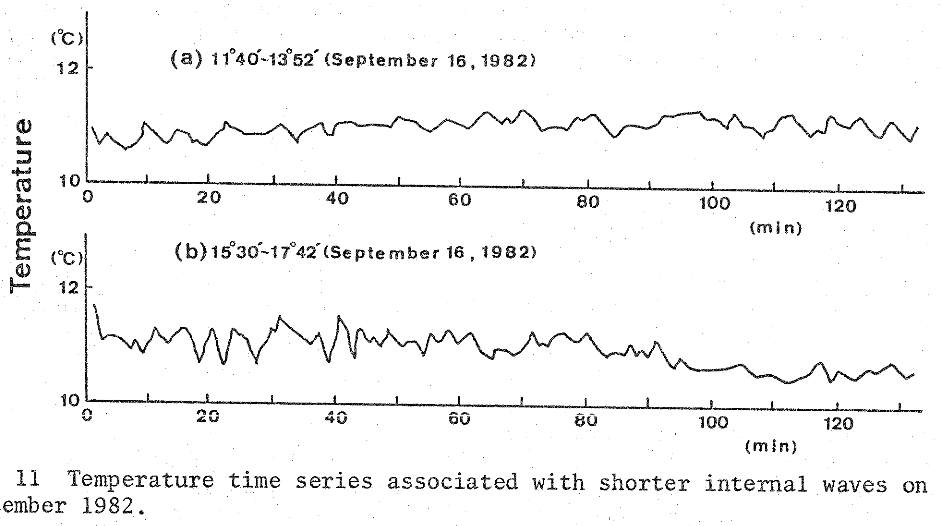

Fig. 11 Temperature time series associated with shorter internal waves on 16 September 1982.

is shown in Fig. 12.

Temperature records detected every 40 seconds show that shorter wave-like trains are superimposed on the mean trends which indicate temperature increase (see Fig. 11 (a)) and decrease (see Fig. 11 (b)) presumably due to internal seiche. Temperature spectra decomposed by FFT are displayed in Fig. 13, in which N on the lateral axis denotes the Brunt-Väisälä frequency in thermocline and it corresponds to 241 seconds in period. Energy peaks of 303, 455 and 481 seconds are easily recognized in the figure. The component of 303 seconds among them is 1.5 times of the Brunt-Väisälä period and is implied to be an internal wave with the same wavelength scale as the magnitude of thermocline thickness.

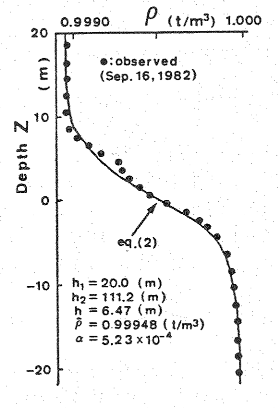

Fig. 12 Vertical density profile at St-2 on 16 September 1982. Total thermocline thickness in the Holmboe's model is about 13 m.

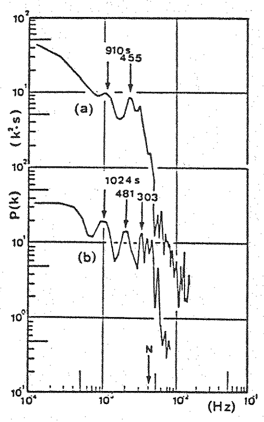

Fig. 13 Spectra with respect to temperature variation. Curves (a) and (b) are based on temperature time series (a) and (b) in Fig. 11, respectively.

Considering these internal waves are of 1st mode in the vertical direction, the Holmboe's model based on parameters given in Fig. 12 produces 40.0, 105.0 and 116.0 m in wavelength, and these are corresponding to the above mentioned periods respectively. As to the waveheight, they can be roughly estimated to be 1.2, 0.7 and 1.3 m from temperature records in Fig. 11 in the same manner as in internal seiche. The shortest wave is reconfirmed to be a typical Brunt-Väisälä wave with a wavelength in the same order of thermocline thickness.

CONCLUSIONS

Data sets obtained in Lake Chuzenji show a typical response of thermal stratification to the wind forcing and subsequent internal seiching motion. Main results in this paper are summarized as follows :

1) Surface layer thickens toward the leeward area in response to wind. However, the wind action seems effective only to break the isotherms near surface and of little importance to erode thermocline. The model of hydrostatic pressure balance can provide a reasonable estimation for the variations of surface layer thickness over thermocline.

2) In the time series of temperature observed at thermocline, we can see about 12 hours period due to uninodal internal seiche. This period can be accurately evaluated by the Holmboe's model. The waveheight of internal seiche just after the occurrence rises up to 5 m and internal seiching motion continues as long as 4-5 days since its generation.

3) Isotherms show clearly another internal wave which has the spatial structure of both 3rd mode in horizontal direction and 2nd mode in vertical one. The period of this wave can be calculated to be 14.7 hours from the Holmboe's model and be confirmed in the temperature time series based on vertical thermal profiles.

4) Internal wave of which frequency nearly equals the Brunt-Väisälä's one is recognized to develop in thermocline. Wavelength of this wave has the same magnitude as the thermocline thickness.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a, b, A	= constants;
C_h	= phase velocity of internal wave;
g	= gravitational acceleration;
h	= characteristic length scale of thermocline;
h_1, h_2	= mean depths of surface and bottom layer;
$\bar{h}_1$	= depth of surface layer induced by surface elevation;
k	= wavenumber;
l	= lake length;
m	= kh;
n	= eigenvalue in dispersion relation;
P, Q	= functions in dispersion relation;
Ti	= waveperiod of uninodal internal seiche;
x	= distance from windward end;
z	= vertical coordinate;
α	= dimensionless parameter in density profile;
$\Gamma(x)$	= Gamma function;
ε	= relative density difference;
η	= surface elevation;

ρ_1, ρ_2	= densities of surface and bottom layer;
ρ_0	= mean density profile;
$\hat{\rho}$	= mean density between surface and bottom layer;
σ	= wavefrequency; and
τ	= wind shear stress.