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SNOW DRAIN SYSTEM
- VELOCITY FORMULA FOR SNOW-LADEN WATER FLOW -

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SYNOPSIS

Based on hydraulic experiments, a velocity formula is established for snow-laden, turbulent water flow. The eddy viscosity is found to be a function of the friction velocity and the average values of the thickness of the water between floating snow lumps and walls, and between snow lumps and bottom of the drain. The formula is applicable to snow drains made of concrete with a smooth surface.

INTRODUCTION

Snow drain systems are used for removing snow from city areas in the Tohoku and Hokuriku Districts of Japan where heavy snowfall occurs. A snow drain is an open channel with a steady supply of water. Snow is deposited into the drain mainly by human power, and transported by water flow to its terminus, which is usually a natural river. The transport capacity of a snow drain system is estimated through empirical formulas [Japanese Association for Mechanization of Construction Work (1), Tanaka (3) and Toura (4)].

The authors have surveyed the dimensions and hydraulic characteristics of the drains in the city of Ojiya, Niigata Prefecture, as well as the method of snow removal operation [Sato and Shuto (2)]. The width of the drains in this city ranges from 0.3 m to 0.5 m, with a mode of 0.35 m. The depth of the supplied water alone ranges from 0.05 m to 0.35 m, with a mode of 0.15 m. The average flow velocity of the water alone is about 1.0 m/s.

It was observed that the snow removal operation was often interrupted by stagnation of the snow-laden water flow in the drains. Once stagnated, a long time is required to open a drain again and the efficiency of the snow removal operation is very much reduced.

There are four main causes of stagnation. The first cause is due to the local increase of friction produced by defects in the drain. The second is due to the incorrect deposition of new, dry, snow which easily adheres to the side wall of the drain. The third cause is due to incorrect operation of snow-removal machines. Snow masses larger than the dimensions of the drain may be deposited suddenly, resulting in the complete closure of a section. These three causes are most likely to occur at a deposition point or at a point in the drain with some defect.

On the other hand, the fourth cause of stagnation can occur anyplace. During snow removal operations, snow can be deposited at any point along the drain. Near the water supply outlet, only a small amount of snow is present in the flow, but the snow ratio continuously increases with distance from the water source. As the amount of snow increases, flow resistance increases and the flow velocity decreases, eventually resulting in stagnation along the entire length of the drain. Thus, the fourth cause is related to an upper limit to the transport capacity of a snow drain. The present paper aims to deduce the resistance law for snow-laden water flow, which may provide a basis for determining the critical condition for the fourth cause of stagnation.

MODEL OF THE VELOCITY DISTRIBUTION OF SNOW-LADEN WATER FLOW

Assumptions

(i) Snow lumps

During snow removal operations in the field, snow lumps of arbitrary size are deposited in the snow drains. There is, of course, a maximum size to these lumps, which is related to the size of the tools used and the limitations of manpower. If a very large snow lump is deposited, then complete closure of a section results. In drains with shallow water, snow lumps slide on the bottom of the drain. Under these conditions, snow removal operations can not be carried out efficiently and are often interrupted by stagnation. In order to obtain high efficiency, it is necessary to deposit snow so that it floats on the water and flows with less resistance. Even with this precaution, the available energy is rapidly consumed and with an increase in the amount of snow, the fourth kind of stagnation can occur.

Snow-laden water flow in a drain consists of two parts; the central part of the drain section (Region S) occupied mainly by snow lumps, and the remainder of the section (Region W) filled with water between the snow lumps and the walls and bottom of the drain. After deposition into the drain, snow lumps absorb water and their pores completely fill with water. In the following analysis, snow lumps are assumed to float on the water maintaining the same distance from the walls and bottom of the drain, respectively. There is no velocity difference in Region S, since snow lumps are solid and relative motion among them is comparatively small. The flow is, therefore, a kind of plug flow.

(ii) Shear stress and eddy viscosity in Region W

If a snow-laden flow is assumed to be uniform, the shear stress in Region W follows a linear law as shown in Fig.1. The eddy viscosity ε' is defined by, analogously to an ideal Bingham plastic fluid,

$$\tau - \tau_c = \rho_w \varepsilon' (du/dy) \qquad (1)$$

where y = the vertical coordinate taken positive upward with the origin on the bottom of the drain; τ = the shear stress at height y; τ_c = the shear stress acting on the lower surface of Region S; and ρ_w = the density of water. The eddy viscosity ε' is assumed to be independent of y, since the height of Region W is usually very small, but it depends on the friction velocity and the height of Region W.

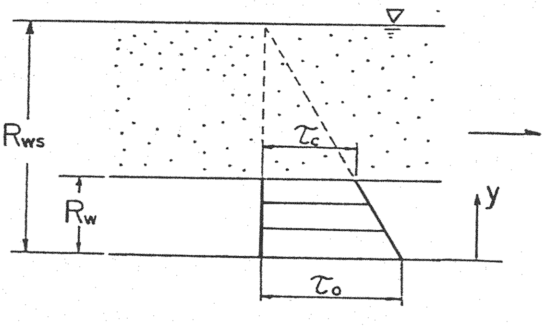


Fig. 1 Distribution of shear stress

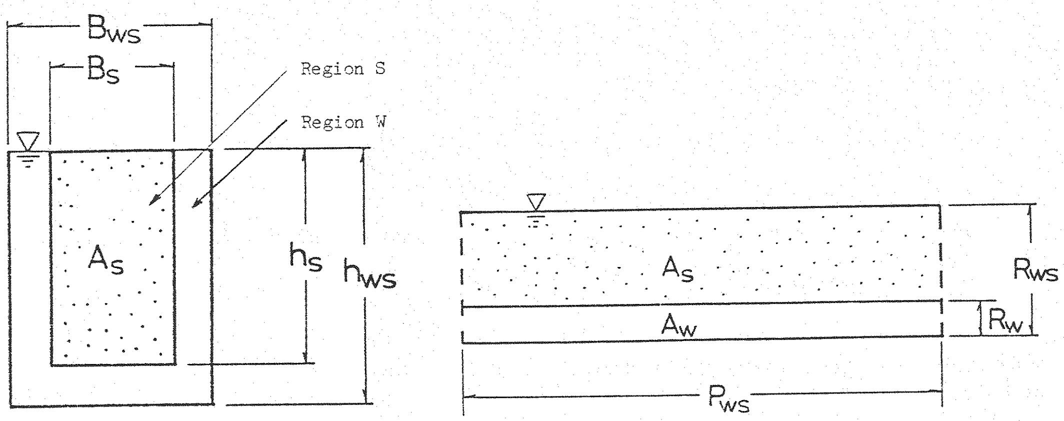
Mean Velocity of the Snow-laden Flow Averaged over a Section

(i) Hydraulic radius and height of Region W

The hydraulic radius R_{ws} for the flow is defined by

$$R_{ws} = A_{ws}/P_{ws} = B_{ws}\, h_{ws}/(2h_{ws} + B_{ws}) \qquad (2)$$

where P_{ws} = the wetted perimeter. Definitions of other quantities are shown in Figs.2(a) and (b).



(a) Actual section (b) Model section

Fig. 2 Definition sketch

The height, R_w, of Region W is defined for convenience as follows, by dividing the area of the region by the perimeter P_{ws}.

$$R_w = A_w/P_{ws} = (A_{ws} - A_s)/P_{ws} = (B_{ws}h_{ws} - B_s h_s)/P_{ws} \tag{3}$$

Neither A_w nor A_s can be easily measured. Therefore, they are estimated from the snow discharge Q_s and the total discharge of the snow-laden water flow Q_{ws}.

$$Q_{ws} = B_{ws}h_{ws}V_{ws} = A_{ws}V_{ws} \tag{4}$$

$$Q_s = B_s h_s V_s = A_s V_s \tag{5}$$

The ratio, K_s, of snow-lump-mixing in the volume is defined by

$$K_s = Q_s/Q_{ws} = A_s V_s/A_{ws}V_{ws} = A_s r_v/A_{ws} \tag{6}$$

where r_v = the ratio of the velocity of snow lumps to the section-averaged velocity of the snow-laden water flow. This ratio is expressed in terms of K_s alone, if the velocity profile is once established, as will be shown later.

With these quantities substituted into Eq. 3 , we obtain the following expression for R_w:

$$R_w = A_{ws}(1 - K_s/r_v)/P_{ws} = R_{ws}(1 - K_s/r_v) \tag{7}$$

(ii) Velocity profile

The linear shear stress distribution is given by

$$\tau = \tau_0 \, (R_{ws} - y)/R_{ws} \tag{8}$$

where τ_0 = the shear stress on the bottom of drain. Integration of Eq.1 with the boundary condition u = 0 at y = 0, after substituting Eq.8, yields the velocity profile u(y) in Region W.

$$u(y) = \int_0^u du = (1/\rho_w \varepsilon') \int_0^y (\tau - \tau_c) \, dy$$

$$= (R_{ws}/2\rho_w \varepsilon' \tau_0) \, \{(\tau_0 - \tau_c)^2 - (\tau_0 - \tau_c - \tau_0 y/R_{ws})^2\} \tag{9}$$

If the shear stress ratio defined by

$$r_\tau = \tau_c/\tau_0 = (R_{ws} - R_w)/R_{ws} = 1 - R_w/R_{ws} \quad (10)$$

is introduced, Eq. 9 becomes

$$u(y) = (\tau_0 R_{ws}/2\rho_w\varepsilon')\ \{(1 - r_\tau)^2 - (1 - r_\tau - y/R_{ws})^2\}, \text{ for } 0 \leq y \leq R_w \quad (11)$$

In Region S, for $y \geq R_w$, the velocity is the same as at $y = R_w$. Then,

$$u(y) = u(R_w) = V_s = (\tau_0 R_{ws}/2\rho_w\varepsilon')\ (1 - r_\tau)^2\ , \text{ for } y \geq R_w \quad (12)$$

(iii) Mean velocity

Averaged over a section, the mean velocity of the snow-laden water flow becomes

$$V_{ws} = (1/R_{ws})\ \{\int_0^{R_w} u(y)dy + (R_{ws} - R_w)u(R_w)\}$$

$$= (1/R_{ws})\ \{(\tau_0 R_{ws}^2/3\rho_w\varepsilon')(1 - r_\tau)^3 + (r_\tau \tau_0 R_{ws}^2/2\rho_w\varepsilon')(1 - r_\tau)^2\}$$

$$= (\tau_0 R_{ws}/6\rho_w\varepsilon')(r_\tau - 1)^2(r_\tau + 2) \quad (13)$$

A simpler form of Eq. 13 can be obtained as follows. If Eqs. 7 and 10 are used, the shear stress ratio is related to K_s and r_v as

$$r_\tau = \tau_c/\tau_0 = (R_{ws} - R_s)/R_{ws} = K_s/r_v \quad (14)$$

Another relationship is obtained between r_v and r_τ from Eqs. 12 and 13:

$$r_v = V_s/V_{ws} = \{\tau_0 R_{ws}(1 - r_\tau)^2/2\rho_w\varepsilon'\}/\{\tau_0 R_{ws}(r_\tau - 1)^2(r_\tau + 2)/6\rho_w\varepsilon'\}$$

$$= 3/(r_\tau + 2) \quad (15)$$

Combining the above two equations, the shear stress ratio can be expressed in terms of K_s as

$$r_\tau = 2K_s/(3 - K_s) \quad (16)$$

In addition, if the flow is assumed uniform, the bottom shear stress τ_0 is given by

$$\tau_0 = \rho_w S_{ws}\ g\ R_{ws}\ I = \rho_w u_{*ws}^2 \quad (17)$$

where S_{ws} = an apparent specific weight of the snow-laden water flow; I = slope of the channel; g = the gravitational acceleration; and u_{*ws} = the friction velocity.

Finally, the mean velocity is

$$V_{ws} = 9S_{ws}\ g\ R_{ws}^2\ I(1 - K_s)^2/\varepsilon'(3 - K_s)^3$$

$$= 9\ u_{*ws}^2 R_{ws}(1 - K_s)^2/\varepsilon'(3 - K_s)^3 \quad (18)$$

which contains a constant ε' which will be determined empirically in the following section.

EXPERIMENT

Hydraulic experiments were carried out in winter in an open channel installed outdoors along the side of the Shiraishi River, Hichigashuku, Miyagi Prefecture.

The channel, made of transparent acrylic resin, was 10 cm wide, 20 cm high and 10 m long. Channel slopes were 0.094 and 0.047.

Water was steadily supplied by pumping up from the Shiraishi River to a constant head tank connected with a triangular weir installed at the upstream end of the channel. Snow lumps were deposited by hand along a stretch from 1 m to 2.25 m downstream from the upstream end of the channel. Deposition continued for 90 seconds in each run. The snow discharge was determined from the shoveling frequency and the size of the snow lumps which had been measured before the experiment began.

Because usual methods of measurement were not applicable to this kind of experiment due to the floating snow lumps, two 16 mm cine cameras were used with the speed of 10 frames per second to measure the temporal variation of water depth. The mean water depth for the steady state was determined from these records. The section-averaged mean velocity V_{ws} was evaluated by

$$V_{ws} = Q_{ws}/B_{ws}h_{ws} \tag{19}$$

The total discharge Q_{ws} is the sum of the water discharge from the triangular weir Q_w and the discharge of ice Q_i, which is equal to the substantial part of the snow discharge Q_s. Therefore, it is calculated as follows,

$$Q_{ws} = Q_w + Q_i = Q_w + \rho_s Q_s/\rho_i \tag{20}$$

where ρ_s and ρ_i = the densities of snow and ice, respectively.

The specific weight S_{ws} is calculated by

$$S_{ws} = (\rho_s Q_s + \rho_w Q_w)/\rho_w Q_{ws} \tag{21}$$

Results of nineteen runs of the present experiments as well as results obtained at the National Research Center for Disaster Prevention are employed in the analysis. The channel used by the latter institution were made of wood and were 10 m long, 30 cm, 40 cm and 50 cm wide. Slopes were 0.043, 0.01 and 0.02. The channels were nearly of the same size as used in the city of Ojiya. Experimental ranges are tabulated in Table 1.

Table 1 Experimental conditions

	Authors	Reference [3]
ρ_s (kg/m^3)	200 - 230	450 - 540
Q_w (m^3/s)	$(4.4 - 8.1)\times10^{-3}$	$(2.4 - 6.6)\times10^{-2}$
Q_s (m^3/s)	$(5.9 - 24)\times10^{-4}$	$(2.1 - 6.6)\times10^{-2}$
Q_s/Q_w	$(7.0 - 48)\times10^{-2}$	$(2.1 - 11)\times10^{-1}$

EDDY VISCOSITY AND MEAN VELOCITY FORMULA

Values of the eddy viscosity left undetermined in Eq. 18 were evaluated from the experimental data for respective runs and they scattered between 10^{-5} and 2×10^{-4}(m^2/s). The maximum size of eddies responsible for the momentum exchange in Region W is restricted by the thickness of the water between snow lumps and the solid boundary of the drain. Therefore, the eddy viscosity may be proportional to the product of the friction velocity u_{*ws} and R_w. In Fig.3, ε' and the product $u_{*ws}R_w$ are plotted. Circles indicate the data of the acrylic resin channel and triangles and squares indicate the wooden channels. On neglecting a slight difference due to the difference in the material of the channels, the following relationship is obtained, which is shown by a straight line in Fig.3.

$$\varepsilon' = 1.9 \times 10^{-2}\, u_{*ws}\, R_w \tag{22}$$

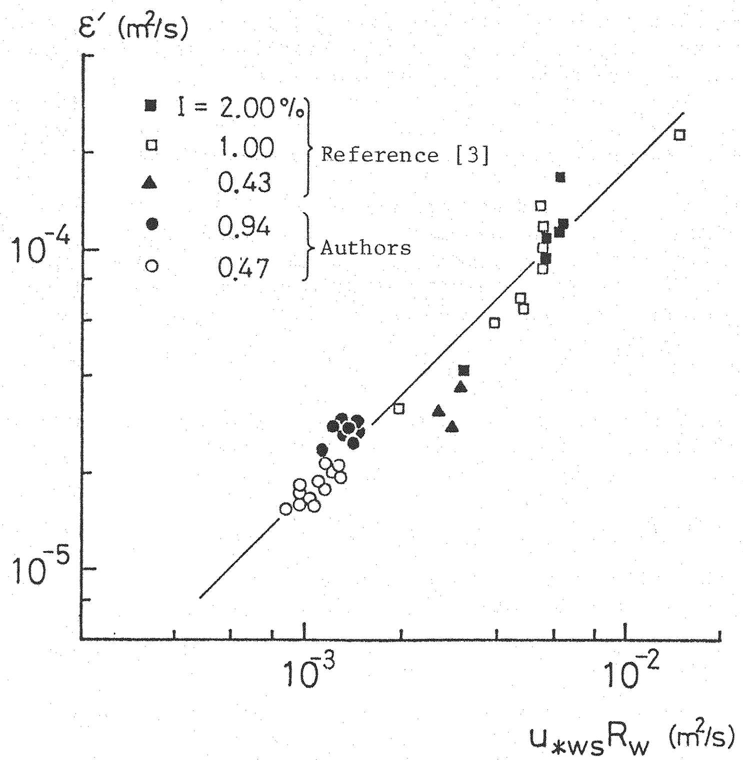


Fig. 3 Relationship between the eddy viscosity and $u_{*ws}R_w$

Substitution of this relationship into Eq.18 yields

$$V_{ws} = 474\ u_{*ws}\ R_{ws}(1 - K_s)^2/R_w(3 - K_s)^3 \qquad (23)$$

Finally, the section-averaged mean velocity of a uniform flow of the snow-laden water is expressed in the following simple form,

$$V_{ws} = 158\ (1 - K_s)\sqrt{g\ S_{ws}\ R_{ws}\ I}/(3 - K_s)^2 \qquad (24)$$

if the ratio R_{ws}/R_w is rewritten as

$$R_{ws}/R_w = (3 - K_s)/3(1 - K_s) \qquad (25)$$

by the use of Eqs. 7, 15 and 16.

Equation 24 means that Manning's roughness n is given in terms of the snow-lump-mixing ratio K_s as follows:

$$n = R_{ws}^{1/6}\ (3 - K_s)^2/158\sqrt{gS_{ws}}\ (1 - K_s) \qquad (26)$$

Since Manning's roughness takes almost similar values for acrylic resin, wood, and concrete with a smooth surface, the derived formula can be applied to practical cases. For an actual drain of standard size, 0.35 m wide and 0.20 m deep, Manning's roughness is obtained as 0.012 in the case of water alone, setting K_s = 0. This is an acceptable value for both experimental channels and for any existing drain in the field made of concrete with a smooth surface.

CONCLUSION

Field observations of snow removal operations in Ojiya revealed that there were four different causes of stagnation of snow-laden water flow. The uppermost transport efficiency of a snow drain is determined by the fourth cause of stagnation, which occurs when the available energy of flow is consumed by the friction of the snow-laden water flow. In this respect, it is essential in the design of a snow drain to know the friction law of the snow-laden water flow. Assuming a uniform flow, the mean velocity is given by Eq. 24. The eddy viscosity is experimentally determined and is given by Eq. 22 as a function of the friction velocity and the thickness of water between the solid boundary of the drain and the perimetric surface of snow lumps. Because the friction characteristics of the material comprising the drain do not explicitly appear in the formula, Eq. 24 is applicable to drains made of concrete with a smooth surface.

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APPENDIX - NOTATION

The following symbols are used in this paper:

A	= sectional area
B	= width
g	= gravitational acceleration
h	= depth or thickness
I	= slope of channel
K_s	= ratio of snow-lump-mixing in volume
n	= Manning's roughness
P	= wetted perimeter
Q	= discharge
R	= hydraulic radius
r_v	= ratio of the velocity of snow lumps to the section-averaged velocity of the snow-laden water flow
r_τ	= shear stress ratio
S_{ws}	= specific weight of the snow-laden water flow
u	= horizontal velocity
u_{*ws}	= friction velocity
V	= section-averaged velocity

y	=	vertical coordinate
ρ	=	density
τ, τ_c, τ_0	=	shear stresses at level y, acting on the lower surface of Region S and on the bottom of drain, respectively

subscript

i, s, w, ws	=	ice, snow, water, and water & snow, respectively