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SAFETY EVALUATION METHOD OF STRUCTURAL SYSTEM AND ITS APPLICATION TO SEISMIC DESIGN OF RC BRIDGE PIER

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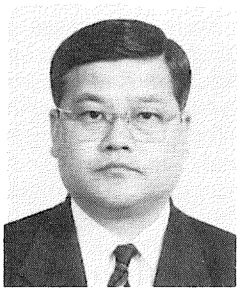

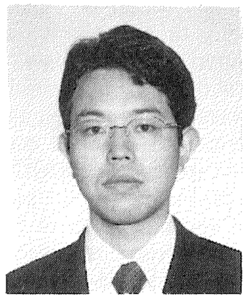

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A new evaluation method for the probability of failure of a structural system based on reliability theory is proposed. Using numerical examples, it is demonstrated that this method is accurate yet simple to implement. The safety of an RC bridge pier against earthquakes is evaluated on the basis of the proposed method. Three limit states – flexural capacity, shear capacity and ductility – are taken into consideration in the analysis. It is shown that the shear/flexural capacity ratio and ground conditions are very important factors in evaluating the seismic performance of RC bridge piers. Finally, a new concept of seismic design for RC bridge piers based on reliability theory is proposed.

KeyWords : *structural safety, reliability theory, safety index, RC bridge pier, sesmic design*

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1. INTRODUCTION

Reliability-based design is founded on the concept that one can estimate the probability of an undesirable event such as a fracture, occurring over the lifetime of a structure, despite the uncertainties involved. It is a design method that provides an assured level of safety by reducing the probability of such an occurence (referred to as the failure probability) below a target value (referred to as the target failure probability). The quality of a design is judged by comparing the failure probability with the target failure probability, and this use of the failure probability makes it possible to evaluate the safety of a structure quantitatively. Above all, this design method enables one to gain a firm grasp of the safety level desired, and to design a structure so as to attain the prescribed reliability independently of other design requirements[1].

Even when based on reliability, however, conventional design methods apply such safety checks separately to each limit state considered relevant by the designer; all likely limit states are not considered simultaneously. Moreover, safety verification is usually carried out only on the limit state which is most likely to arise; i.e. the one with the highest failure probability. For example, according to conventional thinking, in the case of an RC bridge pier designed to undergo flexural failure, it is considered sufficient to ensure that the shear capacity exceeds the shear force at the moment when flexural capacity is reached and to examine safety against flexural failure only. In a case like this, the failure probability of the pier is assumed to be totally unaffected by its shear capacity. However, the ratio of shear capacity to flexural capacity (referred to as the capacity ratio), for example, is closely correlated to the ductility of the bridge pier; if the capacity ratio is low, the bridge pier is subject to brittle fracture[2]. Moreover, the flexural and shear capacities are influenced by differences between the uncertainty levels of models containing equations for calculating the flexural and shear capacities; it can be expected that these values will substantially influence the failure probability of the bridge pier. In general, to use one limit state to represent a failure event which really comprises a plurality of limit states results in a risky evaluation. Even if the design satisfies the target failure probability for that limit state, the resultant structure may not meet the prescribed safety requirements. Thus, in principle, one should appropriately evaluate correlations among a multiplicity of limit states, and compute the probability of failure due to an event which comprises these limit states. In other words, to guarantee that the target failure probability of a bridge pier designed for flexural failure is satisfied, it is necessary to evaluate not only the flexural capacity, but also to attach some conditions to shear capacity.

Adapting this point of view, this study aims, firstly, to propose a safety evaluation method for structural systems which offers ease of calculation, good accuracy in a practical context, and takes into account multiple limit states while remaining an approximate method. Then, based on the proposed method, an assessment related to the capacities and ductility will be adopted as the ultimate limit state for a RC bridge pier (free-standing, single-column type), and the reliability of the RC bridge pier in an earthquake will be analyzed. In doing so, particular importance is attached to the capacity ratio of the bridge pier, and the effect of shear capacity of a flexural-failure-dominant bridge pier on its safety is studied. In addition, based on the knowledge obtained, an outline of a seismic design method based on reliability theory for RC structures is proposed.

2. PROPOSED RELIABILITY EVALUATION METHOD FOR STRUCTURAL SYSTEMS

2.1 Basic Equation of Reliability Evaluation Method

For combinations of events in set theory, if higher-order events represented by the intersection of three or more simultaneous events are ignored, the probability of failure event E, comprising k separate events, is expressed as follows.

$$P(E) = \sum_{i}^{k} C_i k \tag{1}$$

where,

$$C_1 = P(E_1)$$

$$C_2 = P(E_2) - P(E_2E_1)$$

$$C_k = P(E_k) - \sum_{i=1}^{k-1} P(E_kE_i) + \sum_{\substack{m=1,k-2 \\ n=2,k-1}}^{m<n} P(E_kE_m \cap E_kE_n), k > 3$$

In this calculation, there are three forms of failure probability: $P(E_k)$, $P(E_kE_i)$ and $P(E_kE_m \cap E_kE_n)$.
The Rosenblatt transformation[3] is generally the most suitable method of calculating the failure probability $P(E_k)$. Use of the Rosenblatt transformation can deal with cases in which a probability variable in the limit state equation is an unnormalized variable as well as cases in which there is a correlation between probability variables. In addition, even in the case of a limit state equation for a nonlinear function, the failure probability can be calculated to a certain degree of accuracy.

Ditlevsen's narrow bounds method[4] is known as a way to calculate the joint probability $P(E_kE_i)$ of failure events E_k and E_i. Ditlevsen's bounds are the upper and lower limits of the joint probability, obtained under the hypothesis of independence or perfect correlation. Furthermore, by combining the Rosenblatt transformation and Ditlevsen's bounds, the failure probability of a failure event with a plurality of limit states can be approximated in the form of an inequality. For a failure probability thus obtained, however, the upper and lower limit values may differ widely from each other if the value of the failure probability is of the order of 10^{-2}, resulting in cases where the safety of the structural system cannot be evaluated accurately. Other known methods include the PNET method[5] proposed by Ang et al. This too has problems, in that the calculated failure probability heavily depends on the conditions set during calculation, and that, depending on the order of the failure probability, a result erring too far on the side of risk may be produced. Hence the scope of its application is limited.

In this study, therefore, we will take the equation (1) as the basic equation for evaluating the reliability of structural systems and then demonstrate simple calculation methods for the terms $P(E_k)$ and $P(E_kE_i)$ in the equation. We will also propose a method for incorporating the intersection of joint events $(E_kE_m \cap E_kE_n)$ into the calculation. This will solve various problems faced in prior studies and provide a stable technique for calculating failure probability, that is unaffected by the value of the failure probability and by assumptions of probability theory. These calculation methods are given in detail below.

2.2 Calculation Method of $P(E_k)$

Taking into account the type of structural systems to be analyzed, the following assumptions were made: [a] The probability distribution of probability variables representing capacities is either normal or logarithmic normal; [b] There is no correlation between probability variables representing capacities and probability variables representing external force; and [c] There is also no correlation among probability variables representing external force.

Under these assumptions, when the limit state equation $g_k(X)$ representing failure event E_k is expressed using equation(2), the process for obtaining probability variables in a space of independent normal variables obtained by the Rosenblatt transformation may be simplified as follows[6].

$$g_k(X) = g_k(x_1, x_2, \cdots, x_j, x_{j+1}, \cdots, x_n) \tag{2}$$

where,

$x_1, x_2, \cdots, x_j$: probability variables related to capacities (**R**)
$x_{j+1}, \cdots, x_n$: probability variables related to external forces (**S**)

1) It is assumed that in the failure space, the design point which gives the shortest distance from the origin to the failure surface, $x_0^* = x_0$, is the mean value of the probability variables.
2) A design point u_0 in the space of independent standardized variables, and corresponding to the assumed design point, is obtained by the Rosenblatt transformation. The process is as follows.

Using assumption [a] above, for probability variables belonging to $\mathbf{R} = (x_1, x_2, \cdots, x_j)$,

$$u_i = (y_i - \sum_{l=1}^{i} \alpha_{il} U_{i-1}) / \alpha_{ii}$$

where, (3)

y_i: in the case that x_i is a normal variable,

$$y_i = \frac{x_i - \mu_i}{\sigma_i}$$

in the case that x_i is a logarithmic normal variable,

$$y_i = \frac{x_i - \lambda_i}{\zeta_i}$$

α: $\alpha_{11} = 1.0$

$$\alpha_{il} = \rho_{yiy1}$$

$$\alpha_{il} = \left(\rho_{yiyj} - \sum_{l=1}^{j-1} \alpha_{il} \alpha_{jl} \right) / \alpha_{jj} \quad (1 < j < i)$$

$$\alpha_{ii} = \sqrt{1 - \sum_{l=1}^{i-1} \alpha_{il}^2}$$

where,

μ_i, σ_i : mean and standard deviation of the probability variable x_i

λ_i, ζ_i : mean and standard deviation of the probability variable $\ln x_i$

ρ_{yiyj} : correlation coefficient of y_i and y_j and it may be approximated by means of the correlation coefficient of x_i and x_j

On the other hand, under assumption [3], for probability variables belonging to $\mathbf{S} = (x_{j+1}, \cdots, x_n)$,

$$u_i = \Phi^{-1}[F_i(x_i)] \quad (i = j+1, \cdots, n)$$

where, (4)

$F_i(x_i)$: cumulative distribution function of probability variable x_i

Φ : cumulative distribution function of standard normal distribution

3) A value of the Jacobian at the point x_0 is obtained:

$$\mathbf{J} = \frac{\partial(u_1, u_2, \cdots, u_n)}{\partial(x_1, x_2, \cdots, x_n)}$$

(5)

In particular, if probability variables belonging to $\mathbf{R} = (x_1, x_2, \cdots, x_j)$ are normal variables,

$$\mathbf{J} = \begin{pmatrix} \mathbf{J_R} & \mathbf{0} \\ \mathbf{0} & \mathbf{J_S} \end{pmatrix}$$

where, (6)

$\mathbf{J_R}$: For $\mathbf{R} = (x_1, x_2, \cdots, x_j)$,

$$\mathbf{J_R} = \begin{pmatrix} 1/(\sigma_1 \alpha_{11}) & & 0 \\ \varepsilon_{21}/\sigma_1 & & \\ \vdots & \vdots & \ddots \\ \varepsilon_{j1}/\sigma_1 & \varepsilon_{j2}/\sigma_2 & 1/(\sigma_i \alpha_{jj}) \end{pmatrix}$$

$\mathbf{J_S}$: For $\mathbf{S} = (x_{j+1},\cdots,x_n)$,

$$\mathbf{J_S} = \begin{pmatrix} \dfrac{1}{\phi(u_{j+1})}\dfrac{\partial\left[F_{j+1}(x_{j+1})\right]}{\partial x_{j+1}} & & 0 \\ & \ddots & \\ 0 & & \dfrac{1}{\phi(u_n)}\dfrac{\partial\left[F_n(x_n)\right]}{\partial x_n} \end{pmatrix}$$

Moreover,

$$\varepsilon_{ij} = \sum_{l=1}^{i-j}\left(-\alpha_{il}\varepsilon_{lj}\right) \quad (i > j)$$

ϕ: probability density function of the standard normal distribution

If probability variables belonging to $\mathbf{R} = (x_1,x_2,\cdots,x_j)$ are logarithmic normal variables, the standard deviation σ_i can be taken as ζ_{xi}.

4) The product of the limit state equation at the design point u_0 and the gradient vector is obtained. Here, the gradient vector is the vector representing the direction of the tangent plane at the point in a normal variable space at which the possibility of failure is the highest.

5) A new design point u^* is obtained. In the space of the original variables, the design point x^* can be obtained as a linear approximation using the following equation:

$$x^* \cong x_0 + J^{-1}\left(u^* - u_0\right)u^* \tag{7}$$

6) $P(E_k) = \Phi(-\beta) = \left(u^{*t}u^*\right)^{1/2}$ is calculated.

7) Using x^* obtained above as a new design point, steps 2) through 6) are repeated until the design point x^* converges.

2.3 Calculation Method of $P(E_kE_i)$

It was assumed that for Ditlevsen's bounds, the area of overlap between the regions A and B in Fig. 1 is proportional to $P(E_kE_i)$ [7]. When the limit state equations representing failure events E_k and E_i are $g_k = 0$ and $g_i = 0$, respectively, the direction cosine of the angle formed between the hypersurfaces of the two critical state equations of Fig. 1 equals the correlation coefficient ρ_{ki} of the two events. Consequently, the following is obtained as an approximation equation:

$$P(E_kE_i) = \left(1 - \frac{\cos^{-1}\rho_{ki}}{\pi}\right)\left(P(A) + P(B)\right) \tag{8}$$

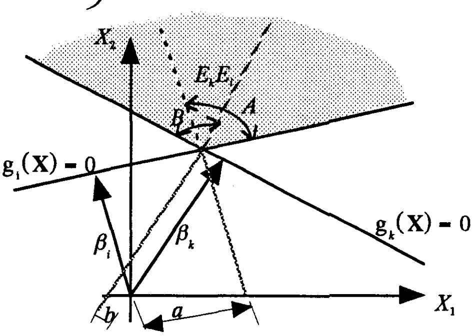


Fig.1 Ditlevesen's Bounds (in the case of normal correlation)

where,

$$P(A) = \Phi(-\beta_i)\Phi(-a)$$

$$= \Phi(-\beta_i)\Phi\left(-\frac{\beta_k - \rho_{ki}\beta_i}{\sqrt{1-\rho_{ki}}}\right) \tag{9}$$

$$P(B) = \Phi(-\beta_k)\Phi(-b)$$

$$= \Phi(-\beta_k)\Phi\left(-\frac{\beta_i - \rho_{ki}\beta_k}{\sqrt{1-\rho_{ki}}}\right) \tag{10}$$

Here, the correlation coefficient ρ_{ki} is given by:

$$\rho_{ki} = \frac{Cov(g_k, g_i)}{\sigma_{gk}\sigma_{gi}} \tag{11}$$

When the limit state equations are nonlinear, a Taylor expansion of each limit state equation at the design point is carried out and the resultant linear approximation is used in equation(11).

2.4 Calculation Method of $P(E_kE_m \cap E_kE_n)$

Unlike a two-order joint probability, $P(E_kE_m \cap E_kE_n)$ cannot be approximated geometrically. Hence, an approximation is obtained by considering the correlation between the failure events. First, parameter Ω representing the correlation between the events is defined as follows.

$$\Omega = \frac{P(E_kE_m \cap E_kE_n)}{\min(P(E_kE_m), P(E_kE_n))} \tag{12}$$

When $\Omega = 1.0$, events E_kE_m and E_kE_n are completely dependent, and when $\max(P(E_kE_m), P(E_kE_n))$, events E_kE_m and E_kE_n are independent. Thus, the range of Ω is as follows.

$$\max(P(E_kE_m), P(E_kE_n)) \leq \Omega \leq 1.0 \tag{13}$$

However, the correlation between events E_kE_m and E_kE_n cannot be obtained directly. Therefore, an approximation is obtained from the respective correlation coefficients of failure events E_k and E_m, E_k and E_n, and E_m and E_n, using the following equation:

$$\Omega = \left(\min(\rho_{km}, \rho_{kn}, \rho_{mn})\right) \times \left(\sum_{all} \rho - \min(\rho_{km}, \rho_{kn}, \rho_{mn})\right) \tag{14}$$

where,

$$\sum_{all} \rho = \rho_{km} + \rho_{kn} + \rho_{mn}$$

Equation (14) represents the ratio of the correlation between the two most weakly correlated events and the other two events.

By applying the calculation methods described in (2), (3), and (4) above, the reliability of a structural system can be evaluated analytically with multiple limit states into account simultaneously.

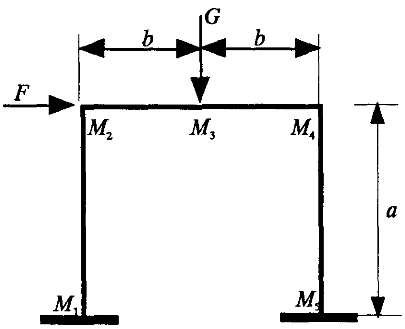


Fig.2 Calculation Example 1[8]

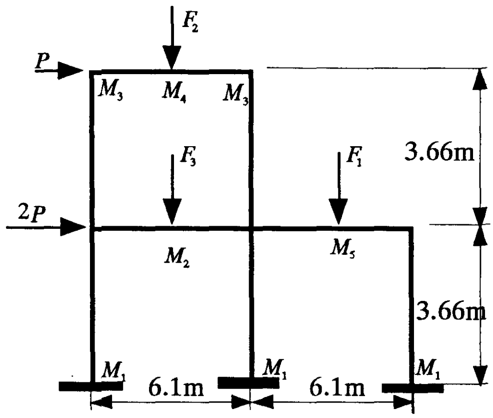


Fig.3 Calculation Example 2[9]

2.5 Comparison between Proposed Equations and Prior Study

When the formation of a mechanism is taken as a collapse, rigid frame structures have a number of limit states, and some of these may occur at a similar loading. When calculating the failure probability, a proper consideration of the correlation between these limit states is necessary. Here, the failure probabilities of two rigid frame structures are calculated from the results of prior studies and also using the equations proposed in this study. By comparing the failure probabilities thus obtained, the accuracy, simplicity, and effectiveness of the proposed equations are evaluated. In the following examples, all probability variables are assumed to be normally distributed. Correlations between probability variables are disregarded and the analysis considers only correlations between failure events.

a) Example 1 (adapted from Ditlevsen[8]).

The 1-level rigid-framed structure shown in Fig. 2 was adopted as an example. The following three equations were worked out as the limit states in mechanism formation:

$$g_1 = M_1 + 2M_3 + 2M_4 + M_5 - Fa - Gb$$
$$g_2 = M_1 + M_3 + M_4 - Gb$$
$$g_3 = M_1 + M_2 + M_4 + M_5 - Fa$$

where,

a, b : fixed values, with the assumption $a = b = 2$

F, G : probability variables representing external forces; mean values assumed to be $\mu_F = \mu_G = 1.0$; standard deviations assumed to be $\sigma_F = \sigma_G = 0.5$

M_i : probability variables representing capacity (plastic moment); mean value assumed to be $\mu_M = 1.0$ and standard deviation $\sigma_M = 0.5$ for all

Failure probabilities $P(E)$ computed with the proposed equations and from the earlier results are as follows.

First-order bounds	$0.173 \leq P(E) \leq 0.316$
Ditlevsen's bounds	$0.173 \leq P(E) \leq 0.264$
PNET method	$P(E) = 0.258$
Monte Carlo simulation (Number of samples n=50,000)	$P(E) = 0.230$
Proposed equation	$P(E) = 0.230$

b) Example 2 (adapted from Salahuddin[9])

The 2-level, 2-span rigid-framed structure shown in Fig. 3 was adopted as an example. The following eight equations were worked out as the limit states for mechanism formation:

$$g_1 = 5M_1 + 3M_2 + 3M_3 + 2M_4 - 10(F_1 + F_2) - 48P$$
$$g_2 = 6M_1 - 36P$$
$$g_3 = 5M_1 + 4M_2 + 2M_3 + 3M_4 + M_5 - 10(F_1 + F_2 + F_3) - 48P$$
$$g_4 = 5M_1 + 3M_2 + M_3 - 10F_1 - 36P$$
$$g_5 = 2M_3 + 2M_4 - 10F_2$$
$$g_6 = M_1 + 3M_5 - 10F_3$$
$$g_7 = 5M_1 + 2M_2 + M_3 + 4M_4 - 10(F_1 + F_2) - 48P$$
$$g_8 = 4M_2 - 10F_2$$

where,

F_i, P: probability variables; mean values are assumed to be $\mu_{F1} = 38, \mu_{F2} = 20, \mu_{F3} = 26$, and $\mu_P = 7$ respectively, and coefficients of variation to be 25% for all the probability variables

M_i: probability variables representing capacity (total plastic moment); mean values are assumed to be $\mu_{M1} = \mu_{M2} = 70$, $\mu_{M3} = 150, \mu_{M4} = 90$, and $\mu_{M5} = 120$ respectively; coefficients of variation are assumed to be 15% for all the probability variables

The results of the calculations are as follows.

First-order bounds	$0.0319 \leq P(E) \leq 0.168$
Ditlevsen's bounds	$0.0319 \leq P(E) \leq 0.132$
PNET method	$P(E) = 0.082$
Monte Carlo simulation (Number of samples n=50,000)	$P(E) = 0.104$
Proposed equation	$P(E) = 0.110$

These calculated examples allow us to verify that the proposed method (referred to hereafter as the reliability evaluation method for structural systems) does not require the integration of values in the calculation process, is as easy to use as any of the earlier methods, and gives results in accord with those obtained by the Monte Carlo simulation.

Now, the failure probability calculated by the reliability evaluation method for structural systems is converted into a the safety index β according to equation (15).

$$\beta \cong \Phi^{-1}(P(E)) \tag{15}$$

Equation (15) gives perfect correspondence in a case where the limit state equation expressing the failure event is linear and all probability variables are normally distributed. The correspondence between failure probability and safety index for such a case is shown in Table 1. Even when these conditions are not met, equation (15) gives good agreement with considerably high accuracy if an accurately calculated failure probability is converted. Further, it should be noted that it is not so much the difference in magnitude of the failure probability that directly affects the design; the difference in safety index magunitude is more noticeably reflected in the design[10]. From this point in the study onwards, this safety index will be used to evaluate the safety of RC bridge piers.

Table1 Relation between Failure Probability and Safety Index

Failure Probability	0.5	10^{-1}	10^{-2}	10^{-4}	10^{-6}
Safety Index	0.0	1.28	2.33	3.72	4.75

In the following sections, the seismic safety of a free-standing RC bridge pier is examined using this new reliability evaluation method for structural systems. An outline of the analysis is given in Section 3, and the analytical results and a discussion are presented in Section 4.

3. OUTLINE OF SEISMIC SAFETY ANALYSIS OF RC BRIDGE PIER

3.1 General

In evaluating the safety of an RC bridge pier using the reliability evaluation method for structural systems as proposed above, a limit state equation has to be developed. In general, this limit state equation is set up as the "capacity term" minus the "external force term". In this study, capacity and ductility were chosen as items to be examined in investigating the ultimate limit state of the RC bridge pier. Accordingly, the capacity term consists of flexural capacity, shear capacity, and ductility, while the external force is the active inertial force produced by the earthquake or response displacement. The following describes the method of calculating these capacities and the ductility, as well as the method of time-history seismic response analysis used to calculate the inertial force and response displacement as the external force term.

When implementing a time-history seismic response analysis, the load-carrying capacity is evaluated prior to analysis. Hence, from a deterministic point of view, examining only the ductility would lead to no design problems. Nevertheless, in reliability design, the margin up to the limit states of capacity and/or ductility is converted into a failure probability, and safety is judged on the basis of this result. This is fundamentally different from conventional design methods, which simply verify whether the estimated capacities and ductility exceed the response values.

3.2 Seismic Response Analysis Model

In this study, the system to be analyzed was modeled as shown in Fig. 4 and dynamic analysis was carried out. The bridge pier and superstructure were modeled in one dimension, and the nonlinear hysteresis characteristics of the bridge pier were taken into consideration. As a nonlinear model, the stiffness degradation tri-linear model was used. One third of the mass of the bridge pier in the one-dimensional model was incorporated into the mass of one span, and the rest was incorporated into the mass of the footing. The damping constant was set at 0.02. Dynamic interaction between structure and ground was based on a proposal made by the Civil Engineering Society's subcommittee for dynamic interaction [11]. The nonlinearity of the ground around the base was taken into account by using the bilinear restoring force model for ground spring by Harada et al. [12].

To calculate seismic response, an incremental method based on Newmark's β method was used (β =1/6). This dynamic analysis was carried out to calculate the active inertial force and displacement of the RC bridge pier; under seismic loading these make up the external force term of the limit state equation. Seismic waves were input in the longtudinal direction which, as a rule, is low in earthquake resistance.

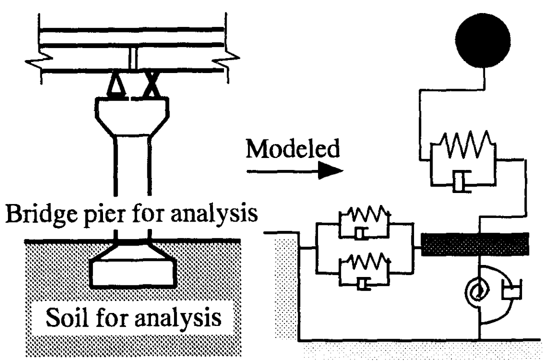


Fig.4 Numerical Analysis Model of the Structural System Used for Analysis

3.3 RC Bridge Piers and Ground Types Used in Analysis

For reliability evaluations such as the ones carried out in this study, it is preferable to choose RC bridge piers designed using the same design

Table2 Bridge Piers Used for Analysis

(a) Bridge Pier A (Capacity ratio = 1.18)

Requirements of bridge pier	Bridge pier	4.0m in diameter, 9.8m in height
	Foundation	Pile
Bridge pier	Section	4.0m in diameter
	Axial reinforcing bars	D51×72
	Hoop ties	D25ctc150mm
Foundation	Section of footing	9.5m×13.25m
	Pile	1.5m in diameter×10

(b) Bridge Pier B (Capacity ratio = 1.32)

Requirements of bridge pier	Bridge pier	4.0m in diameter, 8.5m in height
	Foundation	Pile
Bridge pier	Section	4.0m in diameter
	Axial reinforcing bars	D38×78
	Hoop ties	D22ctc125mm
Foundation	Section of footing	9.5m×11.0m
	Pile	1.5m in diameter×12

(c) Bridge Pier C (Capacity ratio = 1.84)

Requirements of bridge pier	Bridge pier	3.0m×3.5m, 10.5m in height
	Foundation	Pile
Bridge pier	Section	3.2m×3.7m
	Axial reinforcing bars	D32×23
	Hoop ties	D25ctc150mm
Foundation	Section of footing	9.5m×12.0m
	Pile	1.5m in diameter×9

standards, hence the choice of the three single-column RC bridge piers (piers A, B, and C with capacity ratios of 1.18, 1.32, and 1.84,) respectively shown in specifications[13]. All were designed as part of the rehabilitation of road bridges damaged by the Hyogoken-Nanbu Earthquake of 1996 and are capable of being represented by a one-dimensional model. In tables 2 (a) to (c), sectional areas and other particulars of the bridge piers are shown. Tables 3 (a) and (b) show the characteristics of the concrete and reinforcing bars. For the analysis, ground types were selected from a geological map[14] of the Tohoku Shinkansen (Bullet Train line). Four of each ground type classification I, II, and III [15] were selected for seismic design; these classifications are defined in Table 4. This was to avoid bias in ground characteristic values T_G calculated using equation (16). Table 5 shows the characteristic values and other details of the selected ground types.

$$T_G = 4\sum_{i=1}^{n} \frac{H_i}{V_{si}} \tag{16}$$

Table 3 Material Characteristics

(a) Characteristics of Concrete

Compressive strength (kgf/cm^2)	Tensile strength (kgf/cm^2)	Strain under maximum compressive stress	Ultimate strain
240	32	0.002	0.0035

(b) Characteristics of Reinforcing Bars

Yield strength (kgf/cm^2)	Tensile strength (kgf/cm^2)	Yield strain	Strain at initiation of strain hardening	Ultimate strain
3500	5000	0.002	0.02	0.1

Table 4 Ground Type Classification for Seismic Design

Type of Ground	Type Ⅰ	Type Ⅱ	Type Ⅲ
Characteristic value of ground (unit: second)	$T_G < 0.20$	$0.20 \leq T_G \leq 0.6$	$0.6 \leq T_G$

Table 5 Ground Models

Ground model	Type of ground	Characteristic value of ground (second)	N value (weighted)
Ⅰ-1	Type Ⅰ	0.084	34.96
Ⅰ-2	Type Ⅰ	0.122	29.14
Ⅰ-3	Type Ⅰ	0.146	13.42
Ⅰ-4	Type Ⅰ	0.196	15.02
Ⅱ-1	Type Ⅱ	0.246	9.67
Ⅱ-2	Type Ⅱ	0.378	6.63
Ⅱ-3	Type Ⅱ	0.474	18.83
Ⅱ-4	Type Ⅱ	0.541	6.99
Ⅲ-1	Type Ⅲ	0.606	2.46
Ⅲ-2	Type Ⅲ	0.710	4.87
Ⅲ-3	Type Ⅲ	0.817	7.10
Ⅲ-4	Type Ⅲ	0.888	6.36

where,

H_i : thickness (m) of the i th stratum
V_{si} : average velocity of the shear elastic wave in the i th stratum (m/s)
n : total number of strata from ground surface to the top of the bedrock

Although physical characteristics of a particular ground ,such as the velocity of shear elastic waves, can be obtained by density logging using mechanical boreholes etc., the geological map referred to above did not indicate such physical values and so they were estimated on the basis of N values, etc. [16]. The ground characteristic value T_G represents the basic natural period of the subsurface ground in the amplitude range of small strains.

3.4 Input Seismic Waves

In this study, the seismic waveform observed at the Kaihoku Bridge during Miyagiken-Oki earthquake (observation at the bedrock surface; maximum acceleration = 293gal) was used as the seismic input. The waveform was input into the bedrock of the ground being analyzed and the response at the bottom of the footing was estimated using multiple reflection theory. Because the same precast pile foundation was used for the different ground conditions in this study, it was not possible to take the decline of effective input motion into consideration. Therefore, the seismic waveform at the bottom of the footing was used directly when analyzing dynamic responses. The model set up by Kitazawa et al. [17] was used as the nonlinear ground model when implementing multiple reflection theory. This is an average model which considers ground non-linearities with reference to a number of studies.

3.5 Capacities and Ductility of RC Bridge Pier

The flexural capacity part of the capacity term was assumed to be the flexure observed when the compressive strain at the concrete extremities reaches the ultimate strain $\varepsilon_{cu}(=0.0035)$, and was calculated by static elasto-plastic analysis. The following equations were used in computing the shear capacities[18]:
Shear capacity without hoop ties:

When $1.5 \leq a/d \leq 2.5$,

$$V_c = 3.58 \cdot (a/d)^{-1.166} \cdot f_c'^{1/3} \cdot \beta_p \cdot \beta_d \cdot \beta_n \cdot b \cdot d \quad (17)$$

When $2.5 < a/d$,

$$V_c = 0.94 \cdot (0.75 + 1.4d/a) \cdot f_c'^{1/3} \cdot \beta_p \cdot \beta_d \cdot \beta_n \cdot b \cdot d \quad (18)$$

Shear capacity contributed by hoop ties:

$$V_s = A_w \cdot \sigma_{sy} \cdot d \cdot (\sin\theta + \cos\theta)/1.15s \quad (19)$$

where,

$\beta_p = \sqrt[4]{100p_l}, \beta_d = \sqrt[4]{100/d}$
$\beta_n = 1 + 2M_0/M_u$
f': compressive strength of concrete (kgf/cm^2)
p_l: tensile reinforcement ratio
M_u: ultimate bending moment
M_0: critical bending moment producing tensile stress in the section of a member
A_w: sectional area of set of hoop ties in space s
σ_{sy}: strength at yield point of hoop tie (kgf/cm^2)
θ: angle made by a hoop tie to the axis of the member
a: shear span (cm)
b: width of section (cm)
d: effective height (cm)
s: interval between hoop ties

There have been a number of studies in which the primary attention was on the plastic deformation or ductility of RC structures, and a certain number of techniques for evaluating ductility (ductility factor) under cyclic horizontal loading tests have been proposed. This study adopts the following evaluation formula from a WG report of the Special Committee for Investigations and Study of the Hyogoken-Nanbu Earthquake of 1996, which was derived through the extensive research and consolidation of earlier studies [2]:

$$\mu = \frac{N}{N_B} + \left(1 - \frac{N}{N_B}\right)\left\{12\left(\frac{0.5V_c + V_s}{M_u / a}\right) - 3\right\} \tag{20}$$

where,

N : axial compressive force

N_B : axial compressive force when equilibrium breaks down (defined as 'the axial force which causes ultimate compressive strain at the concrete extremities while also causing reinforcing bars at the position of applying the resultant tensile force to reach their yield strength.')

M_u : flexural capacity

V_c : shear capacity without hoop ties, as computed using equation (17) or (18)

V_s : shear capacity contributed by hoop ties, as computed using equation (19)

a : shear span

3.6 Setting Up the Limit State Equation

Based on the capacity term and external force term described above, equations (21) and (22) were set up as the limit state equations to be used in assessing safety with respect to flexural capacity and shear capacity, respectively. For the flexural limit state equations, the secondary moment resulting from displacement was also taken into account. Equation (23) was set up as the limit state equation for ductility.

$$g_1 = \alpha_1 M_u - (P_{max} \cdot a + N \cdot \delta_{max}) P_{max} \tag{21}$$

$$g_2 = \alpha_2 (V_c + V_s) - P_{max} \tag{22}$$

$$g_3 = \alpha_3 \left[\frac{N}{N_B} + \left(1 - \frac{N}{N_B}\right)\left\{12\left(\frac{0.5V_c + V_s}{M_u / a}\right) - 3\right\}\right] - \frac{\delta_{max}}{\delta_y} \tag{23}$$

where,

P_{max}, δ_{max} : maximum values of active inertial force and response displacement obtained from dynamic analysis

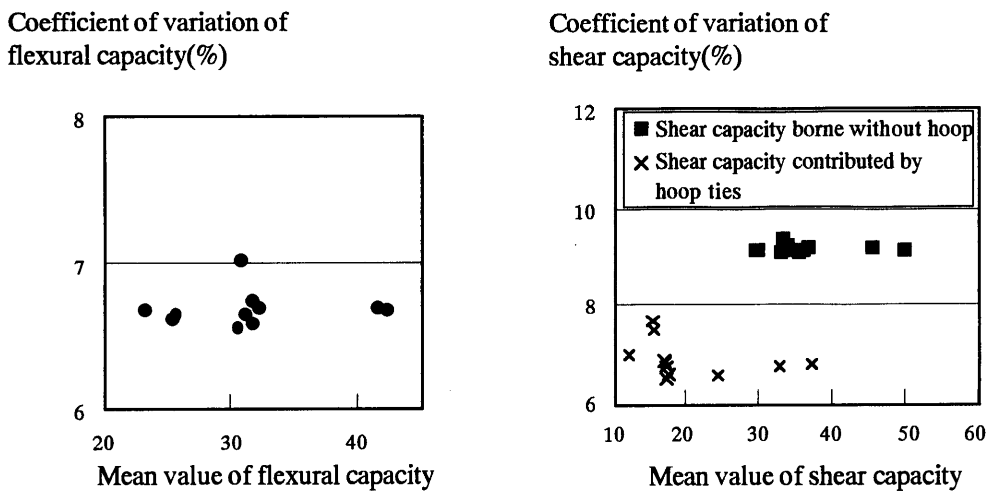
α_1, α_2 : probability variable to deal with variations in the equations for calculating capacities

α_3 : probability variable to deal with variations in the equation for calculating the ductility factor

Since the capacities and external forces included in the above limit state equations contain various uncertainties, the variables $\alpha_i (i = 1 \sim 3)$ were introduced in an attempt to take account of their influence. α_i are also regarded as probability variables.

To calculate the failure probability of the RC bridge pier, it is very important to evaluate the forms of the probability distributions and their variation for these probability variables.

In Section 4, these variables are first evaluated, and then the seismic safety of the RC bridge piers is evaluated using limit state equations (21) to (23).



(a) Effect on Flexural Capacity (b) Effect on Shear Capacity

Fig.5 Effect on Variance in Material Strengths on Capacity

4. SEISMIC SAFETY EVALUATION OF RC BRIDGE PIERS BY THE RELIABILITY EVALUATION METHOD FOR STRUCTURAL SYSTEMS

4.1 Effect of Uncertainties on Capacities

Prior to carrying out the safety evaluation, it is necessary to set coefficients of variation and distributions fot the probability variables. In this section, evaluation of the uncertainties in material strengths contained in the capacity terms and evaluation of uncertainties in the modeling and equations for calculating capacities will be described.

a) Effect of Material Strength Uncertainty on Capacity

The compressive strength of the concrete and the yield strength of the reinforcing bars were adapted as uncertainties affecting material strength. On the basis of survey results, the upper limits of the coefficients of variation ,which represent the degree of uncertainty, were assumed to be 20% for concrete compressive strength and 7% for reinforcing bars yield strength. The variability in material strength was assumed to be normally distributed. To begin with, the effect of variations in material strength on flexural capacity was evaluated. Since flexural capacity is calculated for the moment at which the compressive strain of the concrete extremities reaches its ultimate value, as mentioned above, the computation of capacity is not formulated in positive form. Therefore, a Monte Carlo simulation was used to evaluate the uncertainty in material strength. The procedure for this is outlined below.

1) Selection of specimens
2) Selection of the number of samples and the coefficients of variation of the material strength variables
3) Calculation of material strengths in accordance with the selected probability distribution forms and their characteristic values
4) Calculation of flexural capacities
5) Repetition of steps 1) to 3) for the selected number of samples
6) Calculation of mean values and coefficients of variation from the set of the flexural capacities obtained

A certain number of specimens simulating actual bridge piers were selected and analyzed according to the above procedure. The results are shown in Fig. 5 (a). In this graph, the horizontal axis represents mean values of the flexural capacity (converted into shear force) for the selected specimens. Although each specimen had its own value for the coefficient of variation, in this study, one of flexural capacities were 8% or less without exception. Accordingly, when analyzing the reliability of the RC bridge piers, the calculated flexural capacity was used as a mean value and treated as a probability

Table 6 Form of Distribution for Probability Variables and Parameters of Distributions

Symbol in limit state equation	Form of probability distribution	Parameters of probability distribution	
		Mean value	Coeficient of variation
α_1	Normal Distribution	1.0	10
α_2	Normal Distribution	1.0	20
α_3	Normal Distribution	1.0	40
N	Normal Distribution	Design value	5
δ_v	Normal Distribution	Calculated value	10
P_{max}	Normal Distribution	Result of response	30
δ_{max}	Normal Distribution	Result of response	30

variable having a coefficient of variation of 8%.

Next, the effect of variations in material strength an shear capacity was calculated. As the shear capacity was established as in equations (17) to (19), the respective coefficients of variation δ were computed using the equation below. In the equations to calculate shear capacities without hoop ties, parameters related to bending moment were used. These were analyzed by taking them as probability variables, based on the result obtained by the Monte Carlo simulation.

$$\delta = \frac{\sigma_V}{\mu_V}$$

$$= \frac{\sqrt{\sum_{i=1}^{j}\left[\left(\frac{\partial V}{\partial X_i}\right)\Big|_{\overline{Xi}}\right]^2 \cdot \sigma_{\overline{Xi}}^2}}{V\left(\overline{X}_1,\cdots\overline{X}_j\right)+\sum_{i=1}^{j}\left(\frac{\partial V}{\partial X_i}\right)\Big|_{\overline{Xi}}\left(X_i-\overline{X}_i\right)} \tag{24}$$

where,

X_i : probability variables related to material strength and bending moment

$\sigma_{\overline{Xi}}$: standard deviations of probability variables

$V(\cdot)$: equation to calculate shear capacities

Figure. 5 (b) shows the results of the analysis of coefficients of variation of each equation for shear capacity. From this graph, it was decided to treat shear capacities without hoop ties as probability variables having an 8% coefficient of variation and shear capacities contributed by hoop ties as ones with a 10% coefficient of variation, the calculated shear capacities being taken as mean values.

b) Effect on Capacities of Uncertainty in the Equation and of Uncertainty in the Structural Analysis

The equations for calculating shear capacity without hoop ties and the equations for evaluating ductility were based on experimental results. Moreover, flexural capacity obtained by analyzing static elasto-plasticity was applied to RC members undergoing cyclic loading. As a result, calculated capacity values were influenced by uncertainty in the equations themselves as well as by variations in material strengths. In this study, the influence of such uncertainties is taken into consideration by treating coefficients $\alpha_i (i = 1 \sim 3)$ in the limit state equation as probability variables.

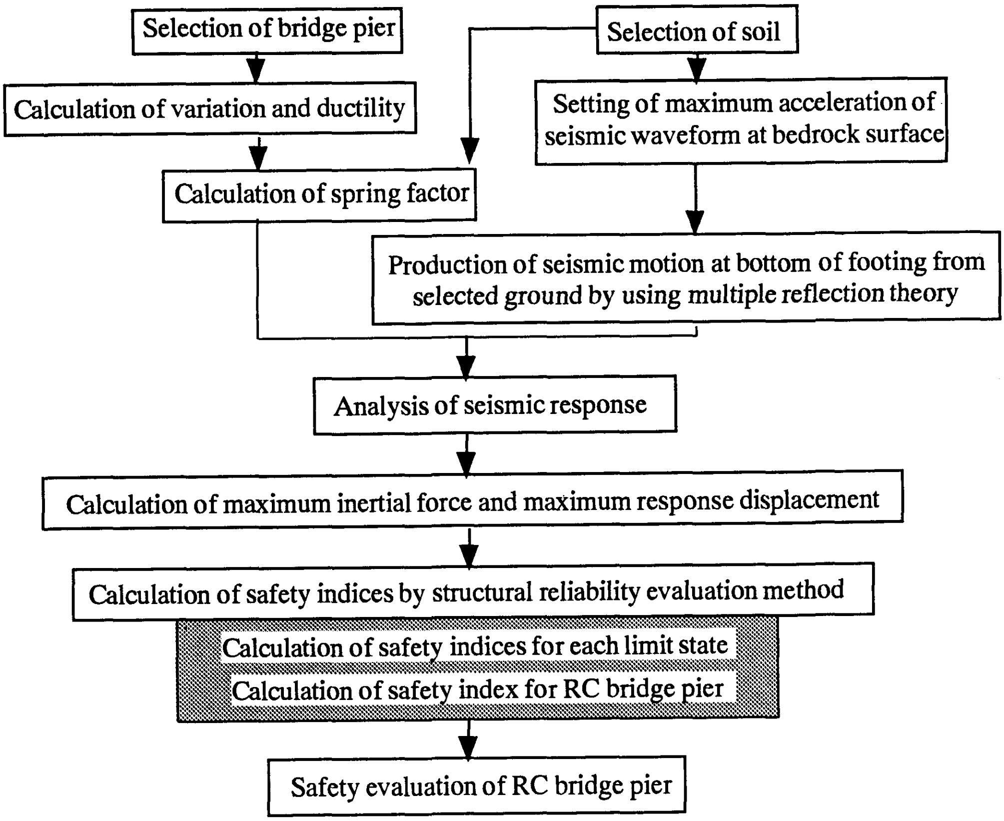


Fig.6 Flow Chart of Bridge Pier Safety Examination

Even when the active inertial force and response displacement due to an earthquake are set at their maximum values, it is not possible to eliminate uncertainties when modeling a structure. Therefore, these should also be taken as probability variables as a matter of course when carrying out reliability analysis. Nevertheless, a satisfactory database for evaluating the influence of such uncertainties, other than those in material strength, has not yet been established. Accordingly, for coefficients of variation other than those representing material strength uncertainties, the assumed probability distributions and parameters listed in Table 6 were used in the safety evaluation of the RC bridge piers as described below, unless otherwise specified. Correlations among the respective probability variables were not taken into account. The coefficients of variation in maximum active inertial force and response displacement shown here, which are the result of earthquake response, simply reflect the uncertainties found in developing the models. In other words, the present study excludes analysis of the degree of seismic risk and so the values do not consider, for example, the scale of earthquakes likely to occur during the structures' s lifetime. We note, therefore, that the results obtained in this study apply only when the seismic waves selected act on the RC piers, and so are in that sense 'conditional'.

4.2 Seismic Safety Evaluation of RC Bridge Piers

Following the procedure given in the safety study flow chart of Fig. 6, a safety evaluation of the RC bridge piers in the event of an earthquake was carried out. Prior to analysis, the seismic waveforms were amplified or attenuated such that the maximum accelerations would be equal at the soil. This was because the bridge piers, foundations, and bedrock to be analyzed were modeled as an integral system, and we aimed to evaluate safety for an event in which identical seismic motion acted on the design bedrock.

(a) Safety Evaluation of RC Bridge Piers Used in the Analysis

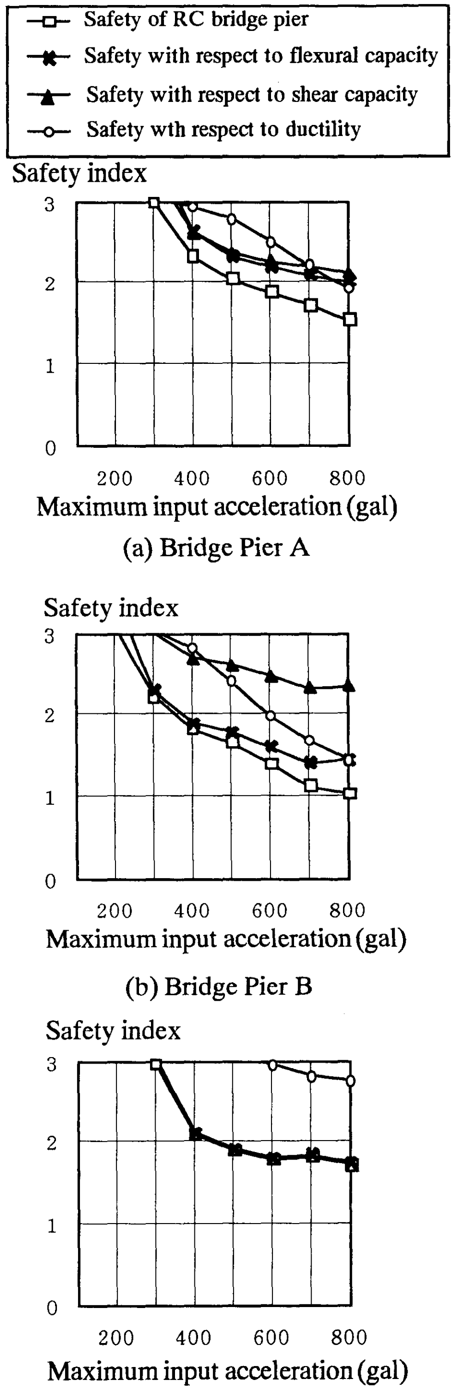
Ground model No. I-1 from Table 5 was selected, and the seismic waveforms were amplified or attenuated to give maximum input accelerations at the bedrock of between 100gal and 800gal. These were input, in 100gal increments, into the models shown in Fig.4. The relationships between maximum acceleration and safety index for bridge piers A, B, and C are shown respectively in Figs. 7 (a), (b), and (c). These graphs individual safety indices obtained by assessing safety with respect to flexural capacity, shear capacities, and ductility, as well as safety indices for the RC bridge piers as calculated by the reliability evaluation method of structural systems proposed in section 2. (Henceforth, the index given by the proposed evaluation method is referred to as the 'RC bridge pier safety index'.)

Figure. 7 (a) shows the results for bridge pier A. Compared to the safety indices of the three limit states, the safety index of the pier is lower for all maximum input accelerations. This means that no dominant limit state exists in this case; pier safety can be evaluated properly only by considering on three limit states at the same time. However, in the case of bridge pier B, shown in Fig. 7 (b), the difference between the safety index with respect to flexural capacity and RC bridge pier index is smaller, while for bridge pier C in Fig. 7 (c), the two are almost the same. Hence, the safety of this final RC bridge pier can be considered as well approximated by flexural capacity. Thus, even when the shear capacity exceeds flexural capacity due to variability in the equations used to calculate capacity, it may not in some cases be possible to ignore an assessment of safety with respect to shear capacity. We believe that the issue of whether safety can be examined for a single limit state, or whether the correlation among multiple limit states needs to be taken into account in the assessment of safety, depends on the capacity ratio of the structure.

(b) Effect of Capacity Ratio on RC Bridge Pier Safety

In this section, the effect of the capacity ratio on RC bridge pier safety is discussed by changing the number of axial reinforcing bars and hoop ties in the RC bridges.

First, each RC bridge pier was modified by increasing the number of axial reinforcing bars in steps, each time by 20% of the original number in each of the cited design examples, while keeping the number of hoop ties unchanged. This increases the flexural capacity and decreases the capacity ratio. Next, the number of hoop ties was raised, each time by 25%, while keeping the number of axial reinforcing bars unchanged, thus increasing the shear capacity and raising the capacity ratio. Ground model No.I-1 was



(c) Bridge Pier C

Fig.7 Relation between Maximum Input Acceleration and Safety Index selected from Table 5, and the relationship between capacity ratio and the RC bridge pier safety index was investigated when seismic waves amplified to 800gal were input into the bedrock. The results of the analysis for bridge piers A, B, and C are shown in Figs. 8 (a), (b), and (c), respectively. Referring to Figs. 8 (a) and (b), when the capacity ratio is raised by increasing the shear capacity (above 1.18 for pier A in Fig. 8 (a) and above 1.32 for pier B in Fig. 8 (b)), the safety index also rises. It can be concluded from this analytical result that for capacity ratios above about 1.7, however, the value of bridge pier safety converges at the level of safety with respect to flexural capacity, and so a further increase in shear capacity does nothing more than impart excessive capacity. Furthermore, if the capacity ratio is decreased by increasing the flexural capacity (below 1.18 for pier A in Fig. 8 (a) and below 1.32 for pier B in Fig. 8 (b)), RC bridge pier safety decrease in all cases, due to rising active shear force during an earthquake and a decrease in ductility of the RC pier.

Since the capacity ratio of bridge pier C is as high as 1.84 even before the number of reinforcing bars is changed, increasing the shear capacity while keeping the flexural capacity fixed has no effect on safety (as represented by the capacity ratio range above 1.84 in Fig. 8 (c)). If the flexural capacity is raised in order to reduce the capacity ratio, as with bridge piers A and B, a tendency for safety to begin fulling when the ratio reaches about 1.7 is seen.

In this analysis, increasing the flexural capacity results in falling pier safety. However, it is demonstrated that increasing the shear capacity while keeping the capacity ratio at a predetermined value allows for effective raising of RC bridge pier safety. We next turned our attention to bridge pier B and developed piers for which the number of axial reinforcing bars was 1.0-fold, 1.2- fold, 1.6-fold, and 2.0-fold greater than in the cited sample. In these piers with varying amounts of axial reinforcement, the number of hoop ties was increased by 25% so as to raise the shear capacity. Ground I-1 in Table 5 was selected. Figure. 9 shows the relationship between safety index and capacity ratio, when seismic motion amplified to 800gal was input into the bedrock. This graph shows only the safety indices obtained using this safety evaluation method for structural systems.

It can be seen from Fig. 9 that an increase in flexural capacity elevates the RC bridge pier safety if the shear capacity is also increased simultaneously. Nevertheless, in this case too, for all combinations of reinforcing bars, safety changes little once the capacity ratio exceeds 1.7 or so. This is thought to be due to the fact that RC bridge pier safety is governed by the assessment of safety with respect to flexural

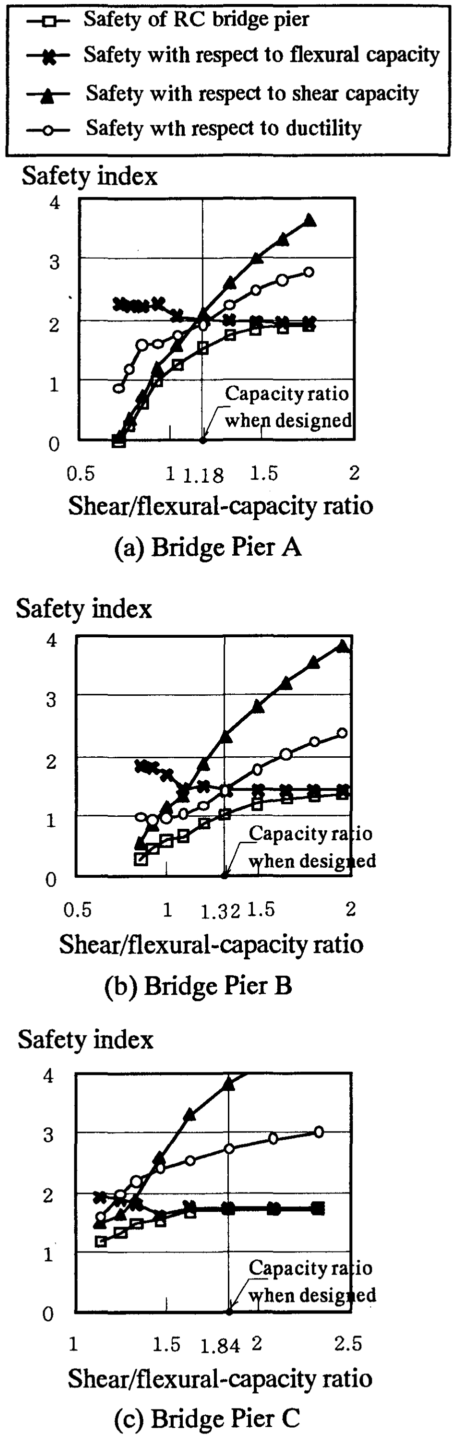
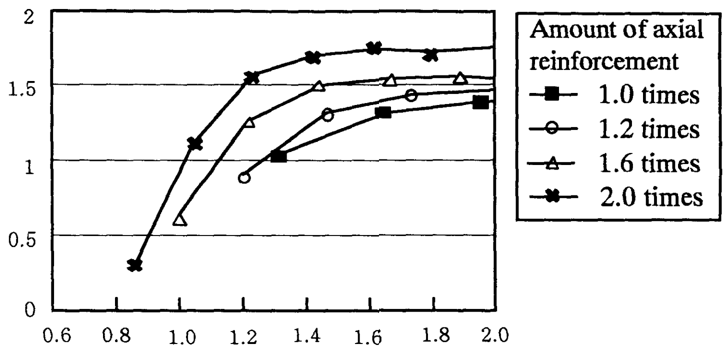


Fig.8 Relation between Capacity Ratio and Safety Index

Safety index



Shear/flexural-capacity ratio

Fig.9 Relation between Capacity Ratio and Safety Index for Different Combinations of Main Reinforcement

capacity because, although not shown in the graph, this value becomes almost constant at a point close to this capacity ratio despite increasing shear capacity. Accordingly, the safety index of RC bridge piers converges to the safety index with respect to flexural capacity (and becomes almost constant) when the capacity ratio exceeds a certain value. Consequently, it may be said that adjusting the amount of reinforcement so as to maintain a the capacity ratio of approximately 1.7 is an appropriate way to enhance bridge pier safety. Judging from this analysis, capacity ratio can be considered a useful index of the earthquake resistance of RC bridge piers.

The value of capacity ratio indicated here depends on the way in which we set up the limit states and on the parameters of the probability variables listed in Table 6. The emphasis in this study is not on the actual value of the capacity ratio, but rather on the fact that capacity and ductility are adopted as two items assessed in considering the limit state of RC bridge piers in an earthquake, and that RC bridge pier safety can be evaluated quantitatively by introducing a reliability evaluation method based on a common yardstick, i.e. the safety index. This allows consideration of whether a structure reaches the target safety level, whether safety must be assessed by considering a multiplicity of limit states, and whether excessive capacity is incorporated, etc.

(c) Influence of Type of Ground on Safety of RC Bridge Piers

In this section, in order to grasp the influence of ground characteristics on the safety of RC bridge piers, analysis is carried out by changing only the ground model. Bridge piers A, B, and C were each combined with all 12 types of ground mentioned earlier, and seismic motion amplified such that the maximum acceleration was equal to 500gal and 800gal was input into the bedrock. For each pier, safety was evaluated for each ground model. The results are shown in Figs. 10 (a), (b), and (c), respectively. In these graphs, the horizontal axis represents the characteristic values of the ground while the vertical axis shows the safety index, meaning the safety of the RC bridge pier as calculated by the proposed reliability evaluation method for structural systems.

It can be seen that bridge pier safety falls almost linearly as the ground characteristic value rises. Therefore, it is to some extent reasonable to categorize ground on the basis of its characteristic value, as is done in current seismic design. However, for type I ground, none of the bridge pier safeties varies much with ground characteristic value, whereas in the case of type II and III grounds, safety varies substantially depending on characteristic value, even for the same type of ground. This means that the safety of identical structures in an earthquake will differ even if, for example, grounds in the same category have different characteristic values. This study models ground for which spread foundations would conventionally be used for the case of pile foundations, causing a lowering of the response. Hence it is expected that, for type I ground, differences due to ground model are less likely to appear. Anyhow, a more detailed ground classification is required in order that designs which maintain a uniform level of structural safety on different ground can be developed. Besides, Fig.10 shows that the extent of the decline in bridge pier safety with increasing ground characteristic value differs by bridge. Therefore, in order to develop a design that ensures a predetermined level of safety against a hypothetical seismic waveform, safety will have to be discussed using a model which treats the bridge pier and ground as one. Of course when this is done, the most direct method of ensuring uniform safety despite differences in ground, etc. will be to identify the degree of safety of each structural system using safety indices as calculated by, for example, the reliability evaluation method for structural systems described in this paper.

(d) Influence of the Form of Probability Distribution on the Safety of RC Bridge Piers

In this section, we evaluate the safety of RC Bridge piers when the form of the probability distribution for each probability variable in Table 6 is changed, and we discuss the effect of such changes. The probability distributions considered include cases where all variables are assumed to be logarithmic normally distributed, where the external force term, i.e. the maximum inertia force P_{max} and maximum response displacement δ_{max}, are assumed to be of the extreme-I type (Gumbel), and where probability variables expressing the capacity term are normally distributed. Bridge pier B was selected as the subject of the analysis, and No. I-1 in Table 5 was chosen as the ground model. Seismic waves suitably amplified and attenuated to be between 100gal and 800gal in maximum input acceleration were input, at 100gal increments, into the bedrock. The result of the analysis is shown in Fig. 11.

There is that no significant difference in the response of in safety index to increasing maximum input acceleration between the case where all probability variables are assumed to be normally distributed and where the logarithmic normal distribution is assumed. This is because, though not shown in the graph, safety index values calculated from the three limit state equations for flexural capacity, shear capacity, and ductility are more or less the same over the whole range of maximum input accelerations. Accordingly, as far as the limit states used in this study are concerned, the assumption that probability variables in the limit state equations are normally distributed and the assumption that they are logarithmic normally distributed can be thought to be almost equivalent when evaluating RC bridge pier safety.

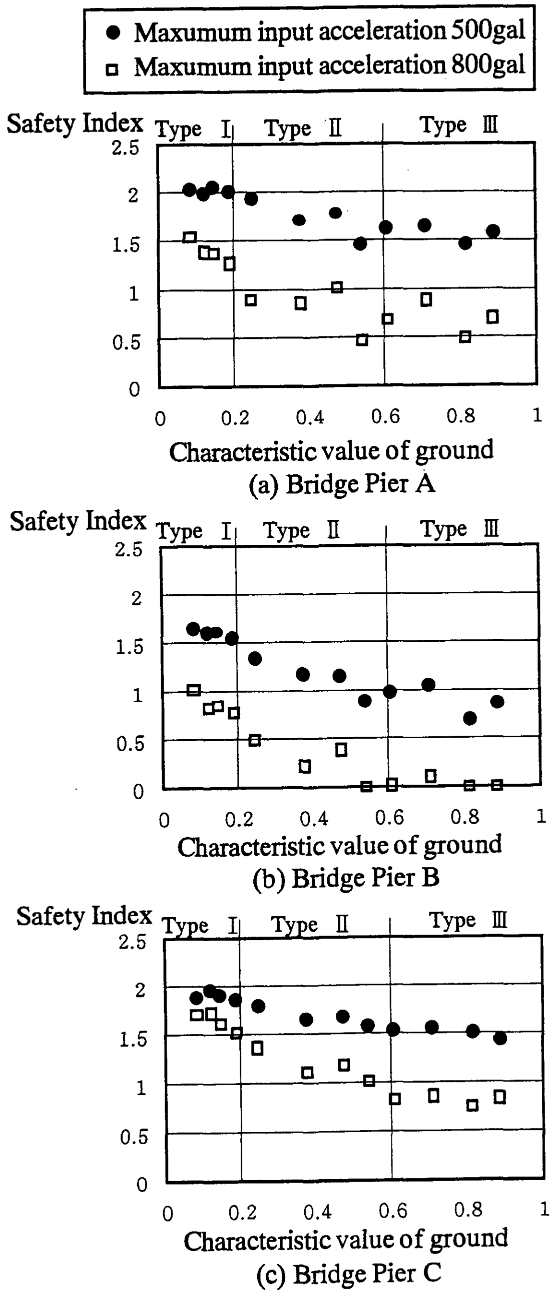


Fig.10 Relation between Ground Characteristic Value and Safety Index

When the capacity term is assumed to be normally distributed and the external force term to be of the extreme-I type, pier safety is lower than for cases where the normal distribution and logarithmic normal distribution are assumed. The lower the maximum input acceleration, the more apparent is this

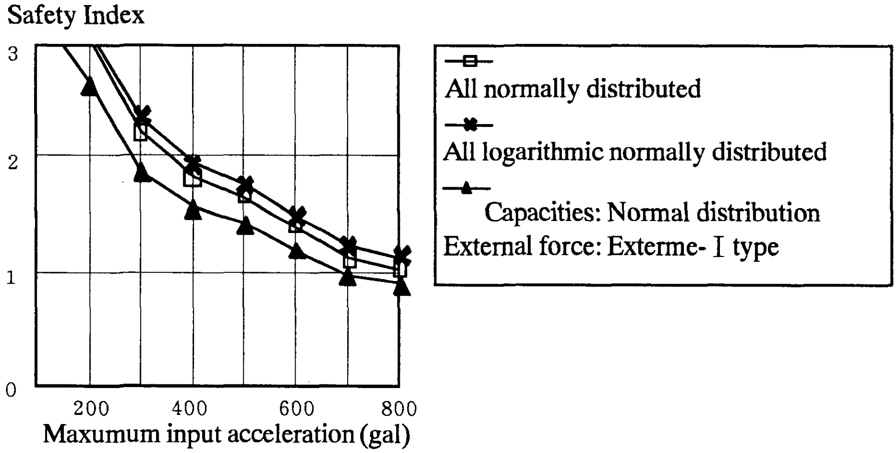


Fig.11 Influence of Form of Probability Distribution on Safety of RC Bridge Pier

difference. In the range of small maximum input accelerations, RC bridge pier safety is adequate and so the safety index is high; that is, the failure probability is held to a low enough level. Generally, in the range of low failure probability, the influence of the distribution edge shape becomes rises and it has been pointed out that the sensitivity of the failure probability to the selected distribution form is high[19]. The case in which the external force term is assumed to be of the extreme-I type is thought to correspond to this situation. In the range of high maximum input acceleration, the effect of the distribution form on RC bridge pier safety is lower. Accordingly, when the acceleration range - which is a problem of seismic design - is estimated, the influence of differences in the form of the probability distribution of the various probability variables used in this study on the safety evaluation is rather small. Hence it is considered that the analytical result mentioned above, obtained under the assumption that all are normally distributed, is generally applicable.

(e) Influence of Uncertainty in the Model and the External Force Term on RC Bridge Pier Safety

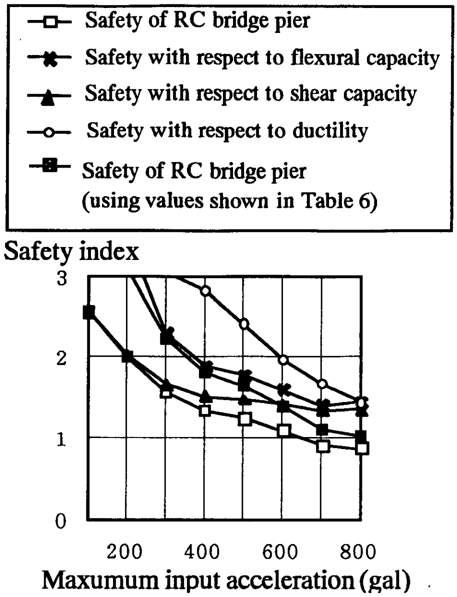
In this section, of the probability variables given in Table 6, the certain coefficients are changed and the effect of such changes an RC bridge pier safety is discussed; those coefficients varied are probability variables α_2 and α_3, to represent variations in the equations for calculating shear capacity and evaluating ductility factor, respectively, and the coefficients of variation of maximum inertial force P_{max} and response displacement δ_{max}. Regarding the probability variable α_1, which represents variations in the equation for calculating flexural capacity, we concluded that its uncertainty had already been taken into account through the values shown in Table 6, and so the coefficient of variation was not changed. All the probability variables were assumed to be normally distributed.

In order to find the relationship between maximum incident acceleration and safety index for the selected probability variables and coefficients of variation, ground No.I-1 in Table 5 was selected as the ground model and seismic waves amplified or attenuated to give maximum input accelerations of between 100gal and 800gal were input, at 100gal increments, into the bedrock. Bridge pier B was used for this analysis. Figure. 12 (a) shows the relationship between maximum input acceleration and safety index when coefficient of variance α_2 was changed from the value of 20% set in Table 6 to 40%. Similarly, the relationship between maximum input acceleration and safety index when coefficient of variation α_3 was changed from the 40% of Table 6 to 50%, and when the coefficients of variation of the maximum inertial force and maximum response displacement were changed from 30% to 40%, are shown respectively in Figs. 12 (b) and (c). Figures. 12 also indicate the safety indices of the RC bridge pier in the case where the coefficients of variation are as set in Table 6.

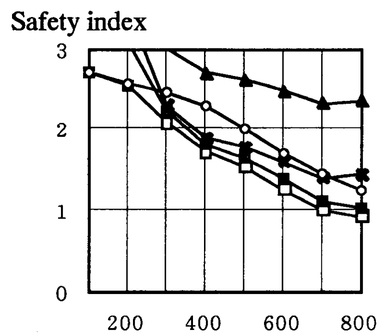
In Figure 12 (a), when the coefficient of variation of α_2 is increased, that is, when the calculated shear capacity is assumed to vary considerably, the examination with respect to safety against shear capacity is nesessary. This results in a decline in RC bridge pier safety. Clearly, though, setting the coefficient of variation representing errors in the shear capacity equation to 40% is excessive. It should be noted, however, that even for bridge pier B which has a capacity ratio as high as 1.32, depending on fluctuations in the shear capacity equation, the examination of safety with respect to shear capacity is necessary. Accordingly, for bridge piers designed to suffer flexural failure, and which have lower capacity ratios, if fluctuations in the shear capacity equation exceed a certain degree, a substantial effect on pier safety can be expected.

Figure. 12 (b) shows the case where the coefficient of variation expressing errors in the ductility equation is raised from 40% to 50%. It can be seen that safety evaluation of bridge pier B is little affected by the accuracy of this equation for calculating ductility. This is because the safety of bridge pier B can be approximated by means of the safety index already calculated by assessing safety with respect to flexural capacity. As a result, Figs. 12 show that when a bridge pier is designed, full consideration should be given only to the limit states which have the greatest effect on bridge pier safety.

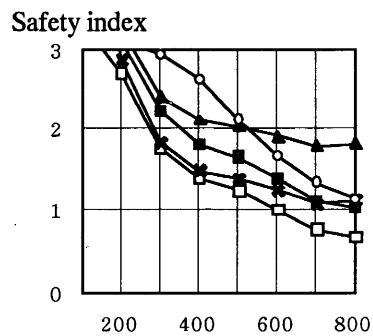
Figure. 12 (c) shows the results of analyzing changes in the coefficients of variation of the maximum inertial force and maximum response displacement. Here, unlike the case of changing the coefficient of variation of the capacity equation, or even compared to Fig. 7 (b) which shows analytical results with the coefficient of variation set at 20%, the relationship between the safety indices calculated from the three limit state equations remains unchanged. Also, the proportion by which the safety index decreases as the maximum input acceleration rises is fixed and is independent of the coefficient of variation. Therefore, if the accuracy of structural analysis indicating the behavior of a bridge pier in an earthquake can be improved, small coefficients of variation can be assumed for the probability variables and this leads to greater bridge pier safety. Thus, it can be concluded that design for a particular targeted safety index can be accomplished more economically.



(a) Coefficient of variation of α_2 set at 40%



(b) Coefficient of variation of α_3 set at 50%



(c) Coefficient of variation of P_{max}, δ_{max} set at 40%

Fig.12 Effect on Safety Index of varying the Coefficient of Variation Representing Uncertainties

5. APPLICATION OF THE RC STRUCTURE SEISMIC DESIGN PROPOSED IN THIS STUDY

5.1 General

In structuring an RC bridge pier in accordance with conventional specifications, the designer has no means of knowing how to achieve a given level of safety with the particular values of safety coefficient and design stress resultant adopted. On the other hand, this analysis has shown that even a bridge pier designed for flexural failure cannot be assigned a safety index approximated on the basis of comparing flexural capacity with design bending moment. Rather, the safety of the pier can be properly evaluated only when safety with respect to shear capacity and ductility are also examined at the same time. In this section, based on the knowledge derived from the above evaluation of the seismic safety of RC bridge piers, we discuss a seismic design method for RC bridge piers which allows the designer to attain the safety level he or she wishes.

5.2 Seismic Design Method of RC Structures on the Basis of the Reliability Evaluation Method for Structural Systems

Using the reliability evaluation method of structural systems as a basis, the most direct method for calculating the capacities and amounts of reinforcement required to attain a given safety index target, is to calculate the safety indices of bridge piers with gradually incresing amounts of reinforcement until, eventually, the target is reached. However, to formulate the problem without resorting to such a trial-and-error method is a very difficult task, for it requires that individual design points be found for multiple limit states, and that such design points be aligned so as to attain the target safety indices without contradictions among the failure events as a whole.

To deal with this situation, RC bridge pier failure events, comprising multiple limit states, were resolved into a problem with a single limit state produced by an assessment of the flexural capacity. And attempt was then made to apply seismic design based on this reliability theory to RC bridge piers. In this application, the following hypotheses were made:

(i) For the capacity ratio of the RC bridge pier, a shear capacity 1.7 times the flexural capacity was assigned.

(ii) The secondary moment produced by displacement is about 10% of the design bending moment. Therefore, in order to simplify the calculations, its effect was ignored when calculating the safety index with respect to flexural capacity. The resultant limit state equation for examining safety with respect to flexural capacity and active bending moment is as follows:

$$g_1 = \alpha_1 M_u - P_{\max} \cdot a \qquad (25)$$

(iii) For all probability variables, the values shown in Table 6 were used for the coefficients of variation.

(iv) All probability variables were assumed to be normally distributed and independent of each other.

From the earlier analysis, it is clear that hypothesis (i) enables us to approximate RC bridge pier safety by using the safety index value calculated by equation (25), and that bridge pier safety for the designed flexural capacity will become optimum. With the reliability evaluation method for structural systems, a technique using the Rosenblatt transformation was proposed for calculating a safety index by the use of a single limit state equation like equation (25). With this method, however, it is not easy to find design points which yield the target safety index. To solve this problem, hypothesis (iv) is introduced. This enables us to use an iterative technique[6] according to the Advanced First-Order Method, which allows easy calculation of the design point. With this method, for each active maximum inertial force in the earthquake hypothesized in the design, the flexural capacity required to obtain a target safety index can be calculated by means of the format shown in equation (26).

$$\overline{M_u} = \gamma \cdot \overline{P_{\max}} \cdot a \qquad (26)$$

where,

$\overline{M_u}$: design flexural capacity
$\overline{P_{\max}}$: maximum inertial force obtained from analysis of earthquake response
γ : design coefficient necessary to secure target safety index

When design flexural capacity is determined by equation (26), the amount of axial reinforcement needed to obtain that flexural capacity is determined, thereby allowing calculation of the design shear capacity of reinforcement other than hoop ties. Then, the following equation, derived from hypothesis

(i) and equation (19), is used to compute the number of hoop ties, A_w, needed to secure the target safety when calculating design flexural capacity.

$$A_w = \frac{1.7\gamma \overline{P_{\max}} \cdot a - V_c}{\sigma_{sy} \cdot d \cdot (\sin\theta + \cos\theta)/(1.15s)} \tag{27}$$

Values of the design coefficient γ, which appears in equation (26), obtained for values of the target safety index β from 0 through 3 are shown in Fig. 13.

For instance, when $\beta = 1.0$ is required as the safety index of an RC bridge pier, a value about 1.2 times the maximum inertial force obtained from the response analysis for an assumed earthquake should be set as the flexural capacity, and this will be sufficient if the number of hoop ties obtained from equation (27).

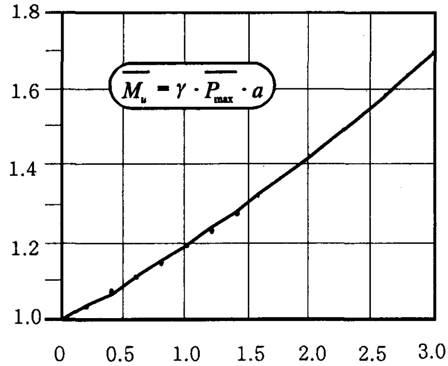


Fig.13 Design Coefficient which Ensures Target Safety Index

Past analysis demonstrates that a safety index expressing RC bridge pier safety as calculated by simultaneously considering multiple limit states to reflect different degrees of uncertainty in each equation for calculating capacity and the possibility of brittle failure in RC bridge piers having low capacity ratios is smaller than the safety index evaluated by simply comparing shear capacity with active bending moment. Therefore, even if the flexural capacity obtained from equation (26) is realized in an RC bridge pier, if the shear capacity is only slightly above the shear force at the time when this flexural capacity is reached, the pier will not have the target safety index considered in equation (26), and the design will be on the risky side. Then, by providing the number of hoop ties which satisfies equation (27), the safety of the pier can be considered by comparing flexural capacity with active bending moment. This will finally allow one to say that the target safety index has been attained in the design of the pier.

The Concrete Standard Specification[20] formulated in 1996 stipulates that the safety of RC structures in an earthquake should be verified by determining that they will not be subject to shear failure and that the deformation of each member remains below its ductility. More concretely, since capacity with respect to the design load is evaluated prior to analysis, safety may be examined only by comparing deformation with ductility, and usually the capacity ratio is made greater than 2. This is because experiments (cyclic horizontal loading tests) have confirmed that a capacity ratio over 2 ensures stable flexural failure and that the ductility of members reaches about 10. On the other hand, this study shows, on the basis of the coefficients of variation assumed in table 6, that bridge pier safety must be judged by safety with respect to flexural capacity when a capacity ratio of 1.7 is given to the bridge pier. Accordingly, the safety evaluation carried out here supports the safety examination method described in the specification. However, as shown by the above analytical result, it is impossible to know what level of safety has actually been secured for the structure, since the bridge pier safety index varies depending on both flexural and shear capacity even if the capacity ratio remains constant and because it is unable to evaluate safety by simply comparing the stress resultant with the capacities and the amount of response displacement with ductility. There are undoubtedly still a number of problems to be solved before reliability-based designs such as the one described in this study can be fully applied to actual design systems. Nevertheless, the introduction of the design method as shown in equations (25) and (26) may solve some of the problems facing the current design system. This will lead to designs that allow clear recognition of the level of safety the designer wants to secure, and we believe the road toward a new design system in which uniform safety, unaffected by design conditions, can be ensured will have been opened up.

6. CONCLUSIONS

The first stage of this study was to establish a method of calculating the probability that a structural system reaches a limit state by considering the multiple limit states with a potential to occur. Next, the assessment of capacities and ductility as limit states of an RC bridge pier in an earthquake was taken up, and the influence of the capacity ratio of the pier and the ground type on its safety pier etc. was studied. Based on the results obtained, the application of the method to actual seismic design was considered.

The main conclusions reached in this study are:
(1) A method for calculating the failure probability of a structural system was established for in a situation where a failure event comprises involves multiple limit states correlated with each other. The proposed technique substantially facilitates calculation; its superiority, both in time and accuracy, over calculation methods in previous studies was confirmed.
(2) The safety of an RC Bridge pier in an earthquake was evaluated using the reliability evaluation method for structural systems proposed in this study. The results showed that when the capacity ratios assigned to the RC bridge piers are in the low range, the evaluation errs on the side of risk if shear force and ductility are not taken into consideration, even in a case of a pier designed for flexural failure.
(3) The safety of an RC bridge pier was analyzed with different capacity ratios by changing the number of main reinforcing bars. The results showed that when the shear capacity is about 1.7 times the flexural capacity, a pier's safety can be ascertained by performing a safety assessment of the flexural capacity and active bending moment; also that for a certain flexural capacity, optimum safety is attained when this capacity ratio is assigned to the bridge pier.
(4) As a result of analysis of ground models selected by considering ground characteristic values, a quantitative understanding of the difference in safety between ground types was obtained.
(5) The influence of uncertainties (variations in the compressive strength of the concrete and in the yield strength of the reinforcing bars) included in the calculated capacities was evaluated. This influence was represented by modifying the calculated capacities by coefficients of variation, and its upper limit was evaluated.
(6) After simultaneously considering safety assessments with respect to flexural capacity, shear capacity, and ductility, a seismic design method for RC bridge piers which secures a predetermined level of safety was proposed.

Also elucidated through this work were the follouwing problems, which are thought to require further discussion in this field of study:
(1) To carry out reliability analysis for bridge piers, the external forces likely to arise during their lifetime need to be estimated by giving thought to occurrence probability and scale of possible events.
(2) Uncertainties in structural analysis and capacity equations need to be compiled into a database.
(3) Since the Hyogoken-Nanbu Earthquake of 1996, it has been emphasized that, regarding seismic design, we must aim at improving the earthquake resistance of the entire bridge system. Accordingly, safety evaluations that include the foundation and shoe, not only the bridge piers, is necessary.
(4) In this study, the analysis was applied to a seismic design method, and was carried out under the restrictive condition that optimum bridge pier safety is obtained by maintaining the capacity ratio at 1.7. In the future, however, it is required to work out, under general restrictive conditions, a method which makes it possible to establish a formula for calculating design coefficients for attaining a target safety, whilst additionally considering economic efficiency, workability, and the importance of the structure, after taking multiple limit states into account.
(5) It is necessary to carry out elaborate calibration before selecting a new standard as the yardstick for safety. This is necessary to ensure that the safety level matches that of existing structures whose reliability has been guaranteed, and to clarify the standard for safety targets.

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