RELIABILITY-BASED OPTIMAL DESIGN IN CONSIDERATION OF STRUCTURAL SYSTEM AND ITS APPLICATION TO EVALUATION OF SEISMIC PERFORMANCE OF RC BRIDGE PIERS

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Performance-based design is set to replace conventional methodologies. In this paper, design flow charts capable of handling any arbitrary level of safety are formulated using reliability-based optimal design theory by taking the structural system into consideration. These charts are then applied to the seismic design of rigid-frame bridge piers and RC bridge piers (single-column types) designed according to the current code so as to examine their seismic performance. It is confirmed that (1) RC bridge piers can be designed with the prescribed reliability using the proposal method and (2) the safety of bridge piers designed according to the current code is not uniform.

Key Words : *system reliability theory, structural optimization, seismic performance, RC bridge pier*

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1. INTRODUCTION

The need to replace conventional methods with performance-based design systems is well recognized. As such systems come into use around the world, reliability theory will be used for the safety evaluation of structures [1], ensuring that uncertainties in design variables (such as variability in material properties and the accuracy of structural analysis) is taken into account. As a result, an assured level of safety will be realized by reducing the probability of undesirable outcomes (referred to as the failure probability) below a target value (referred to as the target failure probability). By employing the concept of a failure probability in this way, it will be possible to evaluate the safety of a structure quantitatively. Above all, this design method enables one to maintain a firm grasp on the safety level desired, and to design a structure so as to realize this prescribed reliability independently of other design requirements.

The first step in developing such a reliability-based design method is to define the limit states. For example, in the case of the rigid-frame bridge analyzed in this study, seismic performance is checked by investigating the limit states of shear failure, ductility, and residual deformation. In conventional (determinate) design, the section may be designed by considering the dominant limit state. However, in general, using just one limit state to represent a failure event which in reality comprises a plurality of limit states can lead to a risky evaluation. Even if the design satisfies the target failure probability for the limit state considered, the structure may in fact not meet the prescribed safety requirements. To ensure the prescribed reliability for such a rigid-frame bridge, there is a requirement to calculate the failure probability in consideration of structural reliability. And because there may be many sets of design variables (solutions) that meet the reliability requirements, criteria for choosing for the most desirable solution must also be laid down. That is, in accordance with the limit condition that system reliability exceeds the prescribed value, an optimal structural design method must be introduced to minimize (or maximize) an objective function.

Adopting this approach, a new design scheme based on the safety evaluation method for structural systems proposed by Suzuki et al. [2] and a method of optimal design using the SQP method [3] is proposed in this study. This scheme makes it possible to ensure that the failure probability of a designed structural system is below a target value while minimizing (or maximizing) an objective function. The scheme is applied to the seismic design of the piers of a prestressed concrete rigid-frame bridge and an RC bridge pier (single-column type). The effect on the designed sections of setting a target probability and the difference in failure probability of the piers of the rigid-frame bridge and the single-column piers is studied.

Analysis of seismic risk is not considered here, so the values do not take into account, for example, the severity of earthquakes likely to occur during the structure's lifetime. We note, therefore, that the results obtained apply only when the level II earthquake motions [4] act on the piers, and so are in that sense 'conditional'.

2. PROPOSED RELIABILITY-BASED OPTIMAL DESIGN IN CONSIDERATION OF STRUCTURAL SYSTEM

2.1 Previous Studies

The objective of structural optimization is to increase the efficiency of the search for solutions that satisfy the initial requirements within the limit conditions. The initial requirements have, until now, been such demands as minimal structural weight or cost, etc.

On the other hand, international design standards such as ISO (International Organization for Standardization) will lead to the introduction of reliability-based design methods. Reliability-based design is founded on the concept that one can estimate the probability of an undesirable outcome, such as a fracture, during the lifetime of a structure, despite the uncertainties involved. It is a design method that provides an assured level of safety by reducing the probability of such an occurrence (the failure probability) below a target failure probability. Firstly, the failure probability is calculated using some safety evaluation method for structural systems; for example, Ditlevsen's bounds [6] or the PNET method [7]. Then random (design) variables are modified as the failure probability of the designed structure approaches the target probability. This modification of random variables is, however, very difficult as it depends on a comparison of the failure probability by the conventional method and the target probability. That is, the designer must find the design point of each random variable for multiple limit states and ensure that the failure probability calculated for these design points is below the target probability.

Further, there may be many sets of solutions that achieve the target probability. Even if the method of safety evaluation proposed by Suzuki et al. [2] is applied to the failure probability method, the same problems arise.
In introducing the optimal structural method mentioned above, a limit condition is defined such that the failure probability of the structure is below a target probability. And an objective function is defined that can be used to determine the best solution from sets of random variables satisfying the limit condition. This makes it possible to provide an assured level of safety by reducing the probability of an unwanted outcome below a target value despite the uncertainties involved. The scheme for finding values of the random variables that minimize (or maximize) the defined objective function is managed using a flow chart.

There have been many previous studies on optimal structural design with limit conditions for failure probability. For example, Mihara et al. tried to formulate the problem of plastic optimization based on a rational reliability constraint for rigid frame structures [8] and Frangopol studied the optimal design of a 1-story rigid frame for minimum weight in consideration of structural system reliability according to Ditlevsen's method [9]. Enevoldsen and Sorensen formulated a number of reliability-based optimization problems: the reliability-based optimal design of structural systems with component- or system-level reliability constraints; reliability-based optimal inspection planning; and reliability-based experiment planning. Kitahara et al. [11] and Sorensen et al. [12] proposed a rational procedure for determining design load combinations and safety formats. In their scheme, load factors or safety factors associated with each design load combination in a safety format are optimized numerically using an objective function. As a result, a uniform level of structural reliability meeting the target, as evaluated by limit state probability, is realized — regardless of the structural type and dimensions. However, the accuracy of structural system reliability as adopted in these studies has never been discussed. In all of these cases, system reliability was represented by the limit state which is most likely to arise; i.e. the one with the highest failure probability or determined by conventional major calculation methods of failure probability like Frangopol [9].

To use just one limit state to represent a failure event which in reality comprises a plurality of limit states results in a risky evaluation. Conventional methods of calculating failure probability also have problems, in that the calculated failure probability depends heavily on the conditions set during calculation; depending on the order of the failure probability, a result that errs too far on the risky side may be produced [13].

Moreover, all the reliability design methods proposed so far have been applied to structures that are too simple. Studies considering the non-linear response of real structures subjected to major earthquakes have been reported only by Akiyama et al. [14] and Shiraki et al. [15]. Akiyama et al. evaluated the horizontal seismic coefficient for cast-in-place piles taking into account uncertainties in the reinforced concrete bridge piers, while Shiraki et al. reported on the seismic safety evaluation of steel rigid-frame piers based on push-over analysis. However they evaluated structural reliability only, and a design method capable of attaining a prescribed level of reliability was not proposed.

In conclusion, previous studies of structural optimal design based on reliability theory all suffer from problems with the accuracy of the failure probability calculation, and none have proposed a design method using reliability-based optimization that seeks to attain a prescribed level of reliability for concrete structures subjected to major earthquakes, such as the Hyogo-ken-Nanbu earthquake (1995).

2.2 Formulation of Structural Optimal Design Based on Structural Reliability Theory

A method of calculating the failure probability of a structural system was established by M. Suzuki for use in a situation where the failure event involves multiple correlated limit states [2] (referred to as the reliability evaluation method for structural systems). This technique substantially facilitates calculation; its superiority, both in time and accuracy, over earlier calculation methods has been confirmed. However, where the failure probability determined by the reliability evaluation method does not meet the target failure probability, the difficulty of calculation means that the designer is unable to modify the design variables so as to achieve the desired reliability. An optimizing method based on the limit condition that system reliability exceeds the prescribed reliability limit is proposed in this study. This optimal structural design method based on the reliability evaluation of structural systems is expressed as follows.

$$\text{find } \{\mathbf{D}\} \tag{1}$$

$$\text{such that } Pf_{sys} = Pf_{sys}(\mathbf{D},\mathbf{X},\mathbf{Z}) \le Pf_{all} \tag{2}$$

$$\text{and } W = W(\mathbf{D}) \to \min \tag{3}$$

$$D_i^l \leq D \leq D_i^u \tag{4}$$

$$f_i(\mathbf{D}) \leq 0, \quad (i = 1, \cdots, m) \tag{5}$$

$$h_j(\mathbf{D}) = 0, \quad (j = 1, \cdots, l) \tag{6}$$

where, W : target function, D : design variables, X : probability variables of capacity and response obtained from structural analysis, Z: fixed variables (such as the pier height), D_i^u, D_i^l : upper and lower limits of design variables, Pf_{sys} : failure probability obtained from the reliability evaluation method for structural systems, Pf_{all} : the target probability, f, h : equality and inequality limit conditions.

The above formulation is a general optimal problem in which none of the design variables aside from equation (2) are taken to be random variables. Equation (2) is the same limit condition as equation (5) in the optimal problem. However, although equation (5) can be evaluated using the results of structural analysis, equation (2) requires reliability analysis in consideration of the variability of $\mathbf{D}$ and $\mathbf{X}$ for judging its condition. Thus, equation (2) is distinct from equation (5). Design variables $\mathbf{D}$ that minimize or maximize the target function W can be computed by ordinary optimal methods except in the case of equation (2), which requires reliability analysis. There is a wide choice of mathematical programming techniques for obtaining optimal solutions, depending on the degree of the structural model [16]. In this study, the SQP (Sequential Quadratic Programming) method [3] is used because it is suitable for nonlinear optimization programming.

The process of optimal structural design using reliability theory may be described as follows.
1) Assign initial values to design variables $\mathbf{D}$ and define the possible limit states given the assumed design load.
2) Define the target probability, target function, and limit conditions.
3) Perform structural analysis to compute the gradients of design variables $\mathbf{D}$ for the target function and limit conditions.
4) Compute the structural failure probability using the reliability evaluation method for the defined limit states using the strengths and loads (responses) calculated in structural analysis. Because the failure probability using the reliability evaluation method for structural systems cannot be expressed explicitly for design variables $\mathbf{D}$, the first-order derivatives of Pf_{sys} are calculated using equation (7).

$$\frac{\partial f_i}{\partial D_k} = \frac{f(D_k + \Delta(D_k)) - f(D_k - \Delta(D_k))}{2 \times \Delta(D_k)} \tag{7}$$

In this study, all first-order derivatives for target functions and other limit conditions are calculated based on equation (7).
5) Calculate the next design variables $\mathbf{D}$ based on the SQP method.
5-1) Give the Hessian matrix for design variables $\mathbf{D}$.
5-2) Minimize equation (8) for $\mathbf{d} \in R^n$ under the linear limit condition $f_i(\mathbf{D}) + \nabla f_i \mathbf{Dd} \leq 0$, $h_j(\mathbf{D}) + \nabla h_j \mathbf{Dd} = 0$.

$$\frac{1}{2}\mathbf{dBd} + \nabla W(\mathbf{D})\,\mathbf{d} \tag{8}$$

Here, Lagrange's multiple vector of solutions $\mathbf{d}$ for equality and inequality limit conditions are assumed to be $\boldsymbol{\lambda}, \boldsymbol{\mu}$ respectively.
5-3) If $\mathbf{d}, \boldsymbol{\lambda}, \boldsymbol{\mu}$ meet the Karush-Kuhn-Tucker condition for minimization under the limit condition, the design process is complete. If not, a distance of travel $\boldsymbol{\alpha}$ is chosen and design variables $\mathbf{D}$ for the next step are calculated using equation (9).

$$\mathbf{D} \leftarrow \mathbf{D} + \boldsymbol{\alpha}\,\mathbf{d} \tag{9}$$

5-4) Steps 3) through 5) are repeated until $\mathbf{d}, \boldsymbol{\lambda}, \boldsymbol{\mu}$ meet the Karush-Kuhn-Tucker condition.

2.3 Example

In this section, the method outlined above is applied to the design of the redundant structures by Frangopol [9] shown in Fig. 1. Frangopol presented a mathematical formulation of this problem as follows.

$$\text{find } \left\{ \mathbf{M}(\mathbf{M}_B, \mathbf{M}_C) \right\} \tag{10}$$

such that $Pf_{sys} = Pf_{sys}\left(\overline{\mathbf{M}}, \mathbf{X}, \mathbf{Z}\right) \leq Pf_{all}$ (11)

and $W = W\left(\overline{\mathbf{M}}, \mathbf{Z}\right) \rightarrow \min$ (12)

$\mathbf{M_B} = (M_3, M_4, M_5)$ (13)

$\mathbf{M_C} = (M_1, M_2, M_6, M_7)$ (14)

where, $\overline{\mathbf{M}}$: probability variables representing capacity (plastic moment), $\mathbf{X}$: probability variables representing external forces P and H, $\mathbf{Z}$: fixed values, with assumption that the length of beams is 10m and the length of columns is 4m.

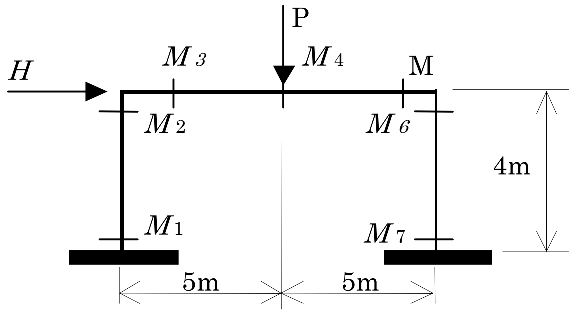


Fig. 1 One-Level Rigid- Frame Structure Used for Analysis [9]

When the formation of a mechanism is taken as a collapse, rigid frame structures have a number of limit states, and some of these may occur at a similar loading. When calculating the failure probability given by equation (1), a proper consideration of the correlation between these limit states is necessary. Here, the failure probabilities used in the proposed scheme are calculated by the reliability evaluation method for structural systems, first-order bounds [13], Ditlevsen's bounds, and the PNET method. The sensitivity of the optimum solution to calculation of the failure probability by these different methods is studied. For the sake of reference, Frangopol used Ditlevsen's bounds and the PNET method to calculate failure probability.

Mean values and coefficients of variation used in calculating failure probability are as follows:

Mean values: $\overline{M_3} = \overline{M_4} = \overline{M_5} = \overline{M_B}$, $\overline{M_1} = \overline{M_2} = \overline{M_6} = \overline{M_7} = \overline{M_C}$, $\overline{P} = 80\text{kN}$, $\overline{H} = 20\text{kN}$

Coefficients of variation : $V(M_i) = 0.10, (i = 1,\cdots,7)$, $V(P) = 0.20$, $V(H) = 0.15$

The purpose here is to obtain the mean values of plastic moments $\overline{M_B}$ and $\overline{M_C}$. Two resistance correlations are assumed: independence among all plastic moments and perfect correlation among all plastic moments. There is no correlation between the gravity and lateral loads. To compare the accuracy of the above methods of calculating failure probability, an example calculation by Ditlevsen's bounds and the reliability evaluation method for structural systems for the portal frame shown in Fig. 1 is given below.

Ditlevsen' bounds	$3.83\times10^{-3} \leq Pf_{sys} \leq 4.92\times10^{-3}$
The reliability evaluation method for structural systems	$Pf_{sys} = 4.10\times10^{-3}$
Monte Carlo Simulation (number of samples n = 50,000)	$Pf_{sys} = 3.99\times10^{-3}$

The above failure probabilities are derived from $\overline{M_B} = \overline{M_C} = 170\text{kNm}$ assuming independence among all random variables. The result given by the reliability evaluation method for structural systems accords with that obtained by the Monte Carlo simulation.

The objective function (equation (12)) given by Frangopol can be expressed by equation (15), which for brevity is called the weight function.

$$W = 10\overline{M_B} + 8\overline{M_C} \tag{15}$$

Based on the assumptions above, Fig. 2 shows values of W versus the ratio $\overline{M_B}/\overline{M_C}$ when the allowable probability of failure is 10^{-5}. This allowable probability is adopted so as to match the analytical conditions with those used by Frangopol. When using first-order bounds and Ditlevsen's bounds, system reliability is approximated by an upper bound or a lower bound. Results based on the adoption of the upper bound method and the lower bound method for both the first-order bounds and Ditlevsen's bounds system are also shown in Fig. 2.

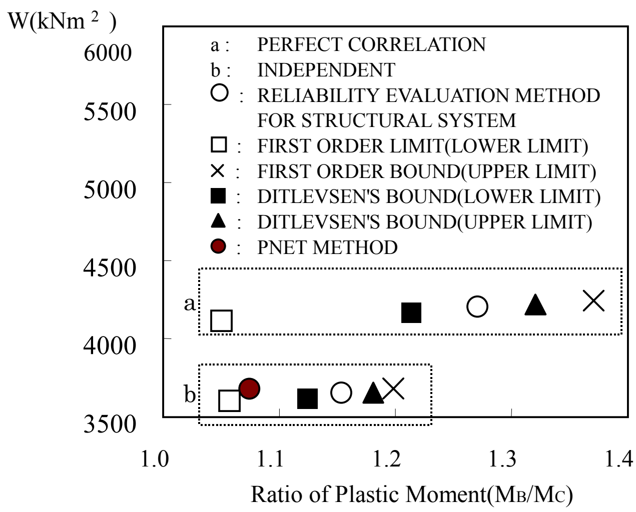


Fig. 2 Sensitivity of Optimum Solution to Changes in Failure Probability Calculation Method

As Fig. 4 shows, the different methods of calculating failure probability have little effect on the optimum weight, while and changes in the strength correlation

cause appreciable variation in optimum weight. However, the ratio of mean beam and column optimum plastic moments, $\overline{M_B} / \overline{M_C}$, is sensitive to the different calculation methods.
The above example and a previous comparison [2] allow us to verify that the reliability evaluation method for structural systems gives better results than other methods and accords with the results obtained by the Monte Carlo simulation. In contrast, a designer who uses Ditlevsen' bounds to evaluate the failure probability may end up with a structural system that does not satisfy the target probability and that has a false distribution of plastic moments of columns and beams. If the order of the failure probability is greater than 10^{-5}, the upper and lower limits of first-order bounds and Ditlevsen's bounds may differ widely from each other. And when the allowable probability of failure reaches 10^{-1} or 10^{-2}, it is confirmed that the ratio $\overline{M_B} / \overline{M_C}$ is much more sensitive to differences in calculation method. In the following section, the target failure probability in the optimal design of a PC rigid frame bridge pier considering level II earthquake motion needs to be greater than $Pf_{all} = 10^{-5}$. The optimum solution is sensitive to the choice of calculation method for failure probability. In the flow chart for the proposed scheme, a change of failure probability calculation method to Ditlevsen's bounds or PNET yields the same solutions as Frangopol's. Therefore, all parts of the process aside from failure probability calculation should give the same results as previous studies.

Even though this was very simple example, it is confirmed that the proposed scheme using the reliability evaluation method for structural systems is a very effective way to achieve structural design with a prescribed reliability. In the following sections, the proposed scheme is applied to the seismic design of the pier of a PC rigid-frame bridge and to an RC bridge pier. Further, and a comparison between the proposed design method and the Japanese specifications for highway bridges is are carried out.

3. APPLICATION OF THE PROPOSED DESIGN METHOD TO SEISMIC DESIGN OF PC RIGID-FRAME BRIDGE PIER

3.1 Rigid-Frame Bridges Adopted for Analysis

PC rigid-frame bridges are analyzed, as shown in Fig. 3. The ground is classified as Group II [4]. The piers are RC structures of height 14m and 28m for Bridge A and Bridge B, respectively. The superstructure of both bridges is as given in "samples for the seismic design of highway bridges [17]". In the longitudinal direction, the superstructure s idealized by a linear beam element and the plastic hinges of piers are idealized by a non-linear rotational spring so as to simulate hysteretic behavior. Other parts of the piers are idealized by linear elements with yield stiffness. In the transverse direction, the rigid-frame piers are idealized as single column piers and the mass of the superstructure is lumped on top of the pier. In fact, variations in axial force must be considered when the flexural stiffness of elements in a rigid-frame structure is evaluated. However, variations in axial force have no effect on the moment-versus-curvature relationship of the rigid-frame bridges analyzed in this study because of their scale and shape. Therefore, the flexural stiffness of each bridge pier element is evaluated based on the moment-versus-curvature relationship calculated from the dead load.

The allowable level of safety is a very important issue in structural design based on reliability theory. The simplest methods of determining are code calibrations performed by fitting. The fitting of codes is typically used when a new code format is introduced and the parameters in the new code are determined such that the same level of safety is achieved as with the old code. In this example, the bridge piers subjected to near-fault motions are first

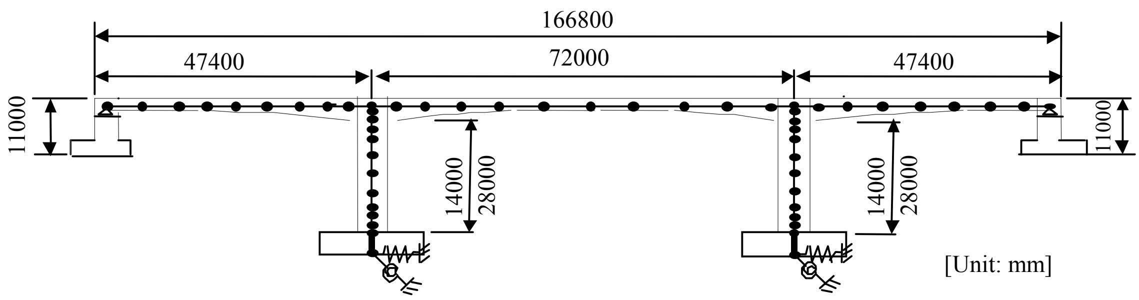


Fig. 3 PC Rigid Frame Bridge Used for Analysis

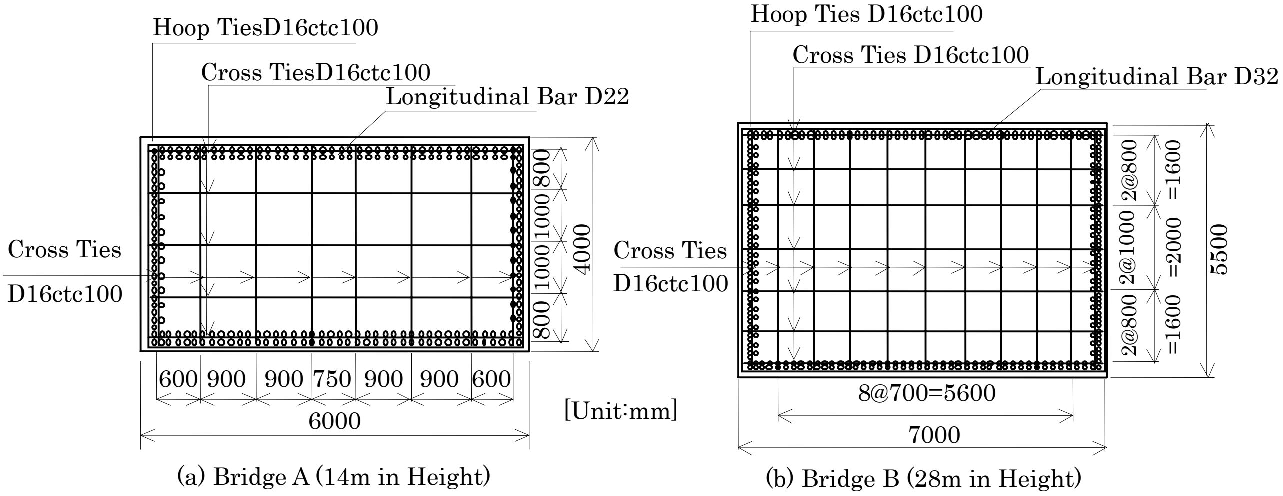


Fig. 4 Sectional Area and Bar A rrangements of the Bridge Piers Designed

designed according to the current specifications for highway bridges [4] to give a design that is neither too safe nor lacking in safety. Longitudinal bars are in a double arrangement, with diameter between 16mm and 32mm. The diameter of hoop ties is between 13mm and 19mm, and the pitch is 10cm or 15cm. Other specifications are based on Ref. 4. The section, axial reinforcing bars, and hoop ties in the longitudinal and transverse directions are modified to satisfy the demands of the specifications. In Fig. 4, the sectional area and bar arrangements of the designed bridge piers designed are illustrated. Other particulars and assumed material characteristics are given in Table 1. The ultimate strength, failure mode, earthquake response, and other characteristics calculated as in Ref. 4 are given in Table 2. Table 2 also shows the failure probabilities (safety indexes) of Bridge A and Bridge B calculated by the reliability evaluation method for structural systems. These failure probabilities are based on the performance function, as described later. The failure probability in this study is much more than in ordinary reliability analysis in consideration of the scale of earthquakes likely to occur during the structure's lifetime. Now, the failure probability Pf is converted into a safety index β.

Table1 Bridge Piers Used for analysis

		Bridge A	Bridge B
Section(m)		4.0 × 6.0	5.5 × 7.0
Longitudinal Bars		D22-228	D32-330
Hoop ties		D16@100	D16@100
Cross ties		D16@100	D16@100
Volumetric Ratio of Ties	LG	0.33%	0.37%
	TR	0.35%	0.33%
Concrete Compression Stress		23.5 (MPa)	
Elastic Coefficient of Concrete		2.45×10^4 (MPa)	
Yielding Stress of Bars		345 (MPa)	
Elastic Coefficient of Bars		2.06×10^5 (MPa)	

$$\beta \cong -\Phi^{-1}(Pf) \tag{16}$$

where, Φ : cumulative distribution function of standard normal distribution.

With these designs, the safety of Bridge B is less than that of Bridge A. That is, the magnitude of the safety margin shown in Table 2 is different in the case of Bridge A and Bridge B. For Bridge B, evaluations of flexural strength and residual displacement in the longitudinal and transverse directions are very close to the allowable values. On the other hand, for Bridge A, the section is designed by evaluation of flexural strength in the longitudinal direction. In practice, the section is usually designed with only one seismic performance evaluation, as with Bridge A. However, when the failure probabilities are calculated in consideration of multiple limit states, failure probabilities such as bridges that multiple evaluations are close to allowable values, are less than bridges that only one evaluation is close to allowable value like Bridge A.

As the seismic performance evaluation focuses only on the piers, failure probabilities are not calculated in consideration of the safety of the superstructure.

Table 2 Evaluation of Seismic Performance based on Ductility Check [4]

		Bridge A	Bridge B
LG	Natural Period (sec)*)	0.68	1.11
	Flexural Strength (MN)	16.2	23.7
	Shear Strength (MN)	26.4	47.4
	Failure Mode	Flexure	Flexure
	Equivalent Design Seismic Coefficient	0.56	0.70
	Seismic Coefficient at Ultimate State	0.58	0.71
	Allowable Residual Displacement (m)	0.16	0.30
	Residual Displacement (m)	0.11	0.28
	Evaluation of Safety	OK	OK
TR	Natural Period (sec)*)	0.55	0.90
	Flexural Strength (MN)	8.6	13.7
	Shear Strength (MN)	27.5	42.4
	Failure Mode	Flexure	Flexure
	Equivalent Design Seismic Coefficient	5.26	5.89
	Seismic Coefficient at Ultimate State	6.40	13.4
	Allowable Residual Displacement (m)	0.16	0.30
	Residual Displacement (m)	0.07	0.29
	Evaluation of Safety	OK	OK
Failure Probability Pf_{sys} (Safety Index β)		4.80×10^{-2} (1.67)	1.00×10^{-1} (1.26)

*) Calculated from Yield Stiffness of Piers

3.2 Setting Up the Performance Function

The seismic performance of a pier is evaluated in terms of shear failure, ductility, and residual displacement in the longitudinal and traverse directions. Performance functions g_i used to calculate failure probabilities are defined as follows ($i = 1 \sim 3$ in the longitudinal directio, $i = 4 \sim 6$ in the traverse direction).

a) Performance Function for Shear Failure

If the calculated strength is assumed to be a fixed value, the failure mode of piers for which shear strength exceeds flexural strength is flexural. In this case, the probability of shear failure is ignored. However, to accurately assess the safety of a designed structural system, the failure probability of a pier in which flexural failure is dominant should be calculated in consideration of shear failure also, since the uncertainty in shear strength is much greater than that in flexural strength. The performance function for shear failure given in equation (17) is used in the calculation of failure probability by the reliability evaluation method for structural systems. The subscript i on the right side is not expressed except in the performance function.

$$g_i = \alpha_1 V_{C,i} + \alpha_2 V_{S,i} - \alpha_3 V_{act,i} \text{ , } (i = 1,4) \tag{17}$$

where, $V_{C,i}$: shear strength without hoop ties [19], $V_{S,i}$: shear strength contributed by hoop ties, $V_{act,i}$: shear force, α_1, α_2 : probability variables to deal with uncertainty in shear strength, α_3 : probability variables to deal with uncertainty in earthquake response.

In this study, the shear forces $V_{act,i}$ in the longitudinal and traverse directions are the lateral forces at the moment when the limit of the pier's flexural strength is reached. Further, it is desirable that failure probabilities are calculated using probability variables that do not include a safety factor. The shear strength proposed by Niwa et al. [19] is used for $V_{C,i}$ in equation (17), as information of mean value and standard deviation etc. was quantitatively disclosed.

b) Performance Function for Ductility

The performance function for ductility given in equation (18) is used for the calculation of failure probability.

$$g_i = \alpha_4 \delta_{U,i} - \alpha_5 \delta_i \text{ , } (i = 2,5) \tag{18}$$

where, $\delta_{U,i}$: ultimate displacement of pier, δ_i : response displacement of equal energy assumption, α_4 : probability variable to account for variations in the process of calculation of ultimate displacement, α_5 : coefficient for estimation of earthquake response displacement.

The ultimate displacement in the longitudinal direction is evaluated based on push-over analysis using the analytical model shown in Fig. 3. In this analysis, a uniform force increment for all pier heights is applied. The ultimate state of the rigid-frame bridge is defined as one with multiple assumed hinges in the piers, which exceed their ultimate rotational ductility. In the section that follows, to study the effect of different definitions of the ultimate state on the scale of the section designed, a further definition consisting of four hinges exceeding the ultimate rotation ductility is introduced. The moment-versus-curvature relationship of the pier is determined using the standard moment-curvature method based on fiber analysis. The concrete stress-strain relationship used in this fiber analysis takes confinement effects into account. This is the model proposed by Hoshikuma et al. [20].

The stress-strain relationship of the steel is idealized by a bilinear model. In this case, material strengths such as the yield strength of bars are increased or decreased from the standard values shown in Table 1 for use in the design of Bridge A and Bridge B, because variations in material strength as described below are taken into consideration. This makes it possible to obtain average response values and to study the probability evaluation. The ultimate displacement δ_U in the traverse direction is evaluated by the same procedure as for the single-column type and described in Ref. 4. The material strengths are also increased or decreased in the traverse direction.

In conventional optimal structural design, structural analyses in the optimizing process affect the overall computation time [21]. Computation time is greatly extended if the earthquake response required in calculating the failure probability is evaluated by dynamic analysis for a rigid-frame bridge at each step of the optimization analysis. For this reason, earthquake response is approximated by push-over analysis and the equal energy assumption. Since the accuracy of the equal energy assumption depends on the characteristics of the earthquake waveform being considered as well as on the structural period, coefficient α_5 is introduced into equation (18) to take into account this variability. This coefficient is described in detail below.

c) Performance Function for Residual Displacement

The performance function for residual displacement given in equation (19) is used in the calculation of failure probability.

$$g_i = \delta_{Ra} - C_R(\alpha_5 \delta_i - \delta_{y,i}),\ (i = 3,6) \tag{19}$$

where, δ_{Ra} : allowable residual displacement, C_R : residual displacement response spectrum [23] , $\delta_{y,i}$: yield displacement.

The allowable residual displacement should be below the level that precludes easy repair after an earthquake. In this study, δ_{Ra} is 1/100 from the bottom of the piers to the center of gravity of the superstructure [4]. The residual displacement resulting from an earthquake is estimated according to the residual displacement response spectrum proposed by Kawashima et al. [23]. This spectrum is derived from 63 earthquake waveforms measured at ground level in Japan. Kawashima et al. used a non-degrading bilinear model of earthquake response to obtain this spectrum. If the ratio of first stiffness (yield stiffness) to second stiffness (post-yield stiffness) is zero, it is reported that the average and standard deviation of the residual displacement response spectrum are approximately $\overline{C_R} = 0.6$ and $\sigma_{CR} = 0.3$, independent of the structural period, ground conditions, and the magnitude of the earthquake response. In this study, the failure probability for the verification of residual displacement is calculated using these values. The hysteretic behavior of a reinforced concrete member is usually idealized using the Takeda degradation model. In this case, as the degraded stiffness is less than the initial stiffness, the reported value of $\overline{C_R}$ for an RC member is less than 0.6 [24]. However, investigations of the residual displacement of damaged piers after the Hyogo-ken Nanbu earthquake has revealed that: (a) no reason could be established for excessive residual displacement that required replacement of the piers; and (b) no explanation could be established for the variations in magnitude and direction of residual displacement, despite statistical efforts focusing on pier shape and other characteristics. Thus residual displacements estimated based only on maximum earthquake response as described above may be error-prone. It may not be an underestimate of safety to calculate the failure probability based on the assumptionof $\overline{C_R} = 0.6$ and $\sigma_{CR} = 0.3$.

3.3 Comparison of Dynamic Analysis with Equal Energy Assumption

As mentioned above, the earthquake response displacement of the rigid-frame pier is estimated in this study not by dynamic analysis but by adopting the equal energy assumption and using push-over analysis in the optimization process. The ratio α_5 of response displacement δ_{dy} by dynamic analysis to the response displacement δ $(= \delta_{dy} / \delta)$ is verified here.

The following conditions are added to the model of push-over analysis. Firstly, the hysteretic behavior of the pier is idealized using the Takeda degrading stiffness model. The damping force is idealized as Rayleigh damping and the damping ratios of each element are: superstructure 3%, pier 2%, and foundation structure 20%. Newmark's β method ($\beta = 1/4$) was used for numerical integration.

Table 3 shows the result of the eigenvalue problem for Bridge B in the longitudinal direction. Because the first-order mode of this bridge is predominant as compared with other modes, as shown in Table 3, the earthquake response is evaluated by the equal energy assumption. Three ground accelerations spectrally fitted to the design response spectrum for Type II ground motion in soil group II [4] are considered here. The value of α_5 is taken as the average of these three accelerations. And since previous studies have shown that α_5 depends on the natural period of a bridge, the bridges that natural period was between 0.40 sec to 1.20 sec were designed such that the heights of Bridge A and Bridge B are changed. The natural periods of all bridges seismically designed using the method proposed below are in this range.

Table3 Result of Eigen value Analysis for Bridge B

Mode No.	Natural Period (sec)	Participation Factor		Mode Damping (%)
		LG	Vertical Direction	
1	1.11	30.0	0.0	9.50
2	0.49	– 0.4	– 3.9	4.74
3	0.33	11.1	– 0.4	5.39
4	0.27	– 0.3	– 15.3	4.09
5	0.23	– 0.8	– 4.4	10.91

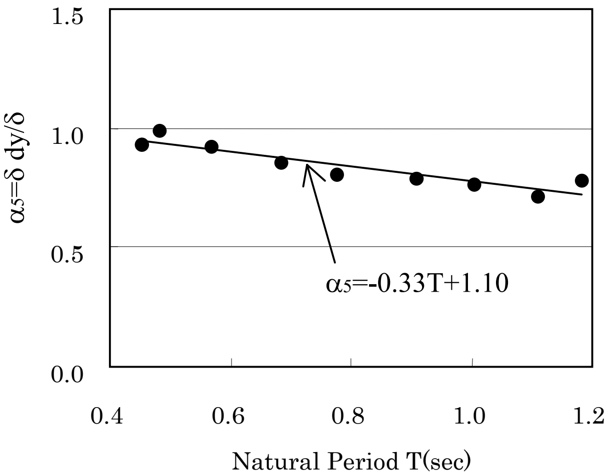


Fig. 5 Relationship between α5 and Natural Period of Bridges

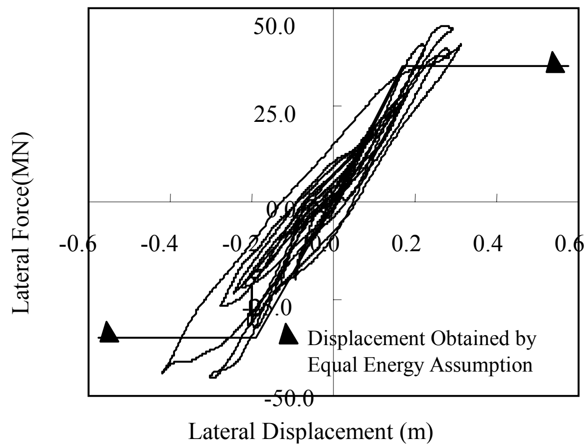


Fig. 6 Example of Relationship between Lateral Force and Lateral Displacement Obtained by Dynamic Analysis and Push- Over Analysis

Figure 5 shows the relationship between α_5 and the natural period of the bridges. Figure 6 is an example where the lateral force (sum of shear forces at the pier top) versus lateral displacement at the top of the pier is compared for the cases of dynamic analysis and push-over analysis. The relative displacement between the gravity center of the superstructure and the bottom of the pier, neglecting the influence of foundation sway and rocking, is used for the verification of ductility in equation 18 and in the calculation of α_5.

Nonlinear displacements based on the equal energy assumption are usually overestimates as compared with the results of dynamic analysis carried out in consideration of foundation damping effects, because the predominant damping mode in calculating the linear response horizontal forces is greater than the 5% used in calculating the response spectrum. In this study, it is assumed that the response displacements determined by this dynamic analysis are the average of the real displacements in the earthquake. These displacements are estimated by the equal energy assumption, with linear response horizontal forces calculated from the response spectrum with first-order damping [28].

The equal energy assumption gives results that accord with the response displacements obtained by dynamic analysis, as shown in Fig. 5, and α_5 is proportional to the natural period T of the bridge, as follows.

$$\alpha_5 = -0.33T + 1.10 \tag{20}$$

In this study, the earthquake displacement δ based on push-over analysis and the equal energy assumption is multiplied by coefficient α_5 for use in calculating failure probabilities during the structural optimization process. This leads to a sharp reduction in computation time required for the optimizing process.

3.4 Probability Distribution and Its Parameters

Table 4 Form of Distribution for Probability Variables and Parameters of Distribution

Symbol in Limit Equation	Form of Probability Distribution	Parameters of Probability Distribution	
		Mean Value	Coefficient of Variation
V_C	Normal Distribution	Calculated Strength	10.0%
V_S	Normal Distribution	Calculated Strength	8.0%
V_{act}	Normal Distribution	Shear Force Obtained by Flexural Strength	4.6%
α_1	Normal Distribution	1.02	8.2%
α_2	Normal Distribution	1.22	14.5%
α_3	Normal Distribution	1.00	30.0%
δ_U	Normal Distribution	Ultimate Displacement	6.1%
δ	Normal Distribution	Displacement Obtained by Equal Energy Assumption	30.0%
α_4	Normal Distribution	1.16	25.0%
α_5		Modification Coefficients for Dynamic Analysis Results	Fixed Value
δ_{Ra}		(h_a *))/100	Fixed Value
δ_y	Normal Distribution	Calculated Yielding Displacement	8.2%
C_R	Normal Distribution	Residual Displacement Response Spectrum (=0.6)	50.0%

*) Distance from the Bottom of Pier to Gravity Center of Superstructure

The variables in equations (17) to (19) must be treated as probability variables because of the variability of material strengths, structural analysis, and strength equations as based on experimental results, etc. The average values and standard deviations of these probability variables are set according to the results of previous studies and other information, as shown in Table 4. In this study, all probability variables are assumed to have normal distributions.

Table 5 the Variation of Material Strength Used for Analysis [29]

Variable	Nominal Value (MPa)	Mean Value (MPa)	Coefficient of Variation
Concrete Compression Stress	23.5	28.2	10%
Elastic Coefficient of Concrete	2.45×10^4	Dependence on Strength	Dependence on Strength
Yielding Stress of Bars	345	414	7%
Elastic Coefficient of Bars	2.06×10^5	2.00×10^5	1%

❈ The correlation is assumed as independence among all variables and they are assumed to be normal distributed.

The standard deviations of V_C, V_S, V_{act}, δ_U, δ_y in Table 4 indicate the effect of material strength variability on calculated strength and ductility. These mean values and standard deviations were evaluated by Monte Carlo simulation with all material variables in Table 5 [29] assumed to be parameters with normal distributions and with no interdependence. It is assumed that the concrete elastic coefficient in Table 5 depends on compression strength (based on Ref. 4). Previous experimental results [19] [29] are used to determine α_1 and α_4, taking into account the variation of the shear strength equation without hoop ties and the calculation process of ultimate displacement, respectively. No evaluation of α_2 is possible in consideration of the variation of the shear strength equation contributed by the hoop ties, because statistical values are not available. The mean value and standard deviation of α_2 are assumed to be the same as those of the shear strength equation in the ductility specification [30].

Coefficients of variation of α_3 and δ in consideration of structural model uncertainty are assumed to be 30%. There is inadequate data to fully evaluate structural model uncertainty, so these values have no solid foundation; sensitivity analysis must be carried out to evaluate the effects of the coefficients of variation shown in Table 4 on the solutions. Where there is difficulty in evaluating probability variables for lack of statistical data, reliability-based design is considered impossible to implement. However, in actual design practice, existing uncertainties such as uncertainty in the structural model are excluded by simply employing certain safety factors. Such an approach is even less adequate because it is difficult to determine a quantitative measure of safety for the designed structure. In reliability-based design, the variations used in the design process are clearly demonstrated and a structure is designed so as to meet the prescribed degree of reliability. Given the growing trend toward performance-based specifications, it goes without saying that reliability-based design is preferable.

3.5 Target Function and Limit Conditions

As stated previously, this study focuses on a search for solutions (that is, sets of design variable values) that yield a failure probability below the target failure probability. A target function, W, is considered a reliable standard by which to judge suitable solutions in place of a simple minimization. In this study, a target function W that minimizes the sum of flexural strength of the top and bottom of pier and shear strength was thought to be most desirable, so this is the definition adopted. Sectional areas, pier heights, and other particulars of the bridge are not changed in the optimizing process; only the longitudinal reinforcement and hoop ties are varied as described below. Also studied was a target function W that minimizes the ratio of flexural strength of the top and bottom of the pier to shear strength. However, this function proved unreasonable since it required more longitudinal reinforcing bars and hoop ties.

Strengths of target function were calculated in longitudinal direction and hoop ties were arranged, as shear strengths of the top and bottom of piers were same. In addition to limiting condition of failure probability, it was also defined that the lower bounds of flexural strength was cracking strength and upper bounds of shear strength was strength calculated for the section such that the hoop tie spacing was 100mm. The optimization process for the piers of this rigid-frame bridge is defined as follows.

$$\text{find } \{\mathbf{D}\} \tag{21}$$

$$\text{such that } Pf_{sys} = Pf_{sys}(\{\mathbf{D},\mathbf{X},\mathbf{Z}\}) \le Pf_{all} \tag{22}$$

$$\text{and } W = \left(V + \frac{M_{up}}{h_S}\right) + \left(V + \frac{M_{do}}{h_S}\right) \to \min \tag{23}$$

$$M_{c,up} \le M_{up} \tag{24}$$

$$M_{c,do} \le M_{do} \tag{25}$$

$$V \le V^u \tag{26}$$

$$\{\mathbf{D}\} = \{M_{up}, M_{do}, V\} \tag{27}$$

where, M_{up}, M_{do}: flexural moments at the top and bottom of pier, $V(=V_C + V_S)$: hear strength, $\mathbf{X}$: robability variables given in Table 4, $\mathbf{Z}$: , δ_{Ra} (fixed values), $M_{c,up}, M_{c,do}$: acking moments at the top and bottom of a pier, V^u: per values of shear strength, and h_S: alf of pier height

Although the design variables are flexural strength and shear strength in equations (21) to (27), during the optimizing process the quantities of longitudinal reinforcement and hoop ties shown in Fig. 4 are varied so as to match the next-step strengths provided by the SQP method. Accordingly, it may be said that the process of minimizing the target function based on equation (23) is to minimize the quantities of longitudinal reinforcement and hoop ties used. As these quantities are modified, the reinforced area at the positions shown in Fig. 4 increases and decreases as a continuous variable. The quantity of reinforcing bars is modified such that the ratio of the new calculated strengths to those obtained by the SQP method is below the allowable error of $\pm 1.0\times 10^{-4}$. The nonlinear rotational spring et al. based on the new arranged section are idealized and the earthquake response displacement δ et al. are calculated using the equal energy assumption. The other variables shown in Table 4 and needed in the calculation of structural failure probability are also updated.

It would be possible to define different target functions, such as weight minimization, initial cost minimization, and life-cycle cost minimization, although the function given in equation (23) is defined for the RC bridge piers studied here. In fact, the target function should be defined in consideration of all design conditions, including intended purpose of the structure and economical efficiency, etc. The resultant structure largely depends on the definition of the target function. The target function given in equation (23) is an adequate assumption because the quantity of reinforcing bars in the bridge piers is minimized, but the sensitivity of solutions obtained by other possible target functions should be studied in future.

Figure 7 presents a flow chart for the seismic design of the piers of a rigid-frame bridge based on the proposed scheme.

4. SEISMIC SAFETY EVALUATION OF PIERS OF RIGID-FRAME BRIDGE

4.1 Evaluation of Target Safety versus Strength Ratio of Pier

With respect to each safety target in equation (22), the design variables and the value of target function W after optimization are calculated. Figure 8 shows the results for Bridge A. The horizontal axis in Fig. 8 represents the safety index, as calculated from that failure probability Pf_{all} according to equation (16) (the target safety index). This concept of a safety index is adopted because the relationship between target safety index and post-optimization design variables (as shown in Fig. 8) is approximately linear and it is easy to understand the relationship between design variables and structural safety [31]. The safety index range indicated in Fig. 8 is converted from failure probabilities in the range 3.0×10^{-1} to 5.0×10^{-3}.

Figure 8 demonstrates that as the target safety index increases, the increment in shear strength is greater than that in flexural strength of the pier top and bottom. The strength ratio of the pier is calculated according to equation (28), and this is shown in Fig. 9 as a function of the target safety index for Bridge A and Bridge B. Strengths in the longitudinal direction are used in calculating the strength ratio. The value of the target function W after optimization is also shown in Fig. 9.

$$\gamma = \frac{V \cdot h_a}{M_{up} + M_{do}} \tag{28}$$

where, h_a : height of pier.

We have previously evaluated the reliability of RC piers and found that the strength ratio was a very important factor in seismic performance [2]. In this study, a rigid-frame bridge is designed by the proposed method based on reliability analysis and the optimal structural design method. Figure 9 shows that the strength ratio corresponds with the safety index, implying that structural safety and, independently of pier height, the strength ratio needed to attain the arbitrary target safety index are equivalent.

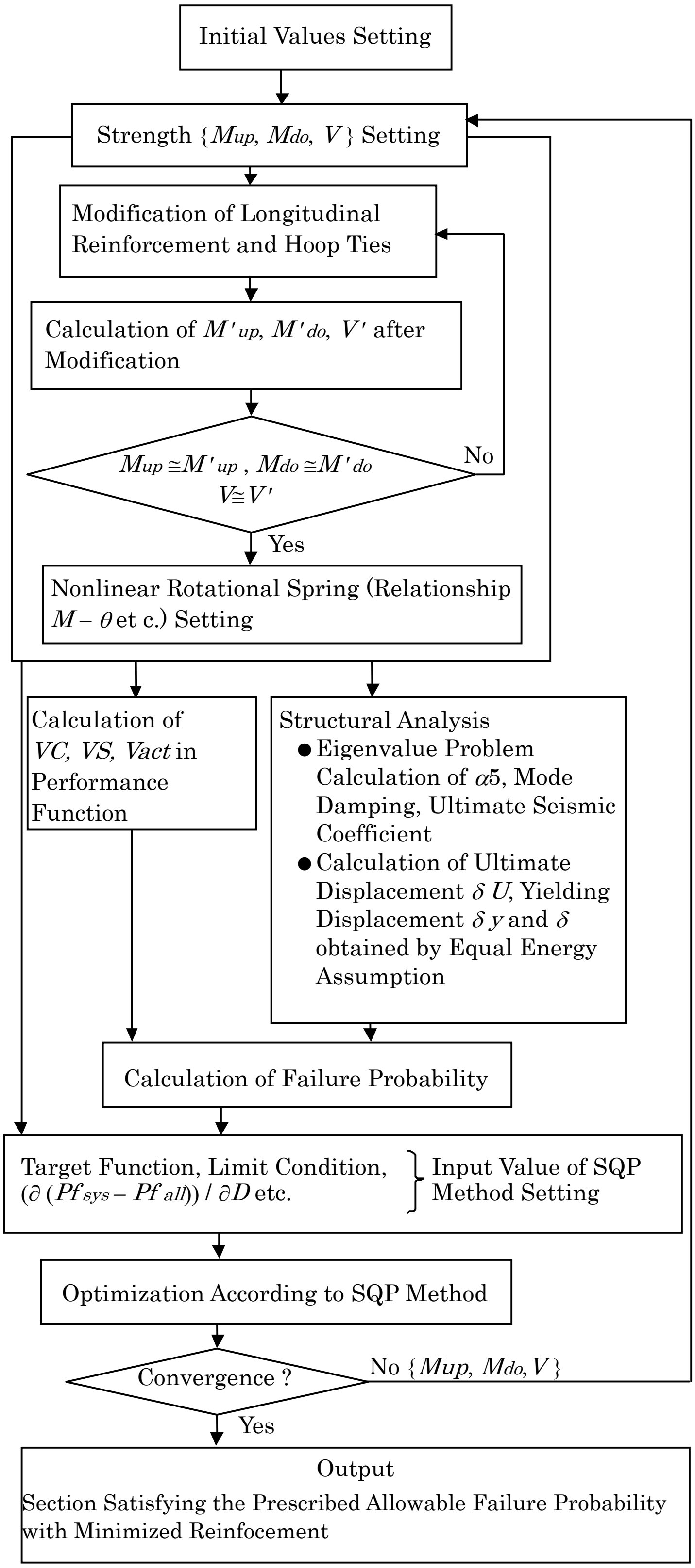


Fig. 7 Flow Chart of Seismic Design Scheme for Rigid- Frame Bridge Pier Based on Proposed Method

The reason for the higher strength ratio requirement when the target safety index is made higher relates to differences between the strength term defined in the performance function (17) to (19) (α_1, V_C, α_2, V_S in

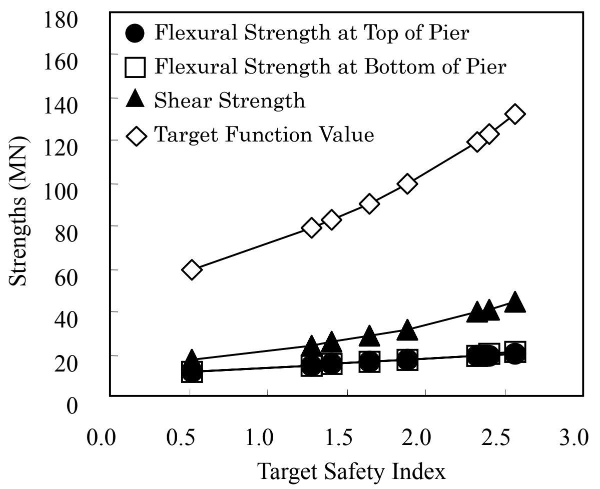


Fig. 8 Relationship beween Target Safety Index and Values of Target Function Value for Various Strength Values as Design Variables

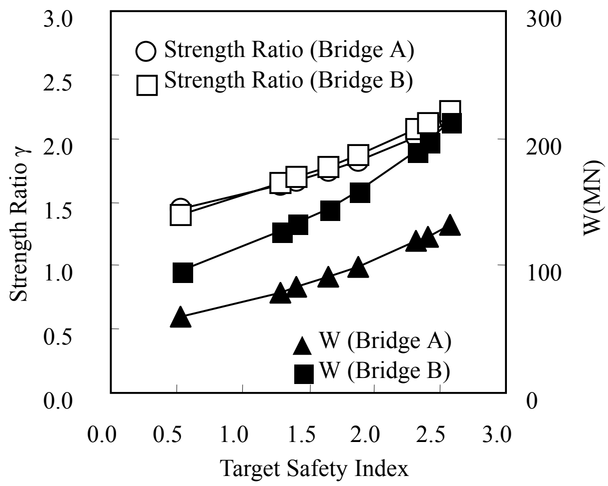


Fig. 9 Relationship between Target Safety Index and Strength Ratio

equation (17), α_4, δ_U in equation (18), and δ_{Ra} in equation (19)) and load term (α_3, V_{act} in equation (17), α_5, δ in equation (18) and C_R, α_5, δ, δ_y in equation (19)) need to increase. Increase of shear strength gains the strength term in performance function for shear failure and, as described above, non-linear rotational springs are idealized by the use of a stress-strain model for confined reinforced concrete in this study. This means that increases in shear strength, i.e. additional hoop ties, gain ultimate rotation of spring and the strength term in performance function for ductility. As increase of flexural strength gains the action shear force, this gains load term in performance function for shear failure. However as this decline the earthquake response displacement, the load term in the performance function for ductility and residual displacement also falls. But then, as the increase in flexural strength decline the ultimate rotation of spring, the strength term in the performance function for ductility also falls. Ultimately, the optimizing process with a higher target safety index results in higher shear strengths so as to ensure safety against shear failure and ductility failure. On the other hand, flexural strengths are minimally increased so as not to decline the safety for residual displacement because of excessive response displacement. Additionally, solutions that minimize the target function defined in equation (23) are found such that the failure probability taking into account multiple simultaneous limit states is below the target probability. Consequently, the optimizing process for greater target safety results in a larger strength ratio.

The strength ratios of Bridge A and Bridge B before optimal design are 2.08 and 2.28, respectively. The values of the target function in equation (23) for sections with the bar arrangements shown in Fig. 4 are W = 95.8MN and W = 166.4MN, and the failure probabilities (as shown in Table 2) are 4.80×10^{-2} and 1.00×10^{-1} for Bridge A and Bridge B, respectively. The strength ratios and target function values W after optimal design when the target probabilities for Bridge A and Bridge B are these initial failure probabilities, respectively, are calculated from Fig. 9. Then, the strength ratios (values of target function W) of Bridge A and Bridge B are 1.74 (W = 90.8MN) and 1.66 (W = 127.1MN), respectively. Thus it is confirmed that the strength ratio and target function values W decrease. Seismic design based on the Japanese specifications for highway bridges does not take minimization of the target function in equation (23) into consideration, and the shear strength in the performance function differs in this code. However, limit states assumed in the specifications and in the method proposed here are fundamentally the same. Nevertheless, the reason for target function values W being so different among structures with the same failure probability is that safety verifications in the Japanese specifications are carried out for each limit state separately; the limit states are not considered simultaneously. A further problem with the specifications arises in the setting of safety factors with respect to uncertainties in the design process, such as material strengths. In the proposed method, the prescribed level of safety is achieved for a structural system by considering multiple limit states simultaneously, and the target function is minimized based on a probabilistic evaluation of these uncertainties. This makes it possible to effectively reduce the volume of reinforcement. Finally, the proposed method enables the designer to gain a firm grasp of the safety level desired and it is possible to equalize structural safety, such as between Bridge A and Bridge B. Despite considering the structural system reliability, proposed method is realization of probabilistic design [1].

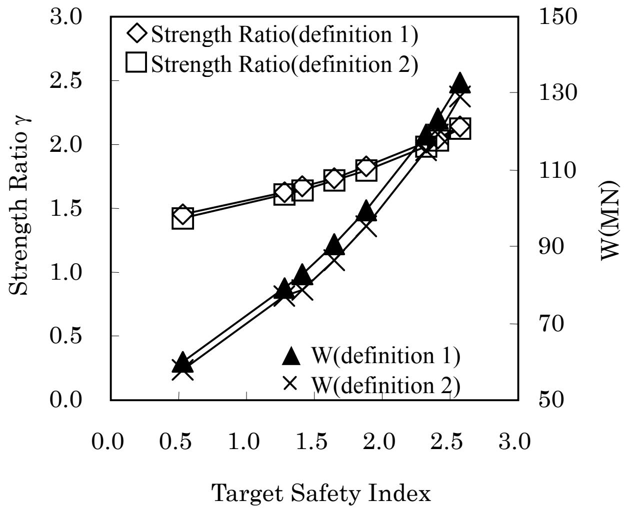


Fig. 10 Effect of Differences in Ultimate State Definition on Optimum Solution (Bridge A)

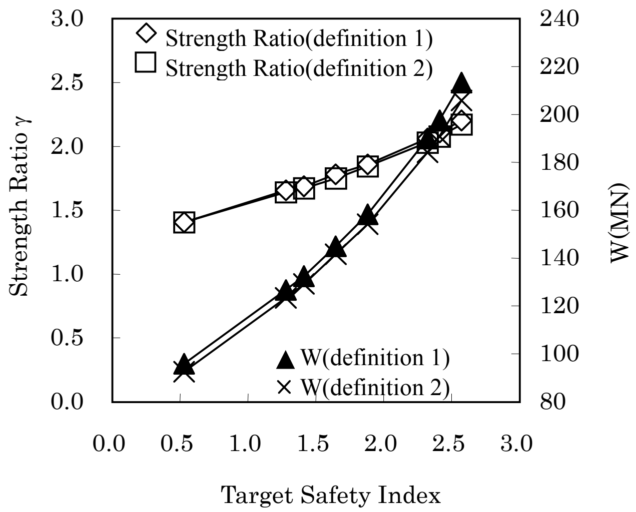


Fig. 11 Effect of Differences in Ultimate State Definition on Optimum Solution (Bridge B)

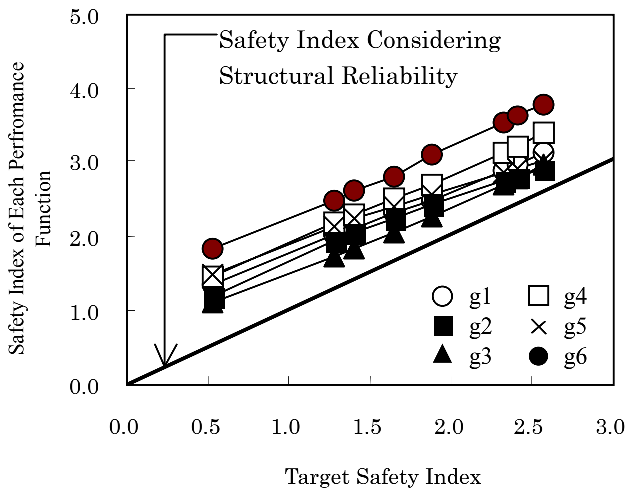


Fig. 12 Safety Index of Each Performanc e Function (Bridge B; Definition 1; one of plastic hinge)

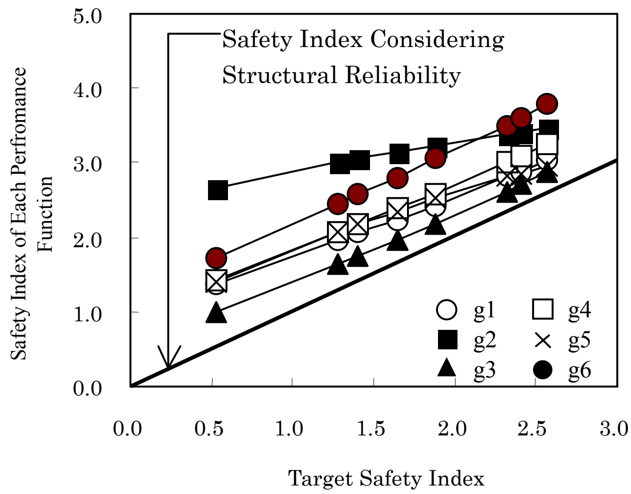


Fig. 13 Safety Index of Each Performance Function (Bridge B; Definition 2; four of plastic hinges)

4.2 Effect of Definition of Ultimate Displacement on Designed Section

Here we look into the effect on optimal solutions of the definition of ultimate displacement obtained by push-over analysis in the longitudinal direction. The ultimate states of a rigid-frame bridge were defined as either one plastic hinge (definition 1) or four plastic hinges (definition 2) at ultimate rotation. It was assumed that the moment-versus-rotation relationship of the non-linear spring was non-degrading when the non-linear rotational spring exceeded the ultimate rotation in the push-over analysis of definition 2. Figures 10 and 11 show the results for Bridge A and Bridge B, respectively; both figures show the relationship between target safety index and strength ratio as well as the target function values. It is clear that the definition of ultimate displacement in both cases has no effect on the optimal solution. For the scale of bridges analyzed in this study, the ultimate displacement obtained by push-over analysis based on definition 2 is about 1.3 times that of definition 1. However, this overestimated displacement has no effect on the ultimate result of optimal design. This is because the limit state approximating structural system reliability is not only g_2 defined as verification for ductility in longitudinal direction. This is expressed using the relationship of safety index obtained for each limit state versus structural safety index (safety index calculated by the reliability evaluation method for structural systems) as follows. Figure 12 shows the relationship between target safety index for definition 1 of Bridge B versus the safety index of each limit state after optimal design. Figure 13 shows the result in the case of definition 2 for Bridge B. The structural safety index plotted from calculations by the reliability evaluation method for structural systems using each safety index (g_1 to g_6) is approximately the same as the target safety index. By comparing Fig. 12 with Fig. 13, it is evident that when the ultimate displacement increases, the safety index of limit state g_2 in the analysis for definition 2 increases by more than that for the definition 1 analysis. As a result, the safety index of limit state g_2 in the analysis for definition 2 has no effect on structural safety. Similar results are also obtained for Bridge A. On the other hand, if limit state g_2 alone is considered and optimal design is carried out such that the failure probability for g_2 is below the target failure probability, the designed section is greatly influenced by the difference in ultimate state definitions. In fact, when optimal design using definition 2 is performed, the ultimate displacement is greater than when using definition 1. Since a consequence of this is that greater response displacement is allowed, it is easier to carry out design based on definition 2 than on definition 1. However, as shown in Fig. 12, the safety index obtained by limit state g_3 for residual displacement in the longitudinal direction has most effect on the structural one. Accordingly, when optimal design based on definition 2 is performed by the proposed method considering multiple limit states simultaneously, in contrast with the case where only limit state g_2 is considered, problems arise in matching the safety index obtained by g_3 with the target index if the flexural strength decreases and response displacement increases. For the same reasons, since the safety index obtained using limit state g_1 for shear failure in the longitudinal direction has an influence on the structural one, despite applying optimal design to definition 2, it is impossible to reduce the shear strength obtained by optimal design for definition 1. Accordingly, even though definition 2 that only increase the safety for limit state g_2 is adopted, because of limitation of limit state g_1 and g_3, sections of bridges after optimal design are the same independently of definition of ultimate state in longitudinal direction.

As this discussion makes clear, since design based on the proposed method is based on reducing the failure probability calculated from multiple limit states to below a target value, the effects of changing allowable values in any limit state are automatically taken into consideration. Thus the results depend on the way in which the limit states are set up and on the parameters of the probability variables listed in Table 4 (as described in the following discussion of sensitivity analysis). However, adoption of this proposed method would make it easy to reflect changes in the definition of ultimate states and in the parameters of probability variables based on future structural and statistical analysis.

4.3 Sensitivity Analysis

a) Residual Displacement Response Spectrum C_R

The effects of changes in the probability variables listed in Table 4 on the designed section are examined here. Firstly, the coefficient of variation of the residual displacement response spectrum C_R is set to 1%, 10%, 30%, and 50%. The effects of these different coefficients of variation on the optimal section for each target safety index are examined. Figures 14 and 15 show the relationship between strength ratio and target function value W for the target safety index.

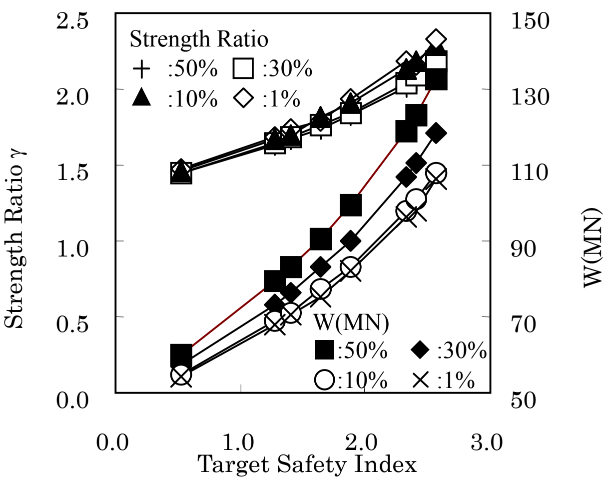


Fig. 14 Effect of Coefficient of Variation of C_R on Strength Ratio and Target Function (Bridge A)

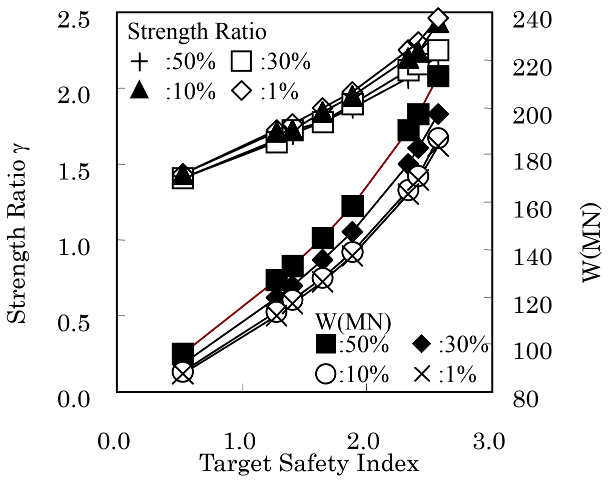


Fig. 15 Effect of Coefficient of Variation of C_R on Strength Ratio and Target Function (Bridge B)

As already noted, it is difficult to estimate the residual displacement using non-linear dynamic analysis because of the existence of several uncertainties. However, if it is assumed that such uncertainties will be reduced as calculation accuracy improves, it will become possible to satisfy the target safety index with less strength, especially in the range of higher target performance as shown in Figs. 14 and 15. As well as the problem of definition of ultimate state as described above, if other probability variables are unchanged, optimal design leads to almost the same section for coefficients of variation of residual displacement C_R below 10%, because of limitations for limit states other than verifications for residual displacement g_3 and g_6. If coefficients of variation of residual displacement C_R are larger, the strength ratios required to satisfy the target safety index are reduced, and vice versa. That is because it is difficult to satisfy the verifications for residual displacement if coefficients of variation of residual displacement C_R are larger and earthquake response displacements need to be made small by providing greater flexural strength.

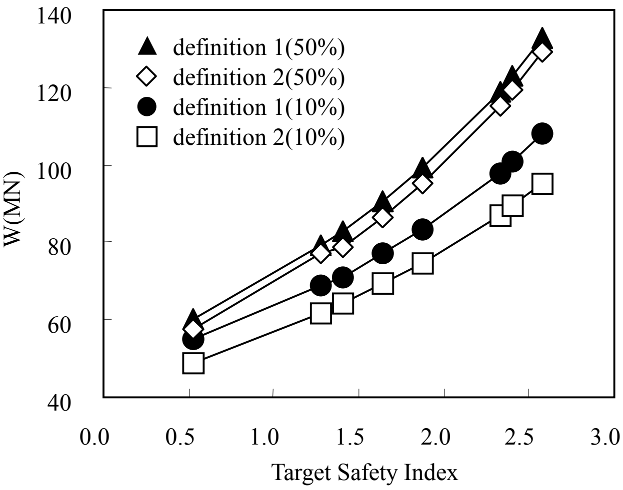


Fig. 16 Effect of Ultimate State Definition and Coefficient of Variation of C_R on Optimum Solution

With the assumed probability variable parameters listed in Table 4, as the limit state for residual displacement governs the structural safety of bridges, the definition of ultimate displacement has no effect on optimal solutions. We examine the case where coefficients of variation of residual displacement C_R are 50% and 10%, and the definitions of ultimate displacement are assumed to be definition 1 and definition 2, as described previously. Figure 16 shows the relationship between target safety index and chosen target function *W* for Bridge A. This demonstrates that the values of target function *W* decrease if it is possible for the coefficient of variation of C_R to be made small and if the definition of ultimate displacement is that of definition 2. This is because limit state g_3 has no effect on structural safety if the coefficient of variation of C_R is made small; since an increase in earthquake response displacement is acceptable, it is possible to reduce the flexural strength. Also, as the shear forces are reduced because of the decrease in flexural strength and the ultimate displacement is evaluated as larger because of definition 2, it is possible to decrease the hoop tie confinement as compared with definition 1; this decreases the shear strength. It is confirmed that the results for Bridge B are almost same as for Bridge A.

For the bridges studied, coefficients of variation of C_R used in the calculation of residual displacement have more effect on the optimal solutions than the ultimate state definition assumed in the push-over analysis for the longitudinal direction. As described above, as the mechanism of residual displacement has not been determined, the variation must be made larger during calculations. The sensitivity analysis described in this study shows that to achieve designs with good and effective seismic performance, it is important to improve the accuracy of residual displacement calculations and reduce their variability.

b) Ultimate Displacement of Pier δ_U

In order to consider variation in the process of calculating the ultimate displacement, the coefficient of variation of probability variable α_4 is changed from the 25% shown in Table 4 to 10% and 40%. Figures 17 and 18 show the relationship of target safety index to value *W* of the target function for Bridge A and Bridge B, respectively.
The difference in the target function value when α_4 is varied from 10% to 25% is only about 6%. The strength ratio needed to satisfy the target safety index remains almost the same throughout this range of values. It is possible to predict these results because definition 2, which results in overestimation of the ultimate displacement, has no effect on the optimal solutions. However, if the coefficient of variation of α_4 is assumed to be 40%, optimal design results in sections that differ widely from the above results. In particular, design becomes difficult if the target safety index is greater than 1.88 ($Pf_{all} = 3.0 \times 10^{-2}$). For this reason, the results for a target safety index of more than 1.88 and coefficient of variation of 40% are not shown in the figure. As shown in Fig. 12, which is the result for Bridge B with a 25% coefficient of variation of α_4, the safety index with respect to limit state g_2 has an effect on structural safety. Further, then the coefficient of variation is assumed to be 40%, the safety index with respect to limit state g_2 is almost the same as the structural safety index. When optimal design

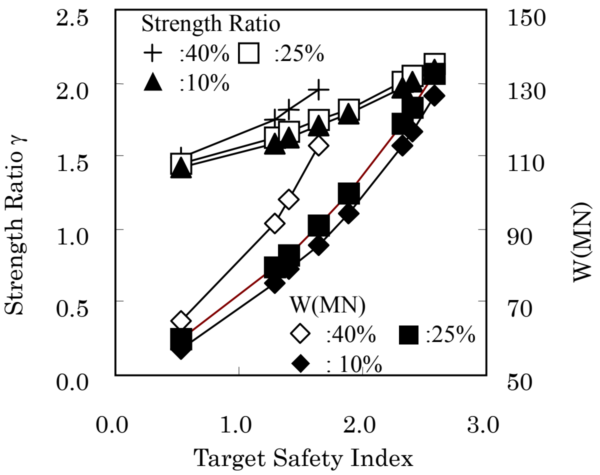


Fig. 17 Effect of Coefficient of Variation of α_4 on Strength Rati o and Target Function (Bridge A)

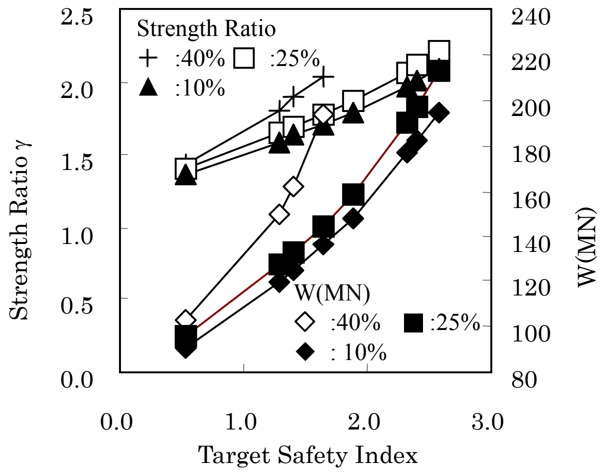


Fig. 18 Effect of Coefficient of Variation of α_4 on Strength Ratio and Target Function (Bridge B)

is carried out under these conditions, the shear strength rises in proportion to the target safety index so as to satisfy safety requirements with respect to g_2. If the target safety index is greater than 1.88, the upper limit of shear strength is reached in equation (26), and consequently it is impossible to satisfy the target safety index.

To illustrate the effects of changing the coefficient of variation of α_4 on limit states governing structural safety, the relationship between coefficient of variation of α_4 and safety index obtained from limit states g_1, g_2, and g_3 for a target safety index of 1.65, a failure probability of $P_{fall} = 5.0 \times 10^{-2}$, and shear strength after optimal design are shown in Figs. 19 and 20 for Bridge A and Bridge B, respectively). As safety indexes calculated from limit states in the transverse direction have no effect on structural safety, these are not shown in the figures. Both figures indicate that when the coefficient of variation of α_4 is made 40%, the limit state governing structural safety becomes g_2 and shear strength approaches the upper limit. Regarding the boundary shown for the coefficient of variation of α_4 in Table 4, if the variation of α_4 falls below this line, the governing limit states changes to g_1 and g_3. The 25% coefficient of variation of α_4 is referred to in a comparison between earlier experiment results and fiber analysis [29]. Even though the accuracy of the calculation method can be expected to improve with time, thus reducing the coefficient of variation of α_4, no reduction in quantity of reinforcement can be expected using the proposed method because structural safety will be governed by limit states g_1 and g_3. To deal with the possibility of higher target safety indexes in future, it will be necessary to reduce the variation in the

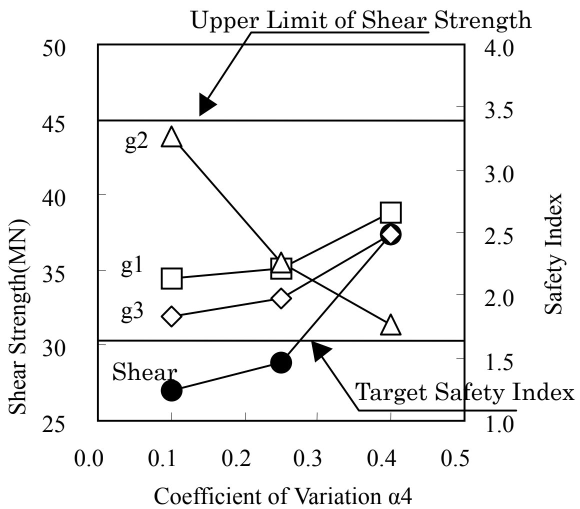


Fig. 19 Relationship between Coefficient of Variation of α_4 and both Shear Strength and Safety Index (Bridge A)

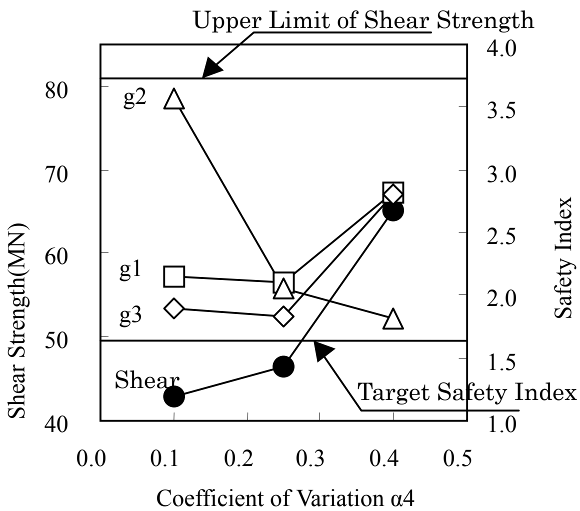


Fig. 20 Relationship between Coefficient of Variation of α_4 and both Shear Strength and Safety Index (Bridge B)

ductility evaluation as well as other factors; for example, the residual displacement response spectrum as described above will have to be taken into consideration.

c) Variation of Probability Variables to Deal with Uncertainty in Shear Strength Contributed by Hoop Ties; α_2

The coefficient of variation of α_2 represents uncertainty in the shear strength contributed by hoop ties. To check sensitivity, the variation is changed from the value of 14.5 shown in Table 4 by 5% and 25%. Figure 21 shows the relationship between strength ratio and target function W for each target safety index in the case of Bridge A. As described above, coefficients of variation of α_2 shown in Table 4 was referred to variation of other shear strength evaluation in Ductility Specifications [30]. Shear strengths in this study, which are based on truss theory, are the same as in the Ductility Specifications [30]; however if the coefficient of variation of α_2 is assumed to be about 25%, the optimal results are influenced a great deal. The same result is also obtained for Bridge B. It is possible to ensure safety with respect to shear failure despite a certain level of variation, because it is known that truss theory underestimates shear strength depending on the number of hoop ties. However, as shown in Fig. 21, if the coefficient of variation of α_2 is excessive, then a considerable effect on reinforcing bars of the member of flexural failure mode that strength ratio is more than 1.0. In future, a database of these variations will be developed and a probabilistic study using the proposed method et al. will be made. It will also be necessary to quantitatively evaluate the effects of shear strength variations on the designed amounts of reinforcing bars and structural safety calculated by reliability theory.

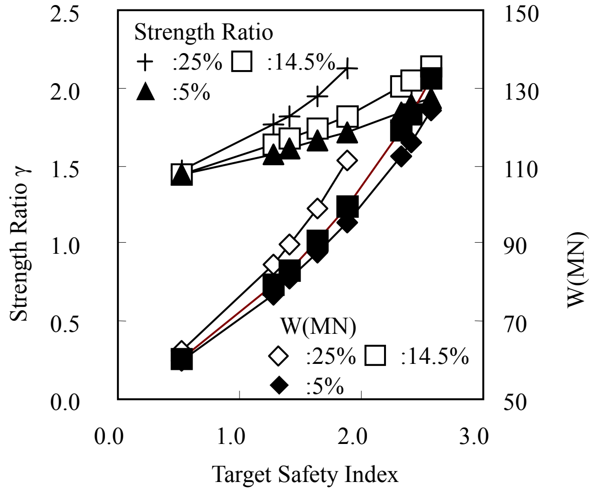


Fig. 21 Effect of Coefficient of Variation of $\alpha 2$ on Strength Ratio and Target Function (Bridge A)

d) Variation of Probability Variables for Action Shear Force and Response Displacement; α_3 and δ

Variables α_3 and δ were set up as the probability variables to deal with action shear force and response displacement. Their coefficients of variation were assumed to be 30%. There are difficulties in developing a database for the kind of variations called “model uncertainties” under assumed seismic loads. Here, the coefficients of variation of α_3 and δ are changed from the value of 30% shown in Table 4 to 20% and 40%. Figures 22 and 23 show the relationship between strength ratio and target function W for each target safety index for Bridge A and Bridge B, respectively. As both of those variables is used in every performance function, they have a significant effect on the optimal solutions. Further, in contrast with the sensitivity analysis described above, changes to both coefficients of variation have an effect on the strength ratio, even in a small range of target safety. The seismic loading assumed in this study is Type II earthquake motion based on the Japanese specifications for highway bridges [4], and no variability is taken into consideration. However, when statistics are available for

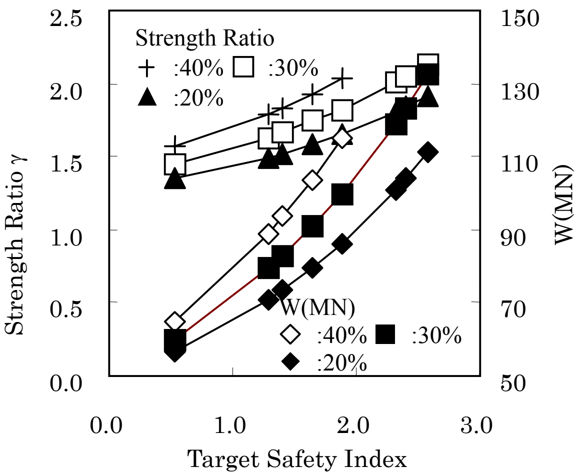


Fig. 22 Effect of Coefficient of Variation of δ, $\alpha 3$ on Strength Ratio and Target Function (Bridge A)

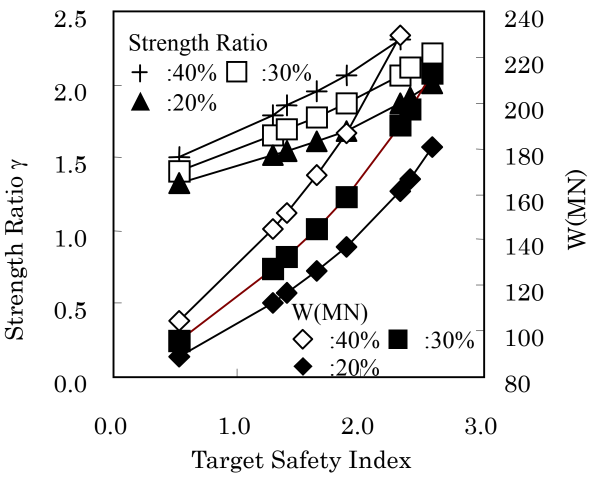


Fig. 23 Effect of Coefficient of Variation of δ, $\alpha 3$ on Strength Ratio and Target Function (Bridge B)

seismic loading, the earthquake response of a structure can also be statistically expressed and the effects reflected in α_3 and δ. Accordingly, since the parameters of probability variables for α_3 and δ are fixed in accordance with the scale of the assumed seismic loading, the frequency of the seismic waveform, and the accuracy of the earthquake response calculation, it is applicable from conditional failure probability shown in this study to optimal design problem considering the failure probability in structural lifetime. In future, these problems must be studied taking seismic risk analysis into consideration. However, for a case where the earthquake response calculated from a determinative seismic load is given in terms of mean values and variations are taken into consideration, the quantity of reinforcement required differs according to the degree of uncertainty in the earthquake response, as shown in Figs. 22 and 23.

5. APPLICATION OF PROPOSED METHOD TO SEIMIC SAFETY EVALUATION OF SINGLE-COLUMN RC BRIDGE PIERS

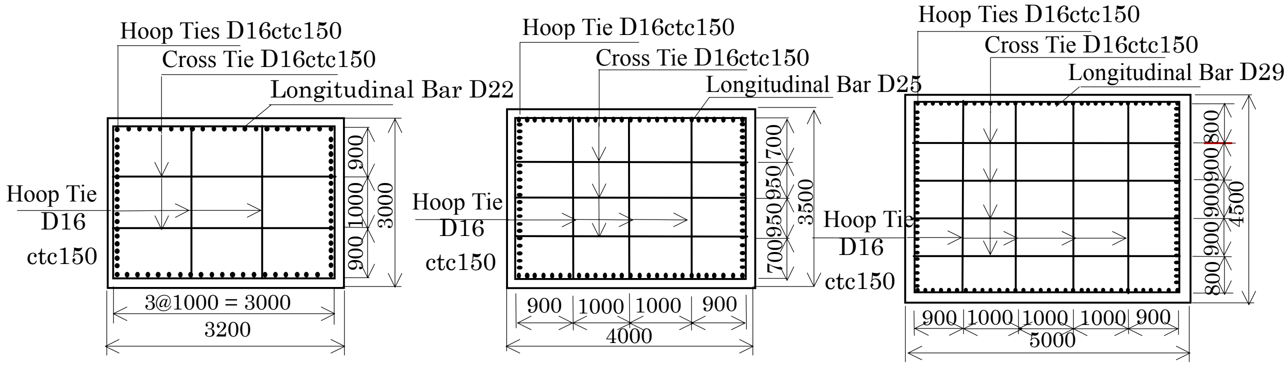
5.1 General

In the proposed design method, the quality of a design is judged by comparing its failure probability with a target failure probability. This use of the failure probability makes it possible to evaluate the safety of a structure quantitatively. Further, adoption of this method enables the designer to gain a firm grasp of the safety level desired, and to design a structure that attains the prescribed degree of reliability independently of other design requirements. The target failure probability used during the process of design is arrived at by a calibration against current specifications or by comparing it with vulnerability to other disasters.

Here, three RC bridge piers meeting the Japanese specifications for highway bridges are re-designed using the proposed method on the assumption that the target failure probabilities are the failure probabilities of Bridge A and Bridge B, as described in Table 2. An attempt is made to ensure a uniform degree of safety among the structures.

5.2 RC Bridge Piers Used for Analysis

It is assumed that the RC piers bear the girder of a bridge. The ground is classified as Group II, the same as in the case of the rigid-frame bridges described above, and the heights of the piers are adjusted such that their natural periods are between 0.4 second and 1.2 second so as to equalize the seismic load. As with Bridge A and Bridge B, the piers are neither over safe nor less than safe for the specifications. During design, the quantity of reinforcing bars and the sectional width are adjusted according to the same arrangements used in the design of the rigid-frame piers, except that the longitudinal reinforcing bars are in a single arrangement. In Fig. 24, the sectional area and bar arrangements of the designed bridge piers are illustrated. The ultimate strength, earthquake response, and other characteristics calculated according to Ref. 4 are given in Table 6. Other particulars and assumed material characteristics are listed in Table 7. Table 7 also shows the failure probabilities (safety index) of the three piers



(a) Pier A (7.5m in height) (b) Pier B (11m in height) (c) Pier C (15m in height)

[Unit: mm]

Fig. 24 Bridge Piers Used for Analysis

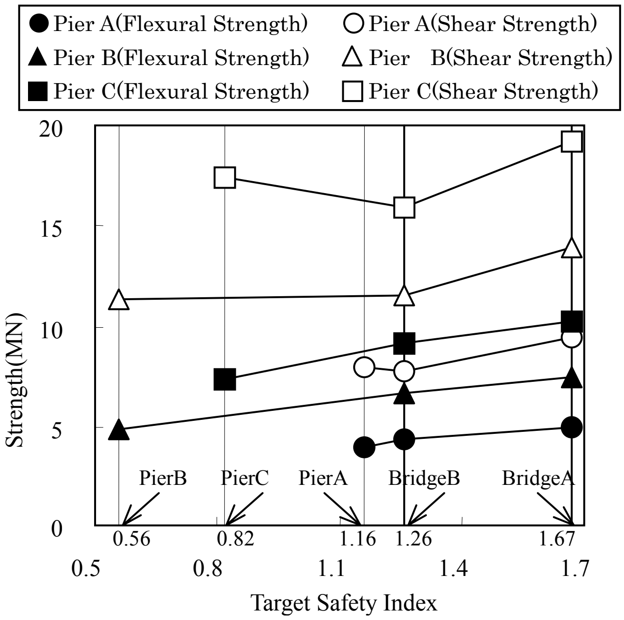


Fig. 25 Strengths Needed to Equalize the Safety Level of Piers A to C with Those of Bridge A and Bridge B

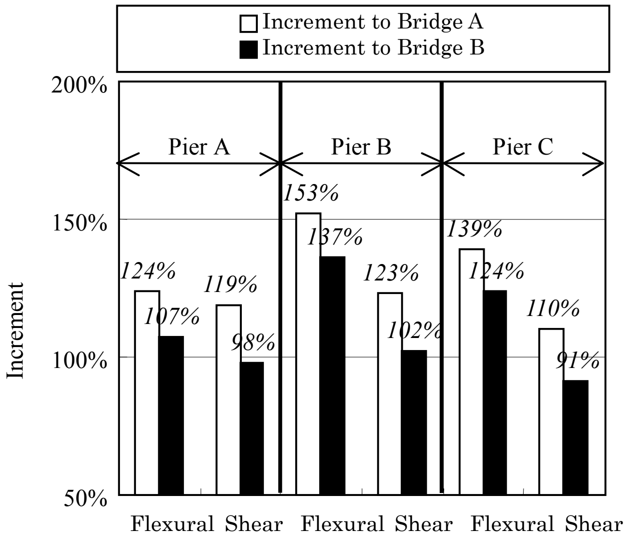


Fig. 26 Strength Increments Needed to Achieve the Same Safety as Bridge A and Bridge B

calculated by the reliability evaluation method for structural systems. As noted above, the failure probabilities depend on the number of limit states affecting the quantity of reinforcement and the sectional width of the designed sections as well as the magnitude of the safety margin for each limit state. In this case, because the three piers have small safety margins defined as the "strength term" minus the "load term" in both longitudinal and transverse directions, the calculated failure probabilities are larger than those of the rigid-frame piers.

5.3 Choice of Performance Functions, Target Function, and Limit Conditions

As with Bridge A and Bridge B, performance functions are expressed using equations (17) to (19) in both longitudinal and transverse directions. However, the shear forces $V_{act,i}$ in the longitudinal and transverse directions are lateral forces at the moment when the flexural strength of the pier bottom is reached. The parameters of probability variables in each performance function are the same as given in Table 4.

The target function is defined so as to minimize the strength (the quantity of reinforcing bars) as in the case of the rigid-frame bridges, and each flexural strength and shear strength is calculated in the longitudinal direction. The lower limits for flexural strength and the upper limits for shear strength are the same as for the rigid-frame bridges. In this case, optimal design of the bridge piers is expressed as follows.

$$\text{find } \{\mathbf{D}\} \tag{29}$$

$$\text{such that } Pf_{sys} = Pf_{sys}(\{\mathbf{D},\mathbf{X},\mathbf{Z}\}) \le Pf_{all} \tag{30}$$

$$\text{and } W = V + \frac{M_{do}}{h_a} \to \min \tag{31}$$

$$M_{c,d0} \le M_{do} \tag{32}$$

$$V_S \le V_S^u \tag{33}$$

$$\{\mathbf{D}\} = \{M_{do}, V\} \tag{34}$$

where, h_a : heights of bridge piers (Pier A: 7.5m, Pier B: 11m, Pier C: 15m).

5.4 Matching the Safety of Single-Column RC Bridge Piers and Rigid-Frame Bridge Piers Using the Proposed Method

The structural failure probabilities Pf_{sys} of Bridge A and Bridge B fulfilling the Japanese specifications for highway bridges [4] are shown in Table 2. These failure probabilities Pf_{sys} are then regarded as target failure probabilities Pf_{all} in equation (3), and the design for the three piers is optimized.

Figure 25 shows the flexural strengths and shear strengths needed to match the safety level of Piers A to C with that of Bridge A and Bridge B. Each value of flexural strength and shear strength shown in the figure is calculated from the average material strengths shown in Table 5, and shear strengths are calculated based on conventional studies used in performance functions. Further, Fig. 26 shows the strength increment entailed in moving from piers fulfilling the Japanese specifications for highway bridges [4] to those matching the safety of Bridge A and Bridge B. This demonstrates that the proposed method can be used to design structural sections with less reinforcement than required to fulfil the Japanese specifications, while still retaining the same failure probability. Accordingly, if the strength increments are calculated based on the section after optimal design, it may be larger than ones shown in Fig. 26. Here, however, piers fulfilling the Japanese specifications are taken as the base level in order to calculate the increasing rates of strengths obtained by sections with the minimum amounts of reinforcing bars to equalize the failure probability of piers to that of rigid frame piers.

Because the safety of Bridge A is better than that of Bridge B, both the flexural strength and shear strength of each pier needs to increase so as to match the safety of Bridge A. Also, because the safety of Pier B is the lowest of the three, it requires a greater strength increment in comparison with the other two piers.

Designing the three piers according to the Japanese specifications yields piers that are less safe than the rigid-frame bridge piers; the resulting strength increments are shown in Figs. 25 and 26. The differences of increasing rates of strengths between piers depend on initial state designed. However, when optimal design is performed with a higher target failure probability, all three piers need increased flexural strength as compared with shear strength for the initial state designed; decrease the strength ratios. Naturally, when the relationship between target safety index and strength ratio is as shown in Fig. 9, all three piers need a higher strength ratio in proportion to the target safety index. The structural sections of piers based on the Japanese specifications are such that shear strength is relatively higher than flexural strength, which contrasts with structural sections based on the proposed method. As described in Section 4.1, this difference arises because the magnitude of the safety margin considered in calculating the shear strength, residual displacement, and other characteristics differs from the probability variable parameters used in this study.

In current design, even though there are many limit states close to the allowable value; which safety margin is narrow, it can't be reflected to designed section. And as the limit state with the large variation such a verification of residual displacement is verified determinately after the various safety consideration, it is impossible to evaluate the safety of a structure quantitatively. Consequently, even though the design process might aim for a design that is neither too safe nor too risky according to the current specifications, the failure probabilities based on structural reliability theory are difference between the presented five structures shown in Fig. 25.

This study has included no discussion of the target failure probability. Since the strength needed to satisfy an arbitrary level of safety while fulfilling other specifications depends on many design conditions, the strength increments indicated in Fig. 26 are nothing but trial values. However, when the safety level of structures fulfilling the current seismic design specification [4] was matched based on failure probability, the need for an increase or decrease in strength for each structure was confirmed. Accordingly, in the case of deterministic seismic design as in this case, the possibility of variations in safety between structures should be kept in mind. In future, the relationship between conventional safety considerations and the magnitude of the inherent uncertainty in design variables will be examined, and efforts must be made to improve current design by incorporating the probability design method presented in this study.

6. CONCLUSIONS

The main achievements and findings of this study are as follows:

(1) A structural design scheme that is able to attain a target failure probability for a designed structural system and minimize (or maximize) a given target function has been presented. The scheme is based on structural reliability theory and an optimal structural method using the SQP method. The difference between the structural failure probability calculation methods was shown to have a significant effect on the optimal solution through an analysis example, and it was confirmed that the proposed method enables one to design a structure so as to attain a prescribed reliability.

(2) Since the first-order mode of the rigid-frame bridge adopted for analysis is predominant, the earthquake response displacement (in consideration of the foundation's damping effects) was estimated by push-over analysis and the equal energy assumption, rather than by dynamic analysis. This made it possible to dramatically reduce the calculation time required by the optimizing process.

(3) As compared to structures resulting from seismic design based on various deterministic safety factors and verified separately for each limit state, which is the method given in the current specifications, a structure designed by the proposed method, which takes into account multiple simultaneous limit states, offers the same failure probability yet its strength values are lower.

(4) It was confirmed that the definition of the ultimate state in the longitudinal direction of the rigid-frame bridge considered in the push-over analysis has no effect on reinforcing bar design if the estimated variation in residual displacement is wide.

(5) In order to successfully carry out rational design for lower values of target failure probability, there is a need to improve the precision of estimates for design (probability) variables in the limit state and also for other variables that affect structural safety (as identified by sensitivity analysis).

(6) When using the deterministic method of seismic design, it is important to bear in mind the possibility that the safety of independent structures may not be equal. To overcome this shortcoming, the current design method needs to be improved by incorporating the probability design method presented in this study.

The adoption of a common measure such as failure probability makes it possible to match the safety level of different structures consisting of varying structural shapes and materials. It also becomes possible to quantitatively verify that the specifications meet the safety target. The proposed method is a realization of these ideas based on the probability method. However, the method is not fully developed, and the following problems are thought to require further discussion.

(1) To systematize this design process, in which a limit condition is set for failure probability, the optimal design needed to define a certain target function must be performed. There are many possible definitions of optimum design, and the amount of reinforcement, etc. depends on the choice of target function. An analysis of the sensitivity of the optimum solution with respect to choice of target function will be required.

(2) The optimal solution is greatly affected by variations in response values obtained by structural analysis, so there is a need for investigations of variability in structural analysis and statistical values (scale, frequency, etc.) for the loadings considered.

(3) Engineers lacking experience with reliability analysis will find it difficult to directly calculate failure probabilities for their designs, so safety factors that simplify the process while achieving structures equivalent to those designed by the proposed method must be examined.

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