

ANALYTICAL STUDY ON CRACK GENERATION AND CRACK WIDTH CAUSED BY REINFORCING BAR CORROSION



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ABSTRACT

The performance of reinforced concrete structures gradually falls as reinforcing bar corrosion proceeds. To forecast the performance of an existing structure, it is necessary to clarify the relationship between the width cracks observed in the reinforced concrete and the degree of corrosion that the reinforcement has suffered. In this paper, a cylindrical model is assumed as the basic calculation model, induced basic calculation formulation on the corrosive amount at the crack generation, and relationship between the corrosive amount of the reinforcing bar and the crack width after the crack occurred. In addition, to clarify the crack generation situation and the relationship between the degree of reinforcing bar corrosion and crack width, finite element analysis is carried out with consideration of the material non-linearity and geometric non-linearity. Based on this finite element analysis incorporating various factors related to concrete cover, reinforcing bar diameter, and the concrete strength, a corrected formulation of the calculation for degree of reinforcing bar corrosion and crack width is proposed. Moreover, by comparing experimental data and results obtained so far in this investigation, the validity of the calculation formulation is demonstrated.

Key Words: *cylinder model, smeared model, discrete model, crack width, degree of corrosion*

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1. Introduction

Concrete structures make up an extremely important part of the social fabric. Given their importance, the functionality of these structures needs to be upheld through proper maintenance. If appropriately designed and constructed, a concrete structure has plenty of durability, and for this reason concrete has been used for large numbers of civil engineering structures. However, in the natural environment of earth, there is no such thing as a permanent concrete structure. To maintain performance, it is important to know the mechanisms by which a concrete structure deteriorates and to come up with countermeasures.

Reinforcing bar corrosion is one of the most important problems in the realm of concrete structure deterioration. The penetration of chloride-ions and concrete neutralization are the main factors that lead to corrosion of embedded reinforcing bars. As corrosion proceeds, the volume of corrosion product increases, and causes expansion pressure on concrete surrounding the reinforcing bar. Cracks then occur and the cover concrete may flake from the surface as a result of the pressure. Once cracks have opened up, corrosion factors penetrate the concrete more easily and the corrosion rate accelerates. Ultimately, the adhesion between concrete and reinforcing bar drops, the sectional area of the reinforcing bar decreases, and material functionality falls dramatically.

In predicting the service life of an existing structure, investigation of the corrosion situation by destructive removal of reinforcing bars from the structure is not a realistic option. Therefore, it is important to be able to clarify the corrosion situation by observing the deteriorated condition of the surface concrete. Without a method of doing this, forecasting the gradual loss of durability or service life of a reinforced concrete structure is impossible.

In the general, the limit condition on maintenance of a concrete structure is whether cracking has occurred at the surface, and there are a variety of maintenances options available if cracks are found to have occurred. However, performance is not lost immediately when a crack occurs. The relationship between crack width and degree of reinforcing bar corrosion then becomes important to the prediction of post-cracking structural performance. For this reason, there is great demand for an analytical method that can take the place of experiment. The use of fracture mechanics to model mechanical behavior according to the generation and progress of concrete cracks is a very promising approach to this problem^{1),2)}.

In this paper, a cylindrical model is assumed to be the basic calculation model, a basic calculation formulation of the relationship between the corrosive amount of reinforcing bar and the crack width after crack occurred, the corrosive amount of reinforcing bar at crack generation are induced. Next, to clarify the crack generation situation and relationship between the degree of reinforcing bar corrosion and crack width taking into account material non-linearity and geometric non-linearity, the smeared crack model (which is able to determine crack generation at any position) is used to analyze the mode of the crack generation. Then, based on mode of the crack generation as determined by the smeared crack model, finite element analysis is performed to determine the relationship between crack width and degree of reinforcing bar corrosion. This correlation is assumed to be a basic formulation, and a corrected formulation for the relationship using numerical analysis to take into account various factors related to the concrete cover, the diameter of the reinforced bar, and the concrete strength is proposed. Moreover, using observed data and results obtained so far using this method, the validity of the corrected formulation is demonstrated

2. Basic formulation

(1) Model for basic formulation of calculation

Expansion pressure resulting from corrosion of a reinforcing bar leads to cracking in the concrete surrounding the reinforcing bar. Both before and after cracks appear at the concrete surface, the characteristics of the concrete material determine the process. Prior to cracking, the degree of reinforcing bar corrosion depends strongly on the concrete cover, the diameter of the reinforcing bar and the ultimate tensile strength of the concrete as well as other factors. On the other hand, after cracking occurs, it dose not depend on reinforcing bar diameter or ultimate tensile strength of the concrete^{3),4),5)}.

Various calculation models for the relationship between the degree of reinforcing bar corrosion and surface crack width have been investigated as well as for the degree of corrosion at crack generation. In this paper, the assumptions are as shown in **Fig.1**. The calculation model described in **Fig.1 (a)** and **Fig.1 (b)** is a cylindrical one

in which the concrete of circular cylinder outer is omitted. It is assumed that the expansion pressure imposed by reinforcing bar corrosion products acts the cylindrical model internal to be same uniform load. In this paper, the calculation formulation for degree of reinforcing bar corrosion at crack generation and the relationship between crack width and degree of corrosion is induced using the simplified cylindrical model shown in **Fig.1**

(2) Calculation formulation at crack occurrence

In this paper, because the stress arising in the direction of cylinder circumference is not uniform, the stress in the direction of the circumference is assumed to be the average stress at crack generation, as shown in **Fig.1 (a)**. Displacement in the radial direction caused in the circular cylinder inner is not considered the influence on the circular direction, but is assumed to be the change of the rust layers. The stress and displacement are described by expression (2.1) and expression (2.2) below.

$$\bar{\sigma} = \frac{q_1 d}{2c} \quad (2.1)$$

$$\delta_1 = \frac{q_1 d \{(2c + d)^2 + d^2\}}{2E \{(2c + d)^2 - d^2\}} \quad (2.2)$$

In which,

σ : mean stress (N/mm²), δ_1 : concrete displacement increment (mm), d: diameter of reinforcing bar (mm)

c: cover thickness (mm), E: Young's modulus of concrete (N/mm²), q_1 : uniform load (N/mm)

(3) Calculation formulation after cracking

The calculation model for post-cracking is shown in **Fig.1 (b)**. Because the concrete displacement increment δ_1 before cracking is very small in comparison with the diameter d of the reinforcing bar, this influence is omitted obtaining the formulation by induction. The average displacement of cylinder in the radial direction caused by expansion pressure is assumed to be the displacement of the corrosion product layer. The displacement where concrete surface assumed to be crack width, the displacement of radial direction and the calculation formulation of the crack width are described by expression (2.3) and expression (2.4) below.

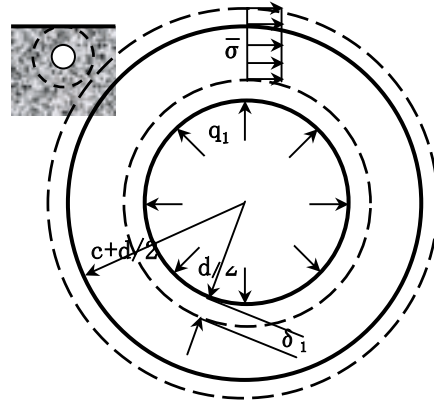
$$\delta_2 = \frac{q_2 (c + d)^4}{16EI} \quad (2.3)$$

$$w = \frac{\pi q_2 (c + d)^3 (5c + 3d)}{16EI} \quad (2.4)$$

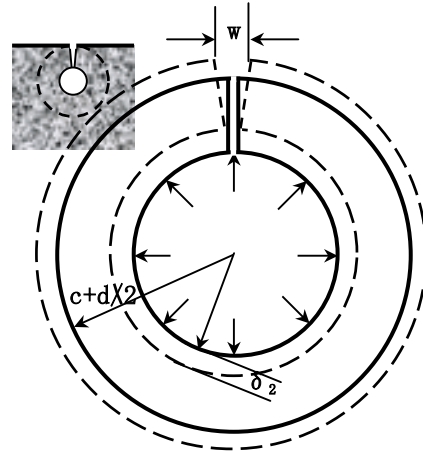
In which,

w : crack width (mm), δ_2 : concrete displacement increment (mm)

I: second moment of section , q_2 : uniform load (N/mm)



(a) Calculation model at crack occurrence



(b) Calculation model after crack occurrence

Fig.1 Dynamic models for basic formulation

(4) Calculation formulation for degree of corrosion

As a reinforcing bar corrodes, its volume increases as compared with the original steel volume. The relationship between corrosion product and iron density is shown in Fig.2. The calculation formulation for degree of reinforcing bar corrosion is given by expression (2.5), when calculate corrosion of unit length as shown in the Fig.2, the calculation formulation for reinforcing bar corrosion is described by expression (2.5)¹⁾.

$$W_c = \frac{\gamma}{(\rho - 1)} u_2 \quad (2.5)$$

In which,

W_c : degree of reinforcing bar corrosion (mg/mm²).

u_1 : radial reduction reinforcing bar thickness (mm)

u_2 : radial increase in reinforcing bar thickness (mm)

γ : density of iron (7.85mg/mm³)

ρ : expansive ratio of corrosion products

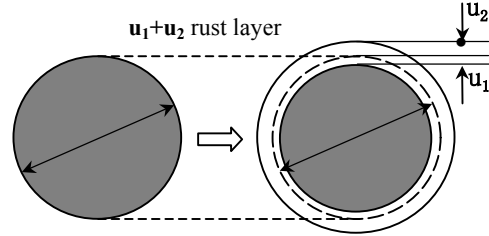


Fig.2 Calculation model for degree of corrosion

(5) Basic calculation formulation for relationship between crack width and corrosive amount

To formulate the calculation for the relationship between crack width and degree of reinforcing bar corrosion, when stress in the cylinder circumferential direction reaches the concrete ultimate tensile strength, the concrete displacement increment δ_1 is assumed to be the radial increment in reinforcing bar size at crack generation, and the concrete displacement increment δ_2 is assumed to be the radial increment in size in the direction of the reinforcing bar after cracking occurs. Using the relationship between concrete compressive and tensile strength⁶⁾, the expression (2.6) of degree of corrosion at crack occurrence and the formulation (2.7) for the relationship between crack width and the corrosion W_{c2} after cracking is obtained by induction based on expressions (2.1) to (2.5).

$$W_{c1} = \frac{0.0673\gamma}{E(\rho-1)} \frac{\{(2c+d)^2 + d^2\}}{(c+d)} f_c^{2/3} \quad (2.6)$$

$$W_{c2} = \frac{\gamma}{\pi(\rho-1)} \frac{(c+d)}{(5c+3d)} w \quad (2.7)$$

Then expressions (2.6) and (2.7) are summarized as the basic formulation for degree of corrosion in expression (2.8).

$$W_c = W_{c1} + W_{c2} = \frac{\gamma}{\pi(\rho-1)} \left[\frac{0.22\{(2c+d)^2 + d^2\}}{E(c+d)} f_c^{2/3} + \frac{(c+d)}{(5c+3d)} w \right] \quad (2.8)$$

In which,

W_c : degree of reinforcing bar corrosion (mg/mm²), W_{c1} : degree of reinforcing bar corrosion at crack occurrence (mg/mm²)

W_{c2} : degree of reinforcing bar corrosion after crack occurrence (mg/mm²), f_c : concrete compressive strength (N/mm²)

w : crack width (mm)

The degree of reinforcing bar corrosion at crack occurrence and the relationship between crack width and degree of reinforcing bar corrosion after cracking occurs can be calculated by using corrosion calculation formulation (2.8). According to research and investigations so far, the crack occurred in the concrete inner may be high. However, in the assumed basic calculation model, these influences the restraining in inner and outer of the concrete cylindrical model do not considered. To

investigate these influences, numerical analysis is performed using the analytical method for crack generation and progress and the analytical method for the crack width.

3. Analytical approach to crack generation and progress

The crack generation modes make contact with concrete cover, reinforcing bar spacing and restraint conditions, etc. according to our research and investigations so far. Using the discrete crack model of concrete, it is possible to calculate crack width. In order to use this model, the determination of crack generation position is very important. In this paper, crack generation is analyzed using the smeared crack model, while the discrete crack model described in Chapter 4 is used to calculate crack width.

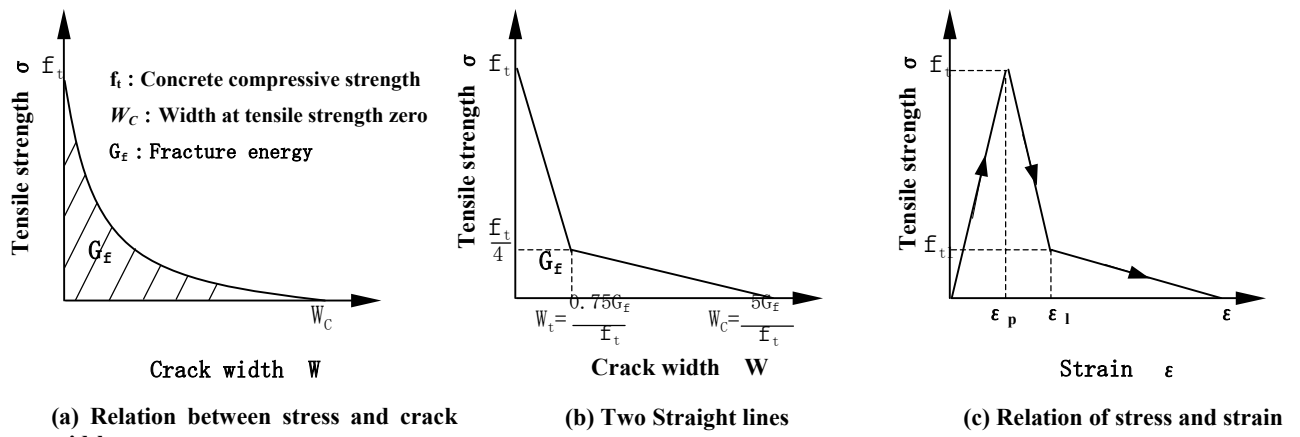


Fig.3 Tensile stiffening characteristic of concrete

(1) Crack modeling

a) Analytical approach

When judging the mode of generation resulting from reinforcing bar corrosion, assume to use FEM considering crack generation in the cross section of the reinforcing bar. The ultimate tensile strength of the concrete does not liberate rapidly after crack occurrence because of the influence of aggregate bonding, etc. Rather tensile strength falls gradually as the cracks open up. In this paper, the relationship between crack width and tensile stress is taken to be that shown in **Fig.3 (a)**, and this is applied to the crack model in the form of the two straight lines shown in **Fig.3 (b)** to express the tensile softening characteristic of the concrete.

b) Composition rule for cracks

In general, a virtual crack model of the relationship between stress and crack width is used directly. This method⁷⁾ is described, for example, in the literature on structural fracture mechanics research by JCI⁷⁾, surveys two nodes at the virtual crack position, assume the point of the crack position along a virtual crack to be input data and perform the secant analysis. The tensile softening curve can be obtained by various experimental methods. There are also various proposals for numerical analysis models. In this paper, the tensile softening behavior of the concrete is applied to all finite elements of the concrete, and the analysis is performed using a 1/4 model^{7,8)}. Because crack width w is converted into the strain through width B of the element, the relation of stress and strain bound can convert the relation of stress-strain as shown in **Fig.3(c)** and in the following expression.

$$(1) 0 \leq \varepsilon < \varepsilon_p$$

$$\sigma = E_E \cdot \varepsilon \quad (3.1)$$

$$E_t = \frac{d\sigma}{d\varepsilon} = E_E \quad (3.2)$$

$$(2) \varepsilon_p \leq \varepsilon < \varepsilon_l$$

$$\sigma = f_t - \frac{0.75f_t(\varepsilon - \varepsilon_p)}{\varepsilon_l - \varepsilon_p} \quad (3.3)$$

$$E_t = \frac{d\sigma}{d\varepsilon} = -\frac{0.75f_t}{\varepsilon_l - \varepsilon_p} \quad (3.4)$$

$$(3) \varepsilon_l \leq \varepsilon < \varepsilon_0$$

$$\sigma = 0.25f_t - \frac{0.25f_t(\varepsilon - \varepsilon_l)}{\varepsilon_0 - \varepsilon_l} \quad (3.5)$$

$$E_t = \frac{d\sigma}{d\varepsilon} = -\frac{0.25f_t}{\varepsilon_0 - \varepsilon_l} \quad (3.6)$$

$$(4) \varepsilon_0 \leq \varepsilon$$

$$\sigma = E_t = 0 \quad (3.7)$$

$$\text{In which, } \varepsilon_p = \frac{f_t}{E_t}, \quad \varepsilon_l = \frac{0.75G_f}{f_t \cdot B} + \varepsilon_p, \quad \varepsilon_0 = \frac{5.0G_f}{f_t \cdot B}$$

In which,

G_f: concrete fracture energy as given in **Fig.3(a)** (0.1N/mm)

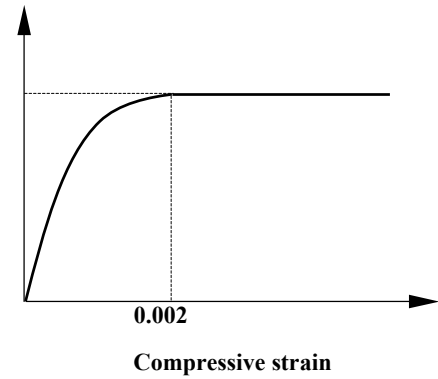
F_t: concrete ultimate tensile strength (N/mm²)

B: width of element (mm)

The rigidity of the formulated element applied to whole rigidity matrix and numerical analysis is performed.

(2) Analytical model

In finite element analysis, it is assumed that the compressive portion of the concrete is elasto-plastic, and this is modeled by a quadrilateral element of four nodes. The analytical method is incremental analysis under displacement control, with displacement applied to the concrete around the reinforcing bar as the reinforcing bar expands. To express secondary original behavior resulting from crack generation, the relationship between average stress and average strain in the direction of the expansion force is used. The softening characteristic after crack occurrence is modeled using the bi-linear curve shown in **Fig.3**. The composite model for concrete in the compressive portion⁹⁾ is represented by the curve shown in **Fig.4** and the expression below. Cracks at the concrete surface are represented by the smeared crack model.



$$\varepsilon \leq 0.002$$

$$\sigma = f'_c \times \frac{\varepsilon}{0.002} \times \left(2 - \frac{\varepsilon}{0.002} \right) \quad (3.8)$$

$$\varepsilon > 0.002$$

$$\sigma = f'_c \quad (3.9)$$

In which,

f'_c : concrete compressive strength(N/mm²)

Using the above analytical models, the crack generation mode resulting from corrosion of the reinforcing bar can be determined.

4. Analytical approach to crack width and degree of corrosion

When assuming that the discrete crack model can forecast crack width, it is necessary to provide the number of cracks and the position at which they are generated. Actually, the generation position of cracks due to reinforcing bar corrosion depends on various factors, including the diameter of the reinforcing bar, the cover thickness, the reinforcing bar spacing, and the restraint conditions, etc. In this paper, using the analytical result achieved with the smeared crack model in Section 3, the crack count, position, and angle are ascertained. Then the crack width analysis is carried out

In finite element analysis taking into account crack generation, the general case in which cracking is judged according to the standard ultimate tensile strength of the concrete has been studied. The crack generation condition is taken to be when the local stress in a node at the virtual crack interface reaches the concrete ultimate tensile strength. When this condition is met, cracks occur. Because a contact element is placed at the interface of crack generation in this paper, it is difficult to apply strain softening to the node at the crack interface is, so once a crack has occurred, it is assumed that the concrete ultimate tensile strength becomes zero. In this paper, the displacement is generated parallel to and a cross the crack surface, as shown in **Fig.5**. The crack opening is calculated using the formulation given by expression (4.1), and this crack opening is assumed to be the crack width at the concrete surface due to corrosion of the reinforcing bar.

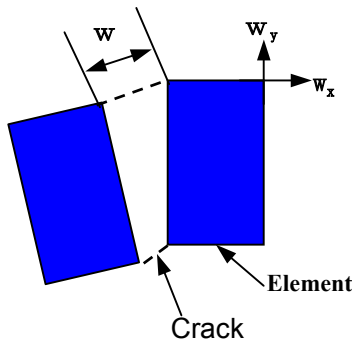


Fig.5 Calculation model for crack width

$$w = \sqrt{w_x^2 + w_y^2} \quad (4.1)$$

In which,

w : crack opening (mm).

w_x : crack displacement in parallel direction (mm)

w_y : crack displacement in orthogonal direction (mm)

Using the above expression, the relationship between crack width and degree of reinforcing bar corrosion can be calculated.

5. Comparison of numerical analysis and experimental result

A basic formulation of the relationship between crack width and degree of corrosion after cracking and degree of corrosion at crack occurrence was given in the 4 section above, and analytical method using FEM. was also described. Here, validity of the analysis is verified through comparison with experiment results obtained up to now.

(1) Object of Analysis

The experimental concrete body^{10,11)} as shown in **Fig.6** is adopted as the object of analysis. It is assumed that the body contains no stirrups and that expansive displacement acts on the concrete due to corrosion expansion of the reinforcing bar. The parameters for the analysis are shown in **Table-1**. To verify the validity of the analytical method as regards progress of cracking and crack generation, the analysis is performed for the experimental specimens so far tested.

As shown in **Table-1**, Case1 and Case3 are square specimens with one embedded reinforcing bar. Case2 is a rectangular beam specimen with two embedded reinforcing bars. Case1 and Case3 are fully modeled in the analysis. Case2 is modeled as a 1/2 of specimen. Displacement in the horizontal direction of symmetry is restrained. The analysis method gives uniform displacement of the concrete surrounding the reinforcing bar.

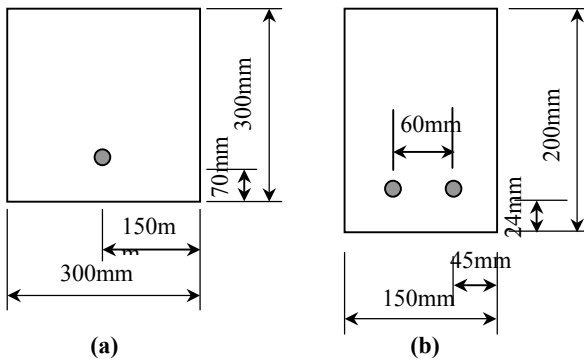


Fig.6 Specimen dimensions for analysis

Table 1 List of parameter used for analysis

Case	Cover c (mm)	Reinforcing bar		Concrete	
		Number	Diameter d (mm)	Compressive strength (N/mm ²)	Tensile strength (N/mm ²)
Case1	70	1	19	30.5	2.63
Case2	24	2	16	35.6	3.2
Case3	25	1	19	30.5	2.63

(2) Analytical result

a) Verification of crack composition model

A typical example comparing the relationship between tensile stress and strain for estimate value of the virtual crack element on circular direction and secondary former element analytical result is shown in **Fig.7**. This figure gives the results of Case1 shown in **Table-1**. In the figure, the relationship between mean stress and strain obtained from FEM analysis is almost the same as the relationship obtained from the composite model described in Section 3.(1). The two correspond closely in the large strain range. From these results, it is judged that the composite model for FEM analysis considering the tensile softening characteristic is appropriate.

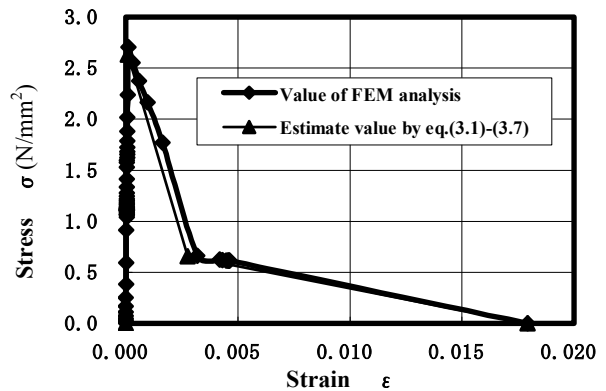


Fig7 Strain-Stress relationship

b) Progress of cracking and verification of occurrence

To verify the validity of analysis as regards crack occurrence and progress, analysis is carried out for specimens containing one reinforcing bar and two reinforcing bars, respectively. The crack occurrence situation is in accord with the experimental result obtained so far, as shown in **Fig.8**. The gray region in this figure indicates where the principal strain is small. The white region is the area of large principal strain. As is clear from **Fig.8**, when the displacement applied to concrete surrounding a reinforcing bar, or in other words the tensile strain ϵ_p of concrete surrounding the reinforcing bar (described in Section 3), is small, the crack progresses gradually towards the concrete surface. Further, when the applied displacement (or tensile strain) is large, the strain of concrete surrounding the reinforcing

bar is larger than ε_p , and the main crack occurs at the concrete surface. The crack generation on the analysis of Case1 is shown in the white area of **Fig.8 (a)**. At the 30th step, compulsive displacement occurs, and radial crack is generated in the direction of the concrete surface. Then, when the crack reaches the concrete surface following the analytical continues until the end. The calculation terminates due to the stress transmission end. At that time, besides the main crack crossing the shortest distance from the reinforcing bar to the concrete surface, another crack also progressed from the reinforcing bar to concrete surface. The crack configuration is almost like across.

Crack generation in the analysis and the experiment for Case2 is shown in **Fig.8 (b)**. The crack progresses toward the concrete surface in the analytical results. It is admitted that this crack occurs among reinforcing bars. According to experimental results¹²⁾ so far, when the reinforcing bar spacing is large, the crack extending from the reinforcing bar to the concrete surface is the main crack. On the other hand, if the reinforcing bar spacing is small, cracks between the reinforcing bars are particularly notable. Crack generation as given by the analysis is similar to the experimental results. In the white region for the 28th step, though gives the compulsive displacement, strain increase is a little, this region becomes gray again in the 80th step. This analytical result shows that, it can judge the crack generation mode taking into account tensile softening characteristics using finite element analysis.

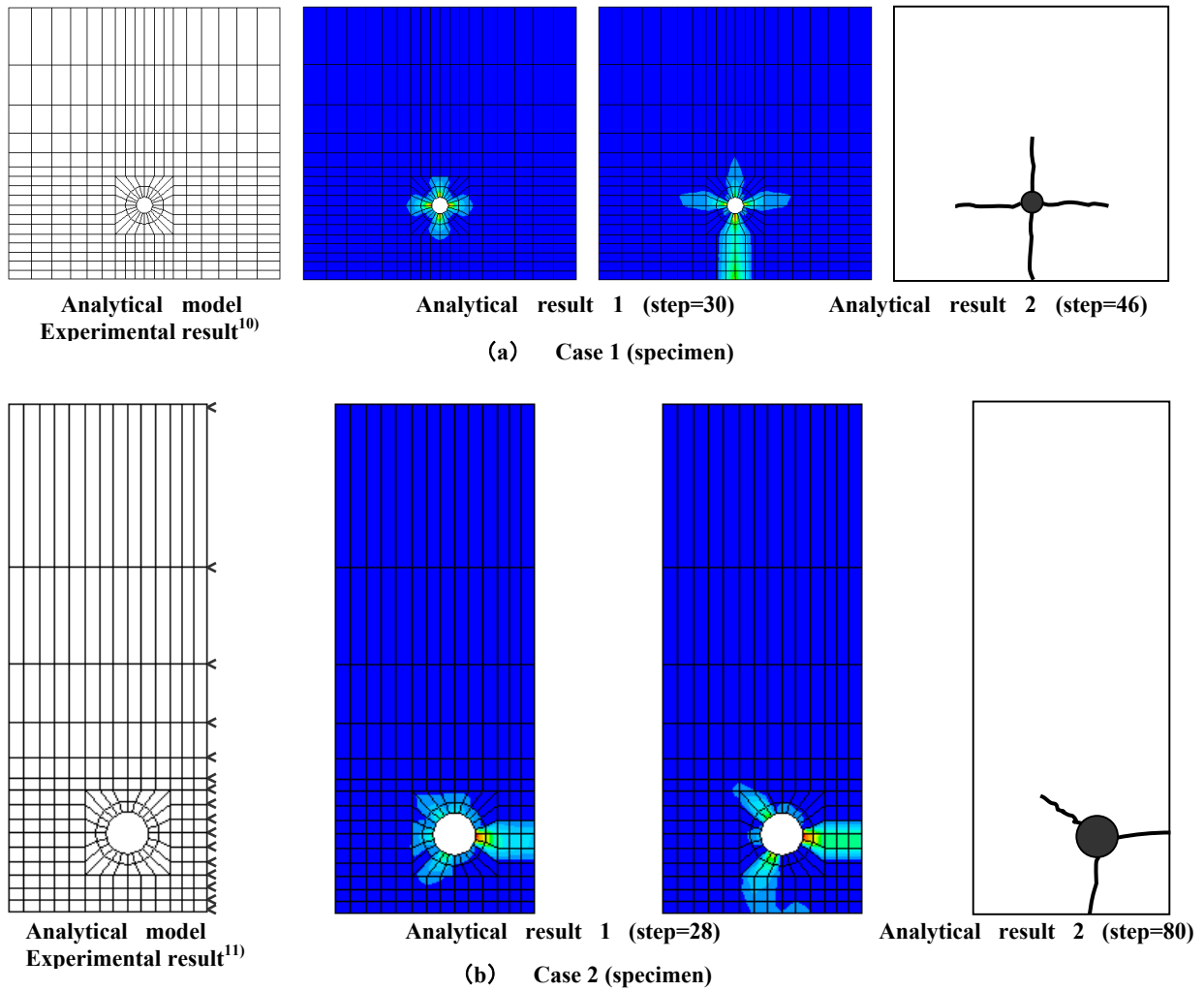


Fig.8 Verification of crack generation mode by finite element method

c) Verification of crack width and degree of corrosion

After determining the crack generation position and the number of cracks using the smeared crack model as described above, the relationship between crack width and degree of reinforcing bar corrosion is verified using the discrete crack model. The parameters used for the analysis are similar to those in **Table 1**. The expansion coefficient of the corrosion product is

assumed to be 3.0^{13}) **Figure.9** is an example of the results, showing Case1. It indicates the progress of cracking as reinforcing bar corrosion. To clearly show changes in crack width, it is intentionally expanded in the figure. A clear and definite crack progresses from within

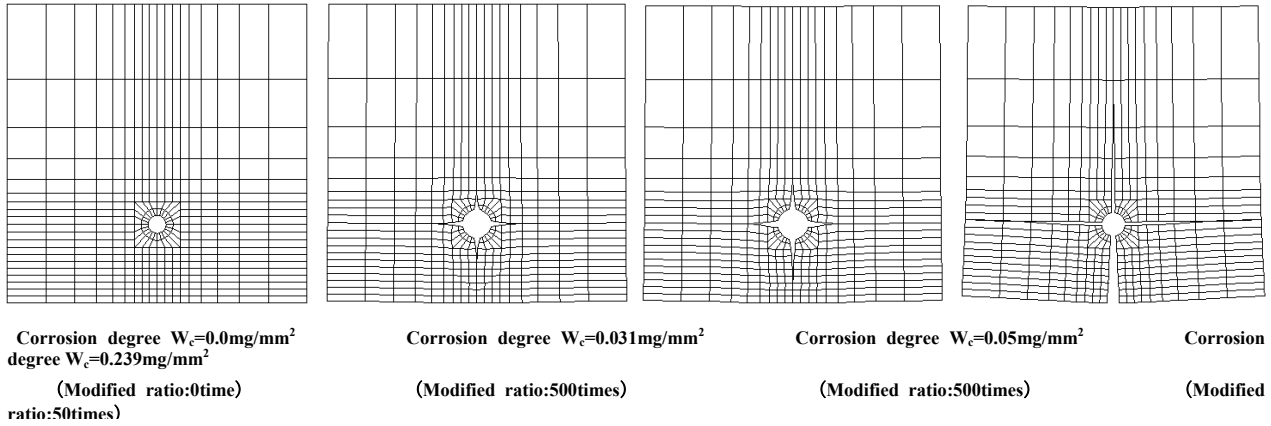


Fig.9 Progress of cracking as reinforcing bar corrosion takes place

the concrete to the surface as reinforcing bar corrosion takes place. Prior to appearance of the crack at the concrete surface, the crack width near the reinforcing bar reaches a maximum (0.088mm).

Once the crack reaches the surface, the width at the surface exceeds that near the reinforcing bar. In this paper, since little experimental data has so far been collected, the relationship between crack width and degree of reinforcing bar corrosion is evaluated only for Case1 and Case3. The results are shown in **Fig.10** and **Fig.11**, respectively. Experimental and finite element analytical values follow similar trends in these figures. Though the tendency of an increase of the crack width and corrosive amount is corresponding to calculated value by using basic calculation formula (2.8), the value is small. Because set up the basic calculation model, was set up without the concrete of the cylindrical model outer, the restraint influence on the cylindrical model is not considered, and calculated values using the basic calculation formula becomes small. Moreover, other cracks that occurred within the concrete during the growth process of the main crack are omitted in **Fig.8** and **Fig.9**. However, these crack are not considered in the basic calculation formula, (In addition,) does not include the influence on concrete tensile softening etc. the corrosive amount has become small in the same crack width.

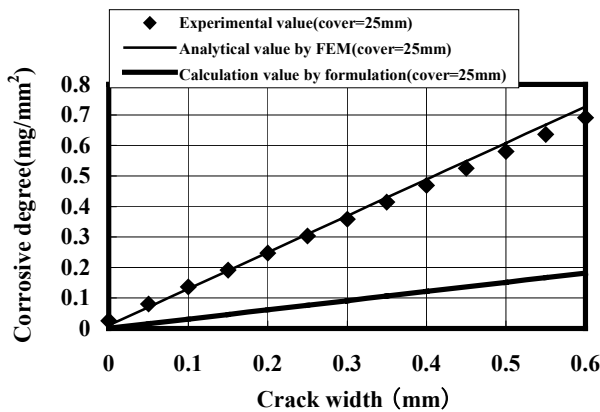


Fig.10 Crack width vs corrosive degree (1)

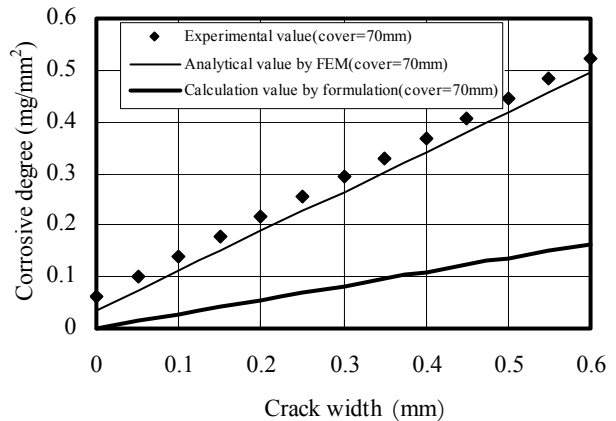


Fig.11 Crack width vs corrosive degree (2)

6.Examination of corrected formulation

The process of crack generation and the relationship between crack width and degree of reinforcing bar corrosion with concrete tensile softening characteristics taken into account is studied respectively due to corrosion of reinforcing bar until 5th

chapter. It was shown that values obtained by finite element analysis agreed well with experiments. However, this type of analysis is very difficult in general, we try to correct expression (2), to offer a simplified formulation. To examine the corrected formulation, parametric analysis is performed.

As shown in **Fig.8**, the number of cracks differs with the number of reinforcing bars. The number of cracks has an effect on the relationship between degree of reinforcing bar corrosion and crack width. In this paper, analysis is carried out for a specimen including one reinforcing bar and the results are compared with experimental data.

The object of analysis objection is the specimen shown in **Fig.6 (a)**. It is assumed that four cracks occurred according to the analytical results in Section 4. The models shown in **Fig.8 (a)** take as the object of analysis. The concrete cover, the reinforcing bar diameter, and the concrete compressive strength are assumed to be this analytical factor, and combined these analytical factors and perform analysis. The value of each parameter used for analysis is shown in **Table2**. The analytical condition is similar to that in Case1 describe in Section 5.

(1) Revision of relationship between crack width and degree of reinforcing bar corrosion

a) Revised formulation for influence of cover thickness an reinforcing bar diameter

The slope of the straight line obtained from finite element numerical analysis for cover thickness of 30,50, and 70mm, reinforcing bar diameters of 12,18, and 24mm, and a concrete compressive strength of 20 N/mm² is used to try to correct the basic formulation. The regression procedure used is first described. The slope of the straight line representing the relationship between crack width and degree of reinforcing bar corrosion obtained by the finite element method, W_{eles} , is divided by the slope obtained using the basic formulation, W_{cals} to give coefficient α_i . The ratio is evaluated for each analytical factor. For instance, for the concrete cover, and a reinforcing bar diameter of 12mm shown in **Fig.12**, the coefficient is described by regression formula α_{12} . Next, using this slope and the cutoff of this regression formula α_{12} , α_{18} , and α_{24} , regression formulation of the influence on the diameter of the reinforcing bar is evaluated. As shown in **Fig.13**, the result is regression formulations a_1 and b_1 for the diameter of the reinforcing bar. The regression formulation (6.1) can be obtained by summarized regression formulation a_1 , b_1 , and regression formula α_{12} , α_{18} , and α_{24} . The regression formulation (6.1) here take as revision coefficient α_1 concerning concrete cover and the diameter of the reinforcing bar.

$$\alpha_1 = (0.0007d - 0.04)c + (-0.0663d + 5.92) \quad (6.1)$$

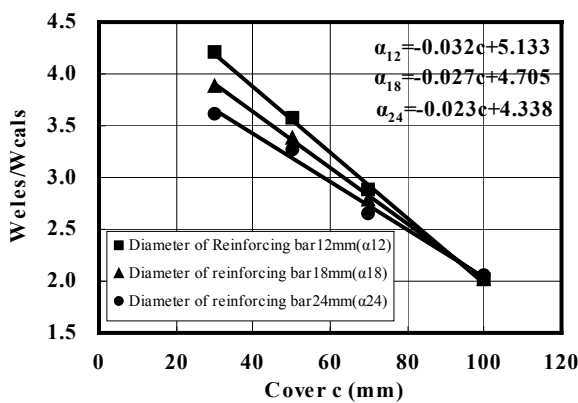


Fig.12 Corrected formulation for cover

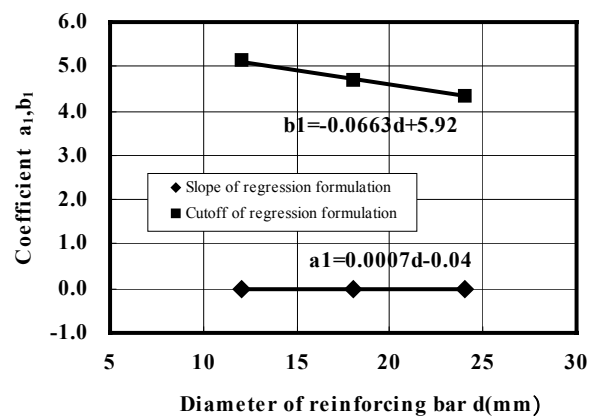


Fig.13 Corrected formulation for diameter

b) Revised formulation for concrete compressive strength

The influence on concrete cover and the reinforcing bar diameter is removed by adopting the degree of corrosion W_{eles} obtained from finite element analysis to $\alpha_1 W_{cals}$ (basic calculation value). The relationship between concrete compressive strength and the ratio W_{eles}/W_{cals} is shown in **Fig.14**. The regression formulation (6.2) obtained here for revision coefficient

β_1 concerning the concrete strength.

$$\beta_1 = -0.0016f'_c + 1.04 \quad (6.2)$$

(2) Revision of corrosion degree at crack occurrence

The cutoff of the straight line obtained from finite element analysis takes as the degree of reinforcing bar corrosion at crack occurrence, and the basic formulation is revised using this numerical value. The regression method is similar to that described in Section 6.1). The results are shown in Figs.15, 16, and 17. The revised degree of reinforcing bar corrosion at crack occurrence obtained in this way is as described by the following expressions (6.3) and (6.4).

$$\alpha_0 = (-0.0005d + 0.028d)c + (-0.0292d + 1.27) \quad (6.3)$$

$$\beta_0 = -0.0055f'_c + 1.07 \quad (6.4)$$

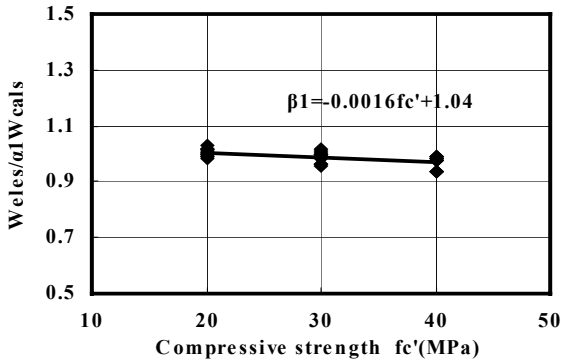


Fig.14 Formulation corrected on compressive strength

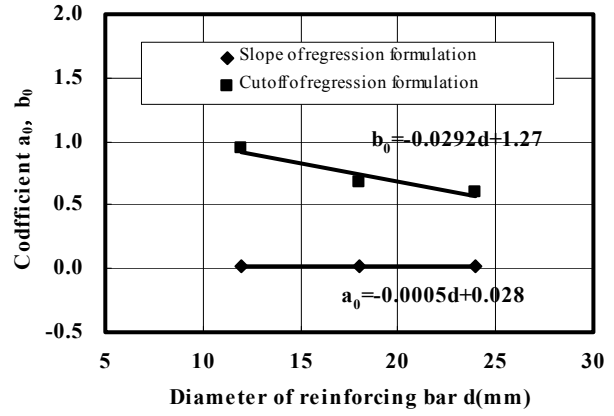


Fig.16 Corrected formulation for diameter

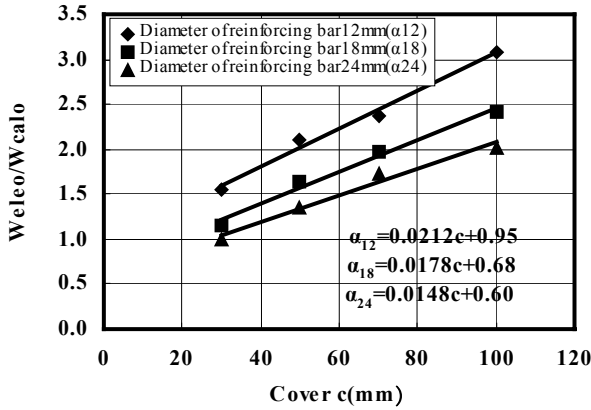


Fig.15 Corrected formulation for cover

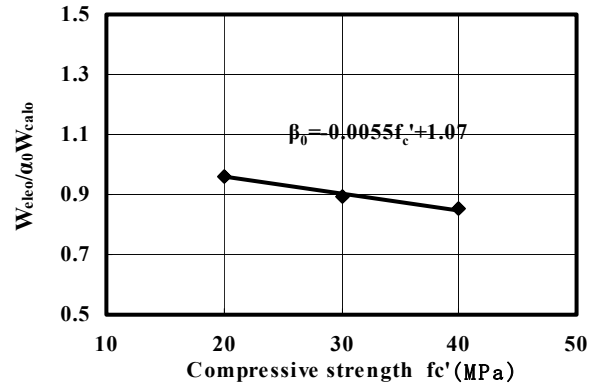


Fig.17 Corrected formulation for compressive strength

(3) New formulation of degree of corrosion

Expression (6.5) can now be written as the corrected formulation for the relationship between degree of reinforcing bar corrosion and crack width based on expression (2.8), and expressions (6.1) to (6.4).

The relationship between degree of reinforcing bar corrosion and crack width can be calculated from this new formulation by introducing concrete cover c , diameter d of the reinforcing bar, and concrete compressive strength f'_c .

$$W_c = \frac{\gamma}{\pi(\rho - 1)} \left(\alpha_0 \beta_0 \frac{0.22\{(2c + d)^2 + d^2\}}{E(c + d)} f_c^{2/3} + \alpha_1 \beta_1 \frac{(c + d)}{(5c + 3d)} w \right) \quad (6.5)$$

In which,

W_c : degree of reinforcing bar corrosion (mg/mm²), γ : density of steel (7.85mg/mm³),

ρ : expansive ratio of corrosion product, c: concrete cover(mm), d: diameter of reinforcing bar(mm)

w: crack width(mm), E: young's modulus of the concrete, f_c : compressive strength of the concrete(N/mm²)

Correct-ion factors at crack occurrence:

α_0 :correction factor for concrete cover and diameter of reinforcing bar $=(-0.0005d+0.028)c+(-0.0292d+1.27)$

β_0 :correction factor for compressive strength of the concrete $= -0.0055 f_c'+1.07$

Correct-ion factors after cracking:

α_1 : correction factor for concrete cover and diameter of reinforcing bar $=(0.0007d-0.04)c+(0.0663d+5.92)$

β_1 : correction factor for the compressive strength of the concrete $= -0.0016 f_c'+1.04$

(4) Verification of corrected formulation

The corrected calculated values and experimental values^{4),10),14)} obtained from expression (6.5) for the relationship between degree of reinforcing bar corrosion and crack width are shown in **Fig.18**. The average ratio of corrected value W_{mocal} to experimental value W_{exp} is 0.9, and the coefficient of variation is 13%. Thus the two sets of results are similar.

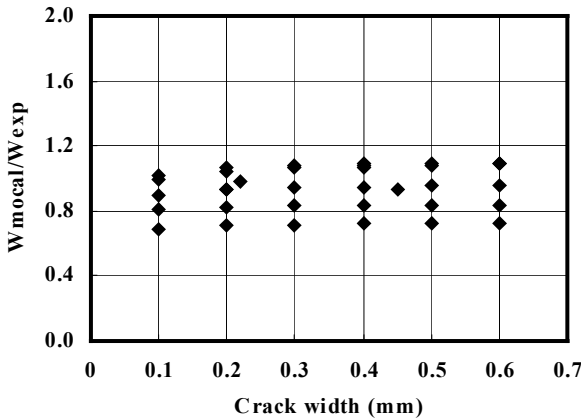


Fig.18 Comparison of new calculated values and experimental values

7.Conclusion

The progress of the crack generation within concrete due to reinforcing bar corrosion, in other words, the numerical analysis on the crack generation position and crack numbers, the relationship between the corrosion amount and the crack width was performed. The results were compared with experimental values obtained to date. This work leads to the following conclusions.

(1) Using the smeared crack model taking into account model tensile softening characteristic, it is possible to analytically clarify the progress of crack generation and use numerical analysis to obtain crack position and crack count. It is confirmed that the crack generation mode can be obtained, and the results agree with experiment.

(2) Crack generation position and crack count, as obtained using the smeared crack model, are assumed to be the cracks that occur in the discrete crack model, so this method make it possible to evaluate the relationship between crack width and degree of reinforcing bar corrosion. It is confirmed that the analytical results indicate a similar tendency as experimental results.

(3) By using the new formulation and incorporating various factors such as the cover thickness, the reinforcing bar diameter, and the concrete compressive strength, it is easy to calculate the relationship between crack width and degree of reinforcing bar corrosion.

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