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INTERACTION BETWEEN FLOW OVER A GRANULAR PERMEABLE BED AND SEEPAGE FLOW - A THEORETICAL ANALYSIS -

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SYNOPSIS

Most of natural rivers are composed of permeable boundaries and the streams must be accompanied with seepage flows. In such cases, appreciable interactions between free surface flow and seepage flow are expected, and subsequently considerable mass and momentum transfers through an interfacial boundary must play an important role in flow structure. In this paper, non-linearity of seepage flow resistance is noticed, particularly in case of large Reynolds number or large permeability, and the anisotropy caused by non-linearity is investigated for seepage flow in a permeable medium beneath a free surface flow. On the other hand, the change of main flow structure is inspected using the concepts of slip velocity and fluctuating transpiration, which may be caused by the presence of anisotropically fluctuating seepage flow.

INTRODUCTION

In natural rivers with sand and gravel beds, it is conjectured that there is an active interrelation between main flow over a bed with free surface and seepage flow in a permeable layer. It is well known in fluid mechanics that steady transpiration (injection or suction) through a boundary considerably affects the structure of main flow, and also in river hydraulics two-dimensional open channel flow with steady transpiration has been investigated by Nakagawa and Nezu (6). On the other hand, even without steady transpiration, hydraulic resistance of flow over a permeable bed is different from that of flow over a solid bed. Lovera and Kennedy (3) have pointed out that the permeability may bring about an increase of flow resistance based on inspection for friction factor of natural rivers without any significant sand waves. In order to explain such an effect of permeable bed or seepage flow in the permeable layer beneath main open channel flow, Nakagawa and Nezu (5) have investigated an induced stress due to the so-called Miles' mechanism (4, 7) after an approach by Chu and Gelhar (1) for turbulent pipe flow with granular permeable boundary. Though they have estimated an induced stress, the following problems remain unresolved and the present study aims to overcome them: (1) "Miles' mechanism" is treated as a kind of black box model, and thus an important parameter involved in a model cannot be determined but experimentally. (2) The effect of the so-called slip velocity at the boundary between main flow and seepage flow has not been considered.

According to the analysis by Chu and Gelhar (1), the presssure fluctuation of open channel flow at the boundary propagates into the permeable layer and it induces velocity fluctuation in seepage flow. In this study, such an induced velocity fluctuation and its role for the interaction between free surface flow and seepage flow are investigated. Particularly, it is considered that an apparent Reynolds stress induced in the seepage flow, which is defined as a correlation between longitudinal and vertical components of velocity fluctuation of seepage flow, governs the velocity distribution profile of seepage flow and subsequently it determines the slip velocity at the boundary, while the amount of transpiration through the boundary is regarded as the vertical velocity fluctuation in seepage flow near the boundary. The slip velocity and the transpiration velocity must reveal an influence of seepage flow on main flow structure. Chapter 2 of this paper is concerned with an evaluation of velocity fluctuation in a permeable layer induced by pressure fluctuation at the boundary; Chapter 3 with the effect of the slip velocity and the transpiration velocity on the structure of free surface flow; and Chapter 4 with the evaluations of slip velocity and behavior of transpiration through the boundary and with an estimation of the change of hydraulic resistance of open channel flow due to the existence of seepage flow.

VELOCITY FLUCTUATION OF SEEPAGE FLOW INDUCED BY PRESSURE FLUCTUATION AT THE BOUNDARY

Navier-Stokes' equation on the local velocity in a permeable medium, $\hat{v}_s$, is written as

$$\rho(\partial\hat{v}_s/\partial t) + \rho(\hat{v}_s\cdot\nabla)\hat{v}_s = -\nabla p + X + \mu\nabla^2\hat{v}_s \tag{1}$$

in which ρ=mass density of water; p=pressure; X=external force; and μ = viscosity of water. For a seepage flow in a permeable medium as loose as sand or gravel layer, Ward (8) has proposed a non-linear Darcy law as for flow resistance as follows:

$$gi = (\nu/K)V + (C/\sqrt{K})V^2 \tag{2}$$

in which g=gravitational acceleration; i=hydraulic gradient; $\nu\equiv\mu/\rho$ = kinematic viscosity of water; K=permeability of the medium; C=experimental constant due to the non-linearity (if C=0, Eq.2 expresses an ordinary Darcy law); and V=average flow velocity in a permeable medium. Applying this non-linear Darcy law as a resistance law, Eq.1 can be rewritten for the average velocity of seepage flow as follows:

$$(1/n)(\partial v_s/\partial t) = -(1/\rho)\nabla p + X/\rho - (\nu/K)v_s - (C/\sqrt{K})|v_s|\cdot v_s \tag{3}$$

in which n=porosity of permeable medium; and v_s=average velocity of seepage flow. Here, we consider a two-dimensional state time average of which can be regarded as a steady state. v_s is decomposed into the longitudinal and the vertical components and into the means and the perturbations, respectively, that is, $v_s = (U_{s1}+u_s, v_s)$, in which U_{s1} = main component of seepage flow in equilibrium region of permeable layer; and u_s, v_s = perturbations of seepage flow velocity in the longitudinal and the vertical directions, respectively, and it is assumed that $U_{s1} >> u_s$, v_s. Then, Eq.3 can be rewritten as follows (see Fig.1):

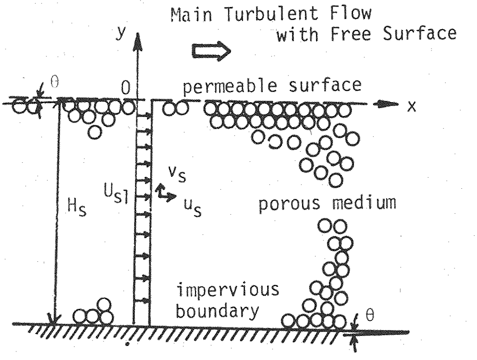

Fig. 1 Definition sketch

$$0 = -(1/\rho)(\partial P/\partial x) + g\sin\theta - (\nu/K)U_{s1} - (C/\sqrt{K})U_{s1}^2 \tag{4}$$

$$0 = -(1/\rho)(\partial P/\partial y) - g\cos\theta \tag{5}$$

$$(1/n)(\partial u_s/\partial t) = -(1/\rho)(\partial p'/\partial x) - (\nu/K)u_s - (2C/\sqrt{K})U_{s1}u_s \tag{6}$$

$$(1/n)(\partial v_s/\partial t) = -(1/\rho)(\partial p'/\partial y) - (\nu/K)v_s - (C/\sqrt{K})U_{s1}v_s \tag{7}$$

in which $p=P+p'$; P, p'=average and perturbation of pressure, respectively; and θ= bed slope. It is assumed that the thickness of a permeable layer, H_s, is constant along the longitudinal direction. The continuity equation is written as

$$(\partial u_s/\partial x) + (\partial v_s/\partial y) = 0 \tag{8}$$

From Eqs.4 and 5, the average seepage velocity is obtained as follows:

$$(C/\sqrt{K})U_{s1}^2 + (\nu/K)U_{s1} - gI_e = 0 \tag{9}$$

in which I_e=energy gradient. When $C \neq 0$,

$$R_K \equiv U_{s1}\sqrt{K}/\nu = (-1+\sqrt{4CR_{K_0}})/2C \tag{10}$$

in which R_{K_0} ($\equiv U_{s1}\sqrt{K}/\nu = gI_eK^{3/2}\nu^{-2}$) corresponds to the case of C=0, which means an ordinary Darcy law. On the other hand, from Eqs.6~8 on the perturbations, we obtain a Poisson type equation as follows:

$$\frac{\partial^2 p'}{\partial x^2} + \frac{\partial^2 p'}{\partial y^2} = -\frac{\rho C}{\sqrt{K}} U_{s1}\frac{\partial u_s}{\partial x} \tag{11}$$

The pressure fluctuation at an interfacial boundary between free surface flow and seepage flow is here to be expressed as

$$p'|_{y=0} = \mathcal{Re}\{p_0 \exp[i(\kappa x+\omega t)]\} \tag{12}$$

in which $\mathcal{Re}\{z\}$ = real part of a complex number z; and i=imaginary unit. Namely, the pressure fluctuation at the boundary is regarded as a sinusoidal wave, the angular wave number, the angular frequency, and the amplitude of which are κ, ω and $2p_0$, respectively. The boundary condition is given by $(\partial p'/\partial y)=0$ at $y=-H_s$. Eq.11 has been already solved by Chu and Gelhar (1), and the solution are written as

$$p'(y) = \mathcal{Re}\{[\cosh\beta\kappa(H_s+y)/\cosh\beta\kappa H_s]\cdot\tilde{p}'(0)\} \tag{13}$$

$$u_s(y) = \mathcal{Re}\{-i[n\kappa/(\alpha_1+i\omega)][\cosh\beta\kappa(H_s+y)/\cosh\beta\kappa H_s][\tilde{p}'(0)/\rho]\} \tag{14}$$

$$v_s(y) = \mathcal{Re}\{-[n\kappa/(\alpha_1+i\omega)\beta][\sinh\beta\kappa(H_s+y)/\cosh\beta\kappa H_s][\tilde{p}'(0)/\rho]\} \tag{15}$$

in which

$$\alpha_1 \equiv n[(\nu/K)+(2C/\sqrt{K})U_{s1}] = n(\nu/K)(1+2CR_K) \tag{16}$$

$$\alpha_2 \equiv n[(\nu/K) + (C/\sqrt{K})U_{s1}] = n(\nu/K)(1 + CR_K) \tag{17}$$

$$\beta^2 \equiv (\alpha_2+i\omega)/(\alpha_1+i\omega) = [(\omega_*^2+\gamma_{12})/(\omega_*^2+\gamma_{12}^2)]+i[(\gamma_{12}-1)\omega_*/(\omega_*^2+\gamma_{12}^2)] \tag{18}$$

$\omega_* \equiv \omega/\alpha_2 = (\omega K/\nu)/[n(1+CR_K)]$; $\gamma_{12} \equiv \alpha_1/\alpha_2 = (1+2CR_K)/(1+CR_K)$; and $\tilde{p}'(0) \equiv p_0\exp[i(\kappa x+\omega t)]$. When $R_K \to \infty$, $\gamma_{12} \to 2$. β is a complex in general, and $\beta \equiv \beta_r + i\beta_i$, in which β_r, β_i=real numbers. When C=0, $\beta=1$ ($\beta_r=1$, $\beta_i=0$). Eqs.13~15 show that the pressure fluctuation at a boundary propagates into the permeable medium and induces velocity fluctuations in seepage flow. And, it implies that there exists instantaneously a transpiration or mass transfer through the boundary and it is expected to cause the change of the structure of free surface flow. The most contributive factor to this change may be $v_s(0)$ and it is given by

$$v_s(0)/u_* = \mathcal{Re}\{-B_*[\beta/(1+i\omega_*)]\tanh\beta\kappa H_s\cdot[\tilde{p}'(0)/\tau_0]\} \tag{19}$$

in which $B_* \equiv [\kappa K u_*/\nu]/(1+CR_K)$; $u_* \equiv \sqrt{ghI_e}$; and $\tau_0 \equiv \rho u_*^2$.

If the pressure fluctuation at the boundary is composed of regular sinusoidal waves with amplitude $2p_0$, angular wave number κ, and angular frequency ω, the variances of u_s and v_s, and the correlation between u_s and v_s can be calculated as follows, which are very instructive for prediction of apparent turbulent intensities and Reynolds stress induced in seepage flow by pressure fluctuation at the boundary.

$$\overline{\{u_s(\eta)\}^2}/u_*^2 = [B_*^2 p_{0*}^2/(\omega_*^2+\gamma_{12}^2)]\cdot f_{uu}(\eta) \tag{20}$$

$$\overline{\{v_s(\eta)\}^2}/u_*^2 = [B_*^2 p_{0*}^2/\sqrt{(\omega_*^2+1)(\omega_*^2+\gamma_{12}^2)}]\cdot f_{vv}(\eta) \tag{21}$$

$$-\overline{u_s(\eta)\cdot v_s(\eta)}/u_*^2 = [B_*^2 p_{0*}^2/\sqrt{(\omega_*^2+1)(\omega_*^2+\gamma_{12}^2)}]\cdot f_{uv}(\eta) \tag{22}$$

in which $\eta \equiv y/H_s$; $\overline{\cdot}$ means time average; and $p_{0*} \equiv p_0/\tau_0$. $f_{uu}(\eta)$, $f_{vv}(\eta)$ and $f_{uv}(\eta)$ express damping tendencies of these quantities of perturbations in a permeable medium, which are given by

$$f_{uu}(\eta) \equiv [\cosh 2\beta_r \kappa H_s(1+\eta)+\cos 2\beta_i \kappa H_s(1+\eta)]/F_* \tag{23}$$

$$f_{vv}(\eta) \equiv [\cosh 2\beta_r \kappa H_s(1+\eta)-\cos 2\beta_i \kappa H_s(1+\eta)]/F_* \tag{24}$$

$$f_{uv}(\eta) \equiv [\beta_i \sinh 2\beta_r \kappa H_s(1+\eta)-\beta_r \sin 2\beta_i \kappa H_s(1+\eta)]/F_* \tag{25}$$

$$F_* \equiv \cosh 2\beta_r \kappa H_s + \cos 2\beta_i \kappa H_s \tag{26}$$

At $y \to -0$ (very near the boundary), $f_{uu}(\eta) \to 1$, and $f_{vv}(\eta) \to 1$ if $H_s \to \infty$ additionally. If $H_s \to \infty$, Eq.25 yields

$$f_{uv}(y) = \beta_i \exp[2\beta_r \kappa y] \tag{27}$$

As seen from these equations, β plays an important role as an anisotropic factor characterizing the non-linearity of the phenomenon. Fig.2 shows the relations among β_i, β_r and ω_* for the case of $R_K \to \infty$, and particularly for the intermediate range of ω_* ($\omega_* \simeq 2$), anisotropy due to non-linearity becomes most appreciable. In Fig.3, some examples of calculations of Eqs.20∿22 are shown. On calculations, the followings are assumed: $R_K \to \infty$, and $\omega_* = 2$. κH_s is changed in three cases: 0.1, 1.0 and 10. It expresses the ratio of the permeable layer thickness to wave length of pressure fluctuation or the so-called length scale of an eddy involved in free surface flow.

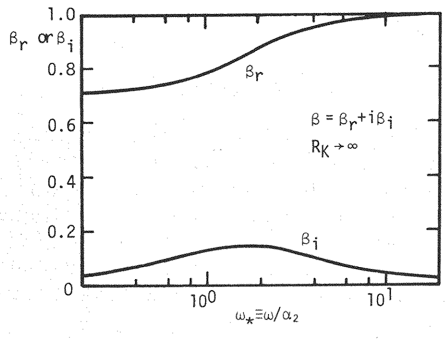

Fig. 2 β_r, $\beta_i \sim \omega_* \equiv \omega/\alpha_2$ ($R_K \to \infty$)

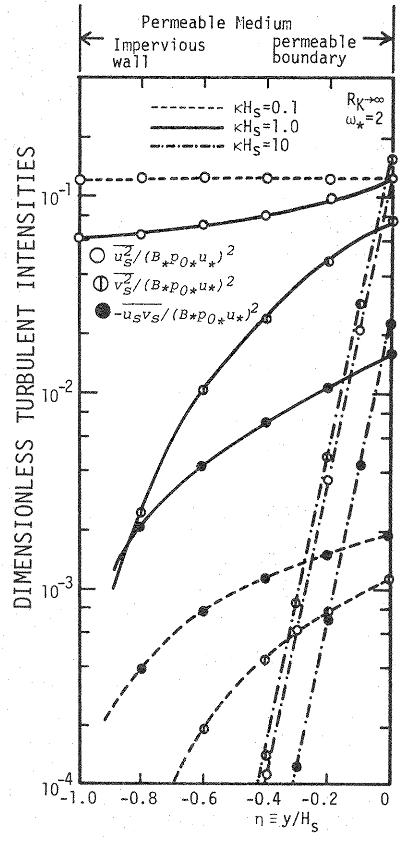

Fig.3 Apparent turbulent intensities induced in seepage flow

EFFECT OF SLIP VELOCITY AND TRANSPIRATION ON MAIN FLOW STRUCTURE

The general boundary conditions for free surface flow on a permeable bed with seepage flow are the presences of slip velocity at the boundary and transpiration through the boundary, and thus, their effect will be investigated based on the assumption of steady uniform flow, for the sake of simplicity.

In the region where the so-called wall law is applicable, the following equation may be valid in case of the flow with transpiration.

$$\tau_0 = -\rho\overline{uv} + \nu\frac{\partial U}{\partial y} - \rho v_0 U \qquad (28)$$

in which U=flow velocity of free surface flow; and v_0=transpiration velocity through the boundary (injection to main flow is expressed as positive transpiration). The left part of Eq.28 is bed shear stress to balance the external force ($\rho g h I_e$; h=depth of free surface flow), the first term of the right part is the Reynolds stress or momentum flux in the vertical direction due to turbulence, the second term is the viscous stress, and the third term corresponds to momentum flux due to the transpiration velocity v_0. Bed composed of granular materials is permeable and rough in general, the so-called viscous sublayer cannot be present. Hence, the viscous term of Eq.28 is neglected. Meanwhile, the so-called mixing length theory is applied for the Reynolds stress. Namely,

$$-\overline{uv} = (\kappa_0 y)^2 (dU/dy)^2 \qquad (29)$$

in which κ_0=Kármán constant ($\ell_0=\kappa_0 y$ = mixing length). Then, the following dimensionless equation can be obtained.

$$(\kappa_0 y^+/v_0^+)\frac{d(1+v_0^+U^+)}{dy^+} = \sqrt{1+v_0^+U^+} \qquad (30)$$

in which $U^+ \equiv U/u_*$; $v_0^+ \equiv v_0/u_*$; and $y^+ \equiv yu_*/\nu$; and the following condition is imposed: $(1+v_0^+U^+)>0$ or $v_0^+>-(1/U^+)$. The boundary condition is that $U^+=U_p^+$ at $y^+=y_0^+$, in which U_p=slip velocity and y_0=the height at which the flow velocity is zero in case of flow over a solid boundary. By integrating Eq.30 considering the boundary condition, we obtain

$$(2/v_0^+)(\sqrt{1+v_0^+U^+} - \sqrt{1+v_0^+U_p^+}) = (1/\kappa_0)\ln(y/k_s) + D_r \qquad (31)$$

in which $D_r \equiv (1/\kappa_0)\ln(k_s/y_0)$; and k_s=equivalent sand roughness. Here, D_r and κ_0 are regarded as universal constants. Solving Eq.31 with respect to U^+, we obtain

$$U^+ = (v_0^+/4\kappa_0^2)[\ln(y/k_s)]^2 + (1/\kappa_0)[(v_0^+/2)D_r+\sqrt{1+v_0^+U_p^+}][\ln(y/k_s)] + [(v_0^+D_r/4)+\sqrt{1+v_0^+U_p^+}]D_r + U_p^+ \qquad (32)$$

This is identified with the so-called bilogarithmic law first deduced by Dorrance and Dore (2).

An integration of Eq.32 along the flow depth gives the following mean flow velocity formula.

$$U_m^+ = (v_0^+/4\kappa_0^2)[\ln(h/k_s)]^2 + (1/\kappa_0)[(D_r-1/\kappa_0)(v_0^+/2) + \sqrt{1+v_0^+U_p^+}][\ln(h/k_s)] + (D_r-1/\kappa_0)[\sqrt{1+v_0^+U_p^+} - v_0^+/2] + (v_0^+/4)D_r + U_p^+ \qquad (33)$$

On the other hand, in case of flow over a solid bed, $v_0=U_p=0$, and then,

$$U_n^+ = (1/\kappa_0)\ln(y/k_s)+D_r \ ; \qquad U_{mn}^+ = (1/\kappa_0)\ln(h_n/k_s)+D_r-1/\kappa_0 \qquad (34)$$

in which the subscript n indicates the quantity without seepage flow.

The flow over a permeable bed will be here compared with the flow over a solid bed under the condition of the same free surface flow discharge and the same energy gradient. Here, Ω is defined as (h/h_n) or (τ_0/τ_{0n}). Because the uniform flow is assumed, $U_m/U_{nm}=1/\Omega$. And,

$$\sqrt{2/f} = U_m^+ = \Omega^{-3/2}\cdot\sqrt{2/f_n} \qquad (35)$$

is obtained, in which f is friction factor and f_n is given for any flow discharge per unit width for free surface flow, bed slope and roughness of the boundary using Eq.34. Consequently, we obtain the following relationship.

$$\sqrt{2/f_n}\ [(1/\Omega)-\sqrt{\Omega+\lambda_1\lambda_2}-(\lambda_1/2\kappa_0)\ln\Omega]$$
$$=(\lambda_1/4)\{(2/f_n)+(1/\kappa_0)[(\ln\Omega)^2+1]\}+(\ln\Omega/\kappa_0)\sqrt{\Omega+\lambda_1\lambda_2}+\lambda_2 \qquad (36)$$

in which $\lambda_1 \equiv v_0/u_{*n} = v_0^+ \cdot \sqrt{\Omega}$; and $\lambda_2 \equiv U_p/u_{*n} = U_p^+ \cdot \sqrt{\Omega}$. The change of resistance due to the presence of seepage flow is evaluated by Ω using Eq.36, if f_n, λ_1 and λ_2 are known. The relationship between Ω and λ_1 for the case of $U_p=0$ ($\lambda_2=0$) is shown in Fig.4-a, while that for given value of f_n is shown in Fig.4-b with a parameter λ_2. According to Fig.4-a, Ω varies with λ_1 more sensitively for smaller value of f_n. Particularly, if the slip velocity is zero, the flow resistance decreases by injection but increases by suction appreciably. Meanwhile, Fig.4-b implies that the slip velocity considerably decreases the flow resistance.

Fig.5 shows typical examples of velocity profile, and Fig.6 reveals that an apparent value of Kármán constant of flow with injection becomes smaller and that of flow with suction becomes larger than that of flow without transpiration.

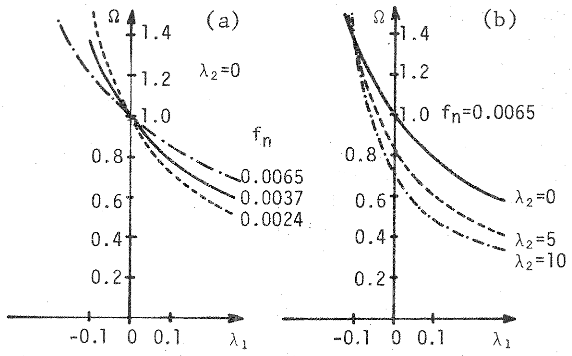

Fig. 4 Relation between Ω and λ_1

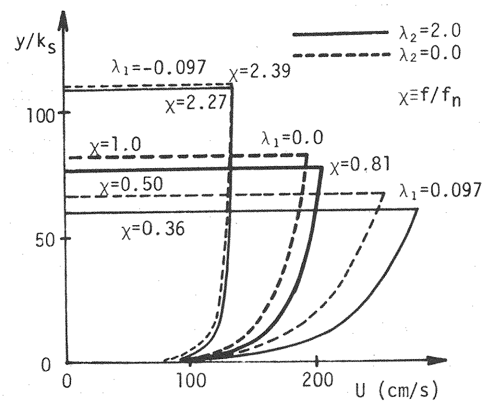

Fig. 5 Velocity profile of free surface flow

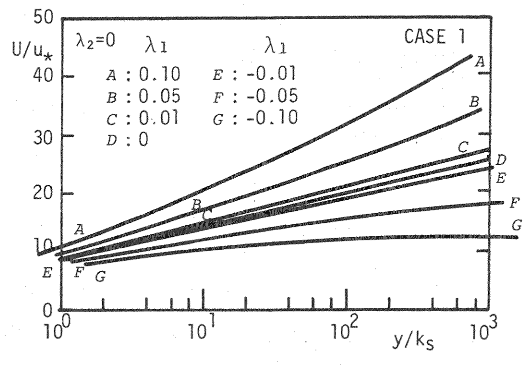

Fig. 6 Apparent change of Kármán constant due to transpiration

VELOCITY PROFILE IN PERMEABLE MEDIUM BENEATH FREE SURFACE FLOW AND RESISTANCE OF FREE SURFACE FLOW WITH SEEPAGE FLOW

In open channel flow with a permeable bed, pressure fluctuation at the boundary induces velocity fluctuations in seepage flow, and subsequently mass and momentum transfers become active through an interfacial boundary between free surface flow and seepage flow. Moreover, a kind of Reynolds stress is brought about in seepage flow as explained in Chapter 2, and the velocity profile of seepage flow may show a kind of shear flow type. If an ordinary Darcy law is applied instead of non-linear one, neither the vertical momentum transfer nor the Reynolds stress appears, and then, the flow velocity profile has a discontinuity at the boundary and that in a permeable medium is uniform. In other words, the induced apparent Reynolds stress makes a continuous velocity profile throughout the free surface flow region and the permeable medium.

When additional velocity of seepage flow which is brought about due to drag by faster free surface flow is represented by U_{s2}, velocity profile of seepage flow $U_s(y)$ $(-H_s<y<0)$ is expressed by

$$U_s(y) = U_{s1} + U_{s2}(y) \tag{37}$$

Applying the mixing length theory, the following can be written.

$$-\rho\overline{u_s v_s} = \rho \ell_p^2 (dU_{s2}^+/dy)^2 \tag{38}$$

in which ℓ_p=mixing length in a permeable medium. The mixing of fluid lump should be constrained by the scale of the porosity of a permeable medium, and then it can be assumed that $\ell_p=\alpha_0\sqrt{K}$, in which α_0=experimental constant. Substituting Eq.22 into Eq.38, we obtain

$$\sqrt{K}(dU_{s2}/dy) = \{B_* p_{0*}/[\alpha_0\sqrt{\Gamma^*(\omega_*)}]\}\cdot\sqrt{f_{uv}(y/H_s)} \tag{39}$$

in which $\Gamma^*(\omega_*)\equiv\sqrt{(\omega_*^2+1)(\omega_*^2+\gamma_{12}^2)}$. If a permeable layer is sufficiently thick, f_{uv} can be approximated by Eq.27, and then the following velocity profile is obtained considering that $U_{s2}\to 0$ for $y\to-\infty$ $(H_s\to\infty)$.

$$U_{s2}^+(y) = \{B_* p_{0*}\sqrt{\beta_i}/[\alpha_0\beta_r\kappa\sqrt{K}\sqrt{\Gamma^*(\omega_*)}]\}\cdot\exp(\beta_r\kappa y) \tag{40}$$

This implies an exponential velocity profile in a permeable medium with faster free surface flow on it, and it is formally consistent with a conclusion of an analysis by Yamada and Kawabata (9) though the approaches are different each other. Fig.7 illustrates the velocity profiles both for free surface flow and for seepage flow, and it demonstrates that U_s is larger than U_{s1} or that the seepage flow discharge is considerably larger than the value predicted by applying an ordinary method. The effect of free surface flow to seepage flow is not less important than the reverse effect, and it is noticed that the presence of free surface flow has an important role for seepage flow structure.

At the boundary $(y\to-0)$,

$$U_{s2}^+(0) = B_* p_{0*}\sqrt{\beta_i}/[\alpha_0\beta_r\kappa\sqrt{K}\sqrt{\Gamma^*(\omega_*)}] \tag{41}$$

and, it is expected that $U_p^+ = U_{s1}^+ + U_{s2}^+(0)$. Namely,

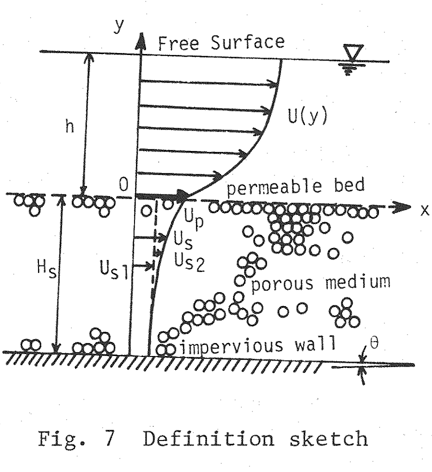

Fig. 7 Definition sketch

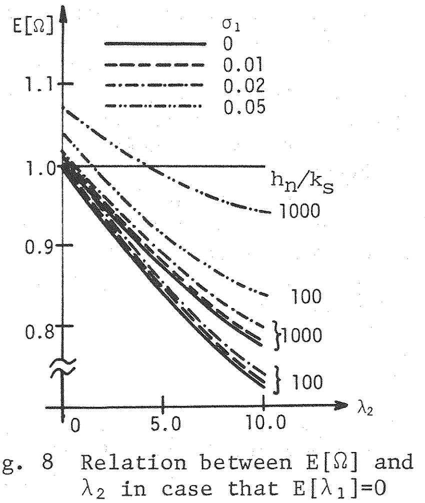

Fig. 8 Relation between $E[\Omega]$ and λ_2 in case that $E[\lambda_1]=0$

$$U_p^+ = U_{s1}^+\{1+[R_* p_{0*}/R_K(1+CR_K)](\sqrt{\beta_i}/\alpha_0\beta_r)[1/\sqrt{\Gamma^*(\omega_*)}] \qquad (42)$$

in which $U_{s1}^+=R_K/R_*$; and $R_*\equiv u_*\sqrt{K}/\nu$. The slip velocity has been determined for given wave of the pressure fluctuation.

On the other hand, the transpiration velocity is given by Eq.19, and it is fluctuating with zero mean. If the pressure fluctuation can be expressed by a regular sinusoidal wave as given by Eq.12, the standard deviation of transpiration velocity, σ_{s*}, is obtained by $\overline{\{v_s(0)\}^2}/u_*^2$, which is given by Eq.21. The pressure fluctuation may be, of course, composed of wide range of wave number and frequency, and thus, the real value of U_p^+ and σ_{s*} may be affected by this fact. The investigation of the statistical characteristics of pressure fluctuation at the boundary and the quantitatively rigorous arguments based on them shall remain as a succeeding research programme. Here, the effect of fluctuating transpiration on the flow resistance of free surface flow will be discussed by a simplified treatment.

If $\lambda_1\equiv v_0/u_{*n}$ follows a normal distribution, means and standard deviation of which are zero and σ_1, respectively, the expected value of Ω, $E[\Omega]$, can be comparatively easily calculated based on the relation between Ω and λ_1, $\Omega(\lambda_1)$, which is assumed to be identified with the relation obtained in Chapter 3 and is given by Eq.36. Though $\sigma_1=\sigma_{s*}\cdot E[\Omega]$ and $E[\Omega]$ is unknown, σ_1 may be possible to be identified with σ_{s*} on calculation. The calculated $E[\Omega]$ is a function of $\lambda_2\equiv U_p/u_{*n}$ $=U_p^+\cdot E[\Omega]$ and f_n. In Fig.8, some examples of calculated results are shown, in which the relationship between $E[\Omega]$ and λ_2 is drawn with parameters σ_1 and h_n/k_s. h_n/k_s is related to f_n by Eq.34. According to this figure, violent fluctuation of transpiration causes an appreciable increase of flow resistance and the slip velocity suppresses the flow resistance.

CONCLUSIONS

The results obtained in this paper are summarized below:

(1) Based on a non-linear Darcy law for hydraulic resistance of seepage flow, a Poisson type equation has been deduced as for the pressure propagation in a permeable medium, and the velocity fluctuation of seepage flow induced by it has been inspected. Particularly, it has been clarified that an apparent Reynolds stress is induced even in seepage flow. According to this analysis, it has been conjectured that there are considerable mass and momentum transfers through an interfacial boundary between free surface flow and seepage flow.

(2) The change of structure of free surface flow due to the slip velocity and transpiration has been investigated based on the bilogarithmic law, and it has been reconfirmed that suction brings about an increase of flow resistance and injection and slip velocity cause a decrease of it. The ratio of bed shear stress in case of flow over a permeable bed to that for flow over a solid bed has been evaluated quantitatively.

(3) The velocity distribution profile of seepage flow in a permeable layer on which a free surface flow exists has been clarified, considering the induced Reynolds stress in the permeable layer. Moreover, based on it, the slip velocity at the boundary has been estimated. Due to the drag by faster free surface flow, the seepage flow velocity profile can be degenerated and be increased.

(4) Even if the time average of transpiration is zero, the accumulation effect exists due to essentially fluctuating transpiration, and the hydraulic resistance of flow with seepage flow can be increased. The fluctuating transpiration can be evaluated as the velocity fluctuation of seepage flow near the boundary induced by pressure fluctuation.

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APPENDIX - NOTATION

The following symbols are used in this paper:

B_* = $(\kappa K u_*/\nu)/(1+CR_K)$;

C = experimental constant involved in non-linear Darcy law;

D_r = universal constant of logarithmic law for rough turbulent flow;

$E[\cdot]$ = expected value operator;

f = friction factor of flow over a permeable layer;

f_n = friction factor of flow over a solid bed;

f_{uu}, f_{vv}, f_{uv} = damping functions of quantities as for turbulent intensities in a permeable medium;

g = gravitational acceleration;

H_s = thickness of a permeable layer;

h = depth of free surface flow;

I_e = energy gradient;

i = imaginary unit;

K = permeability;

k_s = equivalent sand roughness;

ℓ_0 = mixing length in free surface flow region;

ℓ_p = mixing length of seepage flow in permeable medium;

n = porosity of permeable medium;

P = average pressure;

p = instantaneous pressure $(=P+p')$;

p' = pressure fluctuation;

p_0 = half amplitude of pressure fluctuation at the boundary;

p_{0*} = p_0/τ_0;

$\tilde{p}'(0)$ = $p_0\exp[i(\kappa x+\omega t)]$;

R_K = $U_{s1}\sqrt{K}/\nu$;

R_* = $u_*\sqrt{K}/\nu$;

$\mathcal{Re}\{\cdot\}$	= real part of complex number;
U	= velocity of free surface flow with seepage flow;
U_m	= average flow velocity over a permeable bed;
U_n	= velocity of free surface flow without seepage flow;
U_p	= slip velocity at the boundary;
U_s	= seepage flow velocity ($= U_{s1}+U_{s2}$) ;
U_{s1}	= seepage flow velocity in equilibrium region;
U_{s2}	= additional velocity induced in seepage flow;
U^+	= U/u_* = dimensionless velocity;
u_s	= perturbation of longitudinal component of seepage flow velocity;
u_*	= frictional velocity;
v_0	= transpiration velocity through the boundary;
v_s	= perturbation of vertical component of seepage flow velocity;
V_s	= average velocity in permeable layer;
$\hat{V}_s$	= local velocity in permeable layer;
y^+	= yu_*/ν = dimensionless height from the bed;
y_0^+	= dimensionless height to define the slip velocity;
α_1	= $n[(\nu/K)+(2C/\sqrt{K})U_{s1}] = n(\nu/K)(1+2CR_K)$;
α_2	= $n[(\nu/K)+(C/\sqrt{K})U_{s1}] = n(\nu/K)(1+CR_K)$;
β	= $\beta_r+i\beta_i$ = complex number as for anisotropy of permeable medium;
$\Gamma^*(\omega_*)$	= $\sqrt{(\omega_*^2+1)(\omega_*^2+\gamma_{12}^2)}$;
γ_{12}	= $\alpha_1/\alpha_2 = (1+2CR_K)/(1+CR_K)$;
κ	= angular wave number of pressure fluctuation;
κ_0	= Kármán constant;
λ_1	= v_0/u_{*n};
λ_2	= U_p/u_{*n};
ρ	= mass density of water;
μ	= viscosity of water;
ν	= μ/ρ = kinematic viscosity of water;
θ	= bed slope;
Ω	= $\tau_0/\tau_{0n} = (u_*/u_{*n})^2 = U_{nm}/U_m$;
ω	= angular frequency of pressure fluctuation;
ω_*	= $\omega/\alpha_2 = (\omega K/\nu)/[n(1+CR_K)]$;
σ_1	= standard deviation of λ_1;
σ_{s*}	= standard deviation of (v_s/u_*) at the boundary;
τ_0	= bed shear stress $(=\rho u_*^2)$;
τ_{0n}	= bed shear stress in case of solid boundary $(=\rho u_{*n}^2)$; and
η	= y/H_s.