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TIME-LAG APPEARING IN UNSTEADY FLOW WITH SAND WAVES

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SYNOPSIS

The characteristics of unsteady flow with sand waves are very different from those of steady flow under equilibrium condition. Particularly, the bed geometry responds to the fluctuating flow discharge with some time-lag, and consequently the variations of dune dimensions and flow depth draw loops in the processes of increase and decrease of the flow discharge. In this paper, this kind of time-lag system is investigated in comparison with the so-called lag-distance in sand bed instability, and a linear analysis is tried to estimate the responding properties of dune dimensions and flow depth against sinusoidally fluctuating flow discharge. Moreover, in order to predict the time scale with respect to dune development, which is the most effective parameter on unsteady flow with dunes, a simple model is derived.

INTRODUCTION

Bed configurations, flow resistance and sediment transport rate in alluvial streams closely interact each other. The interrelations between them have been eagerly investigated by many researchers to date, and it has become possible to predict reasonably the stage of flow, bed forms and sediment discharge under the condition of fully developed sand waves for laboratory scale.

When the flow is unsteady as seen in floods of natural rivers, however, bed forms usually do not respond uniquely to the change of flow discharge as shown in Fig. 1 (1). This may be caused by the fact that there exists a time-lag between sand wave formation and flow discharge, and in natural rivers the fully developed sand waves can rarely be observed due to the unsteadiness of flow discharge. Only a few researches have been done for unsteady behaviors of flow over alluvial bed with sand waves. The non-equilibrium state of alluvial process is characterized by the time-lag due to sand wave deformation such as growth and decay. Particularly, in lower regime where dunes are dominantly formed, the flow resistance and the sediment transport rate are seriously influenced by bed

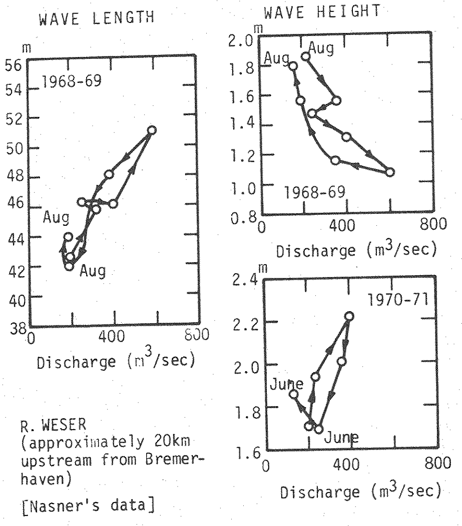

Fig. 1 Examples of observations in rivers (1)

forms. Herein, the characteristics of lag system due to sand wave deformation in lower regime will be investigated under an idealized condition.

Because there are various kinds of patterns in unsteady flow, and their generalized treatment is difficult, "unsteady uniform flow" is assumed here, and the essential properties of the lag system of alluvial streams may still be preserved. In fact, the propagation celerity of flow discharge within the designated region of a stream may be sufficiently faster than the deformation process rate of sand waves caused by the change of flow discharge.

If it takes much time for sand waves to be deformed, it is easily seen that the change of their dimensions against the flow discharge must show a hysteresis curve, as well as the change of flow depth which is much affected by bed forms. When the properties of bed materials and the bed slope of the stream are given, the mean wave length and the height of fully developed dunes are uniquely related to the flow discharge as shown by broken curves in Figs. 2 and 3. However, if the flow discharge is fluctuating in the range shown in Figs. 2 and 3, the dune length or height takes two different values for any flow discharge rate.

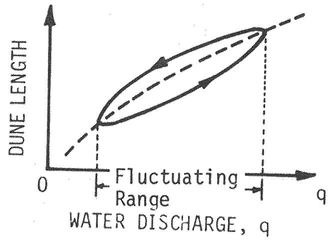

Fig. 2 The relation between dune length and flow discharge

One of the difficulties in quantitative approach to this kind of problem is a complicated interrelation among bed forms, sediment transport process and flow properties as shown in Fig. 4. In order to study this interrelation, the simplest case of "unsteady uniform flow" in alluvial streams, i.e. the process of sand

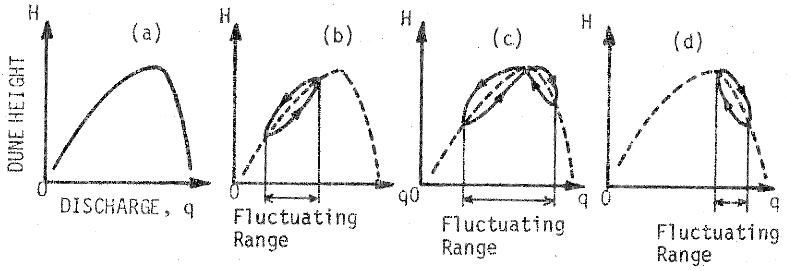

Fig. 3 The relation between dune height and flow discharge

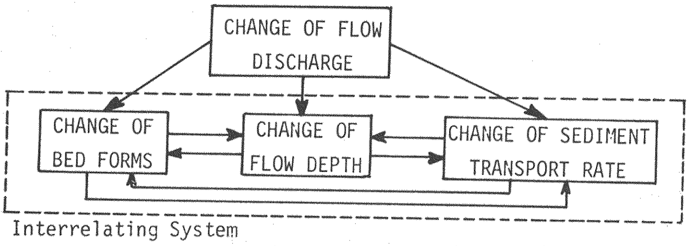

Fig. 4 Interrelating system in alluvial stream

wave development from an initially flattened bed, is considered here. The outline of the time variations of flow discharge, the dune dimensions, and the flow depth, or the total flow resistance ($\tau=\rho g h I$; ρ = mass density of fluid; g = gravitational acceleration; h = flow depth; and I = bed slope) is shown in Fig. 5. Since the "uniform flow" is considered and ρ, g and I are invariant values, the behavior of τ is identical to that of h and can be assumed to be uniform in the region of the alluvial stream under consideration. The variation of sediment transport rate may be small because the increase of the total bed shear stress is predominated by the form drag and the effective shear stress may only be slightly changed.

As clearly seen by the above simple case, a kind of lag is brought about by slow response of bed forms, while the flow responds quickly to the change of bed forms.

Though the interrelating system of alluvial streams is too complicated to solve directly, the study on the responsibilities of the individual subsystems will make possible to develop the model to describe unsteady phenomena in alluvial streams.

In the previous studies on the equilibrium state, it has been intended to estimate bed forms and their dimensions for the given flow discharge at first, and then, to estimate the effective bed shear stress for predicting the sediment transport rate, and to evaluate the form drag for estimating the total resistance and the flow depth. In this procedure, however, the lag behaviors between the dimensions of sand waves and flow discharge are not considered, though these are the most important properties of unsteady flow with sand waves.

If the slower response of bed forms to the change of flow discharge is noticed, the following approach will be favorably approved: the flow resistance and the sediment transport rate are assumed at first to be equal to those for equilibrium state, such as fully developed sand waves or an initially flattened bed, in order to estimate the deformation process of sand waves, and then based on the expected deformation process of sand waves the variation of flow depth and the sediment transport rate are estimated. If necessary, the time variation of the dimensions of sand waves or the deformation process of sand waves can be recalculated for the sake of a better accuracy. This type of approach may be much more effective to analyze unsteady behaviors of dunes.

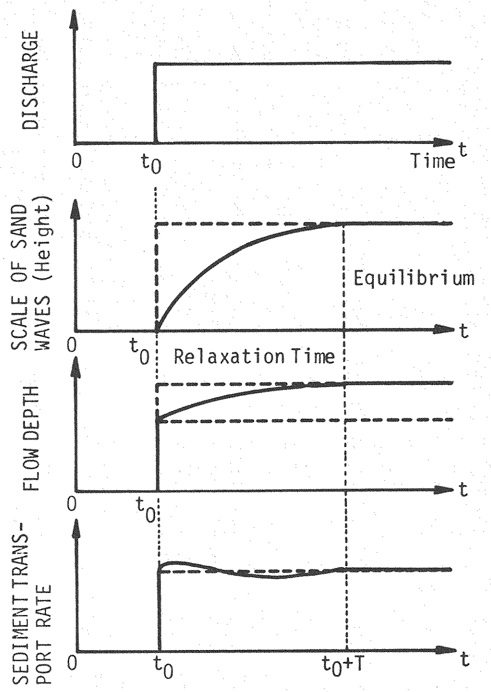

Fig. 5 The simplest case of unsteady behavior of alluvial stream

TIME-LAG DUE TO UNSTEADY BEHAVIOR OF DUNES AND LAG-DISTANCE DUE TO NONEQUILIBRIUM BED LOAD TRANSPORT

In alluvial streams, in addition to the aforementioned time-lag, there exists the lag-distance of sediment transport rate against flow properties represented by bed shear stress, and this lag is well known as a cause of sand bed instability (6, 10). The lag-distance of sediment transport plays a role in transmitting toward downstream the sediment transport state given at a point, while the time-lag noticed in this paper plays a role to transmit the situation of alluvial streams at an instance to that in the future. Therefore, between the both lags, a mathematical similarity in their descriptions and a physical analogy in the mechanism are expected. As for the lag-distance, its mechanism has been considerably well clarified (10, 11), and thus in this chapter, the mechanism of the time-lag due to sand wave deformation is inspected in association with the lag-distance.

In the bed load transport process just downstream of the fixed rough bed as considered in the authors' previous study (10), the spatial change of the sediment pick-up rate $p_s(x)$ and the deposit rate $p_d(x)$ can be expressed as shown in Fig. 6. Based on the Eulerian interpretation of the stochastic model for bed load motion (10), the following equation has been established:

$$p_d(x) = \int_{x_0}^{x} p_s(x') f_X(x-x') dx' \tag{1}$$

where $f_X(\xi)$ = probability density function of step length. The above equation expresses how the deposit rate responds to the spatial change of the pick-up rate.

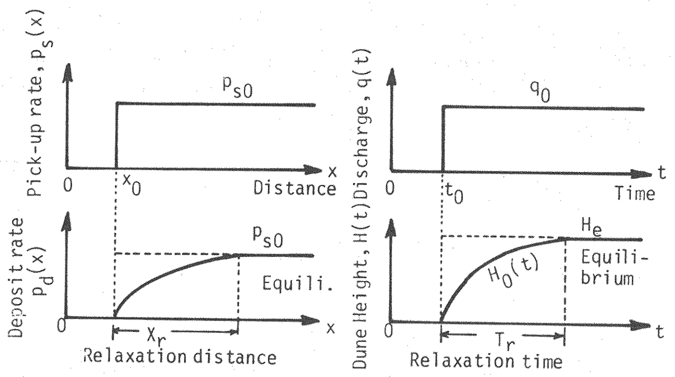

Fig. 6 Comparison between lag-distance of sediment transport and the time-lag of sand wave deformation

For the simplest case as shown in Fig. 6, the spatial variation of the pick-up rate can be expressed as follows:

$$p_s(x) = \begin{cases} p_{s0} & (x \geqq x_0) \\ 0 & (x < x_0) \end{cases} \tag{2}$$

Then, the deposit rate can be expressed as follows:

$$p_d(x) = p_{s0} \cdot \int_{x_0}^{x} f_X(x-x')\,dx' = p_{s0} \cdot F_X(x-x') \tag{3}$$

where $F_X(x)$ = distribution function of step length. Hence, by inspecting the response of $p_d(x)$ for the 'rectangular-shape' variation of pick-up rate as shown in Fig. 6, $f_X(\xi)$ can be determined as follows:

$$f_X(x) = \frac{d}{dx}[p_d(x)]/p_{s0} \tag{4}$$

which is obtained by differentiating Eq. 3 with respect to x.

By cinefilm motion analysis of bed load motion (10) in a laboratory flume, the spatial variation of deposit rate in this simplest case has been experimentally expressed as follows:

$$p_d(x) = p_{s0}[1-\exp(-x/\Lambda)] \tag{5}$$

where Λ = mean step length. And then,

$$f_X(x) = (1/\Lambda) \cdot \exp(-x/\Lambda) \tag{6}$$

Eq. 6 shows that step length follows an exponential distribution as clarified by the tracer tests (9). The probability density function of step length obtained by such a way is an impulse response between $p_s(x)$ and $p_d(x)$, and the response of $p_d(x)$ to various pattern of the spatial fluctuation of $p_s(x)$ can be obtained using the impulse response.

If the above idea is applied to the properties of unsteady response of alluvial bed channels, the response of the scale of dunes H(t) (represented by dune height here) to the temporal fluctuation of flow discharge q(t) (flow discharge per unit width) may be expressed by the following equation:

$$H(t) = \int_{t_0}^{t} q(t')\, g_T(t-t')\, dt' \tag{7}$$

where $g_T(\tau)$ = impulse response of the change of bed forms due to the change of flow discharge, and it may be estimated from the time variation of dune height from an initially flattened bed as shown in Fig. 5, as follows:

$$g_T(t) = \frac{1}{q_0}\frac{d}{dt}[H_0(t)] \tag{8}$$

According to the above arguments, the characteristics of sand wave development from an initially flattened bed should be made clear, which is a basis of the research for unsteady flow with sand waves, and recently the investigations on this subject have been eagerly conducted by some researchers (2, 3, 7, 16, 17).

PHASE SHIFT BETWEEN THE CHANGE OF FLOW DISCHARGE AND THAT OF DUNE DIMENSIONS - A LINEAR ANALYSIS -

As aforementioned, the lag system was described mathematically, though the time-lag has not been determined yet quantitatively. In case of spatial lag of sediment transport, the lag-distance has been determined quantitatively for a small amplitude sinusoidally wavy bed in instability analysis (10, 11). Herein, the phase shift between the change of flow discharge and that of dune dimension will be determined similarly for a small amplitude sinusoidally fluctuating flow discharge. Such an attempt has been already done by Fredsøe (4), and a resemble linear analysis will be conducted here, though some assumptions are modified in this analysis to improve the description of the phenomena.

The temporal fluctuation of flow discharge is assumed to be expressed by

$$q(t) = q_0(1+a_Q \sin\omega t) \tag{9}$$

where q_0 = averaged flow discharge; $a_Q q_0$ = amplitude of fluctuation of flow discharge; and ω = angular frequency of fluctuation of flow discharge ($\omega = 2\pi/T_f$; T_f = time period of flow discharge fluctuation).

The time variation of mean wave height of sand waves from an initially flattened bed under the condition of a constant flow discharge and a constant bed slope may be expressed by using an exponential function, which is not necessarily good approximation but for convenience' sake, as

$$H_0(t) = H_e[1-\exp(-t/T_{EX})] \tag{10}$$

where H_e = equilibrium height of fully developed sand waves for a given flow discharge, bed materials and bed slope; and T_{EX} = time scale of sand wave development in Eq. 10.

Using Eq. 8, the impulse response of the change of bed forms against the change of flow discharge is given by

$$g_T(t) = (H_e/q_0 T_{EX})\cdot\exp(-t/T_{EX}) \tag{11}$$

Moreover, substituting Eqs. 9 and 11 into Eq. 7, the time variation of dune height against the flow discharge fluctuation can be predicted as follows:

$$H(t) = \int_{-\infty}^{t} q_0(1+a_Q \sin\omega t')(H_e/q_0 T_{EX})\exp[-(t-t')/T_{EX}]\,dt'$$

$$= H_e + \frac{a_Q H_e}{[(1/T_{EX})^2+\omega^2]T_{EX}}[(1/T_{EX})\sin\omega t - \omega\cos\omega t] \tag{12}$$

Comparing the above equation with the following equation:

$$H(t) = H_e[1+a_H \sin(\omega t-\phi_H)] \tag{13}$$

where $a_H H_e$ = amplitude of the fluctuation of dune height; and ϕ_H = phase shift of the fluctuation of dune height against that of flow discharge, the following equations are obtained:

$$\sin\phi_H = \omega T_{EX}/\sqrt{1+(\omega T_{EX})^2} \quad ; \quad \cos\phi_H = 1/\sqrt{1+(\omega T_{EX})^2} \tag{14}$$

$$a_H/a_Q = 1/\sqrt{1+(\omega T_{EX})^2} \tag{15}$$

From these relations, it is seen that the phase shift belongs the first quadrant and that the amplitude and the phase shift of dune dimensions are determined by the two time scales, i.e. the time required for dunes to grow up to the fully developed state and the time scale of flow discharge fluctuation.

Next, the variation of flow depth will be considered. When the flow discharge and the dune height have varied from q_0 and H_e by δq and δH (very small quantities) respectively, the change of flow depth δh can be expressed by

$$\delta h = \left(\frac{\partial h}{\partial q}\right)_0 \delta q + \left(\frac{\partial h}{\partial H}\right)_0 \delta H \tag{16}$$

where the subscript 0 represents the quantity for undisturbed equilibrium state for q_0. The first term of the above equation can be calculated without taking account of the change of dune height. Using the Chézy coefficient, C, h is expressed by

$$h = (1/C\sqrt{I})^{2/3} \cdot q^{2/3} \tag{17}$$

And, since C can be regarded as a constant in this calculation,

$$\left(\frac{\partial h}{\partial q}\right)_0 = \frac{2}{3} \cdot (h_0/q_0) \tag{18}$$

The second term of Eq. 16 is independent of the change of flow discharge and it is caused solely by the change of the form resistance τ_f. Then,

$$\left(\frac{\partial h}{\partial H}\right)_0 = \frac{1}{\rho g I}\left(\frac{\partial \tau_f}{\partial H}\right)_0 \tag{19}$$

The form resistance, τ_f, can be crudely estimated by Yalin's method (14) as follows:

$$\tau_f = \frac{1}{2}\rho g h_0 \frac{H^2}{h_0 L} \cdot \frac{U_0^2}{g h_0} \tag{20}$$

where H, L = equilibrium dune height and length for the condition of q_0; and $U_0 = q_0/h_0$, respectively. According to the experimental facts (2, 7, 17), the steepness of dunes is kept nearly constant during sand wave development, and then it is here assumed that

$$\theta \equiv H/L = \text{constant} \tag{21}$$

Thus, the following can be obtained:

$$\left(\frac{\partial h}{\partial H}\right)_0 = \frac{1}{2}\frac{\theta \rho U_0^2}{\rho g h_0 I} = \frac{1}{2}\theta \cdot (C/\sqrt{g})^2 \tag{22}$$

Consequently,

$$\delta h = \frac{2}{3}\frac{h_0}{q_0} \cdot \delta h + \frac{1}{2}\theta (C/\sqrt{g})^2 \cdot \delta H \tag{23}$$

The small quantities δq and δH are expressed by

$$\delta q = a_Q q_0 \sin\omega t \; ; \qquad \delta H = [a_Q H_e/\sqrt{1+(\omega T_{EX})^2}]\sin(\omega t - \phi_H) \tag{24}$$

Then, the amplitude and the phase shift of the fluctuation of flow depth, which are represented by a_h and ϕ_h, respectively, can be obtained by comparing with

$$\delta h = a_h \sin(\omega t - \phi_h) \tag{25}$$

Consequently, the following results have been obtained by using Eqs. 14 and 15.

$$\gamma_h \equiv (a_h/a_Q)^2$$

$$= \{\psi_u^2 H_e\theta/[2\sqrt{1+(\omega T_{EX})^2}]\}^2 + 2\psi_u^2 H_e\theta h_0/\{3[1+(\omega T_{EX})^2]\} + \frac{4}{9}h_0^2 \quad (26)$$

$$\sin\phi_h = \psi_u^2 H_e\theta\omega T_{EX}/\{2\gamma_h[1+(\omega T_{EX})^2]\} \quad (27)$$

$$\cos\phi_h = \{\psi_u^2 H_e\theta/2\gamma_h[1+(\omega T_{EX})^2]\} + \frac{2}{3}(h_0/\gamma_h) \quad (28)$$

where $\psi_u \equiv C/\sqrt{g}$ is termed velocity coefficient.

The results in this chapter indicated by Eqs. 14 and 15, and Eqs. 26∿28 are illustrated in Figs. 7 and 8, respectively.

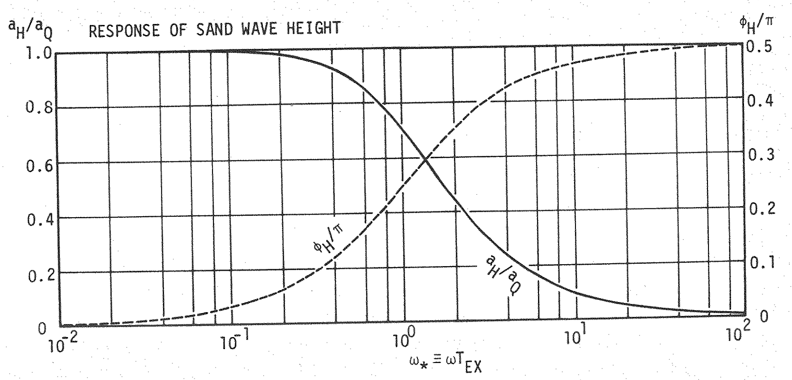

Fig. 7 The phase shift and the responding amplitude of dune height to fluctuation of flow discharge

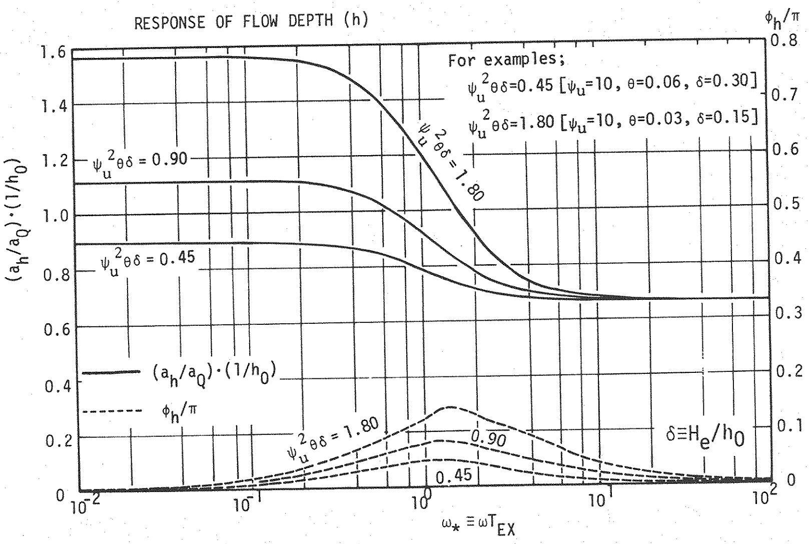

Fig. 8 The phase shift and the responding amplitude of flow depth to fluctuation of flow discharge

According to Figs. 7 and 8, the phase shifts and the amplitudes of the dune height and the flow depth are large when ωT_{EX} is about unity. In other words, the most interesting phenomena may be observed if the time scale of flow discharge fluctuation is comparable to that of dune development. Then, it is important to estimate the time required for dune development.

Additionally, when the amplitudes and the phase shifts of the fluctuating components of two variables, X(t) and Y(t), are expressed by

$$X(t) = a_X \cdot \sin\omega t$$
$$Y(t) = a_Y \cdot \sin(\omega t - \phi_{YX}) \quad (29)$$

The following relation can be obtained by eliminating the variable t.

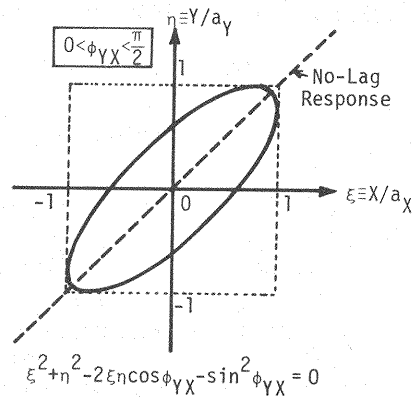

Fig. 9 A simple model of hysteresis appearing

$$(X^2/a_X^2) + (Y^2/a_Y^2) - (2XY/a_X a_Y)\cos\phi_{YX} - \sin^2\phi_{YX} = 0 \quad (30)$$

This equation expresses a loop of an ellips as shown in Fig. 9.

TIME REQUIRED FOR DUNES TO GROW UP TO FULLY DEVELOPED STATE

In the case of dunes or ripples, sands are eroded on the upstream side slope of a sand wave and deposited on the lee side slope of the same wave. Thus, individual waves migrate downstream without changing their dimensions. The dimensions of sand waves are, however, widely distributed, and the migration speeds of individual waves are different from each other according to their dimensions. Then, some waves may be amalgamated, and the mean dimension of sand waves increases with time by repetition of such an amalgamation. The authors derived a model for sand wave development previously (7), and herein the time scale of sand wave development will be estimated by a similar model, in which the followings are assumed based on the experimental observations (7):

1. The wave celerities of individual waves can be expressed by

$$U_{Wi} = q_B/[\hat{s}(1-\rho_0)H_i] \quad (31)$$

where U_{Wi} = wave celerity of individual sand waves; q_B = bed load transport rate; $\hat{s}$ = coefficient of sand wave geometry (≃0.5); ρ_0 = porosity of sand; and H_i = height of individual sand waves.

2. Individual sand waves are geometrically similar and this similarity is preserved independent of the elapsed time.

These assumptions were found to be satisfied for the almost whole stage of sand wave development experimentally. Based on the above assumptions, the following expression for the time variation of dune height has been derived (7):

$$H_0(t) = \sqrt{2f_s(\alpha)} \cdot \sqrt{q_B\theta/(1-\rho_0)} \cdot \sqrt{t} \quad (32)$$

where $f_s(\alpha)$ has been derived in consideration of statistical arrangements of individual waves and can be given by

$$f_s(\alpha) = \int_0^\infty \xi' \int_\xi^\infty [(\xi-\xi')/\xi^3]\, f_L(\xi)d\xi\, f_L(\xi')d\xi' \quad (33)$$

where α = variation coefficient of dune length; and $f_L(\xi)$ = probability density function of the normalized dune length. As for the distribution of dune length, a gamma distribution of which shape parameter is about 4.5 has been recommended by the authors (8), and then $f_s(\alpha)$ becomes about 0.08.

Dune height in this model increases infinitively as shown by the broken line in Fig. 10, contrary to the actual trend indicated by the solid curve in the figure.

Hence, at first putting $H_0(t)=H_e$ in Eq. 32 to estimate the time scale T_m, and then using this time scale, the time required for dunes to grow up to the fully developed state from an initially flattened bed, T, is obtained by

$$T = kT_m = k(1-\rho_0)\theta L_e^2/2q_B f_s(\alpha) \qquad (34)$$

where k = experimental constant. Moreover, $T_{EX}=k_2T$, where k_2 = another experimental constant. In Eq. 34, the equilibrium dune length for fully developed state, L_e, should be estimated. In case of dunes,

$$L_e = k_L \cdot h \qquad (35)$$

where k_L = constant (see Table 1). Furthermore, the steepness of dunes and the sediment transport rate may be expressed as functions of dimensionless bed shear stress. As for the former, some formulas have been proposed recently by Yalin and Karahan (18) and others. The formula by Yalin et al.(18) is expressed by the following equation for the condition that (h/d)>100 and $Re_*>20$ (d = sand diameter; $Re_*\equiv u_*d/\nu$ = grain size Reynolds number; u_* = frictional velocity; and ν = kinematic viscosity):

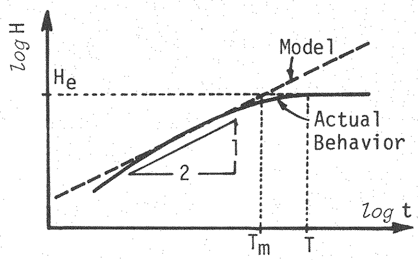

Fig. 10 Time variation of dune height in dune development process from an initially flattened bed

Table 1 Proposed value of k_L

Value of k_L	Researchers
5.0	Yalin (13)
2π	Hino (5), Yalin (15)
4.80	$[L_2/h]$ Nakagawa et al.(8)
6.67	$[L_1/h]$ Nakagawa et al.(8)

Note
$L_1\equiv M_0/M_1$, $L_2\equiv\sqrt{M_0/M_2}$
$M_j\equiv\int_0^\infty k^j S(k)dk$
S(k)=sand wave spectrum; and k=wave number

$$\theta \equiv f_\theta(\tau_*)$$
$$= 0.047(\tau_*/\tau_{*c}-1)\cdot\exp[1-0.078(\tau_*/\tau_{*c}-1)] \qquad (36)$$

where $\tau_*\equiv\tau/(\sigma-\rho)gd$ = dimensionless bed shear stress; σ = mass density of sand; and τ_{*c} = dimensionless critical bed shear stress ($\simeq 0.047$).

On the other hand, as for bed load transport rate, the following empirical formula is available for dune regime as revealed experimentally (7) :

$$\Phi_B(\tau_*) \equiv q_B/\sqrt{(\sigma/\rho-1)gd^3} = 10\,\tau_*^{2.5} \qquad (37)$$

Using the above relations, the time scale of dune development can be estimated. The dimensionless expression of T can be expressed by

$$T_* \equiv \frac{T\sqrt{(\sigma/\rho-1)gd}}{(h/d)\cdot h} = \frac{1-\rho_0}{2f_s(\alpha)}\cdot k\cdot k_L^2\,\frac{f_\theta(\tau_*)}{\Phi_B(\tau_*)} \qquad (38)$$

The result calculated from the above equation is shown in Fig. 11, where some experimental data by the authors (7) and Yalin and Bishop (17) are shown to verify the present model. On the calculation, it is assumed that k=2.0 and k_L=5.0. T_* is modified in Fig. 11 considering that the experimental data by Yalin et al. (17) have been obtained for artificial light material instead of natural sand. The present model can reasonably estimate the time scale of dune development.

As an additional remark, Yalin and Bishop (17) have also derived the time scale of dune development by dimensional analysis without any consideration on the mechanism of dune development. They have regarded the following dimensionless parameter as a similarity criterion of dune development.

$$\beta_* \equiv H_eL_e/q_BT \qquad (39)$$

and, they have derived

$$T = \beta_* H_e L_e / q_B = \beta_* \theta L_e^2 / q_B \qquad (40)$$

where the parameter β_* should be determined by experiments. Comparing the above equation with Eq. 34,

$$\beta_* = k(1-\rho_0)/2f_s(\alpha)$$

$$\simeq 3.75\,k \qquad (41)$$

can be obtained. Yalin et al. (17) have determined β_* by experimental data as $\beta_* k_L \simeq 252$, and, since $k_L \simeq 5 \sim 2\pi$, this implies $\beta_* \simeq 6.4 \sim 10$. Hence, $k \simeq 1.7 \sim 2.7$, and it is almost consistent with the assumption that k=2.0 in the calculation to obtain Fig. 11.

Several examples of the calculation using Fig. 11 are mentioned below:
If h=10cm and d=0.05cm, T≃1.0hr., and if h=2.0m and d=0.1cm, T≃150hr.
The former example may correspond to sand wave development in the laboratory, and the latter to that in rivers, as crude approximations.

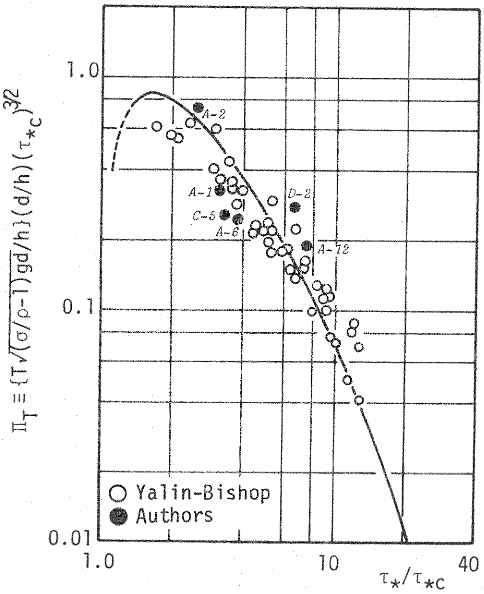

Fig. 11 Estimation of time scale of dune development

CONCLUSION

The results obtained in this study are summarized below:

1. One of the most important characteristics of unsteady flows with sand waves in alluvial streams is the behaviors of time-lag among bed forms, flow properties and sediment transport, and it has been suggested that the time required for sand waves to deform to those corresponding to the given conditions is the main cause of this kind of lag.

2. In order to describe unsteady phenomena in alluvial streams, a description of the deformation process of sand waves is important, and a preferable model has been proposed, where the interrelating system of alluvial streams has been simplified but the essential properties as lag-inducing system are preserved.

3. Simulating the time-lag due to sand wave deformation to the lag-distance of sediment transport, which is a cause of sand bed instability, a simple model to describe the lag behaviors appearing in unsteady flow with sand waves has been proposed.

4. A linear analysis has been conducted to know the outline of the lag behaviors in unsteady flow with sand waves. In this analysis, a small amplitude sinusoidal fluctuation of flow discharge has been considered. The lag behavior has a close relation with a ratio of the time scale of flow discharge fluctuation to the time required for dunes to grow-up from a flat bed to the fully developed state. The hysteresis relation between dune dimension or flow depth and flow discharge is remarkable when these time scales are comparable each other.

5. The most important factor to determine the lag behavior in unsteady flow with sand waves is the time required for sand wave development, and then, a simple model has been proposed to estimate this time scale. The amalgamation of individual sand waves due to the difference of their migrating celerities can be regarded as an elementary event of sand wave development. The obtained formula to estimate the time scale of dune development has been verified by experimental data.

Consequently, the outline of the mechanism of the time-lag appearing in unsteady flow with sand waves has been clarified by this study. From a view point of the sufficient applicability, however, the following problems remain:

1. A linear analysis is not appropriate to describe the behaviors of alluvial

streams during floods, and then the method of non-linear calculation should be developed.

2. The accuracy of the assumed impulse response for dune development is not so good, particularly in case of dune destruction with decreasing flow discharge. A better description for dune deformation process should be obtained based on more detailed experimental observations for some fundamental unsteady flow patterns.

3. A number of data for unsteady behaviors of flows with sand waves should be collected to verify the proposed treatments.

In order to achieve the above topics, the research on this theme should be continued.

Most of this paper is translated from the authors' Japanese paper (12) though some parts are slightly modified.

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APPENDIX - NOTATION

The following symbols are used in this paper:

a_h	= amplitude of fluctuation of flow depth;
$a_H H_e$	= amplitude of fluctuation of dune height;
$a_Q q_0$	= amplitude of fluctuating flow discharge;
C	= Chézy coefficient;
d	= sand diameter;
$f_L(\xi)$	= probability density function of normalized dune length;
$f_s(\alpha)$	= parameter to express the statistical properties of dunes;
$f_X(\xi)$	= probability density function of step length of sand particle;
$f_\theta(\tau_*)$	= steepness of dunes as a function of dimensionless bed shear;
$F_X(x)$	= distribution function of step length;
g	= gravitational acceleration;
$g_T(\tau)$	= impulse response of change of bed forms against the change of flow discharge;
h, h_0	= flow depth; average flow depth;
H, H_e	= mean dune height; equilibrium dune height;
$H_0(t)$	= time variation of dune height from flat bed to fully developed state under a constant flow discharge;
I	= energy slope (or bed slope);
k, k_2	= experimental constants;
k_L	= constant with respect to dune length;
L, L_e	= mean dune length, equilibrium dune length;
p_d	= deposit rate of sand particle;
p_s	= sediment pick-up rate;
q, q_0	= flow discharge per unit width; flow discharge per unit width without fluctuation;
q_B	= sediment transport rate;
Re_*	= grain size Reynolds number ($\equiv u_* d/\nu$);
$\hat{s}$	= shape coefficient with respect to dune geometry;
T, T_{EX}	= time required for dunes to develop from a flat bed to the fully-developed state; time scale of dune development ($=k_2 T$);
T_m	= time scale of dune development obtained by the present model;
T_f	= time scale of fluctuation of flow discharge;
T_*	= dimensionless time scale of dune development;
u_*	= friction velocity;
U_0	= mean velocity of flow;
U_W	= wave celerity of dunes;

α	= variation coefficient of dune dimensions;
β_*	= similarity criterion of dune development proposed by Yalin et al.;
γ_h	= $(a_h/a_Q)^2$;
δh	= increment of flow depth;
δH	= increment of dune height;
δq	= increment of flow discharge per unit width;
θ	= steepness of dunes (=H/L);
Λ	= mean step length of sand particle;
ν	= kinematic viscosity of fluid;
ρ	= mass density of fluid;
ρ_0	= porosity of sand;
σ	= mass density of sand;
τ	= bed shear stress;
τ_f	= hydraulic resistance due to bed forms (form drag);
τ_*	= dimensionless bed shear stress;
τ_{*c}	= dimensionless critical bed shear stress;
ϕ_H	= phase shift of fluctuation of dune height against that of flow discharge;
ϕ_h	= phase shift of fluctuation of flow depth against that of flow discharge;
$\Phi_B(\tau_*)$	= bed load transport rate as a function of dimensionless bed shear;
ψ_u	= velocity coefficient; and
ω	= angular frequency of flow discharge $(=2\pi/T_f)$.