# EFFECT OF PRESTRESS ON SHEAR CAPACITY OF PRESTRESSED CONCRETE BEAMS

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Yasuhiro MIKATA

Susumu INOUE

Kazuo KOBAYASHI

**Tamotsu NIEDA** 

In this study, the effects of prestress level and the distribution of prestress over the section on shear capacity are evaluated using the test results of prestressed concrete beams without shear reinforcement and prestressed reinforced concrete beams. The shear force carried by the concrete is calculated according to the JSCE code, the ACI code, and modified compression field theory, and so on. Calculated values are compared with the experimental results. Based on this, a simple method of taking into account the effect of prestress, including the stress distribution over the section as well as the inclination of the diagonal crack on shear capacity, is proposed.

*Key Words* : prestressed concrete beams, shear force carried by concrete, modified compression field theory, prestress distribution

Yasuhiro Mikata is an engineer at the Kinki branch of Road Management Technology Center. He received his Doctor of Engineering Degree from Osaka Institute of Technology in 2002. His research interest is the establishment of a rational design method for the shear capacity of prestressed concrete members. He is a member of the JSCE and JCI.

Susumu Inoue is a Professor of Civil Engineering and Urban Design at Osaka Institute of Technology, Osaka, Japan. He received his Doctor of Engineering Degree from Kyoto University in 1994. His research interests include the shear and seismic design of concrete structures. He has authored numerous papers on the structural behavior of reinforced and prestressed concrete members. He is a member of the JSCE, JCI, JSMS, and JPCEA.

Kazuo Kobayashi is a Professor in the Department of Civil Engineering and Urban Design at Osaka Institute of Technology, Osaka, Japan. He received his Dr. Eng. Degree from Kyoto University in 1973. He has authored several books and numerous papers on the behavior and analysis of reinforced and prestressed concrete structures. He is a fellow of the JSCE. He is also a member of the JCI, JSMS, and JPCEA.

Tamotsu NIEDA is an Associate Professor of Civil Engineering at Osaka Institute of Technology, Osaka, Japan. His major research fields are creep and drying shrinkage of artificial lightweight aggregate concrete and thermal induced cracks in massive concrete. He is a member of the JSCE, JSMS, and JCI.

# 1. INTRODUCTION

The current Japanese design codes for concrete structures in the civil engineering field take account of the effects of axial compressive force, including prestress, on shear capacity in a general sense by considering decompression moment. This, however, results in the effect of prestress being evaluated only in terms of the compressive stress introduced at the extreme tension fiber of the section. As a result, the effect of stress distribution over the section is not considered. This method is based on the description in CEB-FIP MC 1978<sup>[1]</sup>. However, it is not adopted in the present CEB-FIP MC 1990<sup>[2]</sup>. It has been indicated that evaluations based on decompression moment have some limitations in applicability <sup>[3]</sup>. Therefore, a rational design method able to evaluate the effect of compressive stress as well as stress distribution across the section on shear capacity needs to be established.

In this situation, various researches have been done on the shear capacity of concrete members. For example, the Strut and Tie model is recommended in CEB-FIP MC 1990 for the analysis of the shear resistance mechanism. Collins et al. proposed a modified compression field theory<sup>[4]</sup>, in which the equilibrium and compatibility conditions related to average stress and average strain are used to calculate the shear capacity and shear stress of members. Priestly proposed that effect of compressive force on shear capacity should be evaluated by the inclination of the diagonal compression strut formed by axial force instead of including its effects in the shear force carried by the concrete<sup>[5]</sup>.

In this study, the results of tests on prestressed concrete beams without shear reinforcement and with various levels of prestress and stress distribution across the section<sup>[6,7]</sup>, as well as prestressed reinforced concrete beams with a T-section<sup>[8]</sup>, are reviewed in order to evaluate the effect of prestress on shear capacity. The shear force carried by the concrete is also calculated according to modified compression field theory and the calculated values are compared with the experimental results. From these findings, a simple method of taking into account the effect of prestress, including stress distribution over the section and the inclination of the diagonal strut, on shear capacity is proposed. Finally, the applicability of the method is discussed.

## 2. OUTLINE OF LOADING TESTS

## 2.1 Test variables

The test specimens consist of prestressed concrete (PC) beams and prestressed reinforced concrete (PRC) beams, as shown in Fig.1. The A-series specimens are PC beams without shear reinforment, in which prestressing bars ( $\phi = 19$ mm,  $f_{pu} = 1277$ N/mm<sup>2</sup>) are used.

The test variables for the A series are as follows:

(1) Stress at the extreme tension fiber of the section:  $\sigma_{ct} = 0$ N/mm<sup>2</sup>, 2.0N/mm<sup>2</sup>, 3.0N/mm<sup>2</sup> and 4.0 N/mm<sup>2</sup>. (2) Stress distribution across the section; While maintaining the stress in the extreme tension fiber at 4.0N/mm<sup>2</sup>, the stress at the extreme compression fiber of the section is changed as 0N/mm<sup>2</sup> (triangular distribution), 2.0N/mm<sup>2</sup> (trapezoidal distribution) and 4.0N/mm<sup>2</sup> (rectangular distribution). (3) Compressive strength of concrete:  $f'_c = 40 \text{N/mm}^2$ ,  $80 \text{N/mm}^2$ .

The PC beams have no shear reinforcement or longitudinal reinforcement. The quantity of prestressing bars is determined so that shear failure is likely to occur in all specimens based on the JSCE Standard Specification for Design and Construction of Concrete Structures.

The B- series specimens are PRC beams with a T-section with deformed mild steel D16 bars ( $f_{sv}=320$  N/mm<sup>2</sup>) for the longitudinal non-tensioned reinforcement and D6 ( $f_{sy}$ =410N/mm<sup>2</sup>) for the shear reinforcement. As for prestressing steel, prestressing bars of  $\phi$ =13mm ( $f_{py}$ =1390N/mm<sup>2</sup>) were used.

The test variables for the B series are as follows:

- ① Spacing of shear reinforcement:  $s = \infty mm$ , s = 120mm (shear reinforcement ration  $\rho_w = 0.59\%$ )
- (2) Compressive strength of concrete:  $f'_c = 40 \text{N/mm}^2$ ,  $80 \text{N/mm}^2$ .

③ Stress at the extreme tension fiber of the section:  $\sigma_{ct} = 0$  N/mm<sup>2</sup>, 2.0N/mm<sup>2</sup> and 4.0 N/mm<sup>2</sup>. Details of these test variables are shown in Table 1.



Fig.1 Dimensions of specimens and loading conditions (unit: mm)

## 2.2 Loading test

All specimens are tested under symmetrical two-point loading. The A-series specimens have a total length of 1750mm, a flexural span of 400mm, and a shear span of 450mm. The shear span-effective depth ratio of these beams is a/d=3.21. The B-series specimens have a total length of 2000mm, a flexural span of 300mm, and a shear span of 400mm. The value of a/d is 2.35 in these beams. During the loading tests, the applied load, shear reinforcement strain and deflection at the center of the span are measured.

# 3. ANALYTICAL METHOD

Modified compression field theory <sup>[4]</sup> is used to analyze shear behavior. This theory takes into account the tensile stress between diagonal cracks and the reduction in compressive strength due to the existence of diagonal cracks. Concrete with diagonal cracks is regarded as a continuous element with uniform characteristics, and the shear capacity of a member is calculated from the equilibrium and compatibility conditions within the element.

Modified compression field theory gives the shear capacity as the sum of shear forces carried by the concrete and by shear the reinforcement, as indicated in Eq.(1), which is a function of the inclination of the diagonal strut  $\theta$  and the average principal tensile stress in the concrete  $f_1$ . As a stress-strain model for concrete in diagonal compression, Eq.(2) and (3) proposed by Vecchio et al. are used; this relates the average diagonal compressive stress in the concrete  $f_2$  to the principal tensile strain  $\varepsilon_1$ . As a stress-strain model in diagonal tension, on the other hand, Eq.(4) and (5) recommended by Vecchio et al. are used, in which tension softening is taken into account. (In specimens without shear reinforcement, the shear cracking load is equal to the maximum load in calculation. Therefore Eq. (5) is not used.)

$$V = V_{c} + V_{s}$$

$$= f_1 b_W jd \cot \theta + (A_W f_W / s) jd \cot \theta$$

(1)

$$\mathbf{f}_{2} = \mathbf{f}_{2 \max} \left[ 2 \left( \frac{\varepsilon_{2}}{0.002} \right) - \left( \frac{\varepsilon_{2}}{0.002} \right)^{2} \right]$$
(2)

$$\frac{f_{2 \max}}{f'_{c}} = \frac{1}{0.8 + 170\epsilon_{1}} \le 1.0$$
(3)

$$\mathbf{f}_{1} = \mathbf{E}_{c} \boldsymbol{\varepsilon}_{1} \left( \boldsymbol{\varepsilon}_{1} \leq \boldsymbol{\varepsilon}_{cr} \right) \tag{4}$$

$$\mathbf{f}_1 = \frac{\alpha_1 \alpha_2 \mathbf{f}_{cr}}{1 + \sqrt{500\varepsilon_1}} \left( \varepsilon_1 \ge \varepsilon_{cr} \right) \tag{5}$$

Where,  $f_1$ : principal tensile stress,  $b_w$ : web width, jd: flexural lever arm,  $\theta$ : inclination of diagonal strut (the inclination of diagonal cracks),  $A_w$ : area of shear reinforcement,  $f_w$ : stress in shear reinforcement, s: spacing of shear reinforcement,  $\varepsilon_{cr}$ : principal tensile strain at diagonal cracking,  $\varepsilon_1$ : principal tensile strain,  $\alpha_1$ : coefficient for the bond characteristics of reinforcement ( $\alpha_1$ =1.0 for deformed bars),  $\alpha_2$ : coefficient for loading type ( $\alpha_2$ =1.0 for monotonous loading),  $f_{cr}$ : tensile stress of concrete at diagonal cracking (calculated by Eq.(12)),  $f_2$ : principal compressive stress,  $f_{2max}$ : the maximum principal compressive strength after diagonal cracking, and  $\varepsilon_2$ : principal compressive strain

Although this theory was originally proposed for members subjected to pure shear, it can be used analyze the shear behavior of a member subjected to combined shear, axial force, and flexural force in combination with an ordinary flexural analysis method such as the fiber model. The applicability of this theory to reinforced concrete and prestressed concrete beam members has been proved by Nakamura et al.<sup>[9]</sup> and Gregor et al.<sup>[10]</sup>.

## 4. SIMPLE METHOD OF CONSIDERING THE EFFECT OF PRESTRESS

In modified compression field theory, the effect of prestress is considered in the calculation process of  $\cot \theta$ , in which the inclination of the diagonal strut  $\theta$  is assumed and  $\cot \theta$  is derived using an iteration procedure for axial force equilibrium. That is to say, the effect of prestress on shear capacity is evaluated from the inclination of the diagonal strut  $\theta$ . However, this calculation requires iteration to achieve convergence. In practical design, on the other hand, simplicity is also needed as well as reliability. Therefore, a simple method for calculating the inclination of the diagonal strut without iteration is proposed, in which the effect of prestress on shear capacity is evaluated by  $\cot \theta$ .

From Mohr's stress circle(Fig.2), Eq.(6) is obtained.

$$v\cot\theta = f_1 + f_{cx} \tag{6}$$

Shear stress v at the centroid of the section is given by Eq.(7)

$$v = \frac{V}{b_w jd}$$
(7)

In modified compression field theory, the shear force carried by concrete is given by Eq.(8).

$$V = V_c = f_1 b_w j d \cot \theta$$
(8)

By substituting Eq.(6) and Eq.(8) into Eq.(7), Eq.(9) is obtained.

$$\cot \theta = \sqrt{1 + \frac{f_{cx}}{f_1}} \tag{9}$$



# Fig.2 Mohr's stress circle

In Eq.(9),  $f_{cx}$  represents the average longitudinal stress at the centroid of the cross section, and for simplicity here, it is replaced by the average stress  $\sigma_{cg}$  of the cross section due to prestress. At diagonal cracking, the value of Eq.(8) becomes a maximum and the principal tensile stress  $f_1$  reaches the tensile strength  $f_t$ .

$$\cot \theta = \sqrt{1 + \frac{\sigma_{cg}}{f_{\star}}}$$
(10)

Therefore, the principal tensile stress  $f_1$  is replaced by tensile strength  $f_t$  and Eq. (9) is rewritten as Eq.(10). In the following discussion,  $\cot \theta_1$  means the value calculated using  $f_t$  from the JSCE code (Eq.(11)) and  $\cot \theta_2$  means the value calculated using  $f_t$  from Eq.(12) proposed by Collins et al.

i) JSCE code  

$$f_t = f_{t1} = 0.23 f'_{ck}^{2/3}$$
(11)

ii) Collins et al  $f_t = f_{t2} = {}_{0.33\lambda \sqrt{f'_c}} (\lambda = 1.00 : N/mm^2)$ (12)

where, f'<sub>c</sub>: compressive strength of concrete

#### 5. TEST RESULTS AND DISCUSSION

#### 5.1 Shear capacity in design codes

In the current JSCE code<sup>[11]</sup> and the Design Specifications of Highway Bridges<sup>[12]</sup> (Japan Road Association code), the coefficient ( $\beta_{n, k}$ ), representing the effect of prestress on shear force carried by concrete is given as Eq.(13).

$$\beta_{n} = k = 1 + M_{0} / M_{d} = 1 + 2M_{0} / M_{u} \quad (N'_{d} \ge 0)$$
(13)

if  $\beta_n > 2$ , then  $\beta_n = 2$ 

Where,  $M_0$ : decompression moment,  $M_u$ :flexural capacity,  $M_d$ :design flexural moment, and  $N'_d$ :design axial force.

Here, coefficient  $\beta_n$  is replaced by  $\cot \theta$  (Eq.(10)), and the JSCE  $\operatorname{code}(\operatorname{Eq.(14)})$  is modified to Eq.(15) by  $\cot \theta$ , in which  $V_{c1}$  corresponds to  $\cot \theta_1$  and  $V_{c2}$  corresponds to  $\cot \theta_2$ .  $V_{c1}$ , Vc2 (Eq.(15)) is added to the shear force carried by the shear reinforcement described in the JSCE code in order to calculate the ultimate shear capacity  $P_{us2}$ ,  $P_{us3}$  of PC beams.

The shear force carried by the concrete is also calculated according to the JSCE code, Japan Road Association code, ACI code<sup>[13]</sup>, AIJ code<sup>[14]</sup>, and Eq.(19) proposed by Ito et al<sup>[15]</sup>. All values are compared with the experimental results. In the calculations, values of an safety factors, such as the material factor, and member factor, are set to 1.0. The ACI code and Eq.(19) consider the effect of prestress separately in terms of flexural cracking moment  $M_{cr}$ . On the other hand, Eq.(10) evaluates the effect of prestress as a ratio to the situation without prestress.

i) JSCE Code

$$V_{cd} = \beta_d \beta_p \beta_n f_{vcd} b_w d / \gamma_b \qquad (kN)$$
<sup>(14)</sup>

$$V_{c} = \beta_{d}\beta_{p}f_{vcd}b_{w}d/\gamma_{b}\cot\theta \quad (kN)$$
<sup>(15)</sup>

where,

$$\cot \theta_{1} = \sqrt{1 + \frac{\sigma_{cg}}{f_{t1}}}$$

$$\cot \theta_2 = \sqrt{1 + \frac{\sigma_{cg}}{f_{t2}}}$$

 $\beta_d$ : coefficient to account for the influence of effective depth on shear capacity, and  $\beta_p$ : coefficient to account for the influence of longitudinal reinforcement on shear capacity

$$f_{vcd} = 0.20\sqrt[3]{f'_{cd}}$$
 (N/mm<sup>2</sup>)

ii) Japan Road Association code

$$\mathbf{S}_{c} = \mathbf{k}\tau_{a}\mathbf{b}_{w}\mathbf{d} \quad (\mathrm{tf}) \tag{16}$$

where,  $\tau_a$ : design resistance shear stress

\* \*

iii) ACI Code

$$V_{ci} = 0.05\sqrt{f'_{c}}b_{w}d + \frac{V_{i}}{M_{max}}M_{cr} \quad (kN)$$
(17)

iv) AIJ code

$$Q_{u} = bj \{ \alpha (f_{cs} + 0.1\sigma_{g}) + 0.5_{w} f_{t} (p_{w} - 0.002) \}$$
(tf)  
where,  $\alpha = \frac{4}{\frac{M}{Qd} + 1}$ ,  $j = \frac{7}{8} d$  (18)

b: web width,  $f_{cs}$ : design shear stress of concrete,  $\sigma_g$ : average prestress,  $_w f_t$ : design yield strength of shear reinforcement,  $p_w$ : shear reinforcement ratio, M: design flexural moment, and Q: design shear capacity

v) Ito's equation

$$V_{c} = 0.9\beta_{d}\beta_{p} (f'_{c})^{l/3} b_{w} d + M_{c} / a \qquad (tf)$$
<sup>(19)</sup>

5.2 Failure mode

#### a) Specimens with shear reinforcement

Experimental results are listed in Table 1. The failure mode of tested beams with shear reinforcement changed from shear tension failure to shear compression failure as the amount of introduced prestress increased. In particulars HT-12-40( $\sigma_{ct}$ =4.0N/mm<sup>2</sup>, f'<sub>ck</sub>=80N/mm<sup>2</sup>) failed in flexure.

The measured maximum load of specimens made with high-strength concrete ( $f'_{ck}=80N/mm^2$ ) was approximately 15~40% higher than that of specimens made with ordinary concrete ( $f'_{ck}=40N/mm^2$ ).

In the case of non-prestressed specimens, the ratio of measured maximum load ( $P_u$ ) to calculated maximum load ( $P_{us1}$ ,  $P_{us2}$ ,  $P_{us3}$ ) was 1.24 for T-12-0 and 1.58 for HT-12-0. In the case of prestressed specimens, on the other hand, ratio ( $P_u$  /  $Pus_1$ ) was 1.27 and ( $P_u$  /  $Pus_3$ ) was for T-12-40. Values for HT-12-40 were 1.53 ( $P_u$  /  $Pus_1$ ) and 1.46 ( $P_u$  /  $Pus_3$ ). These results indicate that  $P_{us3}$  using cot  $\theta_2$  gives a closer value to the measured one than  $P_{us1}$ , which uses  $\beta_n$ .

#### b) Specimens without shear reinforcement

Almost all the specimens without shear reinforcement except for beam HP-0-20 showed shear compression failure or shear tension failure. In these cases, the beams did not fail immediately after shear cracking and was able to carry further load due to the formation of an arch mechanism. This led to the fact that the beam HP-0-20 failed finally in flexure. Consequently, the measured ultimate load ( $P_u$ ) became considerably larger than the calculated values ( $P_{us1}$ ,  $P_{us2}$ ,  $P_{us3}$ ). These findings suggest that the proposed evaluation method can be applied to members with shear reinforcement, in which the truss mechanism is dominant. For the members without shear reinforcement, in which the arch mechanism is dominant, further investigation is necessary as regards the applicability of the proposed method.

Series	Specimens	Concrete Strength	Spacing of shear reinfor- cement	Introo pres (N/n	*1 duced tress nm <sup>2</sup> )		Ultimate sh (ca P (k () is	ear capacity al.) N) P <sub>u</sub> /P <sub>us</sub>		*6 Ultimate flexural capacity (cal)	Ultimate load (mea.) Pr	*7 Failure Mode
		(N/mm <sup>2</sup> )	s (mm)	σ'ct	σ <sub>ct</sub>	*2 P <sub>us1</sub>	*3 P <sub>us2</sub>	*4 P <sub>us3</sub>	*5 P <sub>us4</sub>	P <sub>ub</sub> (kN)	(kN)	mode
	P-0-0	41.9		0.0	0.0	36.9	36.9	36.9	52.0	117.5	87.4	SC
	P-0-20	41.9		0.0	2.0	40.7	42.8	44.5	61.0	120.9	98.3	SC
	P-0-30			0.0	3.0	42.7	45.6	47.9	65.4	124.4	105.2	SC
	P-0-40	13.0		0.0	4.0	44.4	48.1	50.9	68.8	126.0	96.3	SC
	P-20-40	43.0		2.0	4.0	44.5	54.0	58.1	77.8	123.1	103.0	SC
Δ	P-40-40		~	4.0	4.0	44.8	58.4	63.5	84.6	120.4	96.4	SC
A	HP-0-0	82.8		0.0	0.0	46.3	46.3	46.3	73.8	171.7	150.1	SC
	HP-0-20	82.8		0.0	2.0	49.6	51.1	53.1	83.6	172.9	148.8	F
	HP-0-30			0.0	3.0	50.5	52.7	55.6	87.2	170.9	148.2	SC
	HP-0-40	70.6		0.0	4.0	52.2	54.9	58.6	91.4	171.5	159.9	SC
	HP-20-40	19.6		2.0	4.0	52.2	60.1	65.5	101.6	171.4	168.5	SC
	HP-40-40			4.0	4.0	52.2	64.1	70.7	109.2	171.3	181.5	SC
	Т-0-0	41.1		0.0	0.0	43.5	43.5	43.5	55.2	187.2	59.1	ST
	T-0-20	10.7	00	0.0	2.0	47.1	49.9	51.7	63.2	190.7	101.1	ST
	T-0-40	40.7		0.0	3.9	50.8	55.7	58.8	70.3	192.2	110.3	ST
	T-12-0	41.1		0.0	0.0	107.2 (1.24)	107.2 (1.24)	107.2 (1.24)	144.2 (0.92)	187.2	133.3	ST
	T-12-20	40.7	120	0.0	2.0	110.7 (1.42)	113.5 (1.38)	115.3 (1.36)	145.1 (1.08)	190.7	156.9	SC
R	T-12-40	40.7		0.0	3.9	114.4 (1.27)	119.3 (1.21)	122.4 (1.18)	146.6 (0.99)	192.2	144.8	SC
	HT-0-0	79.8	79.8	0.0	0.0	54.3	54.3	54.3	77.5	220.1	153.7	ST
	HT-0-20	00.4	~~~~	0.0	2.0	60.8	61.8	64.1	91.7	222.9	165.9	ST
	HT-0-40	90.4		0.0	3.9	64.9	66.5	70.7	99.6	223.0	180.7	ST
	HT-12-0	79.8	.8	0.0	0.0	117.9 (1.58)	117.9 (1.58)	117.9 (1.58)	164.3 (1.13)	220.1	186.4	ST
	HT-12-20	- 90.4	120	0.0	2.0	124.4 (1.45)	125.4 (1.44)	127.7 (1.41)	169.1 (1.07)	222.9	180.2	SC
	HT-12-40			0.0	3.9	128.6 (1.53)	130.1 (1.51)	134.3 (1.46)	173.6 (1.13)	223.0	196.6	F

Table.1 Detail of test specimens and results of loading tests

\*1  $\sigma_{ct}=0$  and  $\sigma'_{ct}=0$  means that prestressing bars are not tensioned

\*2  $P_{us1}=2(V_{cd}+V_{sd}), (V_{cd}, V_{sd} \text{ are calculated by JSCE code using safety factor of 1.0})$ 

\*3  $P_{us2}=2(V_{c1}+V_{sd}), (V_{c1} \text{ is calculated by Eq.(15) and <math>\cot \theta_1, V_{sd} \text{ is calculated by JSCE code})$ 

<sup>\*</sup>4  $P_{us3}=2(V_{c2}+V_{sd})$ , (V<sub>c2</sub> is calculated by Eq.(15) and cot  $\theta_2$ , V<sub>sd</sub> is calculated by JSCE code)

\*5  $P_{us4}=2(V_c+V_s), (V_c, V_s \text{ is calculated by modified compression field theory (Eq.(1) and Eq.(12))}$ 

<sup>\*</sup>6 Values calculated by the fiber model.

\*7 SC: shear compression failure, ST: shear (diagonal) tension failure, F: flexural compression failure (prestressing bars do not yield)

Due to the above mentioned reason, the measured shear cracking load is compared here with the ultimate shear capacity carried by the concrete ( $P_{cs}=2V_c$ ) calculated by several methods.

The measured shear cracking load, and the ultimate concrete shear capacity as calculated by the JSCE code,  $V_{c1}$  and  $V_{c2}$  (Eq.(15)), are listed in Table 2. The ratio of measured shear cracking load for prestressed specimens to that for non-prestressed specimens, the value of  $\beta_n$  in the JSCE code, and the values of cot  $\theta_1$ , cot  $\theta_2$  are also listed the table. From these values, the effects of prestress on the shear capacity of prestressed concrete members were evaluated.

## 5.3 The Effect of prestress on shear cracking load

# a) Effect of prestress at the tension fiber of the section

Figure.3 shows the relationship between measured shear cracking load and introduced prestress at the tension fiber of section for the A-series specimens. The shear cracking loads calculated by several methods are also indicated in Fig.3. Here, the stress at the extreme compression fiber of the section is kept as 0.0N/mm<sup>2</sup> and the stress at the extreme tension fiber of the section is gradually changed from 0.0N/mm<sup>2</sup> to 2.0N/mm<sup>2</sup>, 3.0N/mm<sup>2</sup>, and 4.0N/mm<sup>2</sup>. The measured shear cracking load of P-0-40 was approximately 23% higher than that of P-0-0. That of HP-0-40 was approximately 35% higher than that of HP-0-0. As these results demonstrate, the measured shear cracking load increased with increasing introduced prestress. The same tendency was observed for B-series specimens, as shown in Table 2, irrespective of concrete strength and shear reinforcement ratio.

Series	Specimens	Introd pres (N/n	duced tress nm <sup>2</sup> )	Shear cracking load (Mea.)	Shea	r cracking (Cal.) P <sub>cs.c</sub> (kN)	load	*1 P <sub>csm</sub> /P <sub>cs0.m</sub>	$\beta_n$ (Cal.)	$^*2$ $\cot \theta_1$ (Col)	$^*3$ $\cot \theta_2$ (Col)
		$\sigma'_{ct}$	σ <sub>ct</sub>	P <sub>cs.m</sub> (kN)	JSCE	$V_{c1}$	$V_{c2}$	(Mea.)		(Cal.)	(Cal.)
	P-0-0	0.0	0.0	52.0	36.9	36.9	36.9	1.00	1.00	1.00	1.00
	P-0-20	0.0	2.0	55.4	40.7	42.8	44.5	1.07	1.10	1.16	1.20
	P-0-30	0.0	3.0	54.8	42.7	45.6	47.9	1.05	1.14	1.22	1.28
	P-0-40	0.0	4.0	64.1	44.4	48.1	50.9	1.23	1.19	1.29	1.35
	P-20-40	2.0	4.0	73.7	44.5	54.0	58.1	1.42	1.19	1.45	1.55
Δ.	P-40-40	4.0	4.0	81.7	44.8	58.4	63.5	1.57	1.19	1.57	1.69
A	HP-0-0	0.0	0.0	65.8	46.3	46.3	46.3	1.00	1.00	1.00	1.00
	HP-0-20	0.0	2.0	80.0	49.6	51.1	53.1	1.22	1.07	1.10	1.14
	HP-0-30	0.0	3.0	82.9	50.5	52.7	55.6	1.26	1.11	1.15	1.21
	HP-0-40	0.0	4.0	88.9	52.2	54.9	58.6	1.35	1.14	1.20	1.27
	HP-20-40	2.0	4.0	92.2	52.2	60.1	65.5	1.40	1.14	1.31	1.42
	HP-40-40	4.0	4.0	109.9	52.2	64.1	70.7	1.67	1.14	1.40	1.54
	T-0-0	0.0	0.0	59.1	43.5	43.5	43.5	1.00	1.00	1.00	1.00
	T-0-20	0.0	2.0	65.5	47.1	49.9	51.7	1.11	1.09	1.15	1.19
	T-0-40	0.0	3.9	68.6	50.8	55.7	58.8	1.16	1.17	1.28	1.35
	T-12-0	0.0	0.0	52.3	43.5	43.5	43.5	1.00	1.00	1.00	1.00
	T-12-20	0.0	2.0	70.1	47.1	49.9	51.7	1.34	1.09	1.15	1.19
D	T-12-40	0.0	3.9	71.2	50.8	55.7	58.8	1.36	1.17	1.28	1.35
D	HT-0-0	0.0	0.0	81.1	54.3	54.3	54.3	1.00	1.00	1.00	1.00
	HT-0-20	0.0	2.0	88.4	60.8	61.8	64.1	1.09	1.07	1.09	1.13
	HT-0-40	0.0	3.9	93.8	64.9	66.5	70.7	1.16	1.15	1.17	1.25
	HT-12-0	0.0	0.0	81.4	54.3	54.3	54.3	1.00	1.00	1.00	1.00
	HT-12-20	0.0	2.0	91.6	60.8	61.8	64.1	1.13	1.07	1.09	1.13
	HT-12-40	0.0	3.9	95.5	64.9	66.5	70.7	1.17	1.15	1.17	1.25

Table.2 Shear cracking load

\*1 P<sub>cs0,m</sub>: measured shear cracking load for non-prestressed specimens
\*2 cot θ<sub>1</sub> is calculated by Eq.(10) and concrete tensile strength f<sub>t</sub> is calculated by JSCE code (Eq.(11)).
\*3 cot θ<sub>2</sub> is calculated by Eq.(10) and concrete tensile strength f<sub>t</sub> is calculated by Eq.(12).

Each value of calculated ultimate shear capacity also increases with increasing the prestress. However, the ratio of the increase as calculated by the JSCE code, AIJ code, and Japan Road Association code is smaller than that measured. Consequently, the discrepancy between the calculated shear cracking load and the measured value tended to increase with increasing introduced prestress. Further, the evaluation methods currently adopted in each design code tend to underestimate the shear force carried by the concrete, including the effect of prestress, in the case of monotonous loading. Among the equations investigated in this study, modified compression field theory and Ito's equation gave values that were closer to the measured shear cracking load. The coefficients  $\cot \theta_1$  and  $\cot \theta_2$  proposed in this study provides a more accurate evaluate in of the effect of prestress on shear capacity than coefficient  $\beta_{n}$ , and correctly the increase in shear cracking load with increasing prestress, although the calculated values of V<sub>c1</sub> and V<sub>c2</sub> somewhat underestimate the measured ones.

# b) Effect of stress distribution over the section

Figure.4 shows the effect of the distribution of stress over the section on the shear cracking load. In this case, the stress at the extreme tension fiber of the section is kept at 4.0N/mm<sup>2</sup> and the sectional stress distribution is changed from a triangular distribution to a rectangular distribution.

The measured shear cracking load of P-20-40 (trapezoidal distribution) and P-40-40 (rectangular distribution) was approximately 15% and 27% higher. Respectively, compared with that of P-0-40 (triangular distribution). The measured shear cracking load of HP-20-40 (trapezoidal distribution) and HP-40-40 (rectangular distribution) was approximately 4% and 24% higher respectively, than that of HP-0-40 (triangular distribution). On the other hand, all equations except for the AIJ code yield the same value





Fig.4 Effect of stress distribution on shear cracking load ( $\sigma_{ct}$ =4.0N/mm<sup>2</sup>)

irrespective of stress distribution and the difference in stress distribution is not reflected in the estimation. This is because these design codes consider the effect of prestress on shear capacity only in terms of decompression moment or flexural cracking moment. The effect of prestress is evaluated only by the compressive stress at the extreme tension fiber of the section. The AIJ code does indirectly take into account the effect of prestress on shear capacity by using the average prestress  $\sigma_g$  across the section. In this case, however, the coefficient of 0.1 is multiplied and the ratio of the increase in shear cracking load is smaller than the measured value, although it does increase when the stress distribution across the section is changed from triangular to rectangular. As a result, in the case of a rectangular distribution, the difference between the measured and calculated shear cracking loads increased and shear capacity tended to be underestimated. On the other hand, the ratio of ultimate shear capacity well with the measured value, although the ultimate shear capacity is relatively well with the measured value, although the ultimate shear capacity itself tended to be somewhat underestimated.

#### 5.4 Shear force carried by concrete

Figure.5 shows the relationships between shear force carried by concrete ( $V_c$ ), shear reinforcement ( $V_s$ ), and applied shear force for B-series specimens with shear reinforcements. The shear force carried by the concrete as determined using the JSCE code, and  $V_{c1}$  and  $V_{c2}$  (Eq.(15)) is also indicated in Fig.5.

In Fig.5, the experimented shear force carried by the concrete was calculated by subtracting the measured shear force carried by the shear reinforcement from the total applied shear force. The shear force carried by the shear reinforcement was estimated based on the truss analogy using the measured strain of the shear reinforcement and the measured average inclination of the critical diagonal cracks. (More than two or three critical diagonal cracks occurred in the shear span, as indicated in Fig.6. Therefore, the measured average strains of all shear reinforcement in the shear span, and the measured average inclination of diagonal cracks were adopted here.) The measured crack inclinations are listed in Table 3. Figure.6 shows the final crack distribution after the loading test.

On the other hand, the analytical values of  $V_s$  and  $V_c$  in Fig.5 are calculated using modified compression field theory. In this case, the modified compression field theory was applied to the web area ( $b_w$ jd,  $b_w$ : web width; jd: flexural lever arm). Other areas were analyzed by the fiber model; that is, shear capacity was calculated in consideration of the applied flexural moment. The analyzed section was chosen as 220mm from the support because the shear reinforcement strain was measured at this point.

As seen in Fig.5, the deterioration of V<sub>c</sub> due to diagonal cracking, as well as the increase in V<sub>s</sub> after diagonal cracking, is well estimated by modified compression field theory irrespective of concrete strength. The reduction in V<sub>c</sub> immediately after shear cracking was more significant as the amount of introduced prestress in the specimen was increased with normal concrete, while it was approximately 20% irrespective of prestressing level in specimens with high-strength concrete. The measured V<sub>c</sub> of T-12-0 ( $\sigma$  ct=0) was 19.5kN. That of T-12-40 ( $\sigma$  ct=4.0N/mm<sup>2</sup>) was 28.9kN, about 38% higher than for T-12-0 due to the increase in prestress.

Values of V<sub>c</sub> calculated using Eq.(14) coincide relatively well with measured values for T-12-0 and HT-12-0 ( $\sigma_{ct}$ =0). On the other hand, the measured V<sub>c</sub> for T-12-40 ( $\sigma_{ct}$ =4.0N/mm<sup>2</sup>) is 28.9kN, while the calculated value given by Eq.(14) is 25.4kN and the ratio to the T-12-40 value is approximately 17%. The difference between calculated V<sub>c</sub> by Eq.(14) and measured V<sub>c</sub> increases with increasing prestress, and consequently Eq.(14) underestimates the effect of prestress. On the other hand, the calculated shear force carried by concrete V<sub>c1</sub> and V<sub>c2</sub> (Eq.(15)) for T-12-40 was 27.8kN and 29.4kN, respectively, and these values coincided relatively well with the measured values (28.9kN). The calculated concrete shear resistance V<sub>c1</sub> and V<sub>c2</sub> for T-12-40 was approximately 28% and 35% higher respectively, than the value for T-12-0. Thus, the effect of prestress can be evaluated using V<sub>c1</sub> and V<sub>c2</sub> irrespective of prestressing level, although the results using V<sub>c1</sub> and V<sub>c2</sub> are approximately 10% smaller than the measured values for the specimens with normal strength concrete. In the case of specimens made with high-strength concrete, V<sub>c1</sub> tended to somewhat underestimate the effect of prestress, while V<sub>c2</sub> was able to predict the increase in shear force carried by the concrete with increasing prestress.



Fig.5 Relationships between Vc, Vs and applied shear force

# 5.5 Inclination of diagonal cracks

The measured values of diagonal crack inclination, as well as those calculated by  $\cot \theta_1$  and  $\cot \theta_2$  (Eq.(10)), are listed in Table 3. Figure.6 shows the final crack patterns. The measured value of inclination of diagonal cracks is the angle between the critical diagonal crack and the member axis.

	Inclination of diagonal cracks							
Specimens	$\cot \theta_1$ (Cal.)	$\cot \theta_2$ (Cal.)	Measured () is the average value					
P-0-0	45°	45°	31°					
P-0-20	41°	$40^{\circ}$	38°					
P-0-30	39°	38°	34°					
P-0-40	38°	37°	33°					
P-20-40	35°	33°	35°					
P-40-40	33°	31°	22°					
HP-0-0	45°	45°	35°					
HP-0-20	42°	41°	38°					
HP-0-30	$41^{\circ}$	$40^{\circ}$	40°					
HP-0-40	$40^{\circ}$	38°	32°					
HP-20-40	37°	35°	34°					
HP-40-40	36°	33°	28°					
Т-0-0	45°	45°	41°					
T-0-20	41°	$40^{\circ}$	34°					
T-0-40	38°	36°	26°					
T-12-0	45°	$45^{\circ}$	45°,35° (40°)					
T-12-20	41°	$40^{\circ}$	40°,26° (33°)					
T-12-40	38°	36°	$32^{\circ}, 24^{\circ}$ (28°)					
HT-0-0	45°	45°	34°					
HT-0-20	43°	$42^{\circ}$	37°					
HT-0-40	40°	39°	35°					
HT-12-0	45°	45°	$45^{\circ}, 40^{\circ}$ (43°)					
HT-12-20	43°	$42^{\circ}$	38°,28° (33°)					
HT-12-40	40°	39°	33° .33° (33° )					

Table.3 Inclination of diagonal cracks

## a) Effect of introduced prestress

With specimens in both series without shear reinforcement, the measured inclination of diagonal cracks decreased with increasing prestress. This tendency is also seen with  $\cot \theta_1$  and  $\cot \theta_2$  (Eq.(10)).

In B-series specimens with shear reinforcement ( $\rho_w=0.59$ ), at least three diagonal cracks occurred and their inclination decreased with distance from the loading point to the cracking point. Although these was same scatter, the measured inclination generally decreased with increasing prestress. The discrepancy between measured inclination and the value calculated using cot  $\theta_1$  and cot  $\theta_2$  is approximately  $\pm 4^\circ$ , meaning that they coincided well with each other.

## b) Effect of stress distribution over the section

The measured inclination of the diagonal cracks decreased when the stress distribution over the section was changed from a triangular distribution to a rectangular distribution. Values calculated using  $\cot \theta_1$  and  $\cot \theta_2$  coincided well with the measured ones.

 $\cot \theta_1$  and  $\cot \theta_2$  are evaluated only by the stress equilibrium in the web concrete. In this case, the

effect of shear reinforcement ratio on the inclination of diagonal



cracks is not considered. Consequently, this method is unable to evaluate the precise effect of shear reinforcement ratio on the inclination of diagonal cracks. However,  $\cot \theta_1$  and  $\cot \theta_2$  do reflect the tendency for diagonal crack inclination to fall with increasing prestress.

## 6. CONCLUSIONS

The main conclusions obtained from this study are as follows:

- (1) The measured shear cracking load increases as the amount of introduced prestress increase and when the stress distribution over the section changes from triangular to rectangular.
- (2) The shear capacity of PC members tends to be underestimated by the current Japanese design codes, although they do offer adequate safety. Further, most of the design codes ignore the effect of the stress distribution over section. For this reason, a rational design method able to evaluate the effect of compressive stress as well as stress distribution on shear capacity should be established.
- (3) Using  $\cot \theta_1$  and  $\cot \theta_2$  (Eq.(10)), effect of prestress and stress distribution over the section on shear capacity can be evaluated for prestressed concrete members. Calculations using  $\cot \theta_1$  somewhat underestimate the measured values. However,  $V_{c1}$  and  $V_{c2}$  (Eq.(15)), which are derived from a modification of Eq.(14)(JSCE code) by  $\cot \theta_1$  and  $\cot \theta_2$ , predict the increase in shear cracking load with increasing prestress.
- (4) In the case of specimens with shear reinforcement,  $V_{c1}$  and  $V_{c2}$  (Eq.(15)) predict the effect of prestress on shear force carried by concrete and give results close to the measurements. By adding the value of  $V_s$  in the JSCE code to the value of  $V_{c1}$  or  $V_{c2}$ , the ultimate shear capacity of prestressed concrete members can be safety and appropriately estimated.
- (5) The measured inclination of diagonal cracks decreases with increasing the introduced prestress and also decreases when the stress distribution over the section is changed from triangular distribution to rectangular distribution. The inclination of diagonal crack calculated by Eq.(10) coincides well with the measurements.

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