The aim of this paper is to investigate the influence of pre-existing cracks on the behavior of reinforced concrete beams in shear. Such pre-cracking is sometime inevitable in real RC members exposed to severe environments. An experimental program is implemented, and the experimental results demonstrate significant differences between pre-cracked and non pre-cracked beams. It is found that a pre-cracked beam may exhibit much greater capacity. Moreover, substantial differences in failure characteristics and the load-displacement relationship are also discerned. A rationale based on fundamental mechanics is able to explain this behavior of pre-cracked beams.

**Keywords:** crack interaction, Z-crack, crack arrest and diversion, shear anisotropy, co-axiality principle

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1. INTRODUCTION

Recently, our understanding of reinforced concrete mechanics has advanced considerably as a consequence of comprehensive scrutiny using both experimental and theoretical methodologies\(^1\)\(^2\). However, research related to reinforced concrete has conventionally focused primarily on members and structures with very few initial defects, and it is necessary to recognize that the huge knowledge base at our disposal was mostly acquired in the experimental laboratory where well-controlled conditions can be ensured. Indeed, one of the fundamental assumptions of conventional research methodology is that RC members are devoid of initial defects. As a result, researchers intentionally prepare specimens as perfectly as possible so as to minimize undesirable initial damage. It is only these typically near-perfect RC members for which we can claim to have advanced insight.

As a matter of practical reality, actual reinforced concrete members are hardly ever exempt from initial defects. Some of the differences between real and laboratory members are summarized in Fig. 1. Real RC members have much longer life spans than laboratory ones. In terms of structural behavior, a majority of laboratory specimens are kept for at most 2 months before testing. Moreover, in the young age period, laboratory concrete members are normally so well nurtured that cracking is reduced to a minor problem. On the contrary, real RC members may remain in use for 50 years under ordinary environmental conditions that may be grossly different from those available in the experimental room. This much longer time scale allows the time dependent deterioration mechanisms to work on the concrete. Moreover, during their life time, RC members may be subjected to unexpected non-proportional loading conditions. And since concrete is a highly path-dependent material that memorizes its loading history, this previous loading may result in cracking, stress/strain states, and residual deformation. These characteristics are of anisotropic nature and may play a crucial role in the next loading event.

Cracking is one of the most dominant factors representing the previous loading history of an RC member. This paper aims to clarify the influence of pre-existing perpendicular cracks on the structural behavior of RC members. Normally, structural concern focuses on two major behaviors, namely shear and flexure. It is
commonly accepted that flexure is governed by sectional behavior, and flexural capacity is rooted in the internal couple consisting of compressive force provided by the concrete and tension by the reinforcing bars. In this respect, pre-cracking does not appear to affect flexural capacity since the concrete can still acquire the required compression once the pre-cracks close, while reinforcement tension is barely affected by the presence of pre-existing cracks. On the other hand, shear behavior may offer a different story. Behavior of a member in shear is characterized by the propagation of a diagonal shear crack through a major part of the member, not just a section like flexural case. When such crack propagation takes place, the shear crack will naturally encounter the pre-existing crack planes. This multi-crack situation is the target of interest in this paper.

Previous research works have investigated the shear capacity of RC beams subjected to axial tension\textsuperscript{3,4,5}. In these works, axial tension was applied first and, maintained on the beam, and then shear was superimposed. The first step of inducing axial tension may simultaneously induce pre-cracking as well as pre-existing stress within the RC member. This makes it very difficult to differentiate the influence of pre-cracking alone from that of the pre-existing stress. Unlike these previous investigations\textsuperscript{3,4,5}, the authors aim to experimentally examine the influence of pre-cracking alone in terms of non-orthogonal crack interaction. In the experimental program, no axial tensile stress is introduced into the beam. On the other hand, vertical cracks penetrating the entire section are initially introduced into the RC members by mechanical loading. This method of introducing cracks may differ from the shrinkage and thermal heat mechanism, but the authors consider the mechanical behavior of cracks to be the same regardless of how they are formed. In this paper, the authors focus on the mechanics of crack interaction so as to extract the basic influence of pre-existing cracks. It is assumed that the results of this investigation will be applicable to members containing any kind of pre-existing cracks, including those due to shrinkage, thermal attack, or other mechanical forces.

2. EXPERIMENTAL PROGRAM

(1) Test specimen and material properties

A total of four reinforced concrete beams were tested. The size and dimensions of these beams are shown in Fig. 2 along with the main and web reinforcing bars used. The top and bottom main reinforcement was provided to counteract reversed flexural loading. The main reinforcement ratio was designed to be 1.14%. Tested concrete compressive strength was 26.5 MPa. Tested tensile yield strength of the main bars and tie bars was 338.4 MPa.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{Beam dimension and cross sections}
\end{figure}

(2) Experimental setup

a) Setup for reversed flexural loading

Penetrating cracks may be introduced into beams by a variety of available methods, each of which presents its own difficulties in experimental work. Two possible loading methods for inducing pre-cracking were considered: direct tensile loading and reversed flexural loading. Direct tensile loading limits the size of
beams that can be tested, since a large reaction wall is needed to balance the applied tension in a large specimen. Consequently, we adopted reversed flexural loading due to its relative simplicity. The flexural loading needs to be reversed so as to penetrate cracks across the entire sections. The setup for reversed flexural loading is shown in Fig. 3. This reversed flexure could not be implemented using a single test setup in the experimental laboratory, due to limitations of the testing machine as well as the inherent characteristics of beam testing. To circumvent the problem, the loading procedure was divided into two steps. Initial flexural loading resulted in vertical cracks in the flexural span, as shown in Fig. 3a. The beam was then inverted, i.e., turned upside down (Fig. 3b) and the second (reversed) flexure applied. This second (reversed) flexure caused the vertical cracks to penetrate the entire section, as shown in Fig. 3c.

A two-point loading arrangement was employed for the reverse flexure step. A steel girder was used to transfer the single applied load from the testing machine at two specified points on the beam, as shown in Fig. 3d. This loading arrangement was adopted since it resulted in a pure and constant bending moment over the flexural span without inducing shear. It should be noted in this figure that, there is a possibility of premature shear failure around the two end regions of the beam. In order to prevent this, sufficient web reinforcement was provided in those regions. (see Fig. 2.)

A high level of flexural loading up to two times the yield displacement was intentionally applied to some beams to introduce especially large cracks so that the effect of crack interactions could be clearly observed. Note that the pre-cracking studied in the authors' experiment is considerably more severe than that in previous works\textsuperscript{3-5}. The degree of pre-cracking after reversed flexural loading, which is reported in detail later in this paper, reflects the level of flexural loading used in the experiment.

b) Setup for shear loading

The second stage of the experiment entailed loading to induce shear cracks that propagate across the pre-existing crack planes. To implement this, the loading arrangement used for the first step of reversed flexural
loading was modified. The two bearing supports were moved towards the beam’s mid-span until the ratio of shear span to effective depth became 2.42, as shown in Fig. 4. Shear loading was then applied to the beam by one-point loading at mid-span. Through rotation of beam and re-arrangement of the loading and support conditions, complex non-proportional loading paths were achieved with the available experimental facilities.

Since the main objective of the experiment was to investigate crack interaction and the effects of pre-existing cracks on the beam, web reinforcement was not provided within the shear span nor was axial tension applied during shear loading. The one-point load was transferred from the test machine through a load cell to the beam. Displacement transducers were attached at mid-span and at the two supports. All load and displacement measurements were electronically passed to a data logger connected to a microcomputer for the real time data processing and a graphical display of results.

(3) Sectional capacity

Based on the material properties and cross-sectional geometry, the yield moment capacity is calculated based on sectional analysis. Shear capacity is predicted by the Modified Okamura-Higai equation. These sectional capacities are given for a non pre-cracked beam as a reference case in Table 1.

(4) Test results and observations

a) Observations after flexural loading

Reversed flexural loading resulted in several vertical cracks penetrating the entire section of the beam. As these vertical cracks might play an important role in shear behavior at the next loading step, we carefully recorded the number of cracks, the maximum residual crack width, the crack inclination, and the side of the beam (left or right) that exhibited more severe cracking. Since the number of cracks was basically the same on each side of each beam, the severity of cracking was judged by the width of these pre-induced cracks. This information is graphically summarized in Fig. 5.

Note that the width of pre-induced cracks varies, ranging from a minimum of 0.02 mm up to 5.0 mm. Certain notable features of the initial cracking in each beam should be mentioned here, since they may greatly affect subsequent shear behavior as will be indicated later. First, the degree of cracking as judged by the width of the cracks, was generally not the same on the left and right sides of the beam, even though the number of cracks was more or less the same. Particular attention is directed to Beam 3, which showed considerably severe cracking on the left side. Most of the vertical pre-induced cracks in Beam 2, on the other hand, had approximately the same width. Additionally, in Beam 4, one or two cracks on each side of the beam, specifically the one located near the center of shear span, were noticeably larger than other nearby cracks.
Table 1 Moment and shear capacity of cross section

<table>
<thead>
<tr>
<th>Properties</th>
<th>Reversed Flexural Loading(KN)</th>
<th>Shear Loading(KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear span/effective depth(d/a)</td>
<td>1.45</td>
<td>2.42</td>
</tr>
<tr>
<td>Shear capacity of concrete($V_c$) (Okamura-Higai Equation)</td>
<td>112.1</td>
<td>81.9</td>
</tr>
<tr>
<td>Shear capacity of tie ($V_t$)</td>
<td>199.5</td>
<td>0.0</td>
</tr>
<tr>
<td>Total shear capacity</td>
<td>311.7</td>
<td>81.9</td>
</tr>
<tr>
<td>Yielding Moment capacity($M_y$)</td>
<td>86.9 KN-m</td>
<td>86.9 KN-m</td>
</tr>
<tr>
<td>Yielding Load($P_y$)</td>
<td>385.2</td>
<td>230.4</td>
</tr>
<tr>
<td>Shear Failure Load($P_s$)</td>
<td>623.3</td>
<td>163.8</td>
</tr>
</tbody>
</table>

Table 2 Summary of loading capacity and side of shear failure

<table>
<thead>
<tr>
<th>Beam</th>
<th>Loading capacity(KN)</th>
<th>Percent load increase(%)</th>
<th>Side at which shear failure took place</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam1 (non pre-cracked, reference test)</td>
<td>157.0</td>
<td>0.0</td>
<td>Left</td>
</tr>
<tr>
<td>Beam 2</td>
<td>233.5</td>
<td>48.8</td>
<td>Right</td>
</tr>
<tr>
<td>Beam3</td>
<td>184.9</td>
<td>17.8</td>
<td>Right</td>
</tr>
<tr>
<td>Beam4</td>
<td>217.3</td>
<td>38.4</td>
<td>Left</td>
</tr>
</tbody>
</table>

Note: The percent load increase of pre-cracked beams is computed based on the experimental loading capacity of the reference non pre-cracked specimen.

b) Test results and observations after shear loading

A summary of the test results for each beam is given in Table 2, which shows, for each specimen, the load capacity, percentage load increase compared with the non pre-induced crack case, and the side of the beam where shear failure took place in the experiment. Some brief general comments are first presented here. A detailed discussion of the experimental evidence and a certain peculiarity will be presented separately for each beam. Thereafter, associated phenomena will be explained on the basis of rational fundamental mechanics.

All of the pre-cracked beams exhibited increased loading capacity as compared to the non pre-cracked one (Beam1). Moreover, the experiments demonstrated that the degree of increase is not minor (see Table 2). In Beam 2, in particular, a capacity increase of almost 50% was obtained. Moreover, all pre-cracked beams failed in shear on the side that had the lesser degree of cracking. This fact may support past experimental
investigations\textsuperscript{3,4,5} in which the shear capacity of a beam subjected to axial tension does not decrease as much as expected; these experiments indicate that penetrating pre-existing cracks may elevate the shear capacity. With common acceptance that induced tensile stress reduces shear capacity, as reflected in many design codes\textsuperscript{2,7}, it may be concluded that the influence of pre-cracks and of pre-tensile stresses counteract each other. Previous works\textsuperscript{3,4,5} did not investigate each individual effect separately, hence, the role of pre-cracking and pre-stress was not clarified. As a matter of fact, it is possible that the shear capacity may have increased or decreased, depending on the relative effects of pre-cracks and pre-tensile stresses.

It is not only capacity that increases; displacement ductility also rises, as shown later in Figs. 7-9. Suzuki et al. also reported an increase in displacement ductility of reinforced concrete columns containing slit made of steel plate\textsuperscript{6}. The experimental fact that vertical cracks do not harm a beam in shear is quite interesting, even though it may not fit well with the common belief that cracks are an unpleasant characteristic of reinforced concrete. Therefore, to justify these experimental results, the underlying phenomenon must be explained with suitable mechanics foundation. It is time to reassess our negative attitude towards pre-cracks.

3. GENERAL DISCUSSION FOR EACH BEAM

(1) Beam 1: Non pre-cracked beam (reference case)

The non pre-cracked beam was set up as a reference test. The failure crack in this beam followed the conventional diagonal shear crack as illustrated in Fig. 6. The failure process was initiated by a first diagonal crack appearing at the web near the central region of the shear span. Once this crack emerged, it suddenly propagated towards the loading point and rear support, forming a complete failure path in just a matter of seconds. A permanent decrease in loading capacity therefore resulted. For this non pre-cracked beam containing no transverse reinforcement, the first diagonal crack was the only one to appear, so the failure zone was very closely confined to a narrow zone (See Fig. 6.)

(2) Beam 2: Most pre-cracks quite uniform in width (introduction of Z-crack)

In beam 2, several large flexural cracks were present after reversed flexural loading (see Fig. 5). During the early stages of shear loading, a pair of diagonal cracks was observed forming around each vertical pre-crack, but these did not propagate across the pre-cracks, as shown in Fig. 7. The combination of these diagonal cracks with the vertical pre-cracks gave a crack pattern similar to the letter “Z”, so this configuration is referred to as a Z-crack. Since these diagonal cracks never penetrated through the companion vertical pre-cracks, the beam did not suddenly fail once they occurred.

This crack pattern did, however, result in considerably severe damage to the beam over a wider zone than the localized cracking observed in conventional shear failure, as exhibited by the non pre-cracked beam. Nonetheless, though threatened by such severe damage, the beam was able to continue carrying further load. The constituent diagonal cracks of the Z-crack widened greatly locally. The mode of this opening must be compatible with sliding deformation along pre-cracks. Through direct sliding of pre-cracks as well as compatible opening of associated diagonal cracks, the beam was able to dissipate energy in a stable manner.

It was significant sliding along pre-cracks that minimized the injurious effect of diagonal cracks, and this clearly distinguishes the failure pattern from that of the non pre-cracked beams. Here, vertical pre-crack exerts a significant role also and this reflects in the shape of Z-crack that consists of both diagonal and vertical crack. Without vertical pre-cracks as in the reference case, only a single diagonal crack prevailed and unstable propagation of this crack led to immediate failure. Our experiment demonstrates the significant influence of vertical pre-cracks on the behavior of beams in shear.

The diagonal cracks were unable to propagate in a continuous way, so the beam survived higher applied loading up to yielding of the main bars. It should be noted that this load level is a significant increase from the designed shear capacity of the reference beam without pre-cracks. After yielding, the beam was able to sustain the load through increased deflection at mid-span. The failure crack pattern shown in Fig. 7 occurred on the right side of the beam. The failure pattern is somewhat misleading to the eye, as it looks as if failure...
was accompanied by a single diagonal crack. In reality, though, four independent diagonal cracks can be identified since they did not form concurrently. Experimental observations showed that cracks a and b first appeared at a load of around 128 KN. As already noted, the beam did not fail at this point, but was able to withstand more loading until it yielded at 233 KN. Even after yielding of the main reinforcement, the beam further deformed until finally, cracks c and d formed instantaneously and merged with cracks a and b to finalize the failure process.

(3) Beam 3: Much more severe cracking on the left side

With respect to Beam 3, it was previously indicated that the left side was far more severely cracked by reversed flexural loading than the right side (see Fig. 5). The failure crack pattern of this beam under shear loading is shown in Fig. 8. Note that no diagonal shear crack penetrated through the web portion on the left side of the beam. Several diagonal cracks with a flatter inclination were observed around each large pre-crack instead. It was on the right side, the side with the smaller degree of cracking, that diagonal shear cracking took place, leading to failure of the beam. The relationship between load and deflection is also shown in Fig. 8. The loading capacity was lower than that of Beam 2 but still higher than the non pre-cracked beam. This is reasonable since the pre-cracks on the failure side of this beam were considerably less in width than in the case of Beam 2. By the same reasoning as given previously, pre-cracks that are less in width make less contribution to slip along the pre-cracks, while the normal and shear deformational mode of diagonal cracks became more prominent. Consequently, unstable crack propagation was encouraged more promptly than in Beam 2.
In fact, this behavior was not limited to Beam 3; in all other beams in which the degree of cracking was unsymmetrical between the two sides, shear failure took place on the side which had pre-cracks of less width. This demonstrates that vertical pre-existing cracks are not necessarily harmful to a beam, at least from the viewpoint of structural behavior.

(4) Beam 4: Large spacing of pre-cracks

As noted before, the lower level of reversed flexural loading in the case of Beam 4 meant that only one or two large vertical pre-cracks appeared on each side of the beam prior to application of the shear load (see Fig. 5). These cracks, even if comparatively larger than neighboring ones, were still smaller than those in Beam 2. Furthermore, they were located approximately around the central portion of the shear span. The failure crack pattern is shown in Fig. 9. Z-cracks could be observed but the geometry was noticeably different from that of Beam 2. The Z-cracks in this beam appear to consist of more diagonal cracks than in Beam 2. This is logical, since the width of the pre-cracks in beam 4 is smaller while the crack spacing was greater than in beam 2, as noted before.

The load-displacement relationship is displayed in the same figure. An increase in shear capacity of nearly 40% is clearly seen. As the role of pre-crack sliding is less than in Beam 2, this smaller increase in loading capacity is in line with expectations. However, it is clear that the characteristics of pre-cracking can greatly affect beam behavior. Different pre-crack characteristics may result in different Z-crack geometries, and this leads to different degrees of relative deformation between pre-cracks and diagonal cracks. The relative deformation between diagonal crack and pre-crack is supposed to account for the deviation in structural behavior of pre-cracked beam from the non pre-cracked one.

With regard to the failure characteristics, a difference between this beam and Beam 2 needs to be pointed out. In Beam 2, the geometry of the Z-crack was such that pairs of component diagonal cracks were too far
apart to merge and cause failure. The failure process was thus characterized by the development of new diagonal cracks around the web zone; these suddenly merged with previously formed diagonal cracks, leading to failure of the beam. This process occurred at a very late stage after yielding of the main bars. In Beam 4, on the other hand, the situation was quite different. As shown in the figure, the constituent diagonal cracks were not located very far away from each other, and thus, they were able to merge and finalize beam failure. However, this was not an instant process, and a higher load was required to cause ultimate failure.

4. PHENOMENON OF CRACK ARREST AND DIVERSION

The experimental results demonstrate that vertical pre-cracking significantly affects the shear behavior of a beam in terms of loading capacity, failure characteristics, and load-displacement behavior. The experimentally observed crack patterns indicate that this is because pre-cracks can obstruct the continuous and instant propagation of diagonal cracks. Consider Fig. 10, which illustrates an RC beam with one vertical pre-crack located centrally in the shear span. The reason that diagonal cracks are unable to successfully penetrate across pre-cracks is the low normal and shear traction transfer ability along the pre-crack. The path of diagonal crack propagation is also shown in Fig. 10. Two elements can be differentiated along this path, one near the pre-crack plane and the other far away from it. The difference between these two elements is that the far element can develop adequate stresses for further cracking, while the stresses that can develop in the element near the pre-existing cracks depend on the traction that can be transferred along these cracks. If the pre-cracking is of sufficient width, very little traction is transferred, and the transferred stresses are not enough to lead to further cracking. In other words, the diagonal crack will come to a halt in the element near the pre-crack plane.

Mechanism of crack arrest and diversion

![Diagram of crack arrest and diversion](image)

Experimental Observation

Fig. 10 Mechanism of crack arrest and diversion
We further assert that slip must take place along the pre-existing cracks. Such slip results in relative movement between the left and right parts of the beam. The externally applied loads shown in the figure move up the left part of the beam and move down the right part. With this kinematics and associated slip along the pre-existing cracks, the diagonal crack gives the appearance of being diverted in the direction of the pre-existing cracks. However, this interpretation is strictly incorrect, since in reality a diagonal crack cannot be physically diverted in this way. What crack diversion actually highlights is the contribution of slip along pre-existing cracks. The experimental results reveal the significance of this slip in the form of the Z-crack pattern. The formation of Z-crack shown in Fig. 10 can now be understood and the difference between this and the conventional diagonal crack in a non pre-cracked beam where pre-crack slip is of course zero can also be understood.

The width of pre-existing cracks directly affects the amount of traction that can be transferred and the amount of slip. In other words, pre-cracking width makes a relative contribution in the shape of the ultimate Z-crack. Figure 10 illustrates two Z-cracks with geometrical differences mirroring variations in the contribution made by pre-crack deformation. It should be quite easy to see that the vertical pre-cracking in the left figure must have been larger than in the right figure since the resulting Z-crack exhibits a larger geometric contribution from the pre-existing cracks.

5. DIFFERENCE IN LOAD-DISPLACEMENT RELATIONSHIP BETWEEN PRE-CRACKED AND NON PRE-CRACKED BEAMS

The above discussion makes clear that loading capacity and displacement ductility are greater in pre-cracked beams than in non pre-cracked beams. However, it is also the case that the load-displacement relationships are greatly different. The load-displacement curves for non pre-cracked and pre-cracked beams are shown in Figs. 7-9. Schematic load-displacement relationships for non pre-cracked and pre-cracked beams are shown in Fig. 11 for comparison. The load-displacement behavior of a non pre-cracked beam is characterized by a linear elastic portion up to the initiation of the first flexural crack (point A in Fig. 11a). Thereafter, stiffness decreases while the beam continues bearing the load until the diagonal crack forms at point B. After the emergence of this diagonal crack, the load decreases suddenly and leads to failure characterized by unstable
propagation of the diagonal crack. This typical behavior is well understood and commonly accepted by engineers and researchers.

Conversely, a pre-cracked beam exhibits different and far more complex behavior in the load-displacement relation, and there is much greater non-linearity. A representative load-displacement curve for a pre-cracked beam is shown in Fig. 11b. This beam exhibits non-linearity right from the initial stage of loading, and this is ascribed to the sliding and opening/closing of the pre-existing cracks. This corresponds to the S-curve (line OA) in the load-displacement curve. Point A marks the initiation of the first diagonal crack. Accompanying this crack, the load temporarily drops but increases again since propagation of the diagonal crack is limited by the existence of vertical pre-cracking.

Instead, the beam continues to carry the load. Portion AB of the load-displacement relationship corresponds to this load recovery period. As the figure makes clear, behavior in this period is highly non-linear; it is characterized by mutual contribution of pre-crack deformation and new diagonal crack formation, with strong interaction between the cracks. The load increases with reduced stiffness. Each time a new diagonal crack forms, there is a temporary load drop followed by an increase. Accompanying the formation of these new diagonal cracks is the observed gradual degradation in stiffness. Point B represents the point at which several independent diagonal cracks merge together along the path connecting the loading point to the support. Point B therefore represents the development of a complete failure path. Beams containing pre-cracks exhibit much more severe damage after shear loading because several discontinuous diagonal cracks may form at different times during the loading process. These independent diagonal cracks greatly reduce the stiffness of the member, leading to greater displacement ductility. The result is that considerably more energy is required to cause ultimate failure.

6. BEHAVIOR OF Z-CRACKS

The Z-crack, one of the main observations resulting from this experiment, is the result of combined diagonal cracking and pre-cracking. The behavior of Z-cracks is of considerable importance since it determines the failure characteristics and ultimate capacity of the beam. Experiments have disclosed a diverse variety of Z-crack geometries, depending on the physical and geometric properties of the pre-existing cracks. On one extreme, if there are no pre-existing cracks, then the diagonal crack totally dominates behavior. Failure is then described by unstable propagation of the diagonal crack as already understood.

In beams with pre-cracking, the geometry of the resultant Z-crack governs loading characteristics and thereby plays a major role in beam behavior. This was explained in detail in Section 3. It is inferred that pronounced slip along the pre-existing cracks is closely related to the inability of the diagonal crack to propagate. This seems to be a clear manifestation of crack interaction, and it forms the basis for the mechanics of Z-cracks as well as the overall behavior of the beam. In order to check this notion of crack interaction, crack deformation was experimentally measured in the experiment. A Z-crack is shown in Fig. 12. Note that this Z-crack is dominated by the vertical pre-crack contribution as compared with the diagonal crack. The relationship between load and sliding displacement for the pre-existing crack and the diagonal crack is shown in Fig. 13a. It is clear that sliding along the vertical pre-existing crack is significantly greater than that along the diagonal crack, thus verifying the active mobilization of pre-cracking.

The relationship between load and crack opening is shown in Fig. 13b. The opening of both pre-existing cracks and diagonal cracks is roughly of the same order. But the slip of diagonal crack is considerably less than that of pre-existing cracks. Thus, unlike the slip behavior, diagonal crack is active in the opening mode. The opening of this diagonal crack is associated with the slip of pre-existing cracks according to the deformational compatibility, as shown in Fig. 13c. As slip is very small, this opening of diagonal crack is quite stable and provides the way to dissipate energy out of the beam. In the experiment, large opening of these diagonal cracks can visually be observed.

In sum, it should be recognized that in a pre-cracked beam, there exist two deformational modes. One relates to the pre-existing cracks and the other to conventional diagonal cracks. The kinematics of the pre-existing cracks thus differentiate a pre-cracked beam from a non pre-cracked one.
The two deformational systems interact with each other to result in the Z-crack pattern. As concluded from Fig. 14, several features of pre-cracking, such as the crack width, crack spacing, and crack asperity may affect the relative deformation mode, and this in turn affects the geometry of the Z-crack and thus beam behavior. Figure 15 schematically shows different Z-crack geometries and their experimental counterparts. To aid our insight into their behavior, two groups of beams are illustrated, reflecting different contributions.
from pre-cracking and diagonal cracking. In the case of vertical pre-existing cracks, greater crack width and closer crack spacing enhance the contribution of pre-cracking. This is the left-hand group in the figure. The converse situation is shown in the right-hand group, where diagonal cracking dominates because the pre-existing cracks have less width or wider spacing.

![Smaller spacing](Image1)

![Large crack width](Image2)

Beams in which sliding along pre-crack dominates

![Smaller crack width](Image3)

Beams in which sliding along diagonal crack dominates

Fig. 15 Influence of pre-cracking properties and geometry of Z-crack

7. FAILURE CHARACTERISTICS OF PRE-CRACKED BEAMS AND DEVELOPMENT OF FAILURE PATH

Failure is defined as the process by which a complete crack path develops between the loading point and the support. This discussion aims to illuminate the fundamental differences in this process between non pre-cracked and pre-eracked beams. As shown in Fig. 16, the failure path in a non pre-cracked beam is characterized by unstable propagation of a single diagonal crack, as explained in Section 3. This kind of failure requires minimal energy. On the other hand, a beam with pre-existing cracks shows substantially different behavior, as already described. Failure no longer depends exclusively on the propagation of a single diagonal crack, since a diagonal crack that forms cannot propagate unopposed since it is arrested by one of the vertical pre-existing cracks. This mechanism of crack arrest promotes the independence of multiple diagonal cracks, and relaxes the degree of localization that characterizes propagation failure.

Therefore, in pre-cracked beams, final failure results from the merging of several diagonal cracks that have developed independently rather than the propagation of a single diagonal crack. Typical failure patterns of pre-cracked beams are shown in Fig. 16 (right). In a pre-cracked beam, much more energy is required to complete the failure process.

8. INITIATION OF DIAGONAL CRACKS IN PRE-CRACKED ELEMENT

In the previous sections, the discussion focused on the relative deformational contributions of pre-existing cracks and diagonal cracks. This highlighted the significance of crack interaction to post-diagonal cracking behavior. In contrast, post-diagonal crack behavior in a non pre-cracked beam involves no crack interaction and is thus characterized by abrupt failure. However, pre-cracking not only influences post-diagonal crack behavior, but also affects the generation of new diagonal cracks in a pre-cracked element in addition to the non-linearity it introduces into the pre-diagonal crack period. This is reasonable, since pre-cracking is present initially before application of the shear load, and so its kinematics take effect from the origin.

The experiments also demonstrated that the initiation of diagonal cracks in a pre-cracked beam greatly differed from that in a non pre-cracked one. In a beam with no pre-cracks, elastic isotropy in behavior may
Failure Process

Construction of failure path
transferring force from load point to support

Z-crack in which pre-cracking dominates

Initial crackling state

Diagonal cracks independently formed, merged, caused beam failure

New crack formed along the failure path

Failure process:
Crack a,b formed first --- No propagation
Crack c,d formed --- merged with crack a,b

Construction of complete failure path

Energy consumed for Failure

Z-crack in which diagonal crack dominates

Crack 1 and 2
Independently formed, merged and failed

Deformational Compatibility

Critical Diagonal Crack is delayed.

Critical Diagonal Crack is delayed.

This crack pattern cannot achieve the failure path that transfers load from loading point to support

Initial behavior dominated by pre-crack sliding

Lowest

Intermediate

Highest

Fig. 16 Failure characteristics of pre-cracked beams

be assumed. This isotropy is consequential as it supports the coincidence between principal stress and principal strain direction, which is well known as the co-axiality principle. The co-axiality principle is that the characteristics of isotropic linear elastic materials are defined by two parameters, namely the elastic modulus and Poisson’s ratio. Of course, a non-cracked concrete solid is neither ideally elastic nor isotropic due to the presence of micro-cracks as well as a certain difference between tensile and compressive behavior. Therefore, in an exact sense, the principle of co-axiality is never satisfied for a concrete material. However, it can be supposed that a concrete element behaves in isotropically elastic fashion before substantial cracking takes place.

As to the formation of a new diagonal crack in a non pre-cracked beam (see Fig. 17a), elements in the web portion are most highly strained by shear action. Due to the co-axiality principle, this shear strain induces a corresponding shear stress in the diagonal direction, thus leading to generation of a new diagonal crack once the tensile strength is exceeded. However, in a beam containing pre-existing cracks with any inclination (but of sufficiently great width), diagonal cracks do not form across the pre-existing cracks. The failure crack of Beam 4 in the experiment is shown again in Fig. 17b. The pre-cracked element at the center of the shear span contains one weak plane in the vertical direction. This imparts a strong anisotropy in shear to the element. The imposed total strain to which the element is subjected must therefore be decomposed into a continuum shear strain part and shear slip along the pre-existing crack.

As a result of the low traction transfer across pre-existing cracks, this imposed strain cannot develop enough shear stress on the pre-existing crack surface for the principal stress to rotate with the principal shear strain. In other words, the stress concentration in the diagonal direction is relaxed, and consequently the tensile strength is not exceeded so no new diagonal cracks form. As a result, the contribution of shear slip along the pre-existing crack to total deformation of the element becomes more pronounced. This explanation provides a rationale in terms of the mechanics of a pre-cracked element. Thus the mutual existence of the two deformational modes, one following the pre-existing crack path and the other following the conventional diagonal crack path, which is a pillar of our main discussion, finds logical explanation in fundamental mechanics.
This clarifies the influence of pre-crack width, and provides a rational explanation for it. An element with larger pre-existing cracks exhibit greater shear slip and so more effectively prevents the principal stress from co-rotating with the principal strain. The implication is that the principal stress lags noticeably behind the principal strain orientation. Therefore, it is difficult for new cracks to form in such an element and the deformational mode is basically that of pre-cracking. The converse is true when the crack width is smaller.

For the beam shown in the figure, the above reasoning means that the initiation of a new diagonal crack is not possible at the pre-crack location. Consequently, cracks 1 and 2 must develop independently as they are on opposite sides of the pre-crack plane. Once formed, they propagate separately such that the failure path can be made to join the loading point and the support, hence resulting in the geometry shown in the figure. It is natural that, when a diagonal crack propagates as far as a pre-existing crack, it must stop according to the rules governing crack arrest and diversion as already described. At this point, crack interaction plays a role until the two independent cracks are able to combine, which is the signal for final failure.

This clearly demonstrates that the formation of diagonal cracks in a pre-cracked beam is modified by the presence of pre-cracks. More specifically, the change in diagonal crack generation leads to the initiation of independent diagonal cracks and relaxes the localization that occurs as a diagonal crack propagates in a non pre-cracked beam. As a consequence of this modified crack propagation, the loading capacity of a pre-cracked beam is significantly increased.
9. CONCLUSIONS

An experimental program was conducted to investigate the influence of pre-existing cracks on the behavior of RC beams in shear. Experimental results indicate that there is a significant difference between pre-cracked beams and non pre-cracked beams. RC beams with vertical pre-cracking exhibit considerably higher shear capacity as compared with non pre-cracked beams. The failure characteristics and the load displacement relationship also differ greatly. The width of the pre-existing cracks is identified as the main factor in these differences.

An explanation of the experimental results is provided in terms of the comparative deformatonal characteristics of pre-existing cracks and diagonal cracks in a pre-cracked beam. These relative deformatonal characteristics indicate crack interaction in line with the active crack scheme. The phenomenon of crack arrest and diversion is identified as the cause of the discontinuous propagation of diagonal cracks that underlies the difference in behavior. The Z-cracks observed in the experiments are the logical outcome of the shared contribution of pre-existing cracks and diagonal cracks.

The geometry of the pre-cracking greatly affects the Z-crack geometry that develops, and this in turn influences the shear behavior of the beam. Pre-cracking affects beam behavior not only in the post-diagonal crack range but also before diagonal cracking and in the initiation of new diagonal cracks. The anisotropy of a pre-cracked element, arising because of the low traction transfer along the surface of a pre-existing crack, prevents the principal stress co-rotating with the imposed total strain. This therefore prevents the formation of diagonal cracks where there is pre-cracking. Instead, a large amount of slip takes place along the pre-existing crack. Therefore a rationale based on fundamental mechanics exists to explain the influence of pre-cracks.

ACKNOWLEDGEMENT: The authors would like to express their gratitude to TEPCO foundation for providing financial support to carry out the experimental program.

References

7) Standard Specification for design and construction of concrete structures – 1986 part 1 [design], JSCE.
An experimental program involving the measurement of the local vertical strain distribution within a concrete specimen has been conducted to examine the failure mechanism and localization effect of concrete under uniaxial compression. This leads to a new quantitative approach to localized compressive fracture length. By considering the geometrical parameters and properties of the concrete used, it is found that, when localization occurs, the localized compressive fracture length depends only on the size of the cross-section of the specimen. A new definition of fracture energy under compression in terms of externally applied energy per unit fracture volume is also introduced.

Key Words: fracture mechanics, localization, size effects, fracture energy in compression, localized compressive fracture length

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1. INTRODUCTION

It is now generally accepted that the failure of concrete in tension is localized within a limited zone. Many researchers have studied the localization behavior of concrete in tension, and useful results have been obtained\(^1\). The localized behavior in tension is generally modeled by a stress-crack width relationship based on the tensile fracture energy \((G_t)\). A uniaxial description of the properties is sufficiently realistic, because no major lateral deformations take place simultaneously.

On the other hand, material models for compressive failure of concrete are normally based on a uniaxial compressive stress-strain curve obtained from tests, where uniform deformation throughout the concrete specimen is assumed. This assumption is reasonable for the ascending branch of the stress-strain curve, but is not necessarily accurate for the descending branch. Since it has been found that deformation after the peak stress is localized within certain zones\(^2\), the descending branch of the stress-strain curve becomes size dependent as the measured strain depends on the gage length and position of the gage\(^3\).

Despite a number of studies\(^4\)-\(^5\), the localization behavior and fracture zone of concrete in compression have not yet been clarified. Compressive failure is more complex than tensile failure, because it is always accompanied by significant lateral deformations. These lateral deformations are mainly caused by splitting cracks, which form and expand during the failure process. In addition to these cracks, localized shear bands may also form. Thus, the fracture process in compression is not only determined by localized cracks, as in tension; rather to be realistic, both the local and the continuum components of compressive softening have to be taken into account.

In order to analyze the post-peak behavior of concrete in a more realistic manner, it is necessary to study the behavior of concrete under compression in consideration of crack localization and fracture energy of concrete in compression. Furthermore, a realistic stress-strain curve for concrete under uniaxial compression that takes into account localization behavior will lead to more accurate results in the analysis of bending stresses in reinforced concrete members\(^6\).

2. REVIEW OF PREVIOUS RESEARCH

The problem of accurately determining localized compressive length has often been discussed. Markeset and Hillerborg\(^5\)-\(^7\) took the value of damaged zone, \(L_d\), to be 2.5 times the smallest lateral dimension of the concrete cross-section. This proposed value was based on the finding of other researchers\(^3\)-\(^8\) that the compressive strength, \(f_c\), of a specimen becomes constant when the slenderness ratio reaches a value of about 2.5. This constant value of \(f_c\) is obtained regardless of the test technique and end restraint conditions. However, there has been no direct measurement of the fracture length for the suggested value of \(L_d\).

In 1999, an attempt to measure the localized compressive fracture length was made by Nakamura and Higai\(^4\). They obtained the localized compressive fracture length by considering the local strain, as measured by embedding a deformed acrylic bar fitted with strain gages within concrete cylinder specimens. It was found that the localization behavior could be captured effectively, but no clear evaluation method of localized compressive fracture length was presented and the parameters of the study mainly focused on properties of the concrete used to cast the specimens.

On the other hand, the definition of fracture energy in compression also remains uncertain. Nakamura and Higai\(^4\) defined the local fracture energy, \(G_{fo}\), as the energy absorbed per unit area in the fracture zone. This was estimated from the area under the overall load-deformation curve excluding the elastic unloading part. The compressive fracture energy, \(W_q\), was alternatively defined as the energy dissipated up to the point where the load descends to 1/3 of the peak load; this includes a portion of the elastic strain energy as presented by Rokugo and Koyanagi\(^9\).

In this investigation, an experimental program is conducted in order to clarify the localized behavior of concrete in compression. The experiment is divided into two parts. The first considers the wide range of geometrical parameters such as the height-depth ratio, shape and size of a specimen. The effects of these parameters on localized compressive fracture length and fracture energy in compression of concrete under uniaxial compressive stress are determined while holding properties of the concrete constant.

The effects of concrete properties (i.e. cylindrical compressive strength and maximum size of coarse aggregate) on localized compressive fracture length and fracture energy in compression are subsequently examined in the second part of the experiment. In all tests, high early strength cement is used. Finally, a quantitative judgment of localized compressive fracture length and a definition of compressive fracture energy are introduced.
3. EXPERIMENT

3.1 Part I

The effects of the geometrical parameters height-depth ratio, size and shape of the specimen on failure relating to localized compressive fracture length and compressive fracture energy were examined. Details of the experimental program are listed in Table 1(a).

All specimens were cast using concrete with the same mix proportion and an average cylindrical compressive strength of 45 N/mm². The quantities of materials used are shown in Table 1(b). The tops of specimens were capped with cement paste 6-8 hours after casting to ensure a smooth horizontal surface for loading. Specimens were demolded after one day and cured in water at about 22 °C for 7 to 8 days before testing. Compressive strength tests were also carried out on concrete cylinders φ 100 × 200 mm taken from the same concrete batch.

The technique of measuring strain within a concrete specimen by embedding a deformed acrylic resin bar, as studied by Nakamura and Higai, was found to be effective and was adopted in this experiment. The square-section acrylic resin bar with a cross-section of 10 × 10 mm was first deformed by cutting half cylinder-shaped furrows across the two opposite faces in order to ensure good bonding between the bar and concrete, as shown in Fig.1(a). Strain gages were attached to the bar which was then installed vertically in a position coincident with the specimen axis before casting of the concrete. The strain gages were attached in a vertical orientation to measure the longitudinal local strain at intervals of 40 mm (or 20 mm in the case of 100 mm height specimens). The total deformation of a specimen was externally measured during loading by the use of deflection gages set between the loading plates. Friction at the interfaces between the ends of a concrete specimen and the loading platens was reduced by inserting friction-reducing pad sets consisting of two teflon sheets sandwiching silicon grease. In the case of the standard cylindrical compressive strength tests on the φ 100 × 200 mm cylinder, no attempt was made to reduce friction between specimen ends and the loading plates. The installation of the deformed acrylic bar and the test arrangement are also illustrated in Fig.1. All data were recorded using a data logger.

Post-peak load-deformation curves were captured by one-directional repeated loading in the stress descending range. The initiation and propagation of cracks were also visually observed.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Cross-section (mm×mm)</th>
<th>Height (mm)</th>
<th>H/D</th>
<th>Designation</th>
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<td>C10-10</td>
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</table>

*Average cylindrical compressive strength at 7 days for all cases is 45 N/mm² with the maximum size of coarse aggregate 20 mm.

<table>
<thead>
<tr>
<th>Name of mixture</th>
<th>G_max (mm)</th>
<th>W/C (%)</th>
<th>s/a (%)</th>
<th>Weight per unit volume (kg/m³)</th>
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</thead>
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<td>W</td>
<td>C</td>
<td>S</td>
<td>G (mm)</td>
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<td>20</td>
<td>50</td>
<td>45</td>
<td>185</td>
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</tbody>
</table>

**No admixture; air content is 2.0%.

(a) Acrylic bar with strain gages

(b) Installation of acrylic bar

(c) Loading set up

Fig.1 Installation of strain gages and test arrangement
3.2 Part II

In this part of the experiment, only cylinder specimens $100 \times 400$ mm were used in the testing program. Once again, a deformed acrylic bar fitted with strain gages was embedded inside all specimens. The concrete properties, i.e. maximum size of coarse aggregate, $G_{\text{max}}$, and water-cement ratio, W/C, were varied as shown in Table 2. The specimen preparation process was the same as in Part I, except that instead of capping the tops of specimens, the tops of the specimens were ground.

The testing procedures were also the same as in Part I.

<table>
<thead>
<tr>
<th>Table 2 Experimental program: Part II*</th>
</tr>
</thead>
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</tr>
<tr>
<td>26</td>
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<tr>
<td>27</td>
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</table>

*No admixture was added and the air content for $G_{\text{max}} 13$ and 20 mm is 2.5% and 2.0%, respectively.

4. DETERMINATION OF LOCALIZED COMRESSIVE FRACTURE LENGTH AND FRACTURE ENERGY IN COMPRESSION

After compiling all test results, the overall average of stress-strain curves from external measurements, together with the local stress-strain curves measured by each gage attached to the deformed acrylic bar, were plotted. As an example, the PS10 series of specimens are shown in Fig.2 (a) and (b). Here, $\sigma_{\text{max}}$ and $\varepsilon_{\text{a}}$ refer to the maximum stress and corresponding strain, respectively.

4.1 Localized compressive fracture length, $L_{\text{p}}$

It can be seen from the local stress-strain curves of PS10-40 shown as in Fig.2 (b) (full results including the numerical values for PS10-40 are summarized in APPENDIX A) that, in the stress descending range, some parts of the specimen exhibit softening behavior (increasing strain) while others show unloading behavior (decreasing strain). Thus, in this case, failure was localized into a certain part of the specimen.

As cited before, many attempts have been made to set up the value of fracture length or localized compressive fracture length, $L_{\text{p}}$, on the basis of test results. Most did not directly measure the value of $L_{\text{p}}$, but rather theoretically derived it\(^5\), or else based it on the part of the specimen believed to be subjected to uniform uniaxial compressive stress\(^5\). One attempt to directly measure $L_{\text{p}}$ was made by Nakamura and Higai\(^4\), but the determination of $L_{\text{p}}$ in their study was based on the ability to distinguish between the zone of increasing local strain (the softening zone) and the zone of decreasing strain (the unloading zone), which is somewhat subjective. This problem also became apparent in this study when some parts of a specimen showed unloading behavior at the beginning of the descending path, but then exhibited softening behavior when the load was further increased, as seen in curves for gage numbers 3 to 6 in Fig.2 (b).

---

Fig.2 Typical results from experiment Part I (PS10-SERIES)
Given these problems, a newly developed concept for the determination of \( L_p \) is introduced in this paper. The energy consumed by each portion of a specimen, as calculated from the load-local deformation curve, is used as the criterion for judgment of \( L_p \).

From the load-local deformation curves of a specimen, the energy absorbed by the whole specimen can be calculated by summing up the energy absorbed in each portion of the specimen, assuming that there is constant strain in the intervals between strain gages, as depicted in Fig. 3. Figure 3 (a) shows the portion assumed to have constant strain, while Fig. 3 (b) shows the area beneath the descending part of the load-local deformation, \( P-d_o \) curve for that specimen portion until the load falls to 10% of that at the maximum resistance, \( P_{max} \). The calculated area, i.e. the shaded area, therefore, is the energy consumed within that portion of the specimen; this will be called \( A_{int} \). The summation of \( A_{int} \) over the whole specimen yields the total energy consumed, or \( A_{ext} \), as shown in Eq. (1).

\[
A_{int} = \sum_{i=1}^{n} A_{int_i} \quad \text{(N-mm)} \tag{1}
\]

where, \( n \) is the total number of gages attached to the deformed acrylic bar.

Therefore, a distinct and objective definition of the compressive fracture zone is then obtained as the zone in which the value of \( A_{int} \) is larger than 15 percent of \( A_{ext} \), and the length of this zone is called the localized compressive fracture length, \( L_p \) (Fig. 3 (c)). The 15 percent criterion is selected because calculated values of \( L_p \) correlate well with the experimental observations at this level. In addition, for portions in which \( A_{int} \) is greater than 15 percent of \( A_{ext} \), it can be perceived that the energy absorbed is considerably high and enough to lead to failure.

### 4.2 Compressive fracture energy, \( G_{Fc} \)

The definition of compressive fracture energy has also been suggested by many researchers. Nakamura and Higai\(^4\) defined the local compressive fracture energy in terms of absorbed energy, as provided by an externally applied load up to 20% of \( P_{max} \), within the softening range excluding the elastic unloading path, per unit area of the specimen cross-section. On the other hand, Rokugo and Koyanagi\(^9\) defined the total absorbed energy up to 1/3 (approximately 33%) of \( P_{max} \) in the descending range as the compressive fracture energy. It is clear that a universal definition has yet to be set up. One reason may arise from the lack of reliable data on fracture length, \( L_p \). Hence, in this paper, based on the newly developed criterion for \( L_p \), \( G_{Fc} \) is evaluated from the energy consumed by the specimen up to 10% of \( P_{max} \) within the descending range and the true fracture volume, \( V_p \), resulting from the externally applied load. This concept of \( G_{Fc} \) based on fracture volume, \( V_p \) (not area) is introduced here since, in a realistic interpretation, the externally applied energy would cause a volumetric failure rather than failure at any specific cross-section of a specimen. Further, the value of 10% of \( P_{max} \) is chosen because, from the experiments, it was found to be the level of load at which further loading would cause only a small change in total specimen deformation.

Just as for \( A_{int} \), \( A_{ext} \) can be calculated from the externally measured load-overall deformation, \( P-d_o \) curve. That is, \( A_{ext} \) is the total energy supplied by the external load that causes failure of the specimen. The main place of failure is the localized fracture volume, \( V_p \), which is the product of \( L_p \) and the specimen cross-sectional area, \( A_o \). Therefore, the compressive fracture energy, \( G_{Fc} \), or the applied energy per unit volume of the fracture zone, can be calculated by dividing the obtained \( A_{ext} \) by \( V_p \), as shown in Eq. (2).
Table 3 Summary of test results

(a) Part I

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Section (mm)</th>
<th>A_c (mm²)</th>
<th>H (mm)</th>
<th>H/D</th>
<th>σ_max (N/mm²)</th>
<th>f_c' (N/mm²)</th>
<th>σ_max / f_c' (%)</th>
<th>L_p (mm)</th>
<th>G_Fc (N/mm²)</th>
</tr>
</thead>
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<td>PRISM</td>
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<td>220</td>
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*For PR20-SERIES, D = 200 mm was used.
** Result not available.

(b) Part II

<table>
<thead>
<tr>
<th>Cross-section × length (mm)</th>
<th>Maximum size of aggregate (mm)</th>
<th>Water-cement ratio</th>
<th>σ_max (N/mm²)</th>
<th>f_c' (N/mm²)</th>
<th>σ_max / f_c' (%)</th>
<th>L_p (mm)</th>
<th>G_Fc (N/mm²)</th>
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<td>91.3</td>
<td>118</td>
<td>0.225</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.70</td>
<td>21.8</td>
<td>26.2</td>
<td>83.3</td>
<td>125</td>
<td>0.178</td>
</tr>
</tbody>
</table>

5. CRACKING PATTERNS

Many researchers have reported the effects of end restraint on the measured stress-strain curve of concrete under uniaxial compression\(^3, 8, 10\). Thus, as mentioned above, in all tests here, an attempt was made to reduce friction between specimen ends and the loading platens by inserting friction-reducing pads. From the test results shown in Table 3(a) and (b), the ratios of the maximum specimen stress to cylindrical compressive strength, \(\sigma_{\text{max}} / f_c'\), vary. However, the average values are 71% and 76% when H/D=2 and 4, respectively, which means the friction was effectively eliminated by the friction-reducing pads.

In the case of H/D = 1, the average \(\sigma_{\text{max}} / f_c'\) is 57%. The reason for this low value is that specimens with H/D=1 failed in splitting failure mode, which is completely different from the cases when H/D≥2.

Moreover, from the observation of cracks, it was found that, for specimens with H/D≥2, failure did not commence at the central zone of a specimen but was initiated from one end, meaning that the effects of end restraint were substantially
eliminated. An additional reason arises because of the nature of concrete; it is inhomogeneous and its strength over the specimen length takes the form of a normal distribution. Hence, stable failure throughout the length cannot be expected. Furthermore, certain unavoidable imperfections in the testing arrangement, such as the horizontality of the loading plate, mean that perfectly uniform force transfer from the ends is unlikely to be achieved, especially when the length of the specimen is greater; therefore, the failure is likely to be initiated from one end of the specimen.

5.1 Experiment Part I

From observations of crack occurrence in tested specimens, it can be seen that, for short specimens with H/D=1, failure consisted of splitting from top to bottom of the specimen and the observed value of compressive fracture length, L_p, is, in almost all cases, equal to H. As for the longer specimens with H/D=2 and 4, lots of small visible vertical cracks were observed in a particular region when the peak resistance was reached. At the final stage, in the post-peak region at 0.1P_max, the small cracks coalesced to form the crack zone while a few long vertical cracks penetrated down to the bottom (in a case where the specimen failed from the top). Thus localization occurred only for H/D>2, and later sections discussing L_p and G_f will refer to only the results for specimens having H/D>2. This localization behavior can also be clearly seen in the photographs of the cracking pattern at the final stage shown in Fig.4.

It should be noted that, compared with the work done by Markeset\textsuperscript{5}, these vertical cracks are assumed to be localized into a shear band but positioned vertically in this experiment, because friction at the ends was effectively removed.

5.2 Experiment Part II

Further observations on the effects of maximum size of coarse aggregate and water-cement ratio on localization in compression were carried out. Concrete cylinder specimens with a diameter of 100 mm and a height of 400 mm were selected and tested. Typical results are depicted in Fig.5.

From the observation of cracks during the tests, examples of which are shown in Fig.6 (a) and (b), it is clear that localization occurred in some parts of specimens with a few penetrating long cracks as in experiment Part I. There is no
significant difference in cracking patterns when specimens have different concrete properties.

It is of interest to notice that the slope of each local stress-strain curve shown in Fig.2 (b) (refer to also APPENDIX A) is different. A possible reason for this is that, when loading was applied, the volume of the specimen began to decrease as the length shortened, and this was followed later by lateral expansion. However, due to the residual frictional restraint at each end of the specimen, despite the attempt to eliminate it, and because of the inhomogeneity of concrete itself, expansion at the top and bottom was different. In the case of PS10-40 (Fig.3 (c)), the bottom expansion seems to be larger than that at the top, while more shortening is seen at the top, i.e. it has not yet expanded. Therefore, the ascending curves of local gages positioned near the top of the specimen show significant changes in longitudinal strain, whereas nearer the bottom, and especially at gage 10, only a slight change takes place, as can be seen from Fig.2 (b). On the other hand, after the peak load, the lower portion, where expansion is greater than at the top, fails in the final state; in other words, failure is localized at the bottom. This also confirms the concept that most of the applied energy is absorbed within the localized failure zone. This can be seen from area under the local strain curves, i.e. the higher $A_{max}$ at gages positioned on the lower half as compared with the upper ones.

In conclusion, the test results show in all cases for specimens whose length is greater than $L_p$, the final failure pattern is a combination of a few long penetrating cracks and a zone containing lots of small splitting cracks. The penetrating cracks are, in general, in the diagonal shear band, but in this study the frictional restraint at both ends of the specimens was eliminated in the tests, so the deviation of the crack inclination from the vertical is relatively small. The zone containing the splitting cracks, which indicate volumetric failure, contributes to the failure of most specimens; as a result, in this research, the determination of $L_p$ is based on the length of this zone. Accordingly, $G_{pc}$, which is defined as the energy required to cause compressive failure of a unit volume of the specimen, can then be calculated based on the externally applied energy and the localized failure volume, i.e. the zone containing splitting cracks, as described in detail in the Sections 6 and 7.

6. LOCALIZED COMPRESSIVE FRACTURE LENGTH, $L_p$

6.1 Effects of geometrical parameters

The effects of each geometrical parameter are discussed below for cases except $H/D=1$.

a) Height-depth ratio

By including also the results from experiment Part II, Fig.7 shows that the variation of $H/D$ of a specimen causes no significant change to $L_p$. An average $L_p$ value of almost 120 mm was obtained for both $H/D=2$ and 4 cases. In other words, a change in height of a specimen of a particular cross-section has no significant effect on $L_p$. The results for PR20-80 are excluded from this consideration because at the final stage a long penetrating crack from top to bottom was observed, indicating that the specimen failed with a different failure mode.

b) Size and shape of specimen

From Fig.8, it can be observed that, for specimens having the same type of cross-section, an increase in cross-sectional area leads to a slight decrease in $L_p$ while the square and rectangular cross-section specimens show slightly higher values of $L_p$ compared with cylindrical specimens.

6.2 Effects of concrete properties

The relationship between the experimentally obtained localized compressive fracture length and the cylindrical compressive strength is plotted in Fig.9. It can be seen that, regardless of $f'_c$ and $G_{max}$, $L_p$ is almost constant with an average value of 120 mm and a coefficient of variation of 11%. That means, in comparison with geometrical
parameters, $G_{\text{max}}$ and W/C have little effect on the localized compressive fracture length of a specimen.

6.3 Formulation of $L_p$

As noted above, the height and shape of a specimen and also properties of the concrete used to cast it have virtually no effect on $L_p$, whereas $L_p$ evidently dependent on the cross-sectional area. Therefore, the relation between $L_p$ and the cross-section of a specimen given in terms of $D^*$, the equivalent cross-section width or the square root of cross-sectional area, $A_c$, is considered. The plot of concrete cross-sectional area against $L_p/D^*$ in Fig. 10 (a) shows that, within the range of the tests, an almost constant value of $L_p/D^*$ is obtained when $D^*$ is less than 100 mm ($A_c<10,000 \text{ mm}^2$). An increase in $A_c$ above this leads to a decrease in $L_p/D^*$ in which the rate of decrease gradually falls. In order to simplify the relationship, a constant value of $L_p/D^*$ for $D^*$ larger than 180 mm ($A_c>32,400 \text{ mm}^2$) is assumed.

Finally, the following simplified relationship can be proposed:

$$L_p/D^* = \begin{cases} 1.36 &; D^*<100 \\ = -3.53 \times 10^2D^2 +1.71 &; 100 \leq D^* \leq 180 \\ = 0.57 &; D^*>180 \end{cases} \quad (3)$$

where, $D^* = \sqrt{A_c}$, mm

as depicted in Fig.10 (b).

The effects of various concrete specimen parameters on $L_p$, which were obtained from direct measurements, were also studied by Nakamura and Higai\(^9\). They performed uniaxial compressive tests on concrete cylinders measuring $\phi$ 100 and $\phi$ 150 mm with a range of H/D ratios, and found that H/D has little effect on $L_p$, reflecting the results obtained in this study. However, their results further showed that the cross-sectional area of a specimen has no effect on $L_p$, while $L_p$ is rather affected by $L_p$ and the size and grading of the aggregates. This may be because of the narrower range of cross-sectional area in their test. In addition, $L_p$ was not quantitatively evaluated, which differs from the process presented here. In contrast, according to experimental research done by Rokugo and Koyanagi\(^9\), $L_p$ tended to be constant for concrete specimens with the same cross-sectional area.

7. COMPRESSIVE FRACTURE ENERGY, $G_{\text{Fe}}$

As mentioned above, one-directional repeated loading in the stress descending range was carried out so as to capture the stress-strain curve within the post-peak region. However, for specimens with comparatively large cross-section, a sudden drop in load at the peak point was, for some reasons, unavoidable. Furthermore, $G_{\text{Fe}}$ is substantially dependent on the area under the load-overall deformation curve or the shape of the curve itself, especially in the descending range. Therefore, in order to obtain the most reliable results, the results of the C20 and PS20 series were not included in the consideration of $G_{\text{Fe}}$. Thus, Fig.11 was plotted omitting these.
7.1 Effects of geometrical parameters

Figure 11 (a) shows the relationship between the total specimen volume, $V_c$, and the average value of $G_{Fe}$ for cylindrical and prism specimens. In order to isolate the effects of the geometrical parameters, the results of Part II were not included. It can be seen that there was almost no change in $G_{Fe}$ with changes in the geometry of specimens.

7.2 Effects of concrete properties

Figure 11 (b) shows that the value of $G_{Fe}$ is, in some way, dependent on the cylindrical compressive strength. It is found that when $G_{Fe}$ is divided by $f_c^{1/4}$, an almost constant average value of $0.86 \times 10^{-1}$ with a coefficient of variation of 18% is obtained, as shown in Fig. 11 (c).

On the other hand, from Fig. 12 it can be seen that the maximum size of coarse aggregate has little effect on the magnitude of $G_{Fe}$.

7.3 Formulation of $G_{Fe}$

From Fig. 11(c), the following simplified relationship can be obtained for when localization in compression occurs:

$$G_{Fc} = 0.86 \times 10^{-1} f_c^{1/4}$$  \hspace{1cm} (4)

where, the units of $G_{Fc}$ and $f_c$ are N/mm$^2$.

This formulation is consistent with previous research work$^6$ in that the fracture energy in compression depends on concrete compressive strength regardless of the size and shape of the concrete specimen. Though the coefficient and the power of $f_c$ in the formulation are different because of the difference in the calculation of $G_{Fe}$, but it can be conceived that the relation between $G_{Fc}$ and $f_c$ is nonlinear.

However, the actual cracking pattern of a concrete specimen comprises a zone containing lots of small cracks and a small number of penetrating long cracks. The calculation of $G_{Fe}$ here is based on only the small cracking zone, which is simulated by the localized fracture volume, while the long penetrating cracks and the unloading portion that absorb some part of the external applied energy are not taken into consideration. Note that the results for PR20-80 were also not included here because the final failure pattern of the specimen, which consisted of a long and wide open splitting crack from top to bottom and only few small tensile splitting cracks, is considerably different from the other specimens.

8. CONCLUDING REMARKS

Through the technique of measuring the local strain within a concrete specimen along its axis, localization of the failure in uniaxial compression is shown to occur when the specimen has an H/D ratio greater than or equal to 2. The localized compressive fracture length can be evaluated from the relative amount of energy absorbed by each portion of the specimen. The method presented here offers a new quantitative means of determining the value of $L_p$, and the results obtained are found to agree with observations made during the testing procedure.
The localized compressive fracture length is found to be dependent solely on the specimen cross-section, whereas the specimen height, H/D ratio, and cross-sectional shape are found to have less effect on $L_p$. The cylindrical compressive strength and the maximum size of aggregate of the specimen concrete are found to have virtually no effect on localized compressive fracture length. A relation between $L_p$ and the equivalent section width is proposed.

Subsequently, the fracture energy in compression is calculated by dividing the area under the load-overall deformation curve by the fracture volume. This concept is different from previous research work which, in most cases, did not take into account the effect of the localized failure zone. The test results indicate that $G_{fc}$ varies with changes in concrete cylindrical compressive strength, while changes in geometry of the specimen and the maximum size of aggregate used in casting have less effect on $G_{fc}$. A relation between $G_{fc}$ and $L_p$ is proposed.

The obtained $G_{fc}$ values do not take into consideration the contribution from penetrating cracks and the unloading portion; further studies are needed in order to gain a better understanding of localization behavior in compression.

**APPENDIX A TEST RESULTS FOR PS10-40**

The applied stress-local strain curves for PS10-40 are depicted in detail in Fig. A1. Each curve represents the path obtained from one-directional repeated loading in the stress descending range, and the envelope obtained by connecting the peak points of the repeated curve is also shown.

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Fig. A1 Results for PS10-40 (Gage 1-10)
APPENDIX B USE OF ACRYLIC BAR

The effectiveness of local strain measurements using a deformed acrylic bar was investigated by comparing the deformation measured externally by deflection gages with the accumulated calculated deformation from the local strain gages throughout the whole length of a specimen. It was found that the two sets of results agreed with each other well until the peak load was reached. After the peak load, the calculated deformation from local strain gages was slightly smaller than that indicated by the deflection gages, as shown in Fig.B1. This deviation occurs because, once the maximum resistance is reached, cracking takes place in the specimen; at this point the deflection gages continue measuring the overall-averaged deformation, whereas the calculated deformation from local strain gages is, in some way, evaluated based on the magnitude of the strain measured at the location of the strain gages and is the average figure for the interval between gages. However, the difference is negligibly small, so the use of a deformed acrylic bar with attached strain gages to measure the internal local strain is considered reliable.

References