Journal of Hydroscience and Hydraulic Engineering
Vol. 1, No. 2, November, 1983, pp.93-111.

A STOCHASTIC METHOD OF REAL-TIME FLOOD PREDICTION IN A BASIN CONSISTING OF SEVERAL SUB-BASINS

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SYNOPSIS

Recently, many investigators have proposed stochasic method of real-time flood prediction. In most of the studies, however, there are some assumptions: that the basin dealt with in the study is simple, namely not divided into any sub-basins; that the rainfall prediction is deterministic or perfect; that no missing data are contained in the discharge observations; and that noises incorporated in the rainfall-runoff model to compensate both model incompletion and observation error are white and steady, and so forth.

These assumptions are not always appropriate.

The aim of the present study is to propose methods of coping with difficulties in the case where they are not appropriate.

In this paper, as a case study, the authors deal with the Fukuchiyama basin (1 350 km^2) which consists of several sub-basins and contains four gauging stations, which may have missing observations; it is described as a stochastic state-space model based on the existing flood prediction model of the Yura River in Japan. The noises incorporated are continuous colored system noise and discrete white observation noise. Stochastic rainfall prediction is also taken into account.

It is shown, by using the existing data, that the methods presented here is practical and useful.

INTRODUTION

Flood forecasting is important to prevent or reduce disasters due to flood. In this paper, we propose a stochastic method of real-time flood prediction in a basin consisting of several sub-basins. It is assumed that the total area of the basin is hundreds km^2 or more.

Flood prediction can be executed with rainfall prediction and a rainfall-runoff model; in general, however, the predicted value of discharge is different from the observed value because of both model incompletion and observation error of rainfall and discharge. It is, therefore, reasonable to construct a stochastic rainfall-runoff model incorporating noises which compensate such errors. Meanwhile, it is also necessary to make use of both rainfall and discharge data obtained hourly.

We have investigated stochastic methods of real-time flood prediction based on the state-space description of runoff systems (see Shiiba and Takasao (13), Takasao et al. (14,15,16,17)).

In the methods, Kalman filter theory is applied to predict the system states and outputs. When "prediction" which contains some uncertainty is dealt with, not only in the case of flood prediction, its precision should be quantitatively specified. Using hourly observed data, Kalman filter can compute not only the predicted values but also the covariance matrix of the prediction error in a sequential manner. This is why Kalman filter is often applied to various problems of water resources systems (e.g. (4,22)).

This kind of studies on real-time flood prediction, which have been made by many investigators (for example, Hino (6), Bras (3), Todini and Wallis (19), Wood and Szöllösi-Nagy (21), Shiiba and Takasao (13), Hoshi and Yamaoka (7), Kitanidis and Bras (11), Bolzern et al. (2), Hoshi et al. (8), Takasao et al. (15,16)), contain some assumptions: that the rainfall prediction is deterministic or perfect; that the basin dealt with in the study is simple, namely not divided into any sub-basins; that no missing data are contained in the discharge observations; and that the noises incorporated in the rainfall-runoff model to compensate both model incompletion and observation error are white and steady, and so forth.

These assumptions are not always appropriate.

The aim of the present study is to propose methods of coping with difficulties in the case where they are not appropriate. We made some new attempts as follows:

[1]Basin Scale

We deal with the Fukuchiyama basin (1 350 km^2) consisting of five sub-basins; it must be treated as a multi-input and multi-output (MIMO) system. We show a treatment of both input (rainfall) and output (discharge) in the case of such a basin. Although there have been many papers, it seems that few authors have discussed this kind of subject.

[2]System Output

It is often assumed that there is only one gauging station which has no missing data; in this paper, it is assumed that there are four gauging stations which may have missing data.

[3]Rainfall Prediction

In most of the studies formerly made by many investigators, they seem to have focused on runoff prediction techniques. Rainfall prediction needed for runoff prediction has been often treated in a perfunctory manner; for instance, it is assumed to be given (known). In this paper, we use 3-hour moving average method which is often used in the practical work by the Ministry of Construction of Japan Government; we compute spatial and temporal correlation of hourly rainfalls predicted in each sub-basins and take account of the covariance of their prediction errors. Thus the rainfall prediction is treated as stochastic.

[4]Noises incorporated in the rainfall-runoff model

As for the rainfall-runoff model, we construct a stochastic state-space model based on the existing flood prediction model consisting of six storage functions (of four sub-basins and of two channel systems). Continuous colored system noise and discrete white observation noise are incorporated in the model to compensate both model incompletion and observation error. Steady noise and unsteady noise are presented; and we make a comparison between them in the later chapter.

GENERAL METHOD IN A LARGE BASIN

Synthetic Model of a Large Basin

In a basin whose area is hundreds km^2 or more, time of flood concentration and spatial distribution of rainfall must be considered. Such a large system is, therefore, divided into some smaller sub-systems and treated as a MIMO system.

Let a synthetic model of such a basin be given in the following form.

Let $\underline{x}$ and $\underline{r}$ be a N_x-dimensional state vector and a N_r-dimensional input vector, respectively; where N_x denotes the number of state variables and N_r is the number of inputs.

The state transition in the whole basin is represented as

$$\dot{\underline{x}} = \underline{f}\,(\,\underline{x}\,,\,\underline{r}\,,\,t\,)\quad,\ t \geq t_0 \tag{1}$$

where $\underline{x}$, $\underline{r}$ = column vectors; and t = time (t_0 = the initial time). Symbol '·' denotes differentiation d/dt.

Let $\underline{y}$ be an observation vector

$$\underline{y} = [\ y_1\ ,\ \cdots\ ,\ y_{N_y}\ ]^T$$

where y_n = discharge of the n-th gauging station (n = 1, ···, N_y); N_y = the number of gauging stations. Suppose that the discharge at time t_k can be described as a function of system states:

$$\underline{y} = \underline{g}\ (\ \underline{x}\ ,\ t_k\ )\quad ,\ k = 1,2,\ \cdots\quad \text{and}\quad t_0 < t_1 < \cdots \tag{2}$$

The functions $\underline{f}$ and $\underline{g}$, which are in general nonlinear, will be specified in the later chapter.

Stochastic State-Space Model

In this section, the continuous-discrete state-space model (Eqs. 1 and 2) is re-formed as a stochastic one. Namely, in the same manner as often used, noises are incorporated into the model to compensate model error and observation error:

$$\dot{\underline{x}} = \underline{f}\ (\ \underline{x}\ ,\ \underline{r}\ ,\ t\ ) + \underline{w}(t) \tag{3}$$

$$\underline{y} = \underline{g}\ (\ \underline{x}\ ,\ t_k\ ) + \underline{v}(t_k) \tag{4}$$

where $\underline{w}(t)$ = continuous system noise vector; and $\underline{v}(t_k)$ = discrete observation noise vector. In general, $\underline{w}$ and $\underline{v}$ are assumed to be white and steady.

We consider, however, that the assumption is not always appropriate. If the model structure is complete, in other words, if $\underline{f}$ and $\underline{g}$ in the above equations can perfectly describe actual rainfall-runoff phenomenon, the noises are white. Otherwise, they cannot be white and must be treated as colored. Especially, in the case of short-term rainfall-runoff modelling, the whiteness assumption is not right, because the phenomenon is too large in scale and complex to model it perfectly. As for the steadiness assumption, it is discussed later.

In this study, we employ an exponentially correlated noise as stated below; it is representative of a rather large class of Markov Processes and called occasionally "colored noise".

Let $\underline{p}$ be a N_p-dimensional noise vector. Eq. 3 is rewritten as

$$\dot{\underline{x}} = \underline{f}\ (\ \underline{x}\ ,\ \underline{r}\ ,\ t\ ) + G\underline{p}\quad ,\quad t \geqq t_0 \tag{5}$$

where $G = N_x \times N_p$-matrix. Let the i-th component p_i of $\underline{p}$ be a continuous colored noise described by the linear stochastic differential equation (see Jazwinski (9)):

$$\dot{p}_i(t) = -\ c_{pi}\, p_i(t) + c_{pi}\, q_i^{1/2}\, w_i(t)\quad ,\quad t \geqq t_0 \tag{6}$$

where c_{pi} , q_i = constants; $\{w_i(t)\}$ = zero-mean white Gaussian noise with

$$E\{\ w_i(t) w_i(\tau)\ \} = \delta(t-\tau)$$

where $\delta(t)$ = Dirac delta function.

Now consider the transition of p_i during $t_k(j) \leqq t \leqq t_k(j+1)$, where $t_0 \leqq t_k = t_k(0) \leqq t_k(j-1) < t_k(j) \leqq t_{k+1}$ and $t_k(j+1) = t_k(j) + \Delta t$, $j = 1,\cdots,D$. Let $p_i(t_k(j))$ be independent of $\{w_i(t)\}$ and

$$p_i(t_k(j)) \sim N(\,0\ ,\, q_i\, c_{pi}/2\,) \tag{7}$$

By integrating Eq. 6 from $t_k(j)$ to $t_k(j+1)$, the transition of p_i during the time-interval are obtained as follows:

$$\underline{p}(t_k(j+1)) = M\,\underline{p}(t_k(j)) + \underline{W}(t_k(j)) \tag{8}$$

where $\underline{W}$ = N_p-dimensional column vector; and its i-th component is

$$W_i(t_k(j)) = c_{pi} \int_{t_k(j)}^{t_k(j+1)} \exp\{-c_{pi}\ (\ t_k(j+1) - \tau\ )\}\ q_i^{1/2}\, w_i(\tau)\ d\tau \tag{9}$$

and

$$M = \begin{bmatrix} \xi_1 & & 0 \\ & \ddots & \\ 0 & & \xi_{N_p} \end{bmatrix} ; \quad \xi_i = \exp(-c_{pi}\Delta t) \tag{10}$$

Then

$$E\{\underline{W}(t_k(j))\} = 0 ;$$

$$E\{\underline{W}(t_k(j))\,\underline{W}^T(t_k(j))\} = \begin{bmatrix} s_1^2 & & 0 \\ & \ddots & \\ 0 & & s_{N_p}^2 \end{bmatrix} = Q_{tk(j)} ; \tag{11}$$

$$s_i^2 = c_{pi}\, q_i\, (1-\xi^2)/2$$

Treatment of Rainfall in Stochastic Runoff Prediction

Eq. 5 represents the state transition in the basin. Note that the rainfall, which is the system input, contains observation error because of both its spatial-temporal variation and many immature points left in the technology of rainfall observation. Hence, it must be treated as stochastic variable. In most cases, however, the observed rainfall is assumed not to contain any error; in part because it is difficult, in the state of the art, to evaluate the rainfall observation error (or the estimation error of the areal rainfall in each sub-basin), in part also because few methods of rainfall-runoff analysis which treats rainfall as stochastic input have been considered.

In this paper, we suppose that the observed rainfall does not contain any error and the error of the rainfall prediction from the observation is treated as uncertain. That is to say, it is assumed that the observed rainfall does not contain any error and the predicted value of rainfall contains some error.

Supposing the present time is t_k, the prediction of rainfall from t_k to t_{k+1} is necessary to predict the discharge at t_{k+1} (in Kimura's storage function method, as stated later, when the lead time is shorter than the lag time T_L, the runoff prediction can be executed only with the observed rainfall).

What is complicated in the case of a basin consisting of several sub-basins is that the correlation between predicted values of rainfall in each sub-basin must be taken into account. Furthurmore, when predicting the discharge from t_{k+1} until t_{k+L} ($L \geqq 2$), we must consider not only spatial correlation but also temporal correlation.

In the case where there are J sub-basins, to predict the discharge $L\,\Delta T$ hours hence (ΔT is the constant time-interval of discharge observation), $N_r = J \times L$ predicted rainfalls and the covariance of prediction errors are needed.

Let $\hat{\underline{r}}_j$ be a rainfall prediction vector of the j-th sub-basin. Then

$$\hat{\underline{r}}_j = [\, r_j^1 \,,\, \cdots \,,\, r_j^L \,]^T \quad , \quad j=1,\cdots,J$$

where $r_j^m = m\,\Delta T$-hour-ahead prediction of rainfall in the j-th sub-basin. Let $\hat{\underline{r}}$ be a rainfall prediction vector of the whole basin;

$$\hat{\underline{r}} = [\, \hat{\underline{r}}_1^T \,,\, \cdots \,,\, \hat{\underline{r}}_J^T \,]^T$$

where $\hat{\underline{r}} = N_r$-dimensional column vector. Let $\underline{P}_r$ be the covariance matrix of the prediction error ($N_r \times N_r$).

Though $\underline{r}$ is treated as deterministic value in Eq. 5, when the state transition is predicted, the future rainfall $\underline{r}$ is treated as stochastic. In the prediction step, therefore, Eq. 5 is re-formed into a stochastic differential equation with

$N_{xpr} = N_x + N_p + N_r$ stochastic variables:

$$\dot{\underline{x}} = \underline{f}\,(\,\underline{x}\,,\,\underline{r}\,,\,t\,) + G\,\underline{p}\,, \quad t_k \leqq t \leqq t_{k+L} \tag{12}$$

Local Linearization and Discretization of State Equation

The state equation (Eq. 12) is a stochastic differential equation. Define the augmented state vector

$$\underline{X} = [\ \underline{x}^T\ \ \underline{p}^T\ \ \underline{\hat{r}}^T\ ]^T$$

Then, instead of Eq. 12 the augmented dynamical system is

$$\dot{\underline{X}} = \underline{f}^*(\,\underline{X}\,,\,t\,) + G^*\underline{w}\,, \quad t_k \leqq t \leqq t_{k+L} \tag{13}$$

with deterministic input $\underline{r}$, where $\underline{w} = N_p$-dimensional noise vector whose components are w_i $(i=1,\cdots,N_p)$ given in Eq. 6; and $G^* = N_{xpr} \times N_p$-matrix:

$$G^* = \begin{bmatrix} 0 \\ \hline c_{p1}\sqrt{q_1} & & 0 \\ & \ddots & \\ 0 & & c_{pN_p}\sqrt{q_{N_p}} \\ \hline 0 \end{bmatrix} \begin{matrix} \} N_x\text{-dimensional} \\ \\ \\ \\ \\ \} N_r\text{-dimensional} \end{matrix}$$

Now, at time t_k, we consider how to obtain the stochastic distribution of $\underline{X}$ at t_{k+1}, namely, how to predict the states. Because $\underline{f}^*$ in Eq. 13 is in general a nonlinear function, it is impossible to obtain the transition of the stochastic distribution of the state vector $\underline{X}$ strictly.

Supposing $\underline{X}$ is approximately Gaussian, and linearizing and discretizing $\underline{f}^*$ locally, we successively obtain the transition of the mean $\hat{\underline{X}}$ and covariance P_X of $\underline{X}$.

We first linearize Eq. 13 at time $t_k(j)$:

$$\dot{\underline{X}} = A\,\underline{X} + \underline{b} \tag{14}$$

Subsequently, by integrating Eq. 14 over the proper time-interval Δt, $\underline{X}$ at time $t_k(j+1) = t_k(j) + \Delta t$ is given by the following discretized equation

$$\underline{X}(t_k(j+1)) = F\,\underline{X}(\,t_k(j)) + \underline{d} \tag{15}$$

In such a manner the state equation can be solved from t_k to t_{k+1}.

To linearize Eq. 13 into the form of Eq. 14, either the Taylor series expansion method or the statistical linearization method (5) is often used; the latter is said to be generally more accurate than the former. To discretize Eq. 14 into the form of Eq. 15, for example, the Padé approximation for e^{tA} can be applied (20); it provides

$$F = [\ I - A\frac{\Delta t}{2} + A^2\frac{\Delta t^2}{12}\ ]^{-1}[\ I + A\frac{\Delta t}{2} + A^2\frac{\Delta t^2}{12}\ ]$$

$$\underline{d} = [\ I - A\frac{\Delta t}{2} + A^2\frac{\Delta t^2}{12}\ ]^{-1}\Delta t\underline{b}$$

where I = identity matrix.

From Eq. 9, Eq. 13 consequently becomes linear discrete equation as

$$\underline{X}(t_k(j+1)) = F\,\underline{X}(\,t_k(j)) + \underline{d} + B\,\underline{W}(\,t_k(j)) \tag{16}$$

where $B = N_{xpr} \times N_p$-matrix:

$$B = \begin{bmatrix} 0 \\ I \\ 0 \end{bmatrix} \begin{matrix} \} \; N_x\text{-dimensional} \\ \\ \} \; N_r\text{-dimensional} \end{matrix} \qquad (\text{I is a } N_p \times N_p\text{-identity matrix})$$

From Eq. 11,

$$\begin{aligned} \hat{\underline{X}}(t_k(j+1)) &= E\{\underline{X}(t_k(j+1))\} = F\hat{\underline{X}}(t_k(j)) + \underline{d} \\ P_{t_k}(j+1) &= \text{Cov}\{\underline{X}(t_k(j+1))\} = FP_{t_k}(j)F^T + BQ_{t_k}(j)B^T \end{aligned} \tag{17}$$

Correction of the State Estimate Using Observed Rainfall and Discharge

In this section, we express a state vector as $\underline{x}$; it contains both N_x system states and N_p system noises. Then its dimension is $N_{xp}=N_x+N_p$.

Suppose that given the observed data up to time t_k, the filtered state estimates $\hat{\underline{x}}(t_k|t_k)$ and $P(t_k|t_k)$, the covariance matrix of the error in $\hat{\underline{x}}(t_k|t_k)$, have been stored. At time t_{k+1}, $\underline{r}_{k+1}=[r_1, \cdots, r_J]^T$, observed rainfall intensities in J sub-basins from t_k to t_{k+1}, and $\underline{y}(t_{k+1}) = [y_1, \cdots, y_{Ny}]^T$, observed discharge at N_y gauging stations at t_{k+1}, are obtained. By using these observed data, the state estimates are corrected in the following manner.

We first obtain the state transition from t_k to t_{k+1}. Namely, we integrate the state equation by starting with the initial values $\hat{\underline{x}}(t_k|t_k)$ and $P(t_k|t_k)$ and giving the observed rainfall intensities $\underline{r}_{k+1}$ as input in the same manner as described in the previous section (notice that $\underline{X}$ must be reduced its dimension and replaced by N_{xp}-vector $\underline{x}$ since $\hat{\underline{r}}$ does not need to be considered). Thus, we obtain the corrected state estimates. We express these new estimates as $\tilde{\underline{x}}_0$, $\tilde{P}_0$ (the meaning of the subscript $_0$ (zero) will be specified later).

Using the observed discharge $\underline{y}$, we second estimate the state by means of Kalman filter. Now we are treating N_{xp}-dimensional state vector, we rewrite the observation equation (Eq.4) as

$$\underline{y}(t_k) = \underline{g}^* (\underline{x} , t_k) + \underline{v}(t_k)$$

where $\underline{g}^*$ is in general a nonlinear function. In most cases $\underline{v}(t_k)$ is assumed to be steady or independent of $\underline{x}(t_k)$, and we have ever investigated the case that these are not independent, namely, $\underline{v}$ and $\underline{x}$ are correlated (see Takasao et al. (15,16)). As one method of considering the dependence, $\underline{v}$ is assumed to be a function of the state $\underline{x}$ and the noise $\underline{e}$ which is independent of $\underline{x}$. Linearizing the above equation,

$$\underline{y}(t_k) = H\underline{x}(t_k) + \underline{c} + Z\underline{e}(t_k) \tag{18}$$

where

$$H = \begin{bmatrix} \underline{h}_1 \\ \vdots \\ \underline{h}_{N_y} \end{bmatrix}; \quad \underline{c} = \begin{bmatrix} c_1 \\ \vdots \\ c_{N_y} \end{bmatrix}; \quad \underline{e}(t_k) = \begin{bmatrix} e_1(t_k) \\ \vdots \\ e_S(t_k) \end{bmatrix} \tag{19}$$

$H = N_y \times N_{xp}$-matrix; $\underline{h}_i$ $(i=1, \cdots, N_y)$ = N_{xp}-dimensional row vector; $\underline{c}$ = constant (N_y-dimensional column vector) obtained by the linearization; $\underline{e}(t_k)$ = S-dimensional noise vector with zero-mean and covariance R_k; and $Z = N_y \times S$-matrix.

If some missing data are included in the N_y observed discharges, we cannot filter using observation vector $\underline{y}(t_{k+1})$. Then, making use of $y_i(t_{k+1})$ $(i=1, \cdots, N_y)$ one by one, we estimate the state; and missing observed value is ignored.

Considering that the a priori (that is, before making use of the observed discharges) estimate is $\tilde{\underline{x}}_0$ with $\tilde{P}_0$, we propose the following filter algorithm:

1) Set n=1;
2) Compute the Kalman gain matrix

$$K_n = \tilde{P}_{n-1}\underline{h}_n^T[\underline{h}_n\tilde{P}_{n-1}\underline{h}_n^T + ZR_kZ^T]^{-1}; \tag{20}$$

3) Process the n-th observation $y_n(t_{k+1})$, then updated state estimate is given as

$$\begin{aligned} \tilde{\underline{x}}_n &= \tilde{\underline{x}}_{n-1} + K_n[y_n(t_{k+1}) - \underline{h}_n\tilde{\underline{x}}_{n-1}] \\ \tilde{P}_n &= [I - K_n\underline{h}_n]\tilde{P}_{n-1} \end{aligned} \tag{21}$$

provided, however, that when $y_n(t_{k+1})$ is missing value,

$$\tilde{\underline{x}}_n = \tilde{\underline{x}}_{n-1}\ ,\ \tilde{P}_n = \tilde{P}_{n-1}\ ; \tag{22}$$

4) If $n<N_y$, set n=n+1 and go to Step 2); otherwise,

$$\begin{aligned} \tilde{\underline{x}}(t_{k+1}|t_{k+1}) &= \tilde{\underline{x}}_n \\ \tilde{P}(t_{k+1}|t_{k+1}) &= \tilde{P}_n \end{aligned} \tag{23}$$

and finish this algorithm.

STATE-SPACE REPRESENTATION OF THE EXISTING FLOOD PREDICTION MODEL IN THE YURA RIVER BASIN

Outline of the Yura Basin and the Existing Model

The Yura river, whose stream length is 146 km, flows into the Japan sea; the basin lies in the north of Kinki district and its area is 1 882 km^2 (Fig. 1).

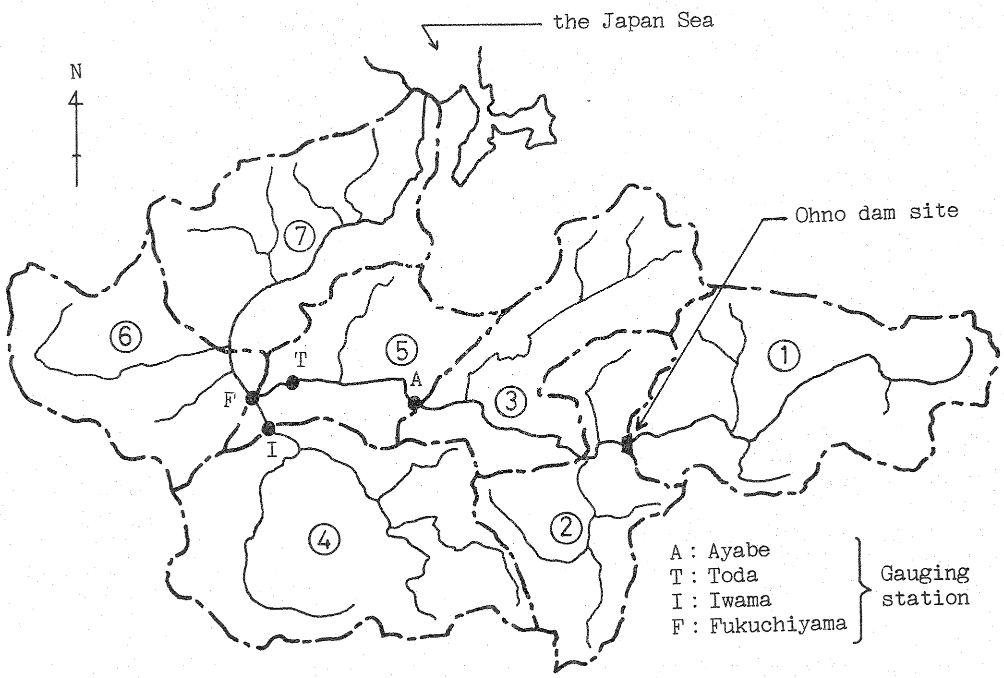


Fig. 1 The Yura River Basin (1882 km^2)

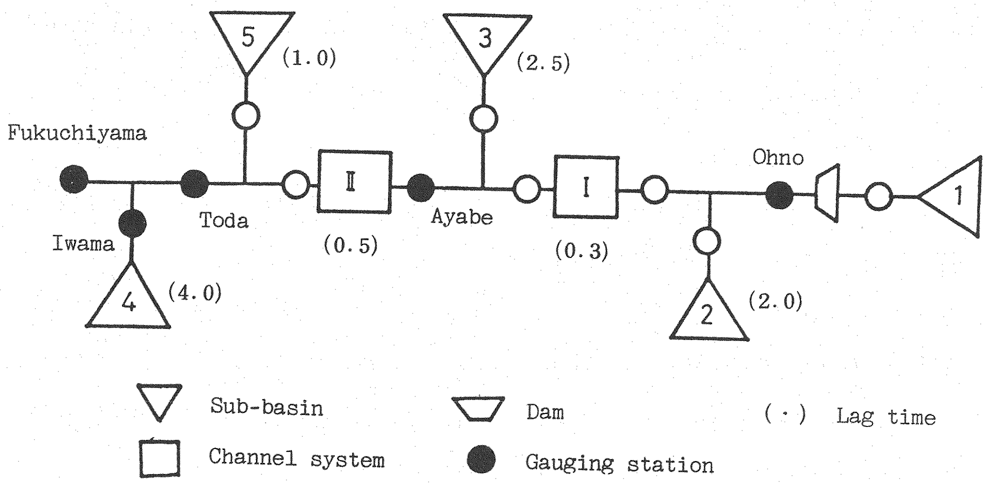


Fig. 2 Schematic Diagram of the Existing Flood Prediction Model of the Yura River

The existing model for predicting flood at Fukuchiyama consists of seven sub-systems, namely, five sub-basins and two channel systems (Fig. 2, (12,18)); and the runoff calculation in each sub-system is carried out by Kimura's storage function method (KSFM) (10). The basic formula of KSFM is:

for each sub-basin

$$s(t) = K\, q(t)^P \tag{24}$$

$$\dot{s}(t) = r_e(t-T_L) - q(t) \tag{25}$$

where $s(t)$ = water storage height (in mm) in the sub-basin at time t; $q(t)$ = direct runoff height (in mm/hr) (the baseflow is assumed to be zero in the Yura basin); T_L, K, and P = constants and T_L is called "lag time"; and r_e = effective rainfall intensity (in mm/hr) given as

$$r_e(t) = \begin{cases} f_1\, r(t) & (R_c(t) \leqq R_{sa}) \\ r(t) & (R_c(t) > R_{sa}) \end{cases} \tag{26}$$

where r = observed rainfall intensity (in mm/hr); R_c = cumulative rainfall (in mm); and R_{sa} ($\geqq 0$) and f_1 ($0<f_1<1$) are constants;

for each channel

$$s(t) = K\, q(t)^P$$
$$\dot{s}(t) = i_c(t-T_L) - q(t) \tag{27}$$

where $s(t)$ = water storage height of channel (in mm); $i_c(t)$ = inflow height (in mm/hr); and $q(t)$ is runoff height (in mm/hr). Water storage height of channel means the amount of channel water storage devided by the total area A (in km^2) upper than the channel; $i_c(t)$ ($q(t)$) is the amount of inflow (discharge) devided by A/3.6.

Table 1 shows the parameters of the model of each sub-systems. Note that in the existing model the runoff calculation of the Ohno sub-basin is not carried out, and the inflow to Channel-I from the Ohno dam is given every hour as the dam outflow.

State-Space Representation

The water storage of each sub-system can be regarded as the state. As the

Table 1 Parameters of the Existing Flood Prediction Model of the Yura River

Sub-systems		Area (km^2)	K	P	T_L (hr)	R_{sa} (mm)
Sub-basin	1	350	-	-	-	-
	2	220	13	0.65	2.0	80
	3	240	30	0.65	2.5	80
	4	370	22	0.65	4.0	80
	5	170	20	0.65	1.0	80
Channel	I	-	9	0.6	0.3	-
	II	-	23	0.6	0.5	-

Ohno sub-basin is being excluded, the existing model has four sub-basins and two channels.

Synthesizing the existing model, we represent it as a state-space model with 6-dimensional state vector.

As the lag time of sub-systems are different, the synthesis is a little troublesome. Let the time of each sub-system be shifted backward as much as the total time lag between the sub-system and the Fukuchiyama base station.

In accordance with the quantitative continuity of water and the interconnection among the sub-systems, the equations of water storage transition during $t_k \leqq t < t_{k+1}$ are:

in the j-th sub-basin (j=2,$\cdots$,5),

$$\dot{s}_j = r_{ej}(t-T_L) - q_j = r_{ej}(t-T_L) - (s_j/K_j)^{1/P_j} \tag{28}$$

where s_j = water storage of the j-th sub-basin; r_{ej} = effective rainfall intensity in the j-th sub-basin; q_j = runoff height from the j-th sub-basin; and K_j, P_j = constants;

in Channels I and II,

$$\begin{aligned} \dot{s}_I &= \frac{3.6Q_1}{A_I} + \frac{A_2}{A_I}\left(\frac{s_2}{K_2}\right)^{1/P_2} - \left(\frac{s_I}{K_I}\right)^{1/P_I} \\ \dot{s}_{II} &= \frac{A_I}{A_{II}}\left(\frac{s_I}{K_I}\right)^{1/P_I} + \frac{A_3}{A_{II}}\left(\frac{s_3}{K_3}\right)^{1/P_3} - \left(\frac{s_{II}}{K_{II}}\right)^{1/P_{II}} \end{aligned} \tag{29}$$

where s_I, s_{II} = water storage heights of each channel (see Eq. 27); Q_1 = discharge (in m^3/sec) from the Ohno dam; K_I, K_{II}, P_I, P_{II} = constants; and A_j = area of the j-th sub-basin (in km^2), especially A_I and A_{II} denote the total areas of the basin upper than the channels I and II, respectively.

The discharges are obtained at four gauging stations;

$$\begin{aligned} Q_{AYA} &= \frac{A_3}{3.6}\left(\frac{s_3}{K_3}\right)^{1/P_3} + \frac{A_I}{3.6}\left(\frac{s_I}{K_I}\right)^{1/P_I} \\ Q_{TOD} &= \frac{A_5}{3.6}\left(\frac{s_5}{K_5}\right)^{1/P_5} + \frac{A_{II}}{3.6}\left(\frac{s_{II}}{K_{II}}\right)^{1/P_{II}} \\ Q_{IWA} &= \frac{A_4}{3.6}\left(\frac{s_4}{K_4}\right)^{1/P_4} \\ Q_{FUK} &= \frac{A_4}{3.6}\left(\frac{s_4}{K_4}\right)^{1/P_4} + \frac{A_5}{3.6}\left(\frac{s_5}{K_5}\right)^{1/P_5} + \frac{A_{II}}{3.6}\left(\frac{s_{II}}{K_{II}}\right)^{1/P_{II}} \end{aligned} \tag{30}$$

where Q_{AYA}, Q_{TOD}, Q_{IWA}, Q_{FUK} = runoff heights at Ayabe, Toda, Iwama, and Fukuchiyama, respectively.

After this, s_2, $\cdots$, s_5, s_I, s_{II} are replaced by x_1, $\cdots$, x_6, respectively; and Q_{AYA}, Q_{TOD}, Q_{IWA}, and Q_{FUK} are also replaced by y_1, $\cdots$, y_4.

Thus, the existing flood prediction model has just rewrited, as deterministic state-space model, into the form of Eqs. 1 and 2 by Eqs. 28-30.

Linearization and Discretization Method Used in This Study

Incorporating noises, we treat the existing model as a stochastic one.
To linearize it, in this study, we apply the statistical linearization method. The nonlinear terms appearing in the existing model are only in the form of $x^{1/P}$. Suppose x is approximately Gaussian, that is,

$$x \sim N(\hat{x}, \sigma^2)$$

Then $x^{1/P}$ is statistically linearized as

$$f(x) = x^{1/P} \cong ax + b \tag{31}$$

where

$$\begin{aligned} a &= [\, E\{xf(x)\} - \hat{x}E\{f(x)\}\,] / \sigma^2 \\ b &= E\{f(x)\} - a\hat{x} \end{aligned} \tag{32}$$

The expectations are

$$\begin{aligned} E\{f(x)\} &= \int_{-\infty}^{\infty} \mathrm{sgn}(x)\, |x|^{1/P} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\{-\frac{(x-\hat{x})^2}{2\sigma^2}\} dx \\ E\{xf(x)\} &= E\{x^{(1+P)/P}\} = \int_{-\infty}^{\infty} \mathrm{sgn}(x)\, |x|^{(1+P)/P} \frac{1}{\sqrt{2\pi}\,\sigma} \exp\{-\frac{(x-\hat{x})^2}{2\sigma^2}\} dx \end{aligned} \tag{33}$$

where the function $f(x)=x^{1/P}$ is extended for negative x by setting $f(x)=-|x|^{1/P}$. For the numerical integration, we apply Hermite-Gauss formura (see, for example, (1)).

As a simple example, consider only the Iwama sub-basin. The nonlinear augmented state equation is

$$\begin{bmatrix} \dot{x}_3 \\ \dot{p}_3 \end{bmatrix} = \begin{bmatrix} -\left(\frac{1}{K_4}\right)^{1/P_4} x_3{}^{1/P_4} + r_{e3} + p_3 \\ -c_{p_3}p_3 + c_{p_3}\sqrt{q_3}w_3 \end{bmatrix} \tag{34}$$

After the linearization, it becomes

$$\begin{bmatrix} \dot{x}_3 \\ \dot{p}_3 \end{bmatrix} = \begin{bmatrix} a^*x_3 + b^* + r_{e_3} + p_3 \\ -c_{p_3}p_3 + c_{p_3}\sqrt{q_3}w_3 \end{bmatrix}$$

$$= \begin{bmatrix} a^* & 1 \\ 0 & -c_{p_3} \end{bmatrix} \begin{bmatrix} x_3 \\ p_3 \end{bmatrix} + \begin{bmatrix} b^*+r_{e_3} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ c_{p_3}\sqrt{q_3} \end{bmatrix} w_3 \tag{35}$$

where

$$a^* = -\left(\frac{1}{K_4}\right)^{1/P_4} a$$

$$b^* = -\left(\frac{1}{K_4}\right)^{1/P_4} b$$

To discretize Eq. 35, we use the Padé approximation.

Prediction of System Inputs

The inputs of the state-space model (Eqs. 28-30) are r_{ej} (j=2, •••, 5) and Q_1. Let T_j be the amount of lag time between the j-th sub-basin and Fukuchiyama. When the runoff prediction lead time L is not greater than T_j, we do not need the prediction of the input of the j-th sub-basin for runoff prediction. If $L>T_j$, it is required.

Fig. 3 shows the relation among T_j, L, and the rainfall needed for the runoff prediction calculation. For example, it shows that in the Wachi sub-basin (the case j=2) the observed rainfall during the past three hours and the predicted rainfall during the future one hour are required to predict the discharge three hours hence. When the flood runoff is predicted in a stochastic manner, it is natural to treat the inputs also in a stochastic manner.

Let RPRE(k+m,j) be the m-hour-ahead prediction of rainfall in the j-th sub-basin. According to Fig. 3, we use RPRE(k+1,5), RPRE(k+2,5), RPRE(k+1,2) and the covariance matrix of their errors to predict the discharge three houres hence; then the dimension of the rainfall prediction vector becomes three. Similarly, in the prediction of the discharge four hours hence, it becomes six. Thus, the greater the lead time becomes, the greater dimension is required.

In this paper, we predict the rainfall by 3-hour moving average method. This simple method has been often used in the practical work of flood forecasting by the Ministry of Construction, for any special trouble has not occured in the conventional flood forecasting. At time k ,it gives

$$\hat{r}_{k+m} = (r_{k-2} + r_{k-1} + r_k) / 3 \qquad (36)$$

where $\hat{r}_{k+m}$ = the predicted value of the rainfall at time k+m; m = lead time (m=1, •••, L*); and r_k = observed ainfall at time k. We assume the maximum lead time of rainfall prediction L* is not greater than that of runoff prediction L.

Analysing the prediction error sequences obtained by the above method from the past data of rainfall about both each sub-basin and each lead time, we obtained the covariance matrix of the rainfall prediction error (Table 2). The data used consist of 25 records of rainfall in the flood period during 1962-1975; they have been arranged by Fukuchiyama Work Office, Kinki Regional Construction Bureau, Ministry of Construction (12).

The outflow of the Ohno dam as well as the rainfall must be predicted. Though

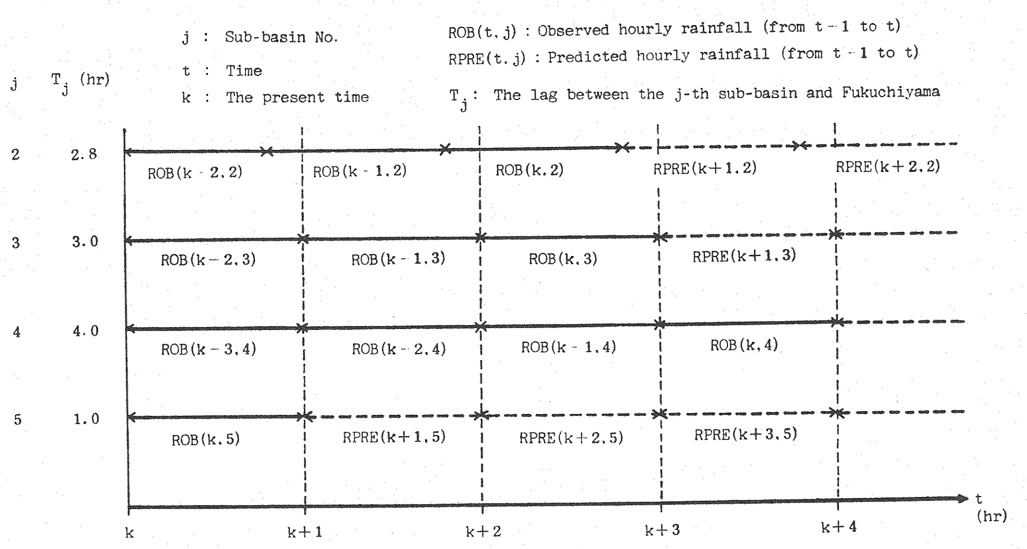


Fig. 3 Relation between the Time Lag and the Rainfall

Table 2 Covariance Matrix of the Rainfall Prediction Error

(i,j)†	(1,2)	(2,2)	(1,3)	(1,5)	(2,5)	(3,5)
(1,2)	13.8	10.5	10.6	6.0	4.8	3.7
(2,2)	10.5	17.9	8.4	7.0	8.9	7.3
(1,3)	10.6	8.4	14.9	8.8	6.1	5.2
(1,5)	6.0	7.0	8.8	13.5	9.9	7.6
(2,5)	4.8	8.9	6.1	9.9	18.1	13.7
(3,5)	3.7	7.3	5.2	7.6	13.7	20.5

† (i,j) denotes i-hour-ahead prediction of the rainfall in the j-th sub-basin

in this study we do not predict the outflow of the Ohno dam, its value and the prediction error variance can be obtained in the following procedure. Before predicting the outflow, we first predict the runoff of the Ohno sub-basin (namely, the inflow of dam) by rainfall prediction and rainfall-runoff model. Then we predict the outflow based on the dam operational rule. As for the covariance among the prediction error of the outflow and those of the rainfall in the other sub-basins, it must be obtained from the correlation among the rainfall prediction error of the Ohno sub-basin and those of the other sub-basins.

In this paper, however, we assume that the outflow can be predicted perfectly and treated as deterministic value; in part because the existing model does not take account of the rainfall-runoff model of the Ohno sub-basin, in part also because incorporating the dam operational rule into the model makes the prediction algorithm more complex.

Filtering and Prediction in the Basin

The state equation of a stochastic rainfall-runoff model is represented by Eq. 12; in the case of the existing model, $\underline{f}$ is given by Eqs. 28 and 29.

As noted above, when the lead time is greater than one hour, rainfall prediction is required. In this case, treating the future rainfall $\underline{r}$ as stochastic variable, we linearize Eq. 12 statistically to obtain an equation corresponding to Eq. 14 and discretize it to obtain an equation corresponding to Eq. 16. Then we estimate the state transition by Eq. 17.

When the lead time is not greater than one hour, rainfall prediction is not needed. In this case, we reduce the dimension to N_{xp}.

The observation equation is generally represented by Eq. 8; $\underline{g}$ is given as in Eq. 30.

Note that there is no lag among Toda, Iwama, and Fukuchiyama; on the other hand, there is 0.5-hour lag between Ayabe and Fukuchiyama. This makes the filtering and prediction algorithm (Eqs. 20-23) a little more complex. It comes to this that; after the third updating in the step 3) (namely, after using three observed discharges at Toda, Iwama, and Fukuchiyama), we start with $\tilde{x}_3$ and $\tilde{P}_3$, and obtain the state transition during $t_k \leqq t \leqq t_k+0.5$; replacing the old values of $\tilde{x}_3$ and $\tilde{P}_3$ with the solution obtained at $t=t_k+0.5$, and using the observed discharge at Ayabe and Eqs. 20 and 21, we obtain $\tilde{x}_4$ and $\tilde{P}_4$; starting with $\tilde{x}_4$ and $\tilde{P}_4$, we obtain the state transition and predict the state at t_{k+1}. If the discharge at Ayabe is missing, we omit the step in which we obtain $\tilde{x}_4$ and $\tilde{P}_4$.

On the Noise Statistics

When a stochastic state-space model and Kalman filter theory are used to filter and predict the system state, it is important to appreciate the noise statistics (in particular the covariance matrix); for, in the algorithm of Kalman filter, the system noise covariance matrix must be given at each time to obtain the covariance matrix of the error in 1-step-ahead estimate; and the observation noise covariance matrix also must be given at each time to compute the Kalman gain matrix.

Usually, the noise covariance matrices are fixed during the flood; in other words, the noises are assumed to be steady. This type of filter is known as "non-adaptive filter". However, the steadiness assumption is suspicious in such a phenomenon as flood runoff; so that some investigators apply "adaptive filter", which estimates the covariance matrix at every observation time. For instance, Hoshi et al. (8) applied it to the Iwaonai basin (331 km^2).

In this study, we deal with two different types of noise as follows:

a) The noise statistics do not change all over the period of flood;

b) The noise statistics depend on the state and thus change with time.

In other words, a) is steady noise and b) is unsteady noise.

We first consider the observation noise. Let $S = K$ in Eqs. 18 and 19. If the observation noise $\underline{v}$ is steady (in the case of a)), suppose Z=I (identity matrix) and the covariance matrix of $\underline{v}$ (that is, $\underline{e}$) is

$$\mathrm{Cov}\{\ \underline{v}(t_k)\ \} = Z R_k Z^T = R \text{ (const.)}$$

In the case of b), Z and R_k may be shaped in various forms; in this study, we suppose

$$Z = \begin{bmatrix} \underline{h}_1\underline{x} + c_1 & & 0 \\ & \ddots & \\ 0 & & \underline{h}_K\underline{x} + c_K \end{bmatrix} \tag{37}$$

and let $R_k = R$ (constant). Then

$$y_i(t_k) = (\ \underline{h}_i\underline{x}(t_k) + c_i\ )\ (\ 1 + e_i(t_k)\ )\ , \quad i=1,\cdots,K \tag{38}$$

This type of noise is called "multiplicative" (15,16). Suppose

$$R = \alpha^2 I \tag{39}$$

where I = identity matrix; and α (>0) is constant. Then all of the observed discharges are assumed to contain about $\alpha \times 100\,\%$ error. The observation noise covariance matrix becomes

$$\mathrm{Cov}\{\ \underline{v}(t_k)\ \} = Z R_k Z^T = \alpha^2 Z Z^T \tag{40}$$

Secondly, we consider the system noise.

The system noise is $B\underline{W}(t_k(j))$ of Eq. 16. The system noise covariance matrix $Qt_k(j)$ is given by Eq. 11. In the case of a), let c_{pi} and q_i be constant all over the period. In the case of b), in this study, we substitute $\beta X_i(t_k(j))$ $(\beta > 0)$ for $c_{pi}q_i$ in the right side of the third equation of Eq. 11. Then the variance of $W_i(t_k(j))$ is in propotion to $X_i(t_k(j))$. Suppose c_{pi} $(i=1,\cdots,N_p)$ are invariant all over the period, then

$$q_i = \beta X_i / c_{pi} \tag{41}$$

Thus q_i changes with time.

Since the state variables appearing in Eqs. 40 and 41 are not known, we replace them with their estimates to approximate $\mathrm{Cov}\{\underline{v}(t_k)\}$ and q_i.

Summarizing, a) is additive noise whose statistics are invariant all over the period of flood. b) is unsteady noise whose statistics depend on the states at each time; and the observation noise is so-called "multiplicative" noise.

Summary of the Stochastic Prediction Method in the Basin

We summarize the stochastic method of flood prediction in the Fukuchiyama basin as follows:

1) Give the initial state estimate and its error covariance matrix;
2) At time k, observe the rainfalls and the discharges;
3) Predict the outflow of the Ohno dam;
4) Using the observed discharge at Toda, Iwama, and Fukuchiyama at time k, obtain the estimate by filtering;
5) Predict the state transition until 0.5 hour hence;
6) Using the observed discharge at Ayabe and the estimate obtained in Step 5), filter the state;
7) Predict the state transition from 0.5 hour hence to one hour hence;
8) Store the 1-hour-ahead estimate and its error covariance matrix for the filtering at time k+1;
9) Predict the rainfalls and give the covariance matrix of the rainfall prediction error;
10) Incorporate the rainfall prediction vector into the state vector and predict the state from k+1 to k+L;
11) Output the predicted discharge at four gauging stations at each time from k+1 to k+L and their error variance (or standard deviation);
12) Finish this algorithm if danger of furthur flood have got out; otherwise, set $k = k + 1$ and go to Step 2).

RESULTS AND DISCUSSION

We applied the above method to the data of rainfall and discharge in the period when the flood had occurred in the Yura River.

In all the cases, we set the initial conditions as follows. The initial storage height of each sub-basin was assumed to be 0.0 mm; as for in each channel, we assumed the initial flow was equal to the initial outflow from the Ohno dam and obtained the channel water storages by the reverse calculation. All of the initial estimates of the system noises were 0.0. The error covariance matrix of these initial estimates was assumed to be diagonal; the variances of the storage heights were $(10.0\ \mathrm{mm})^2$ and those of noises were $c_{pi}q_i/2$ $(i=1,\cdots,6)$.

The computation time-interval $\Delta t = 0.1$ hour and the observation time-interval $\Delta T = 1.0$ hour.

On the Noise Statistics

We studied the two different types of noise, namely type a) and type b). Fig. 4 shows 1-hour-ahead prediction of the flood at Fukuchiyama on Sep. 14 in 1965. In the figure, the solid line is the observed hydrograph and the dashed line is the predicted mean; the shadowed portion shows the region where the prediction error at each time is within $\pm 1\sigma$ from the mean.

The noise variances were given as follows: In the case of a) (Fig.4(A)), they were obtained from the observed peak discharges of the past five floods. The observation noise variances were given corresponding amounts of 10 % of the average of five peak discharges by the reverse calculation. The system noise variances were given corresponding amounts of 10 % of the average of the peak storages; they were obtained from the peak discharges by the reverse calculation in which c_{pi}'s were fixed and q_i's were adjusted. We assumed they were invariant all over the period of flood.

In the case of b) (Fig. 4(B)), we assumed that the observed discharges and the states contain 10 % error and set $\alpha = 0.1$ and $\beta = 0.1$. From the comparison between (A) and (B), the result of Fig. 4(A) is not reasonable, because the prediction error variance during low flow period can be considered relatively small. It is reasonable to estimate the prediction error small during low flow period and large during high flow period as Fig. 4(B).

Considering the above results, we will assume that the noise statistics (variances) depend on the states and change with time after this.

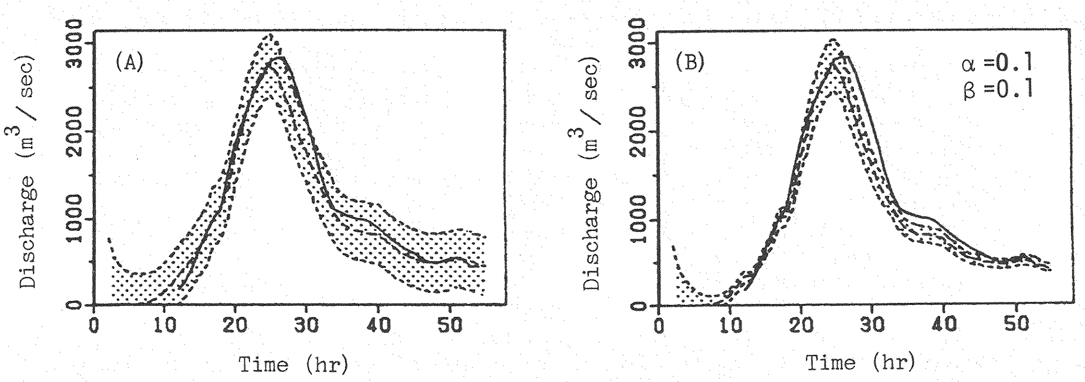


Fig. 4 Comparison of 1-Hour-Ahead Prediction at Fukuchiyama
((A)steady noise, (B)unsteady noise)

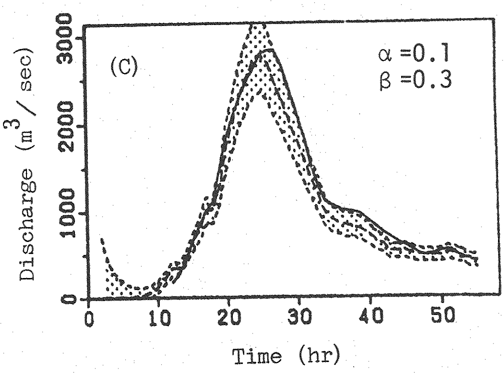
Effects of the Noise Variances on Prediction

As stated in the previous section, the variances of observation noise and system noise are defined by the values of α and β, respectively. Changing these values, we confirmed the effects of the noise variances on the results of prediction.

Fig. 5 shows 1-hour-ahead prediction (at Fukuchiyama) of the same flood as Fig. 4. Pay attention to the difference between the observed value and the predicted value after and quite near the peak, and 1σ prediction error. In the case of Fig. 5(C), the observation error was assumed to be equal to the case of Fig. 4 (B) (α=0.1) and the model error was greater ((B)β=0.1, (C)β=0.3). What is evident on comparing the two results is that the greater β is, the greater 1σ prediction error becomes. As for the predicted mean, however, it is closer to the observed value in the case of (C) than in the case of (B).

When the magnitude of the observation noise is relatively small, the observed output (discharge) at each time is relied much on and contributes so much to state estimation. This is characteristic of Kalman filter. Note that in this way, the prediction depends on not only the absolute magnitude but also the relative magnitude of the observation noise and the system noise.

If we make the observation noise smaller (α=0.01, Fig. 5(E)) than in the case of (C), the predicted value gets near the observed value and 1σ prediction



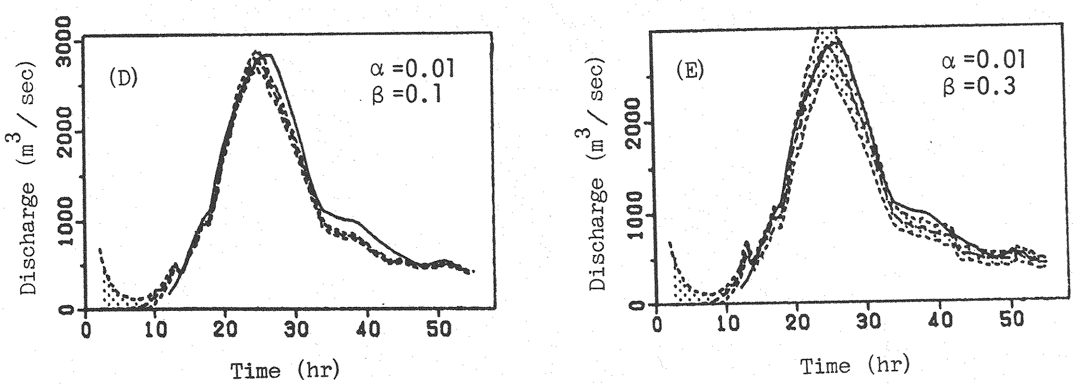


Fig. 5 1-Hour-Ahead Prediction at Fukuchiyama with Various Noise Statistics error becomes small. From the comparison between the results of (B) and (D) the same things can be ascertained.

Paying attention to the correction of the former prediction at the twelfth hour when the discharge was first observed, we recognize that the correction is sharper in the case of (D) and (E) than in the case of (B) and (C). This is also characteristic of Kalman filter.

In Fig. 4 and Fig. 5, all of the predictions after and quite near the peak are lower than the observed values. This shows that the original deterministic model (namely, the existing flood prediction model) do not describe the runoff phenomenon after and quite near the peak of this flood sufficiently.

The magnitude of the observation error varies with each gauging station substantially. Though we assumed 1% amount of observation error in the case of (D) and (E), such a high accuracy is not realistic for the stations of the large basin whose area is more than hundreds km^2. As far as the existing model is concerned, it is best to assume that the model error is 10-20%.

Under the circumstances, we suggest that α and β be 0.05-0.1 and 0.1-0.2,

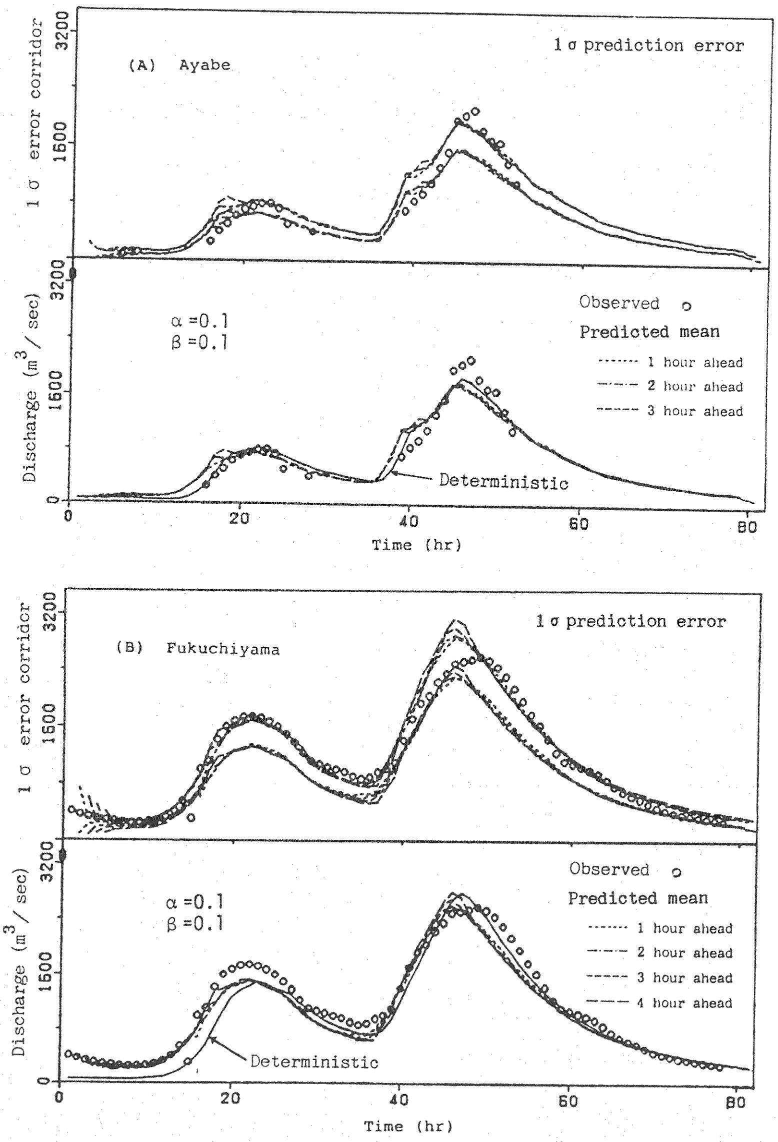


Fig. 6 Prediction at Ayabe and at Fukuchiyama (from 1- to 4-Hour-Ahead)

respectively.

Relation between Lead Time and Prediction Accuracy

Fig. 6 shows the prediction of the flood at Ayabe (A) and at Fukuchiyama (B) on Sep. 18 in 1965. The upper part of each figure shows 1σ prediction error and the observed hydrograph; the lower part shows the observed and predicted hydrographs and the deterministic off-line prediction (which does not use discharge data observed every hour).

The greater the lead time becomes, the greater 1σ prediction error becomes.

Form the comparison between the deterministic off-line prediction and our on-line prediction before the first peak in the lower part of Fig. 6(B); in spite of starting with the same initial conditions, the on-line method corrects error of prediction at once. It is important to make use of discharge data obtained every hour. Since the observation error during the high flow period is greater than the low flow period, the result of prediction is not so close to the observed hydrograph. We assumed that the observation contains about 10% error (α=0.1), so that we consider the prediction is good at least theoretically when the residuals of 1-hour-ahead prediction is about 10% of the observed discharge.

As for 3- and 4-hour-ahead predictions in Fig. 6, they fit the observed hydrograph well.

Apparently the persistence of the prediction residual is recognized in Fig. 4 -Fig. 6. This indicates the fact that these predictions are not optimal and the model identification is not satisfactory. It is important to improve the existing model. It is also important to make the rainfall and discharge observation more accurate. In addition to these points, the rainfall prediction with high accuracy is required to improve the accuracy of several-hour-ahead runoff prediction.

CONCLUSION

We developed a general theory of flood prediction in a basin consisting of several sub-basins, and applied it to the Yura River basin. Our method is useful as a stochastic method of real-time flood prediction.

Flood runoff is a phenomenon in which the magnitude of the states and outputs changes rapidly. When describing it by a stochastic state-space model, it is reasonable to consider that the noises incorporated into the model are unsteady. As one method of considering the unsteadiness, in this study, we assumed they are correlated with the state and outputs, and obtained satisfactory results.

ACKNOWLEDGEMENTS

We are grateful to Fukuchiyama Work Office, Kinki Regional Construction Bureau, Ministry of Construction for providing the data and to Mr. N. Tomisawa and Mr. Y. Kusuhashi for their help in the computation and typing.

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APPENDIX - NOTATION

The following symbols are used in this paper:

c_{pi}, q_i	= constants (see Eq. 6);
$N(m,s^2)$	= Gaussian distribution with mean m and variance s^2;
$\underline{p}$	= colored system noise vector (N_p-dimensional);
$\underline{r}$	= input vector (N_r-dimensional);
sgn(x)	= signum function;
t	= time (continuous);
t_k	= time (discrete), k=0,1,··· ;
T_L	= lag time defined in Kimura's storage function method;
$\underline{v}$	= observation noise vector (N_y-dimensional);
$\underline{w}$	= system noise vector (N_x-dimensional);
$\underline{x}$	= state vector (N_x-dimensional);
$\underline{X}$	= augmented state vector (($N_x+N_r+N_p$)-dimensional);
$\underline{y}$	= observation vector (N_y-dimensional);
Δt	= computation time-interval;
ΔT	= discharge observation time-interval;
α	= constant denoting the magnitude of observation noise;
β	= constant denoting the magnitude of system noise; and
$\delta(t)$	= Dirac delta function.