A STUDY ON THE FUNDAMENTAL CHARACTERISTICS OF LATTICE MODEL FOR REINFORCED CONCRETE BEAM ANALYSIS

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A new modification of the lattice model is described by the authors. This new method depends fundamintally on the calculation of minimum total potential energy for the structure at each calculation increment, starting from the elastic stage up to the failure stage. The angle of inclination of the diagonals and the appropriate discretization of the truss member are very important parameters affecting the results of the lattice model, and these issues are studied in the paper. The applicability of the Modified Lattice Model is examined by comparison with proposed shear strength equations and existing experimental data.

Key Words: *shear-resisting mechanism, modified lattice model, arch element, total potential energy, and subdiagonal element.*

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1. INTRODUCTION

It is generally agreed that the truss analogy is applicable to the analysis of shear resistance in reinforced concrete structures. There are several different truss models that can be used to analyze the shear resisting mechanism in reinforced concrete beams, but each model still has some problems that need to be investigated. For example, the Lattice Model, which was first proposed by Niwa et al. [14] and extended later by the authors into three dimensions [6], [7], has several fundamental points that require clarification while other points need modification. In this model, the arch member is a very important concept, because after yielding of the shear reinforcement the model can explain the increase in shear capacity, while the simple truss model cannot, especially in the case of deep beams. An arch element has certain important effects on shear carrying capacity [14]. The thickness of the arch element is determined by minimizing the total potential energy for the whole structure, but there is no physical explanation for the minimization of total potential energy, and once the value of arch element thickness is determined in the elastic stage, it remains unchanged throughout the loading history. The thickness of the arch element may be changed during the loading stages, but any change is simply neglected.

In this paper, we first clarify this issue and then demonstrate improved accuracy by performing the minimization at every loading stage.

Also studied is the change in thickness of the arch element with the corresponding load carrying capacity at different loading stages. Further, a rational explanation for the strain incompatibility through the width of a beam caused by separating the arch member and the truss member within one beam will be explained. Experimentally, it is found that a two-dimensional stress analyses is not adequate for reinforced concrete members [11]. With this clarified, the fundamental characteristics of the arch element mechanism for the shear resistance of reinforced concrete members are discussed; in particular, the strain values between the arch and diagonal elements in the same cross section are shown to be unequal. The strains may not be uniform in the direction of member width. The third issue to be clarified is the most appropriate direction for discretization of subdiagonal members. The approuch is to determine a suitable procedure for applying the Modified Lattice Model such that a similar response to the experimental results is obtained while changing the spacing of shear reinforcement and the subdiagonal angle. Finally, the "Modified Lattice Model" is used to simulate the shear failure of reinforced concrete beams. The changing stress states of each member inside the beam are investigated.

The authors attempt to give a rational mechanism or rational explanation for all previous problems. Furthermore, based on the modified and rational model, we give some numerical calculation results that may be useful in actual applications.

2. OUTLINE ON THE MODIFIE LATTICE MODEL

The chosen element discretization and structural geometry of the Modified Lattice Model is illustrated in **Fig.1.** The reasoning behind this truss discretization will be verified in the following sections. The reinforced concrete beam is simulated as simple truss components under bending and shear. The compressive stress in the upper part of the beam is resisted by concrete in the form of a horizontal strut with a cross-sectional area equal to the area of the upper rectangle in **Fig.2**. The tensile stress in the lower part is born by the lower steel in the form of horizontal reinforcing members as well as by the horizontal concrete fibers in the lower section with a cross-sectional area equal to the lower rectangular area in **Fig.2**. To resist the shear forces inside the beam, the truss model has diagonal concrete tension and compression members with the areas shown in **Fig.2**; these can be fixed after the value of "t" is determined as will be shown in section 3. For the vertical members, the effect of the concrete is not considered because the resistance of concrete to tension is already incorporated in the diagonal tension member. This is in addition to the vertical steel members, which represent the shear reinforcement in the web. **Figure 2** shows the cross section of a concrete beam modeled as a Modified Lattice Model.

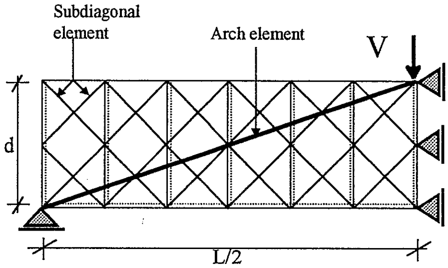


Fig.1 Modified Lattice Model

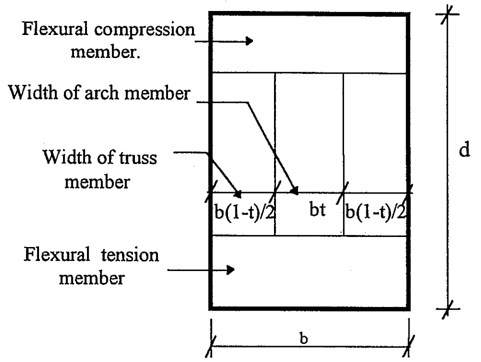


Fig.2 Cross section of concrete beam in the Modified Lattice Model

In **Fig.1**, the thick solid line represents the arch element, which is assumed to be a flat, slender element connecting the nodes at each end of the beam with the area shown in **Fig.2**. In this analysis, the arch element and the diagonal elements are separated and each has its own stress and strain distribution. The reason for this element separation is that structural action is normally a combination of series and parallel couplings between the cracking zones and the uncracked (elastic) zones. In the Modified Lattice Model, we simulate these zones with continuous pairs of tension and compression members. The arch member is considered a very important element in this study, since it represents the core of the beam [13]. The AIJ design code [1] and many other codes [4] assume two dimensional stress fields; but if the member section is wide enough, the stress may not be uniform in the direction of member width. It was also found experimentally by Ichinose [11] that the values of beam strains or stresses are not uniform across the width in the same cross section. This means that the stress or strain diagram is not constant in the direction of beam width [11]. Thus, in this model, we separate the arch element and the diagonal element, and each is given its own stress and strain distribution. The arch element has the ability to resist a large portion of the applied load [13], so it is very important to look for changes in the area of the arch element at different loading stages, as will be shown in section 3. In the Modified Lattice Model, the diagonal tension member of concrete resists the principal tensile stress resulting from shear force. The stress-strain relation of the tension member is assumed to be expressed as in Eq. (1) and Eq. (2) [8], [9] and as shown in **Fig.3**.

For the ascending branch $(\varepsilon_r < \varepsilon_{cr})$

$$\sigma_r = E_c \varepsilon_r \tag{1}$$

For the descending branch $(\varepsilon_r \quad \varepsilon_{cr})$

$$\sigma_r = (1-\alpha) f_t \exp\left[-m^2\left(\frac{\varepsilon_r}{\varepsilon_{cr}}-1\right)^2\right] + \alpha f_t \tag{2}$$

Where ε_r and σ_r are the strain and the stress of the tension element respectively, ε_{cr} is the strain at the time of concrete cracking, and E_c is the modulus of elasticity of the concrete. The stress-strain behavior of concrete in tension is taken to be elastic, as shown in Eq. 1, and gradual softening is assumed thereafter as shown in Eq. (2). In Eq. (2), "m" value can be varied to simulate an appropriate softening slope and the value of α can be appropriately assumed to simulate the appropriate residual stress [3], [10]. Here in this calculation m =0.5 and a = 0.0 are taken based on the fracture energy concept, taking the length of each member as the characteristic length.

The diagonal compression member and the arch member must resist the diagonal compression caused by shear. To account for the compression-softening behavior of crushed concrete, the model proposed by Collins et al. [18] is adopted. In this model, the softening coefficient was made a function of the transverse tensile strain. Thus, the tension and compression members are considered as a pair together. Equation (3) shows the compressive stress-strain relationship of concrete in this study. The stress-strain relationship for reinforcing bars is assumed to be elasto-plastic for the case of tension and compression members.

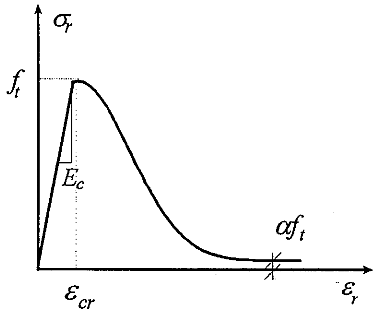


Fig. 3 Tensile stress-strain curve of concrete

$$\sigma_c = -\eta f_c' \left(2\left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2 \right) \tag{3a}$$

Where the peak-softening coefficient is

$$\eta = \frac{1.0}{0.8 - 0.34\left(\frac{\varepsilon_r}{\varepsilon_o}\right)} \le 1.0 \tag{3b}$$

And the strain at peak stress $\varepsilon_o = -0.002$.

3. ADOPTION OF MINIMUM TOTAL POTENTIAL ENERGY

The effect of total potential energy during calculations with the Modified Lattice Model has a significant effect on the final results. It has been found that there is a relation between the area of the arch element and the corresponding total potential energy of the structure. Niwa et al. [14] showed that if the ratio of the width of the arch element is assumed to be "t", the value of "t" is determined by minimizing the total potential energy for the whole structure. In this work however, it is found that this thickness increases gradually during loading from the elastic stage up to complete failure of the beam. This means that the area of the diagonal members falls gradually as the various different loading stages are reached.

A physical explanation for the adoption of minimum total potential energy may be given firstly using a very simple spring model as shown in **Fig. 4**. In this model, the cross-sectional area of the spring consists of two different materials with different moduli of elasticity, E_1 and E_2, under a concentrated load P. Assume the stiffest portion lies in the middle of the cross section with a stiffness value E_1. The total potential energy for this model is obtained by Eq. (4). Substituting for the values of σ and ε, we obtain the total potential energy as a parameter, dependent on the area of each material part within the cross section as shown in Eq. (5).

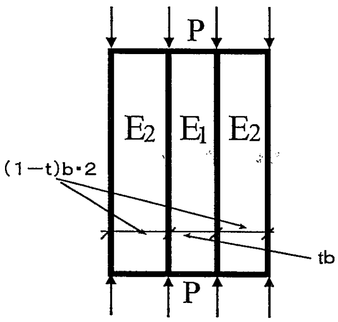


Fig. 4 Cross- section of a spring

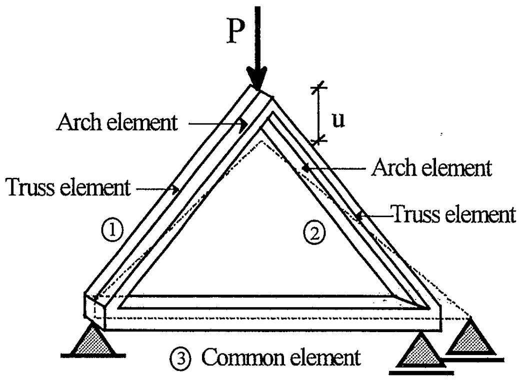


Fig. 5 Triangular model

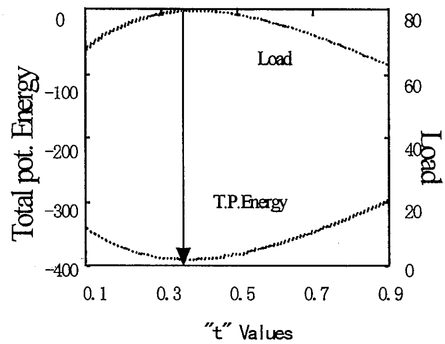


Fig. 6 Behaviour of T.P. energy and applied load for various "t" values

$$\pi = 1/2\int \sigma_1 \varepsilon dv_1 + 1/2\int \sigma_2 \varepsilon dv_2 - Pu \tag{4}$$

$$\pi = -Au^2(1/2)\left[E_1 t/l + E_2(1-t)/l\right] \tag{5}$$

From Eq. (5) the total potential energy is known to decrease monotonically with increasing area of the stiffer portion. Therefore, the stiffer portion should occupy the whole area in order to make the potential minimum. However, our beam element is not exactly in the same category.

So, we illustrate the real situation using the model shown in **Fig.5**. This model is a triangular shape under a concentrated load P. The cross-sectional area of the sides 1 and 2 has been divided into two different materials with two different moduli of elasticity, E_1 and E_2, representing the truss element and the arch element, respectively. The ratio of the width of the arch element is assumed to be "t" to the total width of the member. Member 3 is a common material with a definite modulus of elasticity. The total potential energy of the structure is calculated from Eq. (6).

$$\pi = 1/2\int \sigma \varepsilon dv - Pu \tag{6}$$

Where, u is the vertical displacement at the loaded point of the structure under the applied load "P". Take $\partial\pi/\partial t = 0$ to obtain a "t" value corresponding to the minimum total potential energy and substitute it in the energy

Table 1 Outline of experimental data.

No	Cross Sec.	b cm	h cm	d cm	a/d	f_c MPa	A_s cm^2	f_y MPa	A_w cm^2	f_{wy} MPa	s cm
1	R	20.3	50.8	42.5	2.15	31.0	23.1	530	1.42	530	13.3
2	R	20.3	45.7	38.9	2.00	24.6	24.5	320	1.42	320	18.3
3	R	30.0	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0
4	T	30.0 15.0	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0
5	R	45.0	60.0	52.5	2.86	43.9	95.7	383	1.43	355	25.0
6	R	45.0	60.0	52.5	2.86	66.2	95.7	383	1.43	355	15.0

*In this table, R means rectangular section and T means T-shaped section. No.4: flange width=30.0cm, flange depth=7.5cm and web width =15cm.

equation. **Figure 6** shows the relation beween the total potential energy and applied load "P" for different values of "t". From this figure, we find that the point corresponding to minimum total potential energy corresponds to the maximum applied load at a particular value of "t". This means that, if this value of "t" is used, we obtain the stiffest beam with a minimum potential energy. So, in the Modified Lattice Model, the total potential energy by applying Eq. (7) is calculated for different values of "t" starting from 0.1 ~ 0.9 with a very small increment.

$$T.P.E. = \sum_{i=1}^{n} f_i \delta_i - p\Delta \tag{7}$$

In Equation (7), T.P.E. is the total potential energy of the structure, where f_i is the internal force inside each member of the structure, δ is the displacement of each individual member, n is the number of truss members, p is the value of the external concentrated load, and Δ is the displacement at the loaded point. By minimizing these values of total potential energy we can obtain the corresponding "t" value. From this value, we can calculate the area of the arch element and the subdiagonal elements at each step of the calculation. So, when we consider the total potential energy and the corresponding area of the different elements, we obtain stiffer calculated results with the real response of the beam, which converges with the experimental results along the different loading stages, as we will see in the next section.

4. APPROPRIATE DISCRETIZATION METHOD FOR TRUSS MEMBER

In this section, a clarification of the appropriate discretization of lattice members, which may initially predetermined to have a 45-degree angle, is given. To investigate the extent of the discretization, three different truss models with different numbers of diagonal pairs along the depth of the beam are investigated. The three different forms are shown in **Fig.7**. To determine the most appropriate discretization model among the three previous forms of truss model in **Fig.7**, six different reinforced concrete beams are investigated as examples and the results of the Modified Lattice Model are compared with the experiment results. An outline of the experimental data utilized in examining the Modified Lattice Model using the three different forms of truss in **Fig.7** is presented in **Table 1**.

The calculated results using the Modified Lattice Model and the normal lattice model can be compared clearly in **Fig.8** (Leonhardt's experiment [12]; No.4 in **Table 1**) and **Fig.9** (Ohuchi's experiment [16]; No.5 in **Table 1**). The number of subsection diagonal members increases from model (1) to model (2) and to model (3). After cracking, the neutral axis of the beam starts to move upward during the development of cracks. The height of the cracks depends on the cross

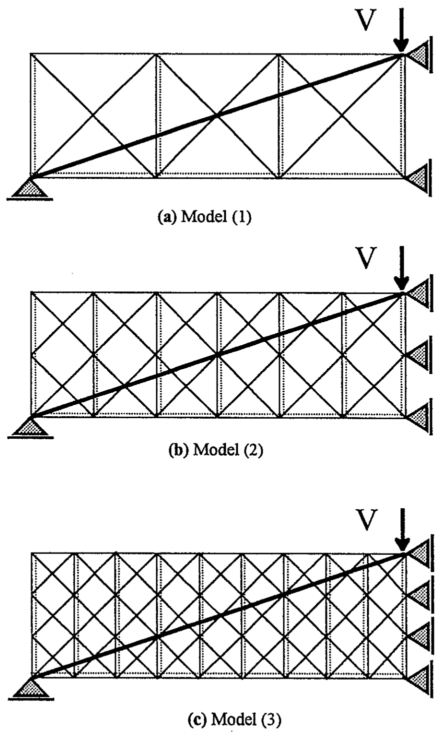


Fig.7 Forms of model of simulation

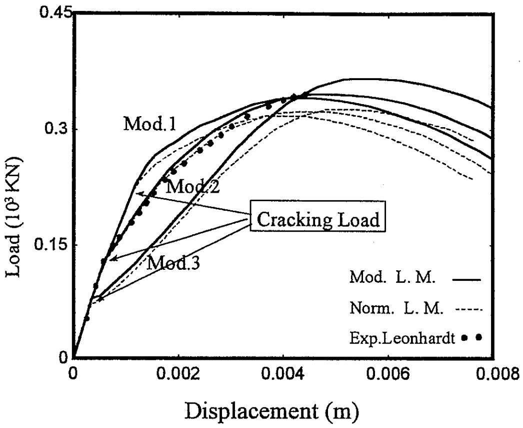


Fig.8 Comparison with experiment (No. 4)

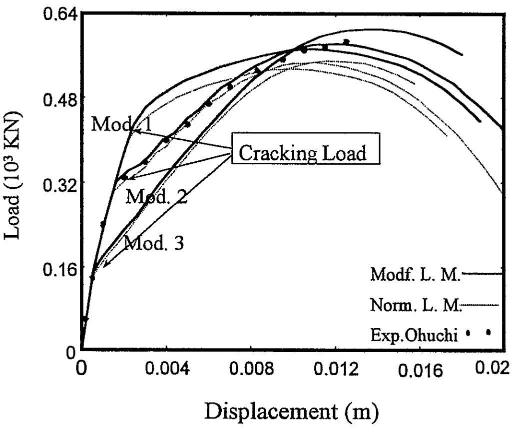


Fig.9 Comparison with experiment (No.5)

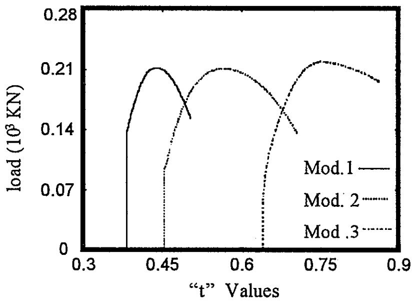


Fig.10 Comparison of "t" values for the three models (No.2)

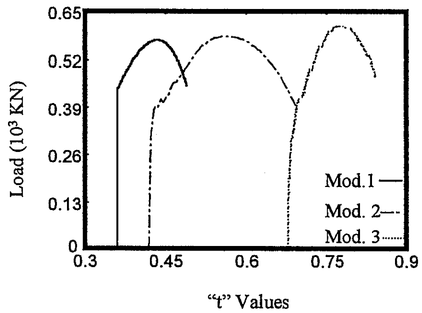


Fig.11 Comparison of "t" values for the three models (No. 5)

section of the beam and the steel reinforcement ratio. So, in the case of model (1), if a crack occurs, it means that the depth of the crack equals the overall depth of the beam. In this case, complete failure takes place suddenly. However, it is not realistic because experimentally failure does not occur suddenly. In the case of model (2), if cracks occur, it means the depth of the crack equals half of the depth of the beam and failure is not experienced suddenly as in the previous case. This situation looks logical and is close to the experimental behavior of reinforced concrete beams. In the case of model (3), the depth of the first crack equals 1/3 of the overall depth of the beam. In this case, the development of cracks is not like the experimental behavior of the beam. Thus we conclude that the numerical results are closest to the experimental results in the case of model (2), as shown in **Fig.8** and **Fig.9**. The distances of the vertical shear reinforcements are changed among the three different models; that which has a nother effect on the outpot results. So in conclusion, model (2) is the preferable means of implementing the Modified Lattice Model. Comparing the results for the three models, we find the cracking load decreases in order from model (1) to model (3). In the case of model (1), the elastic energy of the failed elements is much higher than that in model (2). Also in the case of model (2) it is much higher than in the case of model (3). The reason for this is the increasing of number of subsection diagonal members. The strain energy falls with the decrease in the original length of the failed elements. However, the ultimate loads using these three models are almost the same because of the similarity in fracture energy for the three different models. From these experimental results, we can conclude that the Modified Lattice Model can capture displacement behavior adequately and it exhibits almost the same response as the original beam. In particular, the displacement at the peak is closer to the experimental results than any other truss model.

The changing thickness of the arch element is illustrated in **Fig.10** (Clark's experiment [5]) and **Fig. 11** (Ohuchi's experiment [16]) for beams No.2 and No. 5 in **Table 1**, respectively. According to the Modified Lattice Model analysis, the thickness of the arch element increases gradually from the elastic stage, in which it remains constant, until complete failure of the beam. After the peak point, the depth of the arch element decreases due to crack extension. Thus, the arch element thickness increases gradually in order to maintain the effectivness of the arch element up to the failure point. As has been discussed above, the most appropriate truss discretization is model (2). This suggests that the probable arch width is around 0.4b early in loading, increasing with the load up to 0.7b.

5. APROPRIATE SUBDIAGONAL ANGLE FOR THE MODIFIED LATTICE MODEL

To study the direction of initial cracking in each of the solved beams according to the three different models in **Fig.7**, three different values of subdiagonal inclination angle in the Modified Lattice Model are suggested. These are 51, 45, and 26 degrees, as shown in **Fig.12 (a), (b),** and **(c)** respectively. **Figurte 13** and **Fig.14** show the load-displacement diagram for beam No.2 [5] in **Table 1** for these subdiagonal angles and also for the different models

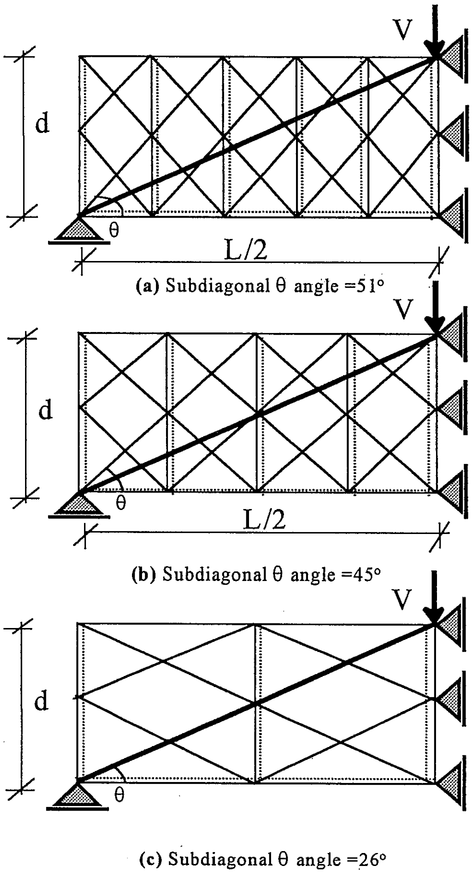


Fig.12 Three different subdiagonal angles

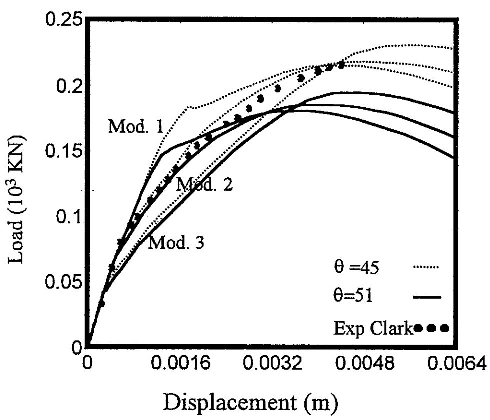


Fig.13 Load-displacement diagram in case of θ=51° and 45° (No.2)

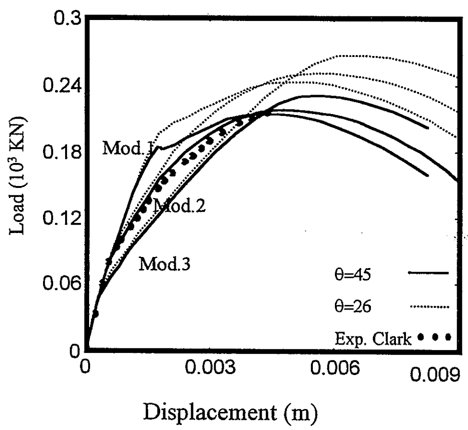


Fig.14 Load-displacement diagram in case of θ=26° and 45° (No.2)

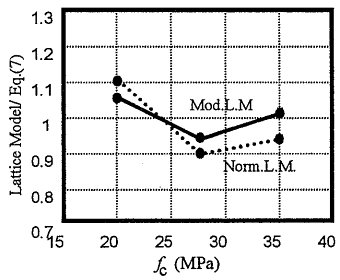
mentioned in **Fig.7**. The numerical results using the Modified Lattice Model are compared with experimental results. It is found that the results using model (2) with a diagonal angle 45 degrees are very close to the experimental results. With any other inclination angle, the load-displacement relationship diverges up or down from the experimental results. In the case of a 51-degree angle, the diagonal members are shorter than in the 45-degree case, so the load-displacement relatipnship follows a lower path than the relation using a 45-degree diagonal angle, as shown in **Fig.13.** But in the case of the same model using a 26-degree angle, the results follow a higher path than in the case of a 45-degree angle, as in **Fig.14**. Actually, this happens because the increasing length of the diagonal member means more elastic energy and greater structure stiffness. This behavior is common to all three different models, and also for each angle of inclination.

6. EXAMINATION OF THE APPLICABILITY OF THE MODIFIED LATTICE MODEL

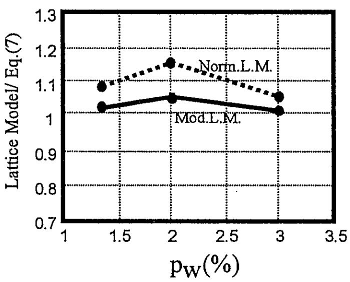
To examine the applicability of the Modified Lattice Model, many different beams are calculated numerically using the second model in **Fig.7 (b)**. The shear strength is calculated for each beam and compared with the basic shear strength equation.

6.1 For beams with web reinforcement

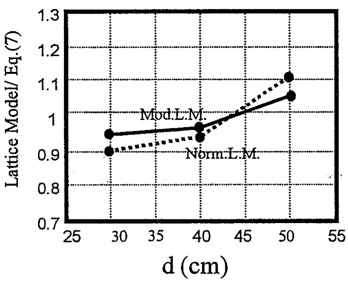
Various reinforced concrete beams with different parameters are analyzed using the Modified Lattice Model. The shear strength of each beam is compared with Eq. (8), which is considered the basic method of calculating shear strength in the truss analogy [17].



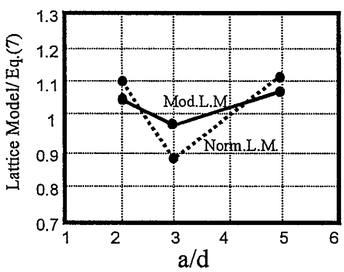
(a) Shear capacity with f_c (MPa) for beams with web R.



(b) Shear capacity with p_w (%) for beams with web R.



(c) Shear capacity with depth for beams with web R.



(d) Shear capacity with a/d Ratio

Fig.15 Change of shear carrying capacity for beams with web reinforcement

$$V_y = V_c + V_s \tag{8}$$

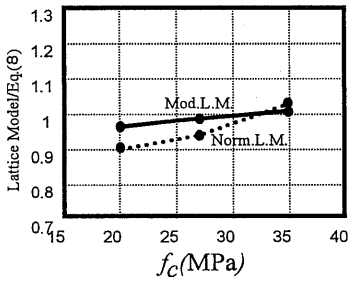
Where, V_c is the shear capacity of linear members without shear reinforcement, obtained by Eq. (9) [15], and V_s is the shear capacity of the shear reinforcing steel and obtained by Eq. (10) [7].

$$V_c = 0.20 f_c^{1/3} p_w^{1/3} d^{-0.25}\left[0.75 + \frac{1.4}{a/d}\right] b_w d \tag{9}$$

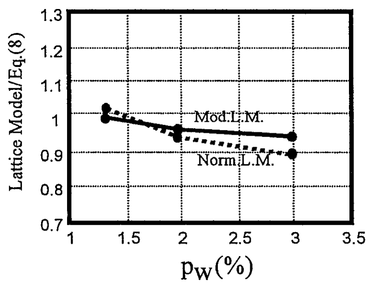
$$V_s = A_w f_{wy} d \frac{z}{s} \tag{10}$$

Where, f_c is the compressive strength of concrete (MPa), p_w is the reinforcement ratio ($=100 A_s/(b_w d)$), d is the effective depth of the concrete beam (m), a/d is the shear span-effective depth ratio, A_w is the area of shear reinforcement over the intervals, f_{wy} is the yield strength of the shear reinforcement, and $z = d/1.15$. To examine the applicability of the Modified Lattice Model to beams with web reinforcement, comparisons are carried out using Eq. (8) and also with the normal Lattice Model [14].

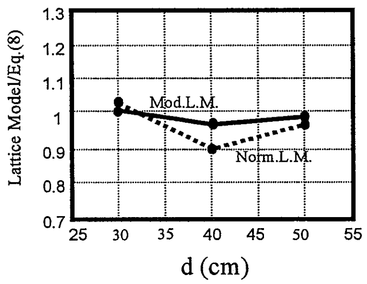
Figure 15 shows how the results of predicted shear capacity using the normal Lattice Model, the Modified Lattice Model, and Eq. (8) change with some parameters, taking beam No.1 [2] in **Table 1** as a definite example



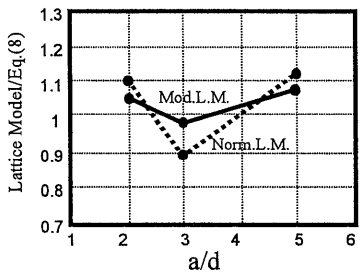
(a) Shear capacity with f_c (MPa)



(b) Shear capacity with p_w (%)



(c) Shear capacity with the depth



(d) Shear capacity with the ratio (a/d)

Fig. 16 The change of shear carrying capacity for beams without web reinforcement for cross-sectional area. These parameters are the concrete strength, reinforcement ratio, effective depth, and also the shear span-depth ratio of the beam. As seen in **Fig.15**, the shear capacity as calculated by the normal Lattice Model is generally smaller than that by Eq. (8), but the predicted results using the Modified Lattice Model are much closer to Eq. (8) and are admissible. Also from **Fig.15**, the tendency of the prediction results using the Modified Lattice Model is still not exactily similar to Eq (8); hence the maximum difference varies from 0.96 to 1.08, but in the case of the normal Lattice Model the range is from 0.88 to 1.17.

6.2 For beams without web reinforcement

To examine the applicability of the suggested Modified Lattice Model to concrete beams without web reinforcement, numerical calculations are performed and compared with Eq. (9) [15] which has been accepted as a basis of the design equation for the JSCE. The results are compared with Eq. (9) and also with the normal Lattice Model [14]. **Figure 16** shows how the results for predicted shear capacity using the normal Lattice Model, the Modified Lattice Model, and Eq. (9) change, with some different parameters. Taking the dimension of the cross-sectional area of beam No. 6 [16] in **Table 1** as an example. These parameters are concrete strength, reinforcement ratio, effective depth, and the shear span-depth ratio. As seen from **Fig.16**, the shear capacity predicted by the normal Lattice Model is smaller than that by Eq. (9), but the variation for the Modified Lattice Model is much smaller and is admissible. Also from **Fig.16,** the tendency of the predicted results using the Modified Lattice Model is also still not similar to Eq. (9); since the maximum range varies from 0.96 to 1.03, but in the case of normal Lattice Model the range is from 0.88 to 1.1. The predicted shear failure mode by the Modified Lattice Model is failure of the diagonal tension member, which corresponds, to the experimental results. Consequently, it can be concluded that the prediction of shear capacity by the Modified Lattice Model is adequate.

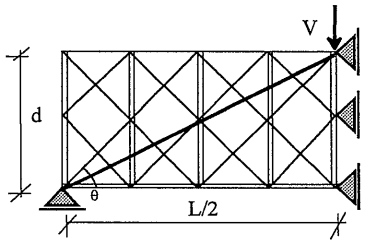
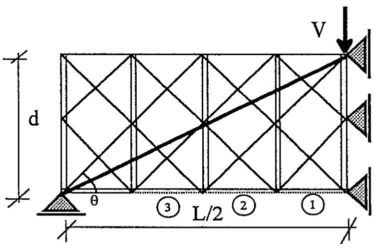


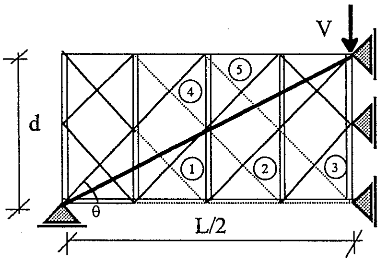
Fig.17 Illustrative beam example (No.2)

7. APPLICATION OF THE MODIFIED LATTICE MODEL TO SHEAR FAILURE SIMULATION

To investigate the changing stress states in each member within a reinforced concrete beam using the Modified Lattice Model, Clark's experiment (No.2 in **Table 1**) was chosen as the subject for a solved example. **Figure 17** shows the Modified Lattice Model for reinforced concrete beam No. 2 in **Table 1**. The stresses in diagonal concrete members and stirrups and the stress in the arch member are examined. From the results of a simulation of this beam using the Modified Lattice Model, it is found that at the primary cracking stage, the concrete elements in the bottom chord start to crack first as shown in **Fig.18 (a)**. Then diagonal cracking initiates as shown in **Fig.18 (b)**. The stirrups then begin to yield. Although the stirrups begin yielding and the diagonal



(a) Cracks in bottom members



(d) Cracks in diagonal members

Fig.18 The propagation of cracks

tension elements have cracked, the beam still continues to carry a load up to complete failure. This is because of the existence of the arch element, which continues to bear the load up to the end of loading with some stirrups. At the final stage, all the stirrups yield. At that time, the arch element is immediately crushed. From this simulation of beam failure, we can categorize it as a shear failure.

According to this simulation, and considering the objectivity of the post processing of the calculated results and the simple representation of the shear resisting mechanism, the Modified Lattice Model can be said to simulate the shear failure mode very smartly. Although the Modified Lattice Model is a simulation in which the compatibility condition, the equilibrium condition, and the constitutive equations used are much simplified as compared with normal FEM, it is able to capture the shear behaviour of concrete beams reasonably well throughout changes to the shear resisting mechanism.

8. CONCLUSIONS

In the proposed Modified Lattice Model, a reinforced concrete beam subjected to shear force is converted into simple truss and arch members by considering the minimum total potential energy for the structure at each step of loading. A nonlinear incremental analysis is performed. The conclusions obtained from this research are as follows:

1. By minimizing the total potential energy of the reinforced concrete beam, we obtain only a single value for the thickness of the arch element, and this corresponds to the stiffest case for the structure. This is quite similar to the original response obtained in experimental analysis.
2. The Modified Lattice Model tends to estimate beam stiffness closer to the experimental results. Furthermore, the predicted displacement at the peak is almost identical to the experimental results.
3. The thickness of the arch member, which plays a very important role in the Modified Lattice Model, increases gradually as the displacement of the loading point increases once diagonal cracks are initiated and until complete failure of the beam.
4. The applicability of the Modified Lattice Model has been examined for beams with and without web reinforcement under different parametric conditions. Predictions of shear strength by the Modified Lattice Model are very close to the basic shear strength equations as accepted by the Standard Specification of JSCE. Also, comparing with the experimental results, it gives satisfactory accuracy.
5. In the case (2) with 45-degree subdiagonal members, the position of the neutral axis is in reasonable agreement with experimental work. So, model (2) with two pairs of subdiagonal members and with a 45-degree angle of inclination is an appropriate discretization in implementing Modified Lattice Model analysis.
6. Using different forms of the Modified Lattice Model, the ultimate load remains almost constant but the cracking load decreases depending on the strain energy of the cracked element.

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