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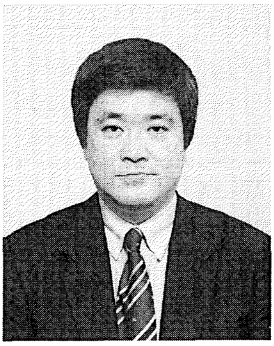
EVALUATION BY LATTICE MODEL OF ULTIMATE DEFORMATION OF RC COLUMNS SUBJECTED TO CYCLIC LOADING

(Translation from the proceedings of JSCE, No.641/V-46, February 2000)


Atsushi ITOH　　Junichiro NIWA　　Tada-aki TANABE

This study extends the lattice model, which offers accurate evaluations of the shear resisting mechanism, into a cyclic model. The inelastic deformation ability of RC columns subjected to reversed cyclic loading is evaluated using this cyclic lattice model. It is confirmed that the cyclic lattice model is able to evaluate the inelastic deformability of six RC column specimens having different amounts of transverse reinforcement to good accuracy as compared with experimental results. Furthermore, the resisting mechanism of RC columns until the ultimate state is analytically evaluated by observing stresses in the members of the lattice model.

Key Words: lattice model, cyclic loading, ultimate deformation, shear reinforcement ratio, shear-resisting mechanism

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Atsushi Itoh is a Doctoral student in the Department of Civil Engineering at Nagoya University, Nagoya, Japan. He is a member of the JSCE, and JCI.

Junichiro Niwa is a Professor in the Department of Civil Engineering at Tokyo Institute of Technology, Tokyo, Japan. He obtained his D.Eng. from the University of Tokyo in 1983. His research interests are related to the fracture mechanics and shear capacity of reinforced concrete. He is a member of the JSCE, JCI, ACI, fib, and IABSE.

Tada-aki Tanabe is a Professor in the Department of Civil Engineering at Nagoya University, Nagoya, Japan. He obtained his D.Eng. from the University of Tokyo in 1971. His research interests include concrete mechanics, seismic design of concrete structures, and thermal stress problems. He is a member of the JSCE, JCI, ACI, RILEM, fib, and IABSE.

1. INTRODUCTION

When failure of flexural-reinforced concrete (RC) members in flexural mode accompanies yielding of the longitudinal reinforcement in the tension zone, considerable inelastic deformation may occur while sufficient flexural capacity is retained, and much energy may be absorbed up to failure. To make the use of this excellent deformability and energy absorption, a ductility design method that permits considerable inelastic deformation during earthquakes while limiting maximum deformation has been developed. The JSCE specifications for seismic design of RC structures adopt a design strategy that permits shear failure after flexural yielding and limits plastic deformation within certain bounds. This type of design relies on calculation of the plastic deformability of RC members subjected to reversed cyclic loading to good accuracy. This led to past attempts to develop an experimental formulation of the plastic deformability of RC columns [1], [2]. In recent years, however, RC structures have increased in size. And as RC structures exhibit a significant size effect, some doubts have arisen with respect to applying these experimental formulas based on limited-size specimens too much larger real structures. However, with the size of specimens that can be tested limited by available experimental tools, consideration needs to be given to the evaluation of plastic deformability by numerical methods.

In the past, numerical studies based on nonlinear FEM analysis, such as those by Okamura et al. [3] and Nakamura et al. [4] have successfully evaluated the plastic deformability of RC members. However, it is found to be very difficult to evaluate all aspects of the real behavior of RC members up to their ultimate state, particularly when the failure mode is shear after yielding. This is because shear failure involves material and structural failures, and all factors related to shear failure are affected by each other. And as for RC structures subjected to cyclic loading, the influence of cover spalling after cracking and other effects should be also taken into account.

The objective of this study is to predict the plastic deformability of RC columns subjected to reversed cyclic loading using a simple numerical model. Since the ultimate failure mode is expected to be shear, the lattice model [5] is used for analysis. The lattice model has proven to be an effective method of handling shear failure problems in RC beams subjected to monotonic loading [5]. In this study, cyclic analysis is carried out on six RC columns with different transverse reinforcement ratios and the ability of the lattice model to effectively predict variations in inelastic deformation of RC columns in terms of transverse reinforcement ratio is examined.

2. ANALYTICAL MODEL

The lattice model [5], as developed by Niwa et al., is extended to handle cyclic loading for use in this study. The major characteristic of the lattice model is that the shear resisting mechanism is clearly expressed, and the results of analysis are objective because RC members are modeled into only one type. Since a RC member is considered as an assembly of truss members in the lattice model, the total degrees of freedom and computational time are expected to be significantly less than in finite element analysis.

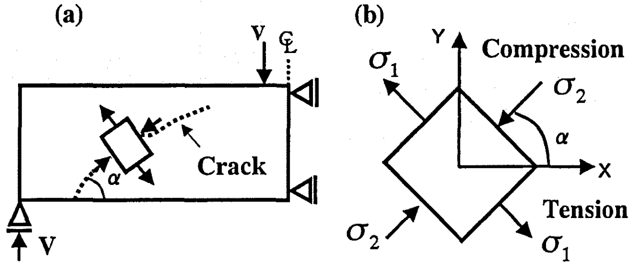


Figure 1. Stress State of Concrete Element

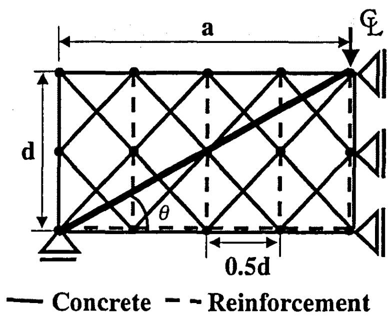


Figure 2. Concept of Lattice Model

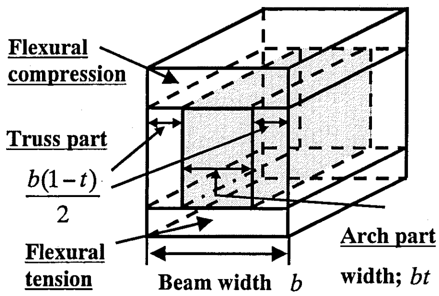


Figure 3. Cross Section of Lattice Model

2.1 Lattice Model

Figure 1(a) shows a reinforced concrete beam with diagonal cracks. It is assumed that a diagonal crack propagates at an inclination of α degrees to the beam axis. The shear force, which acts along the crack surface, is neglected in the lattice model. Based on these assumptions, in an infinitesimal element parallel to the crack direction, as shown superimposed on **Figure 1(a)**, bi-axial compressive and tensile stresses act as shown in **Figure 1(b)**. These stresses acting in an infinitesimal element correspond to the principal compressive stress σ_2 and the principal tensile stress σ_1, respectively, and the angle between the beam axis and the principal compressive stress direction is α.

By assuming that the above compressive and tensile stresses act in web concrete and the crack direction α is 45 degrees, a RC beam, which can be considered a continuum, may be modeled as an assembly of truss members. The configuration of the lattice model is shown in **Figure 2**, and the components of the model are treated as follows. The concrete portion is modeled into flexural compressive and tensile members, diagonal compressive and tensile members, and an arch member. The reinforcement is modeled into horizontal and vertical members. The lattice model differs from the traditional truss model in that it includes the diagonal concrete tensile members and an arch member. The diagonal concrete tensile members make it possible to express shear-resisting behavior before and after diagonal cracking. Moreover, by incorporating an arch member, it is possible to express the redistribution of stresses in each member after yielding of the shear reinforcement while the angle of the diagonal members remains 45 degrees. The arch member, as shown by the thick line in **Figure 2**, is arranged along the line of internal compressive force. In the case shown in **Figure 2**, the arch member is a long member connecting the loading point with the support point. This is based on the assumption that the plane stress condition may be violated, as well as on consideration of the range affected by the stirrups in the web concrete. In fact, the deformation of an arch member may not occur independently of truss members, but it can be considered that the stress field will experience a far from plane stress condition that is normally assumed to form in a beam. A further significance of the arch member is that lattice model analysis implicitly becomes three dimensional although it is actually a two-dimensional model.

2.2 Cross Section of Lattice Model

Figure 3 shows a cross section through a RC beam in the lattice model. The web concrete of the lattice model is divided into truss and arch parts, and the widths of each part are determined to be $b(1-t)$ and bt, respectively, when the ratio of the arch member's width to the beam width (b) is $t(0<t<1)$.

2.3 Determination of t

The only parameter to be determined in the lattice model is t. The value of t can be determined by using the principle of minimum potential energy. According to this theory, when a member is in an elastic state, the real displacement among kinematically permissible displacements minimizes the total potential energy. In order to obtain the maximum stiffness of the RC structure in the initial state, the total potential energy $\pi(t)$ is calculated by changing the value of t in small increments from 0.01 to 0.99. The optimum t value can then be determined from Eq. (1) based on the principle of minimum potential energy.

$$\frac{\partial \pi(t)}{\partial t} = 0 \tag{1}$$

It is obvious that if the deformation of a structure progresses and non-linearity of the material appears, the optimum value of t also varies as the potential energy changes. Therefore in this study, the above-described method was used as a first approximation of t. Generally, the value of t was found to be about 0.3~0.8 for shear members, while for flexural members $t<0.3$ was obtained empirically.

2.4 Definition of Cross-Sectional Area for Lattice Model Members

The cross-sectional area of lattice members is determined as follows. Since the height of the lattice model is taken to be equal to the effective depth (d) of the beam, the height of a set of X-shaped truss members becomes $d/2$ and the height of the arch member is d. By this model, the thickness of the diagonal members as seen from the beam side is equal to $d/2 \cdot \sin 45°$ and the thickness of the arch member is $d \cdot \sin\theta$, where θ is the angle of the arch member (see **Figure 2**). The cross-sectional area of the flexural compressive part is obtained by multiplying the depth of the compressive concrete in the ultimate state, x, by the beam width, b, where $x = (A_s \cdot f_y)/(0.68 f_c \cdot b)$. And the cross-sectional area of the flexural tensile zone is assumed to be the value obtained by multiplying twice the cover depth by b. Analytical results demonstrate that these assumptions of the thickness of flexural compressive and tensile members do not affect the magnitude of shear capacity obtained from the calculation [5]. The areas of longitudinal reinforcement and transverse reinforcement are determined so that the longitudinal and transverse reinforcement ratios of the lattice model and the test specimens are identical.

2.5 Extension of Lattice Model for Cyclic Loading Analysis

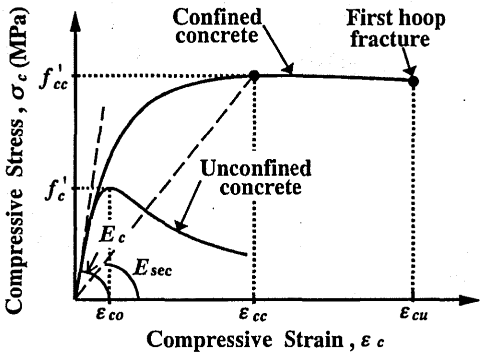
Two modifications were made to extend the lattice model to the analysis of RC columns subjected to reversed cyclic loading, while retaining the concept of the original lattice model for monotonic loading.

1) Considering the symmetry of the structure, the cross-sectional areas of flexural compressive and flexural tensile members, which are vertical concrete members, were made equal to the cross-sectional area of the flexural compressive concrete members.
2) On the understanding that the flow of compressive force in a structure is reversed when the structure is subjected to reversed cyclic loading, arch members were crossed and arranged symmetrically (see **Figure 9**).

Assumption 1) is applicable to the analysis of RC columns subjected to cyclic loading. Incidentally, it is considered that even with this assumption in place, the shear and flexural capacities obtained from calculation are little affected; in fact, flexural capacity is almost completely determined by the yield strength and amount of longitudinal reinforcement arranged in the concrete. Regarding the second assumption, a compressive and tensile arch resisting mechanism operates in the analysis. Although it is not clear that such a tensile arch resisting mechanism exists in real structures, it is considered that its presence has little influence on the analytical results because it is a long, flat concrete member.

2.6 Material Models

The material models for concrete and reinforcement used in this study are described below.



$$\sigma_c = \frac{f'_{cc}\, x\, r}{r - 1 + x^r} \tag{2}$$

$$x = \frac{\varepsilon_c}{\varepsilon_{cc}} \tag{3}$$

$$f'_{cc} = f'_c\left(2.254\sqrt{1 + \frac{7.94}{f'_c}} - \frac{2f'_l}{f'_c} - 1.254\right) \tag{4}$$

$$f'_l = K_e \rho_w f_{wy} \tag{5}$$

$$\varepsilon_{cc} = 0.002\left[1 + 5\left(\frac{f'_{cc}}{f'_c} - 1\right)\right] \tag{6}$$

$$r = \frac{E_c}{E_c - E_{sec}} \tag{7}$$

$$E_{sec} = \frac{f'_{cc}}{\varepsilon_{cc}} \tag{8}$$

Dimension; (MPa) in eq. (2) (4) (5) (7) (8)

Figure 4. Stress-Strain Relationship of Concrete for Compression

a) **Concrete Model for Compression**

When concrete is reinforced with transverse reinforcement for shear, a confining effect may be expected in the core concrete. In this study, in order to take into account this confinement effect, the stress-strain relationship proposed by Mander et al. [6] is adopted. **Figure 4** shows the conceptual view of the stress-strain relationship proposed by Mander and the relevant equations. The nomenclature in Eqs. (2) through (8) is as explained in **Figure 4**. The value of K_e, which is a confinement effectiveness coefficient, is equal to 0.75 for a rectangular cross section. The values ρ_w and f_{wy} are the transverse reinforcement ratio and the yield strength of the transverse reinforcement, respectively. One of the advantages of this model is that it can be applied to any cross-sectional shape as well as to any amount of transverse reinforcement. The value ε_{cu}, shown in **Figure 4**, was determined from the ultimate strain in the transverse reinforcement. However, since experimental observations did not show any failure of the transverse reinforcement, the value of ε_{cu} was actually not used in this analysis. The model proposed by Mander et al. was originally used for stresses in the axial direction, but in this study we have applied it to diagonal concrete members, at 45 degrees to the axial direction.

According to experimental results obtained by Vecchio and Collins, when concrete is subjected to bi-axial stresses in compression and tension, it exhibits a stress-strain relationship differing from that in uniaxial compression. They took account of the reduction in compressive strength due to cracking by using a softening coefficient η, determined as Eq. (9) [7].

$$\eta = 1.0 / \{0.8 - 0.34(\varepsilon/\varepsilon_0)\} \le 1.0 \tag{9}$$

where, ε_0: Yield strain under uniaxial compression
ε: Tensile strain in tensile member perpendicular to each compressive member

In this study, when a diagonal concrete member is under compressive stress, a stress-strain relationship that combines these two models is assumed. The value of the softening coefficient is assumed to vary from a maximum of 1.0 to a minimum of 0.1. Actually, it is unrealistic to assume that the concrete strength might decrease by up to 10%. But in this study, when the amount of transverse reinforcement is not very great, the degree of deterioration in concrete strength due to cyclic loading can be easily evaluated using these values.

A flexural concrete member under compressive stress is assumed to be the following stress-strain relationship:

$$\sigma_c = f'_c\left\{2(\varepsilon_c/\varepsilon_0) - (\varepsilon_c/\varepsilon_0)^2\right\} \tag{10}$$

where, ε_c: Compressive strain in each compressive member
f'_c: Compressive strength of concrete

Along the unloading path, the stress is assumed to decrease with an initial gradient. On the other hand, along the reloading path, the stress is assumed to increase to the maximum stress-strain state in the previous strain history. It is also assumed that concrete has no tensile stress in the compressive strain region.

b) Tension Stiffening Model

When flexural concrete members are subjected to tensile force, a tension stiffening effect can be expected because of the bond effect between concrete and reinforcement. In this study, a tension-stiffening model (**Figure 5**) proposed by Okamura et al. [3] is adopted. This model represents the average stress-strain relationship, and as a result of the bond effect, the tensile stress in concrete does not suddenly drop to zero after cracking. It is assumed that the stress changes linearly upon cracking, while after cracking the tensile stress follows a descending branch, which can be determined by Eq. (11).

$$\sigma_t = f_t(\varepsilon_t / \varepsilon)^{0.4} \tag{11}$$

where, f_t is the tensile strength of the concrete. The strain (ε_{cr}) at which a crack starts to develop is assumed to be 0.0001. Unloading loops are assumed as the stress and strain return to the origin (**Figure 5**). Reloading loops are assumed in the same manner as the concrete model for compression.

c) Introduction of Fracture Energy

The diagonal tensile members of the concrete sustain the tensile stress arising from the applied shear force. Concrete is assumed to be an elastic material before cracking, and the strain (ε_{cr}) at which cracking starts is assumed to be 0.0001. After cracking, the 1/4 tension-softening model [8], which is widely used in the field of fracture mechanics, is adopted as the tension-softening curve for concrete. In this study, in order to apply the model to numerical analysis, the crack width (w) is divided by the length of the concrete diagonal member (L) for conversion into tensile strain. Consequently, the tensile stress-strain relationship can be obtained and ε_1, ε_2 as shown in **Figure 6** can be described as follows.

$$\varepsilon_1 = \varepsilon_{cr} + 0.75G_F / Lf_t \tag{12}$$

$$\varepsilon_2 = \varepsilon_{cr} + 5.0G_F / Lf_t \tag{13}$$

G_F : fracture energy (N/m)

In this study, the fracture energy is assumed to be 100N/m, which is a commonly used value in this type of analysis. The parameters 0.75 and 5.0 in Eqs. (12) and (13) are determined using the 1/4 model proposed by Rokugo et al. Unloading loops are assumed as the stress and strain return to the origin. Reloading loops are assumed in the same manner as in the concrete model for compression. It is also possible to express the size effect by introducing the concept of fracture energy into the concrete model.

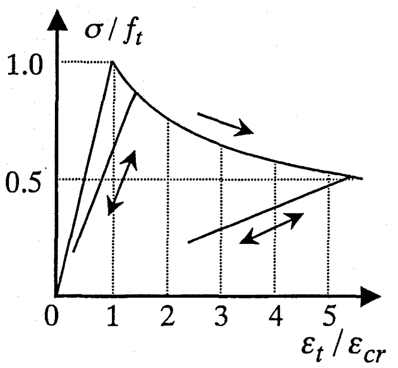


Figure 5. Tension Stiffening Model

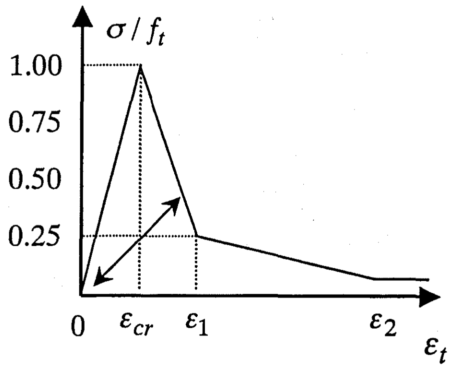


Figure 6. Tension Softening Model

d) Reinforcement Model

The stress-strain relationship of reinforcement is expressed using a bilinear model in which the tangential stiffness of the longitudinal reinforcement is 1/1000*E after yielding and that of transverse reinforcement is 1/100*E (**Figure 7**). E is Young's modulus. In the unloading and reloading phases, the stress changes according to the initial stiffness. The value of 1/100*E used for transverse reinforcement means that the stress increases gradually because the yield strength of the transverse reinforcement is comparatively low and the behavior after yielding may not be perfectly elasto-plastic.

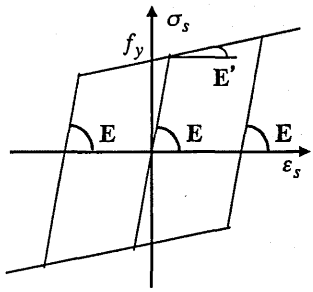


Figure 7. Stress- Strain Relationship of Reinforcement

3. OUTLINE OF EXPERIMENTS AND NUMERICAL ANALYSIS

3.1 Characteristics of Specimens

The experiments were carried out by Puri et al. [9]. The characteristics of the RC column specimens subjected to analysis are shown in **Table 1** and **Figure 8**. The column is a type of cantilever beam and has a relatively small section, 140mm×140mm. The compressive strength of the concrete is approximately $f_c^{'} = 25MPa$. Deformed bars (diameter: 10mm; nominal cross-sectional area: 78.5mm^2; yield strength: 400MPa) act as longitudinal reinforcement in the four corners of the cross section. For transverse reinforcement, round bars (diameter: 4mm; nominal cross-sectional area: 12.5mm^2; yield strength: 280MPa) are used. Effective depth is 115mm. The ratio of shear span to effective depth is 4.35 and in the experiments these columns were subjected to cyclic loading in the absence of axial force. Three cycles of loading were applied at each displacement level ($\pm\delta_y, \pm 2\delta_y, \pm 3\delta_y \cdots$, δ_y; yield displacement). The influence of pull-out of the longitudinal reinforcement is difficult to take into account. However, it is considered negligible, since significant reinforcement is present in the footing. The six specimens have different arrangements of transverse reinforcement to vary the transverse reinforcement ratio from 0.06% to 0.51%. All specimens were designed to fail in shear mode after yielding.

3.2 Lattice Modeling of Specimens

The lattice model used to analyze the specimens, which are illustrated in **Figure 8**, is shown in **Figure 9**. The role of each truss member is given for the case of a horizontal force applied from the upper left of the model. The arch members cross each other to model cyclic loading. The

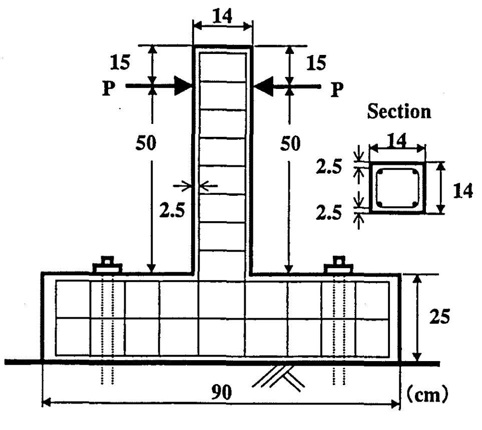


Figure 8. Specimen Dimensions

Table 1. Outline of Column Specimens

Specimen No.	γ_w %	s cm	f_{wy} MPa	A_w cm^2	f_c' MPa	f_y MPa	A_s cm^2
pm1	0.06	28.50	280	0.25	23.9	400	1.57
pm2	0.13	14.00	280	0.25	26.0	400	1.57
pm3	0.19	9.50	280	0.25	25.0	400	1.57
pm4	0.26	7.00	280	0.25	25.0	400	1.57
pm5	0.32	5.50	280	0.25	25.5	400	1.57
pm6	0.51	3.50	280	0.25	25.6	400	1.57

γ_w: stirrup ratio
s: spacing of stirrup
A_w: area of one stirrup
f_{wy}: yield strength of stirrup
f_c': concrete compressive strength
f_y: yield strength of longitudinal bar
A_s: area of longitudinal bar

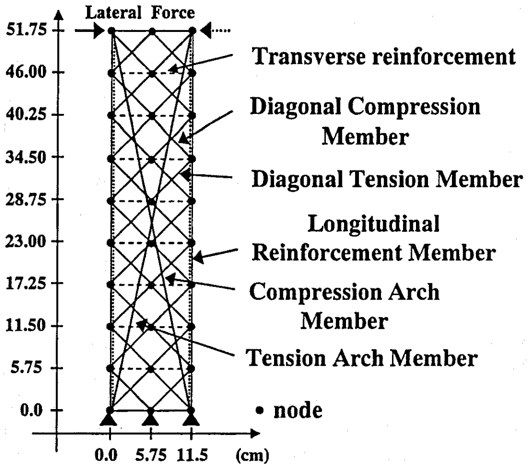


Figure 9. Lattice Model

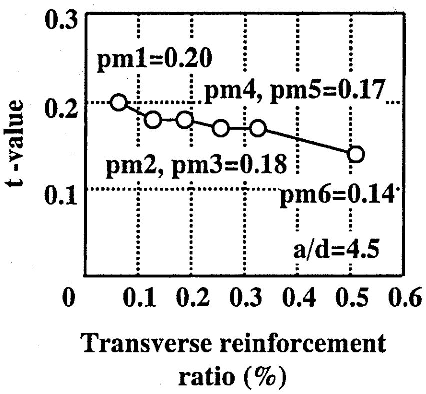


Figure 10. Variation of 't' value

arch members bridge the loading point and the support point in this loading state. In a different loading state, an arch member would need to be provided parallel to the line of internal compressive force.

In the lattice model, since the horizontal spacing of longitudinal flexural members is designed to accord with the effective depth d and the crack direction is assumed to be 45 degrees, the vertical interval of X-shaped diagonal truss members corresponds to $0.5\,d$, as shown in **Figure 9**. Therefore, the shear span of the lattice model is not necessarily equal to that of the specimens. That is the analysis was conducted using a lattice model with a shear span a/d equal to 4.5 to reflect the test specimens (a/d =4.35). In addition, there is a difference in the spacing of transverse reinforcement between the actual structure and the lattice model. In the analysis, however, the cross-sectional areas of transverse reinforcement were adjusted such that the transverse reinforcement ratios matched. The effect of longitudinal reinforcement pull-out was not considered in this analysis, as already noted.

3.3 t Value for Analysis

A preliminary analysis was carried out to determine the optimum t value, the ratio of arch member width to cross section, by elastic analysis. The potential energy of the lattice model was calculated under infinitesimal loading for t values from 0.01 to 0.99 and the optimum t value determined according to the principle of minimum potential energy. The resulting t values are shown in **Figure 10**. As can be seen, the t value tends to decrease as the transverse reinforcement ratio increases due to the fact that flexural deformation of the columns increases as more transverse reinforcement is added.

3.4 Numerical Calculation Program

The lattice model, as extended for use in cyclic stress situations, was used for incremental calculation using the obtained t values by implementing the displacement control method. The modified Newton-Raphson method was used for the iteration procedure until the ratio of unbalanced force to the equivalent nodal force stood at 0.1% or less.

4. RESULTS OF ANALYSIS

4.1 Comparison between Experimental and Analytical Results

The load-displacement relationships for each specimen are shown in **Figure 11**. The test results are plotted as enveloping the load-displacement relationship, while the lines show the analytical results obtained with the cyclic lattice model. The maximum displacement in the test results corresponds to the displacement where the loading capacity decreases suddenly during cyclic loading. Before this displacement was reached after yielding, test specimens suffered spalling of the cover concrete, large diagonal cracks, and crushing of concrete in the column base region. The maximum displacement in the analytical results corresponds to the displacement where the loading capacity could not be maintained due to crushing of diagonal concrete members in the column base region after yielding.

Analytical results are all in good agreement with test results except for the case of specimen *pm1* and the lattice model is able to model the tendency of plastic deformation to increase with transverse reinforcement ratio. The reason for failure to evaluate the actual behavior of *pm1* may be the following. In the analysis, only the effect of transverse reinforcement ratio on the plastic deformation of the member was taken into account, while the effects of actual spacing of the transverse reinforcement arrangement were not considered. In the lattice model, the transverse reinforcement was arranged uniformly at a spacing of $d/2$ so that the transverse reinforcement ratio of lattice model was equal to that of test specimens. Thus, in specimen *pm1* with a low transverse reinforcement ratio, the cross-sectional area of the transverse reinforcement members in the lattice model was very small, at 0.05cm^2. This assumption of the lattice model may be inappropriate for analysis of the *pm1* case. In the analysis, diagonal cracking occurred before and after yielding of the longitudinal reinforcement in the column base region just as in the experiments. In the experiment, the transverse reinforcement (cross-sectional area: 0.25cm^2) arranged in the column base region was expected to be able to prevent the progress of this diagonal cracking. On the other hand, in the analysis, the transverse reinforcing members were unable to prevent the

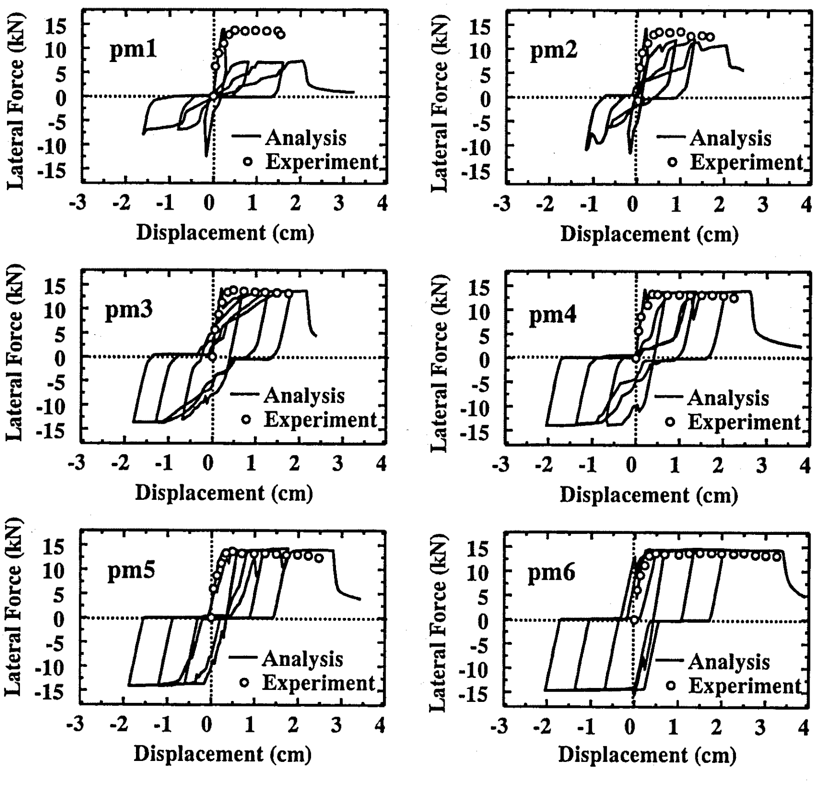


Figure 11. Comparison of Experimental and Analytical Results

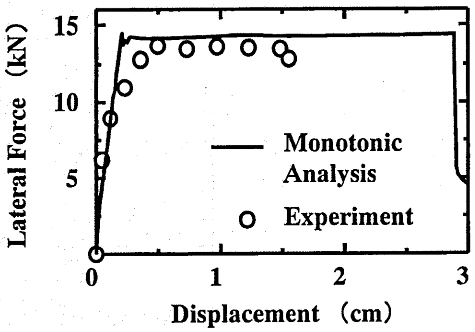


Fig. 12. Result of Monotonic Analysis (*pm1*)

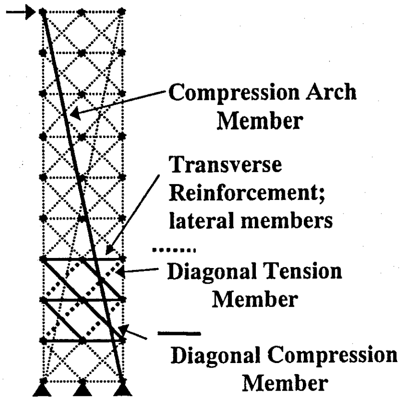


Figure 13. Members for which Average Stress is calculated

progress of this diagonal cracking, diagonal cracking progressed rapidly, and loading capacity decreased very quickly.

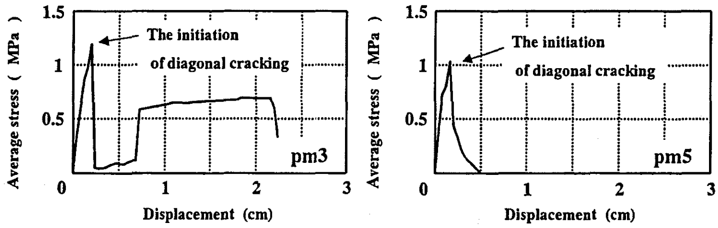
As a result of this, another monotonic analysis was conducted for specimen *pm1* only. Rather than arranging the transverse reinforcing members uniformly, they were arranged at their actual spacing. The load-displacement relationship obtained from this monotonic analysis is shown in **Figure 12.** This figure shows that the energy absorption in the member increases depending on the arrangement of transverse reinforcement, even for an identical transverse reinforcement ratio. Further, it is conjectured that plastic deformation of the member improves when it is subjected to monotonic loading.

4.2 Internal Resisting Mechanism of Specimens

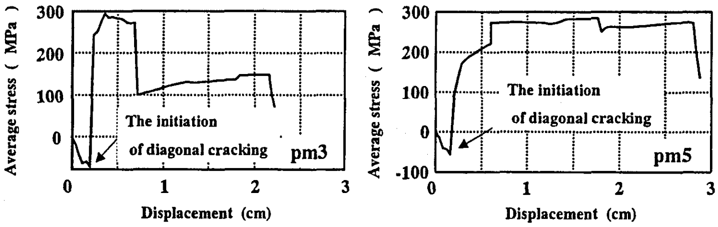
The internal resisting mechanism of RC columns subjected to cyclic loading was evaluated by checking the variation in average stress at a specific member section. In order to calculate the average stress, stresses in the diagonal tensile and compressive members, transverse reinforcement, and arch member (as shown by the thick solid line in **Figure 13**), were used. **Figure 14** shows the relationships between average stress in the lattice model members and displacement at the loading point. The line in **Figure 14** envelops the variation in average stress under cyclic loading. Since specimens *pm1*~*pm3* and *pm4*~*pm6* tended to exhibit similar resisting mechanisms, the results for *pm3* and *pm5* are shown as representatives of the respective groups. The common features of the internal resisting behavior in all specimens are discussed as follows. As **Figure 14** shows, the average stress in diagonal tensile members suddenly decreases after diagonal cracking. On the other hand, the average stress in the transverse reinforcement and diagonal compressive member increases as the external force is resisted once diagonal cracking takes place. The average stress in transverse reinforcement members takes the form of compression before the occurrence of the diagonal cracking, a feature also observed in the experiments. It may be considered that the reason for this compressive stress before cracking is the stress intensity around the loading point and supporting point. The horizontal force becomes almost constant due to yielding of longitudinal reinforcement and the average stresses in the transverse reinforcement and the diagonal compressive member also become constant.

In specimens *pm1*~*pm3*, the stress in the arch member increased even after yielding of the longitudinal reinforcement, ultimately exhibiting strain softening behavior. However, the increase in stress in the arch member was canceled out by the decrease in average stress in the diagonal tensile members. The ratio of arch width to cross-sectional width is less than 20%, so the contribution of the arch member to the shear resisting mechanism can be neglected in this case. In specimens *pm4*~*pm6*, the increase in stress in the arch member is small because the shear strength of the cross section increases, and it was observed that the influence of the arch member on the load resisting mechanism tends to decrease gradually.

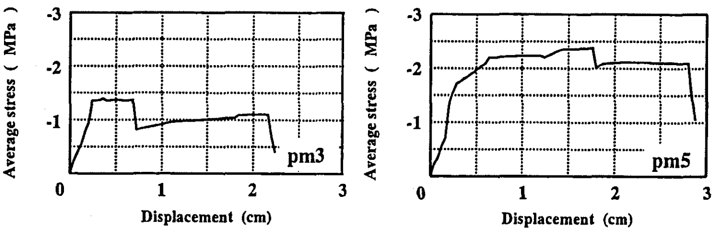
It is also found in **Figure 14** that differences in the amount of transverse reinforcement greatly affect the internal behavior. In the case of specimen *pm3*, the average stress in the diagonal tensile members suddenly decreases at one point due to the occurrence of diagonal cracking. Thereafter, as deformation progresses, the average stress in the diagonal tensile members recovers. On the other hand, in specimen *pm5*, the average stress in the diagonal tensile members also decreases suddenly due to the occurrence of diagonal cracking, but it never recovers. The meaning of this stress behavior in diagonal tensile members can be explained as follows. In specimen *pm3*, initial diagonal cracking did not occur throughout the section where the average stress was calculated, but took place just like localization in the column base region. In addition, along with diagonal cracking in the column base region, stress unloading can be observed in the diagonal tensile members surrounding the crack region. Consequently, the average stress decreases suddenly at one point. Thereafter, as deformation progresses, cracks become more and more localized while the stresses in the diagonal members around the localized crack recover to resist the horizontal load.



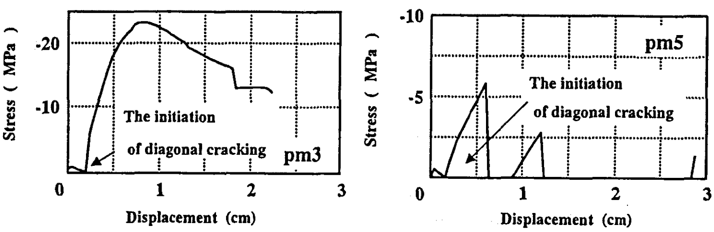
(a) Average stress in diagonal tension member



(b) Average stress in transverse reinforcement



(c) Average stress in diagonal compression member



(d) Stress in arch member

Figure 14. Variation in Average Stress in Lattice Model Members

On the other hand, in specimen *pm5*, the transverse reinforcement is sufficient to disperse diagonal cracking widely over the section where the average stress was calculated. Thus, the average stress in the diagonal members decreases and never recovers. This leads to the conclusion that in the case of *pm1* ~ *pm3*, where the transverse reinforcement was insufficient, diagonal cracking tended to be localized, while in the case of *pm4* ~ *pm6* with sufficient transverse reinforcement, it tended to be dispersed. These results are in good agreement with the experiments, so it is considered that the new cyclic lattice model is able to properly evaluate the internal behavior of RC columns. In addition, it is inferred that the transverse reinforcement members and diagonal compressive members in specimen *pm3* did not operate effectively due to the localization of diagonal cracking.

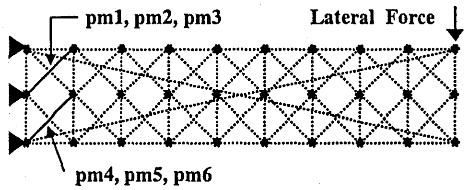


Figure 15. Members related to Failure

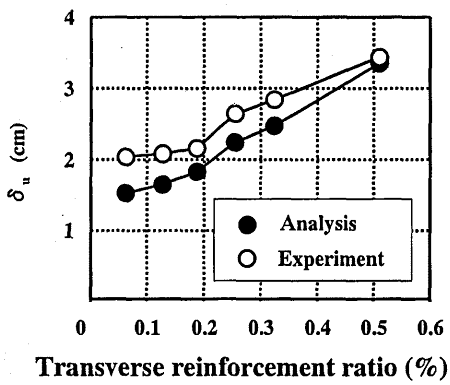


Figure 16. Comparison of Ultimate Displacement

4.3 Failure Component of Specimens

The failure mode of all specimens, as predicted by the cyclic lattice model analysis, was an increase in plastic deformation after yielding of the longitudinal reinforcement. Ultimately, the specimens failed when the diagonal concrete member provided in the column base region to sustain compression fails to work effectively due to crack propagation. In other words, since the compressive strength of the diagonal concrete members decreases due to crack extension (so-called compression-softening behavior), plastic deformation increased in the column base region and this led to specimen failure. As observed in the analysis, this phenomenon means that the diagonal cracks go through the specimens. This failure mode is correctly by the analysis and matches the experimental observations.

Figure 15 shows which members were the main factors leading to ultimate failure. In this analysis, the model proposed by Vecchio et al. [7] was introduced and used to describe the compression-softening behavior of concrete. This gives the degradation in compressive strength of concrete in the diagonal compressive member due to increasing tensile strain in the diagonal tensile member (which is perpendicular to the diagonal compressive member). The first diagonal crack, as predicted by the lattice model, occurred to the left of the column base region (**Figure 15**). In specimens *pm1* ~ *pm3*, the propagation of this crack was not prevented by the transverse reinforcement. Consequently, compression-softening behavior progressed rapidly in the diagonal compressive member provided at the left column base region, and once the compressive strain reached the strain-softening range, the stress dropped significantly. This caused total failure of the members. But for specimens *pm4* ~ *pm6*, the propagation of the initially formed crack was prevented by sufficient transverse reinforcement. Thus, it was confirmed that higher stress and more effective resistance to horizontal force was brought to diagonal compressive members as the amount of transverse reinforcement increased from *pm4* to *pm6*, even after the compressive strain reached the strain-softening range. Finally, the progress of strain-softening behavior in the diagonal compressive member provided to the left of the column base region (**Figure 15**) led to total failure of the members.

4.4 Relationship between Amount of Transverse Reinforcement and Plastic Deformation

Figure 16 shows the relationship between transverse reinforcement ratio and ultimate displacement (δ_u) obtained through experiment and the lattice model analysis. As this figure shows, the cyclic lattice model analysis correctly evaluates the changes in plastic deformation with respect to transverse reinforcement ratio. Further, as mentioned in 4-(2) and 4-(3), it is confirmed that lattice model analysis can also evaluate the effect of variations in transverse reinforcement ratio on crack pattern and internal behavior at the failure point. Thus, in spite of the cyclic lattice model being a simplified method, it can correctly evaluate the influence of transverse reinforcement ratio on the plastic deformability of RC columns under cyclic loading.

5. CONCLUSIONS

In this study, the effects of transverse reinforcement ratio on the plastic deformability of RC columns subjected to cyclic loading were evaluated. The following four major conclusions were reached:

(1) It was found that the behavior of RC columns under cyclic loading could be modeled to good accuracy by extending the monotonic lattice model to a cyclic stress field and using non-linear stress-strain relationships taking into account the cracking and confining effect of concrete.

(2) By observing the average stress variations in the structural members of the lattice model, it was confirmed that the lattice model analysis was able to evaluate the effect of differences in transverse reinforcement ratio on the cracking pattern and internal behavior.

(3) It is possible to predict the failure mode of RC members by checking the stress-strain relationships of members in the lattice model.

(4) Cyclic lattice model analysis can evaluate effectively the influence of transverse reinforcement ratio on the plastic deformability of RC columns subjected to cyclic loading.

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