INFLUENCE OF CORRELATIONS BETWEEN STRUCTURAL VARIABLES ON THE SEISMIC SAFETY OF RC BRIDGE PIERS

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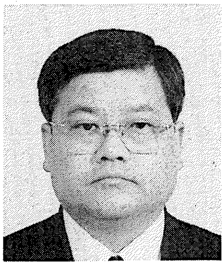
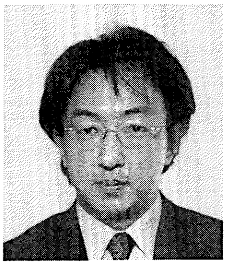
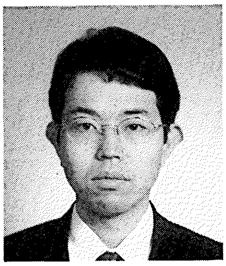


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Although the safety of a structure can be quantitatively evaluated based on reliability design, the result depends on the evaluation method itself. We propose an evaluation method for the safety of a structural system considering the correlations between structural variables and evaluate the safety of RC bridge pier in a major earthquake considering this correlation to clarify its influence. The results show that when a limit state affected by a correlation controls the safety of the structural system (pier), the safety of the pier is increased if the correlation is considered in case of low safety level. Furthermore, significant considerations related to the seismic design of RC structures are pointed out on the basis of the proposed new safety evaluation method.

Key Words : *reliability, safety index, correlation, RC bridge pier, seismic design*

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1. INTRODUCTION

The purpose of structural designs is to determine such particulars as sectional areas such that the probability of events that lead to a loss of desired performance is confined within certain limits in consideration of various uncertainties in the loadings that will be applied over the structure's lifetime. In doing this, it is necessary to consider economic efficiency, safety, durability, and other conditions, while deriving a design that satisfies the required conditions. Designs, which are conscious clearly of qualities required to a structure and supposed external factors can be possible by satisfying these necessities.

Conventionally, structural design has been based on safety coefficients, etc, for the evaluation of uncertainties, as determined over many years of experience. Consequently, design variables have been treated determinately, and designers are unable to clarify exactly how safe the designed structure is. In response to this, efforts have been made to establish a design methodology based on reliability design, using a practical form of probability theory to evaluate the safety coefficients there are decided subjectively in the conventional design system [1].

Since reliability-based design introduces the failure probability as the safety index of a structure, and aims to keep the probability of undesirable events within a target range fixed at the design stage, it should be possible to develop designs which approach the target level under various constraints, such as safety and economic efficiency. In another words, reliability-based design is a methodology which yields structural designs meeting the designer is safety requirements, and that retain a uniform level of safety independent of design conditions and so on.

In an attempt to apply this reliability-based design methodology, the authors have established a method of calculating the probability that a structural system will reach a limit state (or the probability of an undesirable event occurring) by considering multiple potential limit states at the same time. This methodology has been applied to the seismic design of RC bridge piers [2]. In conventional limit state design, the target performance is satisfied for typical limit states among those with the potential to occur. However, studies by the authors and others have shown that the conventional safety examination method overestimates the safety of structures, and it is necessary to evaluate the occurrence probabilities of undesirable events in proper consideration of the correlation among each limit state.

As noted above, reliability-based design allows for a quantitative understanding of the safety of a structure, which has not been possible in the past. On the other hand, the determined safety of a structure depends greatly on the method of calculating the probability, as well as on the degree of uncertainty in each design factor. The methods the authors demonstrate, here are appropriate for assumptions based on probability theory, but the correlations between structural variables, such as axial compressive force and flexural capacity, are ignored.

In this study, RC bridge piers are the focus, so we first calculate the correlation among structural variables from nineteen sets of bridge pier data. Then, based on the reliability evaluation method for structural systems proposed by the authors, we develop a flow chart for reliability evaluation in consideration of the correlations between structural variations in each limit state. Finally, we carry out added an examination of how the correlations between structural variations influence the reliability-based design of RC bridge piers.

2. CORRELATION BETWEEN STRUCTURAL VARIABLES IN RELIABILITY-BASED DESIGN OF RC BRIDGE PIERS

2.1 Evaluation Method for the Failure Probability of a Structural System based on Reliability Theory in Consideration of the Correlation between Structural Variables [2]

The hope is to improve the accuracy with which the failure probability of a structural system is calculated by taking appropriate account of the correlations between each potential limit state and the correlations between structural variables (the correlations between limit states). Further, it is hoped that the calculation method itself will be simple.

Adopting this point of view, the authors' method of evaluation is explained, an improvement on Ditlevsen's limit value is added so that higher-order joint events can be considered, and a flow chart is developed for the reliability evaluation of structural systems in consideration of the above correlations between structural variables.

If we ignore joint events expressed as intersections of three or more simultaneous events, the probability of failure event E with a voluntary failure event number k, or the probability of failure $P(E_k)$, is expressed as follows.

$$P(E) = \sum_{i}^{k} C_i \tag{1}$$

Where,

$$C_1 = P(E_1), \quad C_2 = P(E_2) - P(E_2 E_1),$$

$$C_k = P(E_k) - \sum_{i=1}^{k-1} P(E_k E_i) + \sum_{\substack{m=1,k-2 \\ n=2,k-1}}^{m<n} P(E_k E_m \cap E_k E_n) \qquad (k \geq 3)$$

In this calculation, there are three forms of failure probability $P(E_k)$, $P(E_k E_i)$, and $P(E_k E_m \cap E_k E_n)$. Here, $P(E_k)$ is the probability of failure event E_k, $P(E_k E_i)$ is the joint probability of failure events E_k and E_i, and $P(E_k E_m \cap E_k E_n)$ is the joint probability of the intersection of failure events E_k and E_m, and E_k and E_n.

The correlations between structural variables considered in this study are examined using $P(E_k)$, and the Rosenblatt transformation is used in the probability calculation. Under these conditions, the calculation can be simplified by making the assumptions below.

(a) Since design variables such as material strengths, sectional areas, and other particulars are random variables and fit a normal distribution or log-normal distribution, the distribution of characteristics dependent on these random variables is also expected to be close to these forms of distribution. Therefore, we assume that random variables expressing some form of "capacity" are normal distributions or log-normal distributions.

(b) We assume that capacity and active external forces are independent of each other and that there is no correlation between random variables which express capacity and external force.

(c) We assume that active external forces are independent of each other and there is no correlation between random variables which express external forces.

Adopting these assumptions, we set up a limit state equation $g_k(X)$ which expresses the probability of failure E_k as follows.

$$g_k(X) = g_k(x_1, x_2, \ldots, x_j, x_{j+1}, \ldots, x_n) \tag{2}$$

Where, $x_1, x_2, \ldots, x_j$: random variables related to capacities ($\mathbf{R}$) and $x_{j+1}, \ldots, x_n$: random variables related to external forces ($\mathbf{S}$).

The procedure for calculating $P(E_k)$ from equation (2) is indicated below.

1) Assume a point of failure, $x_0^* = x_0$.

2) A design point u_0 in the space of independent normal variables, and corresponding to the assumed design point, is obtained by the Rosenblatt transformation as follows. Using assumption (a) above, for random variables belonging to $\mathbf{R} = (x_1, x_2, \ldots, x_j)$,

$$u_i = \left(y_i - \sum_{l=1}^{i} \alpha_{il} U_{i-1} \right) \Big/ \alpha_{ii} \tag{3}$$

Where, y_i : in the case that x_i is a normal variable, $y_i = (x_i - \mu_i)/\sigma_i$ and in the case that x_i is a logarithmic normal variable, $y_i = (x_i - \lambda_i)/\zeta_i$, and α : $\alpha_{11} = 1.0$, $\alpha_{i1} = \rho_{y_i y_1}$

$$\alpha_{ij} = \left(\rho_{y_i y_j} - \sum_{l=1}^{j-1} \alpha_{il}\,\alpha_{jl} \right) \Big/ \alpha_{jj} \qquad (1<j<i), \quad \alpha_{ii} = \sqrt{1 - \sum_{l=1}^{i-1} \alpha_{il}^2}$$

Where, μ_i, σ_i: mean and standard deviation of the random variable x_i and λ_i, ζ_i : mean and standard deviation of the random variable $\ln x_i$.

Also, $\rho_{y_i y_j}$ is the correlation coefficient between y_i and y_j. If x_i and x_j are normal or log-normal distributions, $\rho_{y_i y_j}$ can be approximated by the correlation between these and variations [3]. And this $\rho_{y_i y_j}$ is the term that has not been evaluated and on which assumptions such as $\rho_{y_i y_j} = 0$ have been set in conventional safety evaluations to which reliability theory have been applied.

The calculation method for correlation coefficients between structural variables in the reliability-based design of RC bridge piers as considered in this study will be discussed in the next chapter. On the other hand, under assumption (c), for random variables belonging to $\mathbf{S} = (x_{j+1},\ldots,x_n)$,

$$u_i = \Phi^{-1}[F_i(x_i)] \qquad (i = j+1,\ldots,n) \tag{4}$$

where, $F_i(x_i)$: cumulative distribution function of random variable x_i and Φ : cumulative distribution function of standard normal distribution.

3) The value of the Jacobian at the point x_0 is set based on assumption (b).

$$\mathbf{J} = \frac{\partial(u_1,u_2,\ldots,u_n)}{\partial(x_1,x_2,\ldots,x_n)} = \begin{pmatrix} \partial\mathbf{R}/\partial\mathbf{x} & \mathbf{0} \\ \mathbf{0} & \partial\mathbf{S}/\partial\mathbf{x} \end{pmatrix} \qquad \text{(under assumption (c))} \tag{5}$$

4)The product of the limit state equation at the design point u_0 and the gradient vector is obtained.

5) A new design point u^* is obtained. In the space of the original variables, the design point x^* can be obtained as a linear approximation using the following equation:

$$x^* \cong x_0 + \mathbf{J}^{-1}(u^* - u_0)u^* \tag{6}$$

6) $P(E_k) = \Phi(-\beta) = (u^{*t}u^*)^{1/2}$ is calculated.

7) Using x^* obtained above as a new design point, steps 2) through 6) are repeated until the design point x^* converges.

The calculation of $P(E_k E_i)$ and $P(E_k E_m \cap E_k E_n)$ is carried out according to reference [2].

Further, the relationship expressed by equation (7) exists between the probability of failure P_f of a structural system as obtained by this type of method and safety index β.

$$\beta \cong -\Phi^{-1}(P_f) \tag{7}$$

Equation (7) gives a complete correspondence if the limit state equation shown as equation (2) is linear and all random variations are normal distributions. The relationship between β and P_f in this case is shown in Table 1. However, even if this condition is satisfied, equation (7) almost holds good if transformed from an accurately calculated probability of failure. Also it is pointed out that it is not the probability of failure magnitude that is reflected directly in designs, but rather it is the magnitude of the safety index that has an influence on designs [4]. Therefore, we decide that in evaluating the safety of RC bridge piers in an earthquake we should transform the

Table 1 Relation between Failure Probability and Safety Index

Failure probability	0.5	10^{-1}	10^{-2}	10^{-4}	10^{-6}
Safety index	0.0	1.28	2.33	3.72	4.75

calculated probability of failure into a safety index using equation (7) and evaluate safety according to the magnitude of this safety index.

As an examination of the ultimate limit state of RC bridge piers, we take up three characteristics: flexural capacity, shear capacity, and ductility. We set up limit state equations for the flexural and shear capacities as equation (8) and equation (9), respectively, and the limit state equation for ductility as equation (10), using the ductility factor evaluation equation from a WG report of the Special Committee for Investigations and Study of the Hyogoken-Nanbu earthquake of 1996 [5]. There are in fact many other proposed equations for calculating ductility. The reason for adopting this particular are that it is proven to be accurate and when reliability analysis is practiced in consideration of the correlation between multiple limit states, they can be calculated because same random variables of the flexural capacity or the shear capacity are contained mutually in each limit state equation.

$$g_1 = \alpha_1 M_u - (P_{\max} a + N\delta_{\max}) \tag{8}$$

$$g_2 = \alpha_2 (V_c + V_s) - P_{\max} \tag{9}$$

$$g_3 = \alpha_3 \left[\frac{N}{N_B} + \left(1 - \frac{N}{N_B}\right)\left\{12\left(\frac{0.5V_c + V_s}{M_u/a}\right) - 3\right\}\right] - \frac{\delta_{\max}}{\delta_y} \tag{10}$$

Where, M_u : ultimate flexural capacity (at the ultimate strain of the concrete), V_c : shear capacity without hoop ties [6], V_s : shear capacity contributed by hoop ties [6], $P_{\max}$: maximum value of active inertial force during earthquake, $\delta_{\max}$: maximum value of response displacement during earthquake, a: shear span, N: axial compressive force, N_B: axial compressive force when equilibrium breaks down, δ_y: yield displacement (defined as the displacement at which reinforcing bars located where the resultant tensile force is applied reach their yield strength), α_1, α_2 : random variables to deal with variations in equations for calculating capacities, and α_3 : random variable to deal with variations in equations for calculating the ductility factor.

2.2 Calculation Method for the Correlation between Random Variables

In this study it is necessary to correctly understand the correlations between structural variables because the safety evaluation of an RC bridge pier takes those correlations into account. Therefore, we examine the correlation between structural variables present in the limit state equations for the capacity and ductility of RC bridge piers. As the calculation method for correlation, we use the correlation analysis below.

$$\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y} \tag{11}$$

Where, σ_X, σ_Y : standard deviations of random variables X, Y, and μ_X, μ_Y : mean values of random variables X, Y.

Also, if equation (11) is discrete, the correlation coefficient can be written as below.

$$\hat{\rho} = \frac{1}{n-1} \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y} = \frac{1}{n-1} \frac{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}{s_x s_y} \tag{12}$$

Where, $\bar{x}$, $\bar{y}$: sample means of X and Y, and s_x, s_y : sample standard deviations of X and Y.

The value of $\hat{\rho}$ varies between −1 and +1, and it expresses the extent of the linear relation between the two variables X and Y [7].

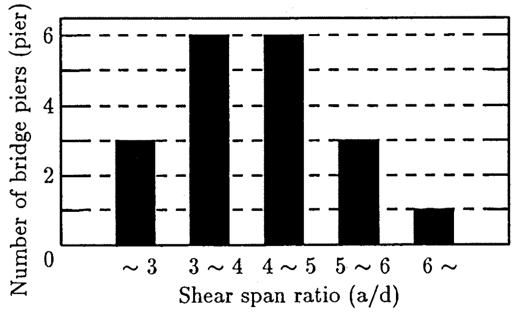
The covariance $Cov(X,Y)$ represented by the equation above expresses the extent of the (linear) relationship between the variables X and Y. The physical meaning of the correlation $Cov(X,Y)$ is (i) that if $Cov(X,Y)$

has a large absolute value and is positive, X and Y have a tendency to get large or small at the same time as their mean values, (ii) if $Cov(X,Y)$ has a large absolute value and is negative, X has a tendency to be larger than its mean value if Y is smaller than its mean value, and conversely X has a tendency to be smaller if Y is larger, (iii) and if $Cov(X,Y)$ is small or 0, there is almost no linear relationship between X and Y.

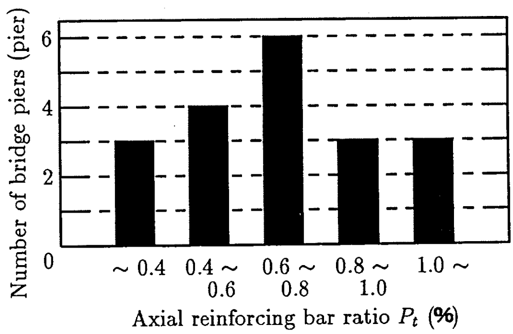
Ultimately, correlation coefficient $\hat{\rho}$ is what covariance $Cov(X,Y)$, which expresses the extent of (linear) relationship between variables X and Y is normalized to.

2.3 Calculation of Correlation between Structural Random Variables in Reliability-based Design of RC Bridge Piers

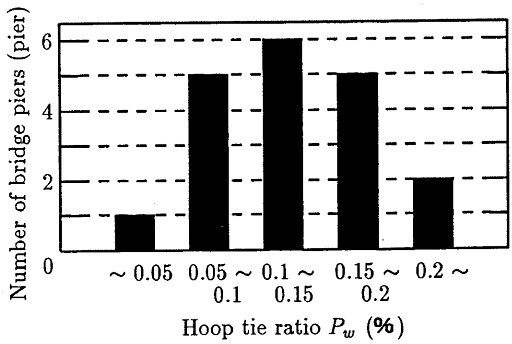
We calculate the correlation coefficients between structural variables contained in each limit state equation for RC bridge piers as shown in equation (8) through equation (10). Each correlation coefficient is calculated from the ultimate flexural capacity (M_u), shear capacity without hoop ties (V_c), shear capacity contributed by hoop ties (V_s), axial compressive force (N), axial compressive force when equilibrium breaks down (N_B), and yield displacement (δ_y) under assumptions (b) and (c) as shown in 2.1. We use data for the nineteen bridge piers shown in references [8] to [12] for the calculations of these correlation coefficients from equation (12). The distributions of shear span ratio, axial reinforcing bar ratio, and hoop tie ratio for each RC bridge pier are shown in Fig.1(a), (b), and (c) respectively. Also, the correlation coefficients used when the safety evaluations for the RC bridge piers in Chapter 4 are carried out need to be calculated from the population to which each bridge pier belongs. Therefore, fundamentally, it is preferable to calculate correlation coefficients from bridge pier data taken from designs based on the same design standard as that used for the bridge piers being analyzed. However, since there are no data bases of such bridge pier data at present, in this study to the safety of RC bridge piers in earthquakes is investigated using correlation coefficients calculated from Fig.1. The results are shown in Table 2. Since the data in Fig.1 is for bridges designed using various design standards, it needs to be noted that the correlation coefficients given in Table 2 may be smaller than if they were calculated from the same population. Also, the reason that the correlation coefficient for $V_c - V_s$ is not shown in Table 2 is because the equation for calculating ductility used in this study is a regression equation from experience, and V_s/M_u and V_c/M_u were made independent of each other.



(a) Distribution of Shear Span Ratio



(b) Distribution of Axial Reinforcing Bar Ratio



(c) Distribution of Hoop Ties Ratio

Fig.1 Distribution of Sectional Areas and Other Specifications

Table2 Correlation Coefficient between Capacity

Variables	Correlation coefficients	Variables	Correlation coefficients
M_u-V_c	0.95	N-V_s	0.57
V_c-N_B	0.94	δ_y-N_B	0.55
M_u-N_B	0.93	V_s-N_B	0.50
N-M_u	0.77	δ_y-N	0.49
N-N_B	0.72	δ_y-M_u	0.46
N-V_c	0.65	δ_y-V_c	0.36
M_u-V_s	0.60	δ_y-V_s	0.0027

Now we can see that the correlation coefficients of $M_u - V_c$, $V_c - N_B$, and $M_u - N_B$ are large. It is thought that a strong linear relationship arises because (i) in making assumptions when calculating capacity values, the structural design factors such as sectional area and concrete compressive strength are held in common, (ii) flexural and shear capacities without hoop ties are calculated according to the axial compressive force. The other correlations shown in Table 2 are smaller than 0.8, indicating a weak linear relationship, and the safety of the structural system is barely influenced when reliability-based evaluation is taken these correlation into account. Therefore, we treat correlation coefficients below 0.8 in Table 2 as if there is no correlation.

Adopting this point of view, in Chapter 4 we will carry out safety evaluations in consideration of three correlations, $M_u - V_c$, $V_c - N_B$, and $M_u - N_B$, at the same time. These three correlations are contained only in the limit state equation for ductility shown in equation (10), and the values of $P(E_k)$ calculated from the limit state equations for flexural and shear capacities are the same as when correlation is not considered.

3. SEISMIC SAFETY ANALYSIS OF RC BRIDGE PIERS

Here, using the previously described reliability evaluation method for structural systems, a practical method of seismic evaluation is implemented in consideration of the correlation between structural variables of RC bridge piers. First, the flow of the analysis from choosing RC bridge piers for analysis to their safety evaluation in consideration of the correlation between structural variables is shown in Fig.2. Each condition in this analytical flow is explained below.

First, as RC bridge piers for analysis in this study, we showed three design examples of single-column RC bridge piers which can be reduced to one-dimensional models in Fig.3. There are designed using the same design standard as examples given in "Specification for Retrofit of Road Bridges Dameged in Hyogoken-Nanbu 1995 Earthquake" [plan] [8]. The sectional areas and other particulars of each bridge pier are shown in Table 3. As the input seismic waveform, that recorded at Kaihoku bridge during the Miyagiken-Oki Earthquake (on bedrock; maximum acceleration = 293 gal) and that taken at Port Island during the Hyogoken-Nanbu Earthquake (on bedrock; maximum acceleration = 621gal) are used. And, after inputting these two seismic waves to the bedrock of the ground chosen voluntarily (classfied as type-I ground since ground characteristic value Tg is 0.084 (sec) which represents the basic natural period of the subsurface ground in the amplitude range of small strain [13]), the waveform at the bottom of the footing was assumed by multiple reflection theory. Further, shape of each seismic waveform when input to the bedrock is shown in Fig.4, and its acceleration response spectrum is shown in Fig.5.

Next, it is necessary to evaluate uncertainties in loads, material strengths, and structural analysis rationally and quantitatively besides the correlation coefficients between the random variables described earlier so as to do the reliability evaluation for structural systems of RC bridge piers in consideration of the correlation between structure variables. So the coefficient of variation and the probability distribution of each random variables what consists of the limit state equations were set up.

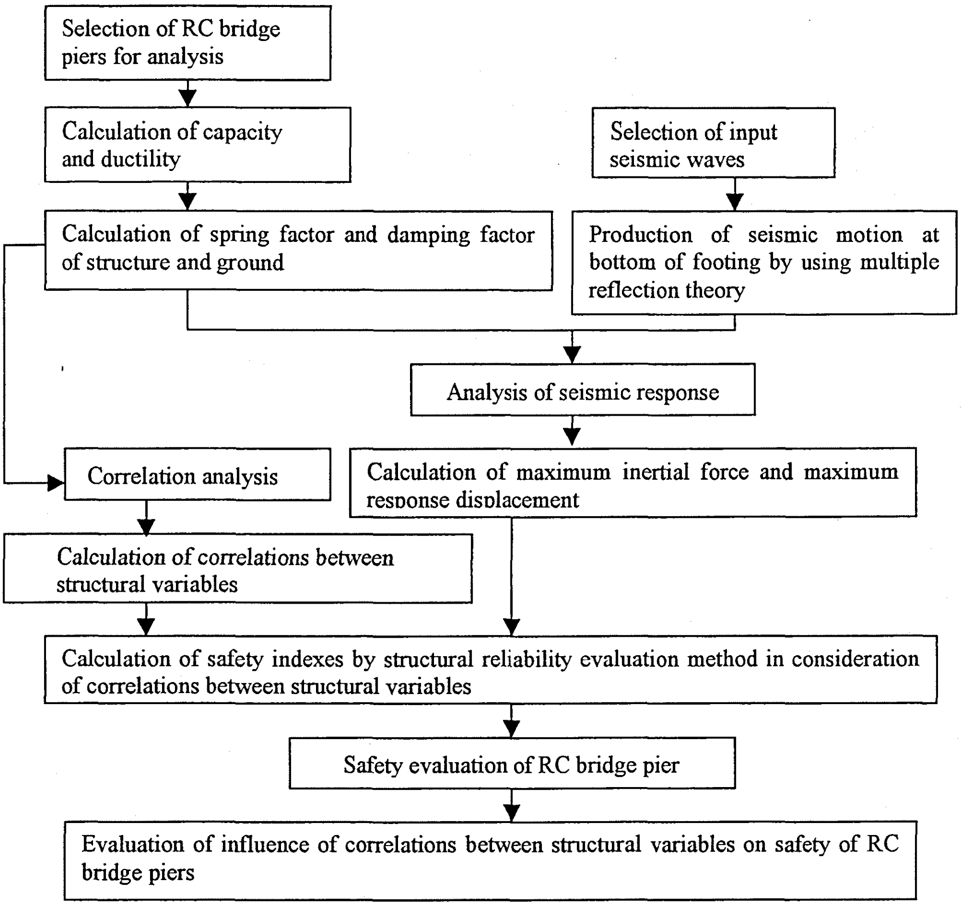


Fig.2 Flow Chart of Seismic Safety Examination for RC Bridge Pier

First, we suppose that the coefficients of variation of the concrete compressive strength and the reinforcing bar yield strength which is considered as the uncertain factor of material strengths are 20% and 7% respectively against each normal value and follow the normal distribution. As a result of an analysis using Monte Carlo simulation, we decide to treat the calculated flexural capacity as a random variation with a coefficient of 8% because of variations in material strengths [2]. Next, to evaluate the influence of variations in material strengths contained in the shear capacity, we calculate the coefficient of variation δ using the following equation:

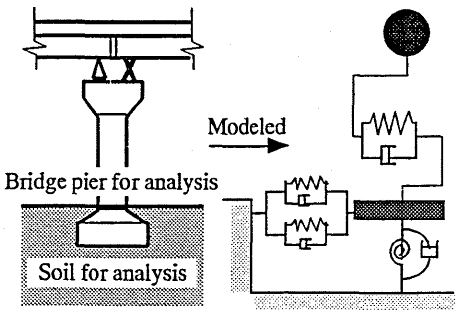


Fig.3 Model Used for Analysis

$$\delta = \frac{\sigma_V}{\mu_V} = \frac{\sqrt{\sum_{i=1}^{j}\left[\left(\frac{\partial V}{\partial X_i}\right)\Big|_{\bar{X}_i}\right]^2 \sigma_{\bar{X}_i}^2}}{V(\bar{X}_1,\ldots,\bar{X}_j) + \sum_{i=1}^{j}\left(\frac{\partial V}{\partial X_i}\right)\Big|_{\bar{X}_i}(X_i - \bar{X}_i)} \tag{13}$$

Where, X_i : random variables related to material strength and bending moment, σ_{X_i} : standard deviations of

Table 3 Bridge Piers Used for Analysis

(a) Bridge Pier 1 (Capacity Ratio=1.18)

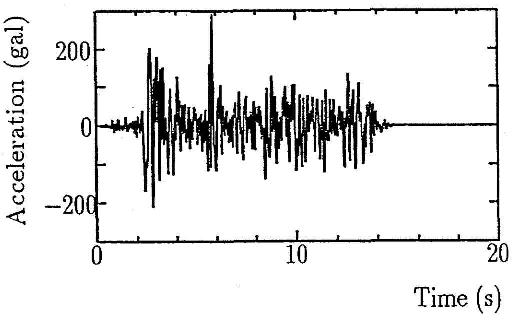
Requirements of bridge pier	Bridge pier	4.0m in diameter, 9.8m in height
	Foundation	Pile
Bridge pier	Section	4.0m in diameter
	Axial reinforcing bars	D51×72
	Hoop ties	D25ctc150mm
Foundation	Section of footing	9.5m×13.25m
	Pile	1.5m in diameter×10

(b) Bridge Pier 2 (Capacity Ratio=1.32)

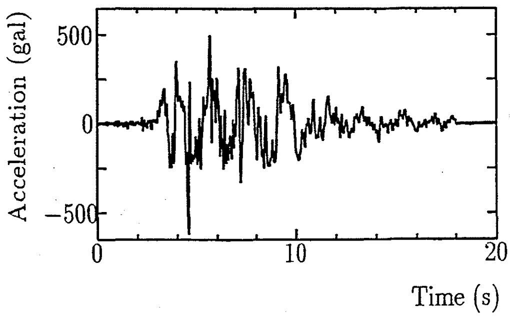
Requirements of bridge pier	Bridge pier	4.0m in diameter, 8.5m in height
	Foundation	Pile
Bridge pier	Section	4.0m in diameter
	Axial reinforcing bars	D38×78
	Hoop ties	D22ctc125mm
Foundation	Section of footing	9.5m×11.0m
	Pile	1.5m in diameter×12

(c) Bridge Pier 3 (Capacity Ratio=1.84)

Requirements of bridge pier	Bridge pier	3.0m×3.5m, 10.5m in height
	Foundation	Pile
Bridge pier	Section	3.2m×3.7m
	Axial reinforcing bars	D32×23
	Hoop ties	D25ctc150mm
Foundation	Section of footing	9.5m×12.0m
	Pile	1.5m in diameter×9

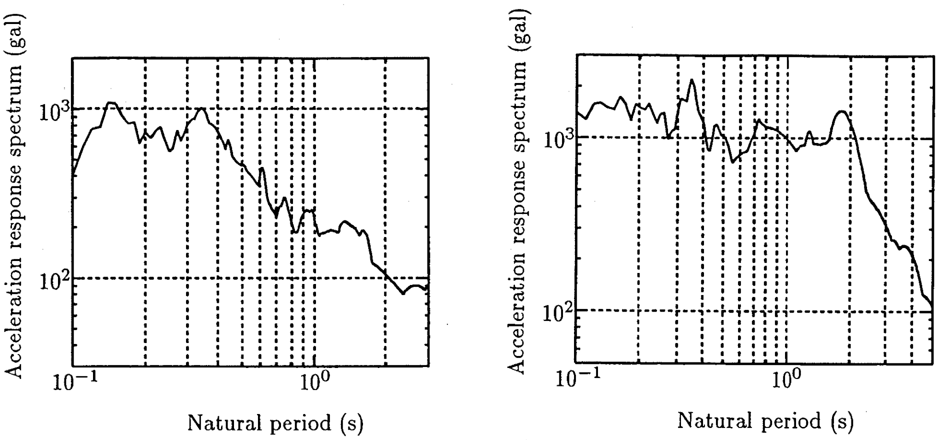


(a) Miyagiken-Oki Earthquake



(b)Hyogoken-Nanbu Earthquake

Fig.4 Standard Seismic Waveforms at Bedrock Used for Analysis



(a) Miyagiken-Oki Earthquake (b)Hyogoken-Nanbu Earthquake

Fig.5 Response Spectrums of Seismic Waveforms at Bedrock in Analysis

Table 4 Parameters of Random Variables

Random variables in limit state equation		Parameters of random variable	
Random variable	Symbol	Mean value	Coefficient of variation
Random variable (Flexural capacity)	α_1	1.0	10 %
Random variable (Shear capacity)	α_2	1.0	20 %
Random variable (Ductility factor)	α_3	1.0	40 %
Axial compressive force	N	Design value	5 %
Yield displacement	δ_y	Calculated value	10 %
Maximum inertial force	P_{max}	Result of response	30 %
Maximum response displacement	δ_{max}	Result of response	30 %

random variables, and $V(\cdot)$: equation to calculate shear capacities.

As a result of the calculation, we decide to treat the coefficients of variation of shear capacity without hoop ties as 10% and the coefficients of variation of shear capacity contributed by hoop ties as 8% [2].

In the values calculated from various capacity calculation equations, the influence of uncertainties in the calculation equations themselves is included as well as the influence of variations of in material strengths. In this study, we incorporate these influences of uncertainties as random variables $\alpha_i (i = 1,2,3)$, which are treated as random variations in limit state equations (8) to (10). As for the maximum value of the active inertial force and the maximum response displacement due to earthquake, since they contain uncertainties brought about by modeling the structure, it is necessary to treat them as random variables. Therefore, we consider the result of response as the mean value, and give it an appropriate coefficient of variation [2]. We treated these random variables as normal distributions. The results are shown in Table 4.

In general, when a seismic safety evaluation is carried out on a structure system, we can think of analysis of the degree of seismic risk which probablitically evaluates an input seismic motion, analysis of the probability of failure of a structure against an input seismic motion, and seismic reliability analysis which is done by summarizing the two analyses above. We should note here that in this study, since the probability of the seismic waveform occurring in the lifetime is not considered, only the probability of failure of the structure under the condition that the input seismic waveform occurs is analyzed.

4. INFLUENCE OF CORRELATION BETWEEN STRUCTURAL RANDOM VARIABLES ON THE RELIABILITY OF RC BRIDGE PIERS

4.1 Safety Evaluation of Each RC Bridge Pier

We carry out a seismic safety evaluation of RC bridge piers 1 to 3 in Table 3 in consideration of the correlation between structural variables.

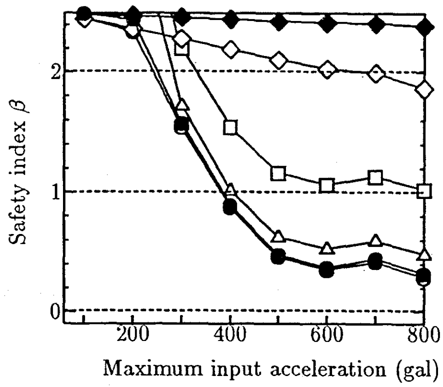
Here, as already noted, we input the Miyagiken-Oki Earthquake and Hyogoken-Nanbu Earthquake waveforms, and we magnify or reduce the maximum input acceleration to 100 gal to 800 gal. They are input into bedrock of type-1 ground (Tg<0.2 (sec)), which is the ground model. We carry out safety evaluations of RC bridge piers in consideration of the correlation between structural variables for each size of maximum input acceleration.

Shown in Fig.6 and Fig.7 are, for the two seismic waves, safety index (β) which expresses the safety of the RC bridge pier which is calculated from equation(1), and safety indexes (β_M, β_V, and β_δ) which are calculated from limit state equations for flexural capacity, shear capacity, and ductility in the both cases that no correlation is considered, and that the correlation of the three pairs of structural variables, $M_u - V_c$, $V_c - N_B$ and $M_u - N_B$, are considered at the same time. And as described before, the considered correlation are contained only in the limit state equation for ductility, not in the limit state equations for flexural and shear capacities. Therefore, the safety against flexural and shear capacities does not change whether or not there is correlation.

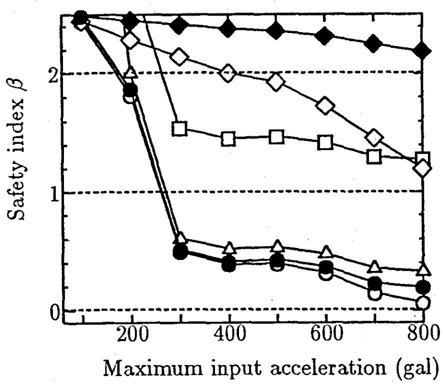
From Fig.6 and Fig.7, when correlation is considered, the safety against ductility rises more than when correlation is not considered. This coincides with what Chou pointed out: that the value of the safety index β is high when the limit state equation contains a lot of structural variables and when the partial differential coefficient of structural variables have different signs and the correlation efficient is high [14]. That is, multiple structural variables are contained in the limit state equation for ductility used in this study, and three structural variables in particular, M_u, V_c, and N_B, have the relationship of denominator and numerator, and as is shown in Table 2, three correlation coefficients for $M_u - V_c$, $V_c - N_B$ and $M_u - N_B$ are high. Therefore, it is thought that the safety against ductility rises by considering these three correlations.

Also, since Fig.6 and Fig.7 shows clearly the changes in safety indexes calculated from all the limit state equations, the difference of the safety index for bridge pier according to whether or not the correlation between structural variables are considered is expressed by a very small number. However, in the range where the safety index $\beta = 0 \sim 1$, as is clear from Table 1, the difference in failure probabilities, which corresponds to fluctuations of the safety index β, increases. Therefore, it can be said that the difference in safety indexes of structural variables according to whether or not they correlate cannot be ignored in the range where the safety index is below 1. As described in the next chapter, when we pay attention to safety index β around here, in the case of bridge pier 2, when correlation is not considered, a significant difference occurs when we calculate the necessary amount of axial reinforcing bars to be evaluated that the bridge pier has the same safety level (its calculated safety index has the same value) as one which was designed by the safety evaluation method in which correlation was considered. This depends on the limit state equation which rules the safety of bridge pier 2. This is because in the case of bridge pier 1, which has a capacity ratio of 1.18, even the safety of a bridge pier designed to suffer flexural failure is ruled by the safety which is calculated from the both limit state equations for the flexural and shear capacities. Also, in the case of bridge pier 3 which has a capacity ratio of 1.84 even if various uncertainties are considered, the safety of the bridge pier is represented only by the safety calculated from the limit state equation for the flexural capacity. Therefore, in the cases of bridge piers 1 and 3, influence on the safety of a bridge pier as a whole is small which the difference of the safety index calculated from the limit state equation for ductility on which the correlation between structural variables influences have. On the other hand, in the case of bridge pier 2,

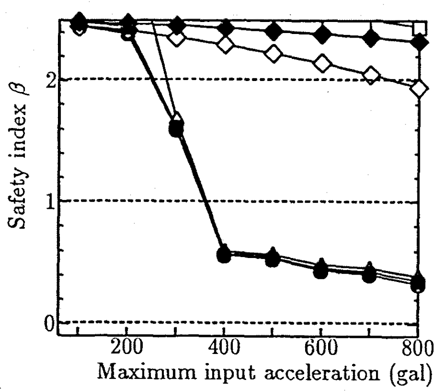
○ : No correlation · β , △ : No correlation · β_M
□ : No correlation · β_V , ◇ : No correlation · β_S
● : Correlated · β , ◆ : Correlated · β_S



(a) Bridge Pier 1

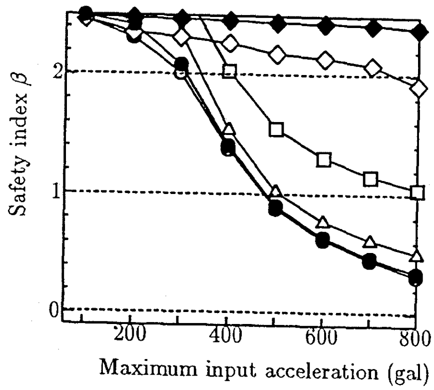


(b) Bridge Pier 2

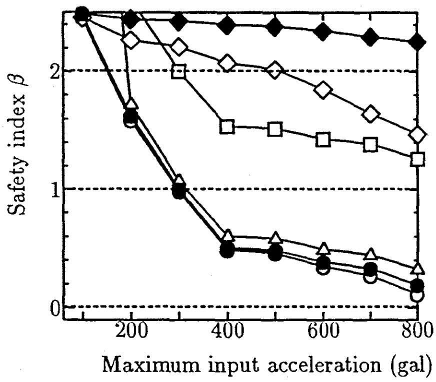


(c) Bridge Pier 3

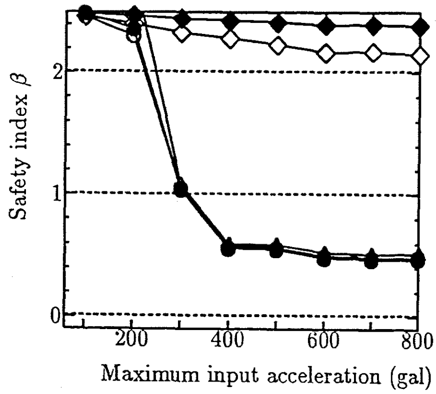
Fig.6 Maximum Input Acceleration and Safety Index (Miyagiken-Oki)



(a) Bridge Pier 2



(b) Bridge Pier 2



(c) Bridge Pier 3

Fig.7 Maximum Input Acceleration and Safety Index (Hyogoken-Nanbu) if correlation is not considered, the safety examination against the limit state for ductility influences the safety evaluation for the bridge pier. Therefore, since safety against ductility rises by considering the correlation between structural variables, the safety of a bridge pier can be highly evaluated. However, this is the case when the calculated safety index β is between 0 and 1. When seismic motion in the low acceleration range is considered, or when the occurance probability of input seismic motion is considered as is described in Chapter 3, the safety index of a bridge pier is overevaluated. If the probability of the input seismic motion occurring is considered, the safety index of the bridge pier is highly evaluated. In this case, since the change in the probability of failure corresponding to the fluctuation of the safety index β becomes small, when application to an actual design is considered, it can be expected that the difference in safety index according to whether or not the correlation between structural variables is considered gets small.

From the discussion above, under the limit state equations set up in this study, when the safety examination against ductility effects on the safety of the whole bridge pier, there are cases where even the safety of a RC bridge pier designed to suffer flexural failure can be highly if the correlation between structural variables is considered. In that case, the capacity ratio of the bridge pier is a standard. On the other hand, if the safety index β falls in a safer range, it can be said that the correlation between structural variables does not affect the safety of the bridge pier. Also, this result holds good under the limit state equations set in this study and the correlation between the structural variables shown in Table 2. Therefore, if another ductility factor equation is used for the limit state equation of ductility, for example, it is necessary to note that the result will be different. However, the significance of this study is the main point it clarifies: that when this methodology is applied to reliability-based design, similar to that the safety of a structure changes according to the sizes of the various working loads, there are cases when safety evaluations of the same bridge pier differ with safety level according to whether or not the correlations between structural variables are considered. Since reliability-based design entails satisfying certain safety requirements by changing sectional areas and other specifications, it is necessary to pay attention to such things as correlation set up in the process to calculate the probability of failure like this.

In the next chapter, we examine the influence on sectional areas and other specifications by considering the correlation between structural variables in concrete terms.

4.2 Consideration of Correlation between Structural Variables

Here, we examine how much the reinforcing bar requirement for a structural design with a target level of safety varies according to whether or not the correlations between structural variables are considered. We set the safety level of a bridge pier when the correlations are not taken into account as the target safety, and calculate the amount of reinforcing bars needed in the case that the bridge pier has the same level of safety as a bridge for which correlation is considered. The calculation flow is shown in Fig.8. Bridge piers with different amounts of reinforcing bars are interpreted as structures with different seismic performance from the viewpoint of structural mechanics, but are considered to have the same level of safety if the evaluation method on the treatment of the correlation between structural variables is different. That is, the difference in amount of reinforcing bars between bridge piers can be said to be an advantage arising from the consideration of the correlations between structural variables. Also, fundamentally, as the target safety matched the existing design standard and safety level, it is

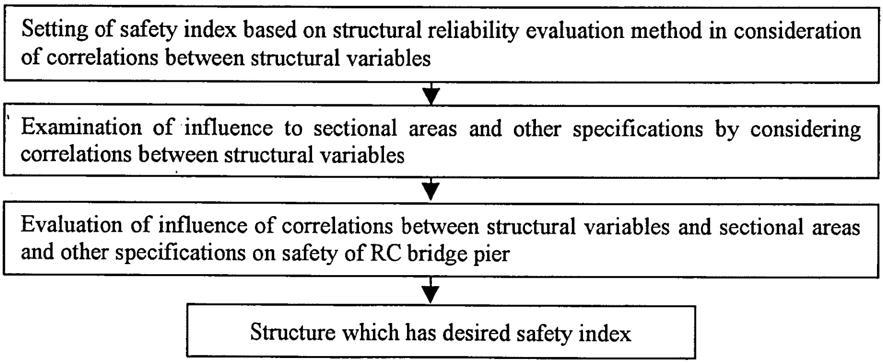


Fig.8 Flow Chart of Structural Design

Table 5 Increase and Decrease in Necessary Amount of Reinforcing Bars

Seismic waveform		Miyagiken-oki earthquake	Hyogoken-Nanbu earthquake
Bridge Pier 1	A_s	1.01 times	0.96 times
	A_w	1.02 times	1.00 times
Bridge Pier 2	A_s	0.95 times	0.98 times
	A_w	1.20 times	1.13 times
Bridge Pier 3	A_s	1.03 times	0.97 times
	A_w	1.01 times	1.00 times

necessary to set up a yardstick for safety of the new standard (calibration). However, since in this study we pay attention only to the influence of the correlations between structural variables, we introduce the concept of failure probability as the index for judgement of relative safety, and set up the target safety as described before. Also, the result shown here is the difference of the amount of reinforcing bars calculated under the condition that the input seismic wave acted on the ground which is used in analysis. Therefore, since when a seismic safety evaluation which includes the occurrence probability of the input seismic motion, or in case of some scale of earthquake assumed in design, the influence according to whether or not the correlation are considered gets small in company with the increase of safety index, the result presented here is not reflected directly when it is applied to reliability-based design.

Under the assumptions above, for bridge piers 1 to 3, we calculate the ratio of the increase and decrease in the number of axial reinforcing bars ($\mathbf{A}_s$) and hoop ties ($\mathbf{A}_w$) necessary to maintain the target safety when the correlations between structural variables are not considered. The result is shown in Table 5, which gives the increase and decrease needed to exceed the target safety when the maximum input acceleration of each seismic waveform is amplified to 800 gal and input to the bedrock.

When the number of axial reinforcing bars is increased, the safety against flexural failure rises, but because of the increase in active shear force and the decrease in ductility expressed as equation (10), the safety of the bridge pier as a whole does not necessarily rise. Therefore, there are some cases such as the number of axial reinforcing bars shown in Table 5 that is below 1.0. However, the increase in the amount of the hoop ties for each seismic waveform is input to bridge pier 2 shows a greater value than the others. This is because in the case of this pier which has a capacity ratio of 1.32, the ratio of failure probability by the limit state equation for ductility greatly influences the failure probability of the pier. In particular, since the examination of ductility is more important when a large non-linear displacements occur in the high-acceleration range, it becomes necessary to make the increase in hoop ties still greater which leads to improved safety against ductility. Moreover, another factor is that when the value of the target safety index is small, the increased failure probability accompanying the increase in safety index is great because of the correspondence of failure probability P_f and safety index β as described before, so it is difficult to improve safety. As a result, when the safety index β calculated from the limit state equation for ductility which is influenced by the correlation between structural variables accounts for a large part of the safety index of a structural system like bridge pier 2, it is possible to make more effective designs by considering the correlations between structural variables properly. On the other hand, even for the same bridge pier 2, the meaning of considering the correlations between structural variables is reduced in the area where safety at the low-acceleration range is good. Therefore, when reliability-based design is introduced to other structural systems, it is necessary to take in the correlations between structural variables properly by the calculated size of the safety index when there exists correlation between structural variables which are related to the limit state which represents the safety of the structural system.

5. CONCLUSIONS

The results obtained from this study are summarized below.
(1) The calculation flow of the safety of RC bridge piers by means of a reliability evaluation method for structural

systems in consideration of the correlations which exist between structural variables such as axial compressive force and flexural capacity is shown.
(2) Using correlation analysis, the correlations between structural variables in each limit state equation are calculated. As a result, $M_u - V_c$, $V_c - N_B$, and $M_u - N_B$, which are influenced by concrete strength, sectional size, etc., showed high correlation.
(3) A seismic safety evaluation of RC bridge piers in consideration of the correlations between structural variables in each limit state equation considered in this study was carried out. As a result, because the safety against ductility is high, it was shown that the safety of the whole bridge pier, especially an RC bridge pier with a 1.32 capacity ratio, is good in area where the safety index is small.
(4) An analytical flow for implementing structural designs to a target safety was demonstrated. By adopting the amount of reinforcing bars as sectional areas and other particulars, the difference in the amount of steel needed to achieve a particular safety level between cases when the correlations between structural variables are considered and when they are not considered for bridge piers with different capacity ratios was clarified.

There is demand for structural designs to clearly indicate the seismic performance of the finished structure, and to insure that safety is within the allowable risk (probability of failure) for a given seismic input. Performance examination is done with taking margin of safety in consideration of various uncertain factors into account. It is now possible for reliability-based design based on probability theory to be applied quantitatively by introducing the failure probability to such performance examinations. In the post-Hyogoken-Nanbu Earthquake era, improve dynamic calculation accuracy in the evaluation of seismic performance, is required in structural design. For example, predicting the response of structures in earthquakes by dynamic analysis is being examined in various ways. It will be necessary to examine the reliability evaluation method in order to reflect the design margin of safety the finished structure in consideration of uncertain ties inherent in structural design. In this study, we have analyzed the influence of correlations between structural variables on safety evaluation, and showed there are some cases in which the evaluation of performance differs according to the capacity ratio and safety level of a bridge pier.

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