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PROPOSAL OF A NEW METHOD FOR DETERMINING THE RADIUS DISTRIBUTION OF AIR VOIDS AND AIR CONTENT IN HARDENED CONCRETE ON THE BASIS OF PROBABILITY THEORY

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The air-void component has a dominant effect on the frost resistance of concrete. However, a method for determining the radius distributions of air voids has not been determined. The present paper proposes a new method forwards this purpose. Its principle is based on the relationship between the radius distribution of spheres and that of their cut sections. The procedure consists of the measurement of the cut sections' diameters, and statistical analysis. The validity of the method has been verified by numerical calculations and experiments. The number of air void sections to be observed and the area to be examined are also suggested.

Key words : hardened concrete, measurement method, radius distribution of air voids, air content, air-void parameters, probability theory

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1. INTRODUCTION

Revealing details of the air-void component of hardened concrete is essential in researching its frost resistance. However, the method for determining its basic characteristics, such as radius distribution and number of air voids, has not been determined. For instance, the linear traverse method now in use [1] only gives a hypothetical size of the air voids on condition that all the voids are isometric spheres.

This paper proposes a method for determining the actual size distribution by measuring the radii of circles observed on a sawn and ground surface of hardened concrete. The number of circles and the area of the cut section required to obtain precise results will also be discussed.

2. METHOD PRINCIPLES

When a piece of hardened concrete is sawn, large number of circles appear on the surface as the sections of air voids. These circles will be called "apparent circles". Since each of the apparent circles belongs to a distinct air void, the void is named a "source sphere". In this paper, the radius of an apparent circle is expressed by x, and that of an air void is expressed by r.

It is impossible to derive the value of r directly from x, but in the case where the air voids comprise an infinite set of spheres with radius r, the probability that the radius of an apparent circle will be x is expected to obey a definite law of probability. Naturally the circles with a radius larger than r would never appear, so when a set of source spheres is a mixture with various radii, the radius of the largest source spheres can be known from the largest radius of the apparent circles. Provided the above mentioned law of probability is known, the radius distribution of the apparent circles having originated from the source spheres with the largest radius can be calculated through the law. And besides, as one apparent circle corresponds to one source sphere, the total number of the apparent circles originating from the source spheres with radius r is equal to the number of source spheres with radius r. Thus, the number of source spheres with the largest radius and the radius distribution of apparent circles originating from them can be determined statistically.

When thus obtained radius distribution is subtracted from the originally observed distribution, the second largest radius of the original distribution becomes the largest of the residual secondary distribution. By treating it in the same way, the number of the second largest source spheres is determined. If this process is repeated, the radius distribution of the set of source spheres will entirely be revealed. It is rather easy to calculate the radius distribution of the air-void component of concrete from that of the source spheres.

3. THEORETICAL ANALYSIS

3.1 Probability density function of the radius of apparent circles in the case where all air voids are isometric spheres

Concrete contains a myriad number of various sized air voids. At first, only the voids with radius r are considered, and the radius distribution of the apparent circles stemming from them will be analysed.

Supposing that spheres with radius r exist randomly in concrete and the number of apparent circles n is sufficiently large, the distribution of their radius x may be the same as the radius distribution of n plates of thin disks which will be formed by cutting a single sphere with n sheets of parallel planes at equal intervals. Consequently, the probability of the radius of an apparent circle being x is equal to the probability of the radius of a disk picked up randomly out of those which have been formed in the previously described way being x.

Considering this concept, a sphere is singled out, and x, z axes are set as in Fig. 1. Then the relation between z and the radius of a cut section x is expressed by the following equation.

$$z = r - \sqrt{r^2 - x^2} \tag{1}$$

Let $z + dz$ correspond to $x + dx$,

$$dz = (\frac{dz}{dx})dx = \frac{x}{\sqrt{r^2 - x^2}}dx \tag{2}$$

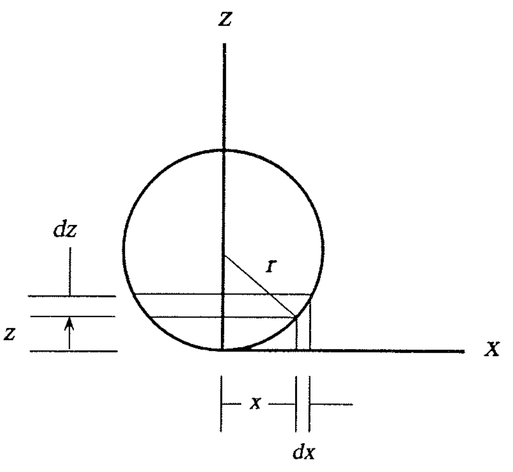

Fig. 1

Consequently, the probability P_r that the radius of a section will be $x \sim x + dx$ is

$$P_r = dz / r = g(x)dx$$

$$g(x) = \frac{x}{r\sqrt{r^2 - x^2}} \quad (0 \leq x < r) \tag{3}$$

As $g(x)$ is the probability density function of x, the cumulative distribution function $G(X)$ is given by

$$G(x) = \int_0^x g(x)dx = 1 - \frac{\sqrt{r^2 - x^2}}{r} \quad (0 \le x < r) \tag{4}$$

3.2 Relation between the radius distributions of apparent circles and air voids

a) Relation between the number of apparent circles and air voids per unit volume of concrete

A rectangular prism of area A and height H (Fig. 2) is considered as a specimen. The measurement is to be performed on the entire upper surface. Supposing the specimen contains N_{tr} particles of air voids with radius r, the number of air voids per unit volume N_{or} is given by formula (5), and the total number of the voids apparent at the surface N_{sr} is estimated by formula (6).

$$N_{or} = N_{tr} / \mathrm{AH} \tag{5}$$

$$N_{sr} = 2rN_{tr} / \mathrm{H} \tag{6}$$

By eliminating N_{tr} from formulas (5) and (6), formula (7) is obtained as

$$N_{sr} = 2r\mathrm{A}N_{or} \tag{7}$$

Denoting by n_{xr} the number of apparent circles which satisfy the conditions of both having a radius of $x \sim x+dx$ and of having originated from a source sphere with radius r, N_{xr} is given as the product of the total number of source spheres N_{xr} and the probability of radius being $x \sim x+dx$. Thus, N_{xr} is obtained as follows.

$$n_{xr} = N_{sr}P_r$$

$$= 2\mathrm{A}N_{or}\frac{x}{\sqrt{r^2 - x^2}}dx \tag{8}$$

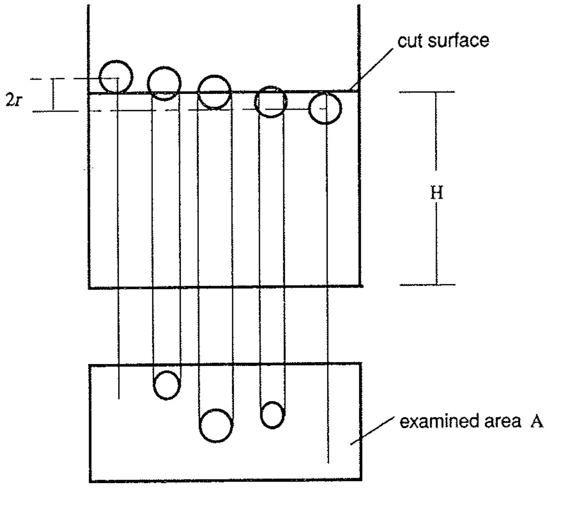

Fig. 2

b) Relation between the radius distributions of apparent circles and air voids

If the density function of the radius distribution in terms of the number of air voids per unit volume of a specimen is given by the function $f(x)$, $N_0\{r \sim r+dr\}$, the number of air voids with radius $r \sim r+dr$, will be

$$N_0\{r \sim r+dr\} = \mathrm{N}_0 f(r)dr \quad (0 \le x < r) \tag{9}$$

in which N_0 is the sum total of air voids contained in a unit volume of the specimen. By combining Eq.(8) with Eq.(9), the number of the apparent circles $n(x,r)$, which at once have a radius of $x \sim x+dx$ and have originated from source spheres with radius $r \sim r+dr$, can be obtained as follows.

$$n(x,r) = 2\mathrm{AN}_0 f(r)\frac{xdx}{\sqrt{r^2-x^2}}dr \quad (0 \le x < r, 0 < r < \infty) \tag{10}$$

Consequently, $n(x)$, the number of apparent circles with radius $x \sim x+dx$, and n, the total number of apparent circles are expressed by

$$n(x) = 2\mathrm{AN}_0\int_0^\infty \frac{xf(r)dx}{\sqrt{r^2-x^2}}dx \tag{11}$$

$$n = 2\mathrm{AN}_0\int_0^\infty\int_0^r \frac{xf(r)}{\sqrt{r^2-x^2}}dxdr$$

$$= 2\mathrm{AN}_0\int_0^\infty rf(r)dr = 2\mathrm{AN}_0\bar{r} \tag{12}$$

The symbol $\bar{r}$ in Eq.(12) is the mean value of the distribution $f(r)$, that is

$$\bar{r} = \int_0^\infty rf(r)dr \tag{13}$$

Thus, the probability P that the radius of an apparent circle will take a value between x and $x+dx$ is

$$P = \frac{n(x)}{n} = \frac{x}{\bar{r}}\int_0^\infty \frac{f(r)dx}{\sqrt{r^2-x^2}}dx \tag{14}$$

$$= p(x)dx$$

$$p(x) = \frac{x}{\bar{r}}\int_0^\infty \frac{f(r)}{\sqrt{r^2-x^2}}dx \tag{15}$$

in which, $p(x)$ is the distribution density function of the apparent circles having originated from a group of air voids with a radius distribution $f(r)$ and an average radius $\bar{r}$.

c) Relation between the total number of apparent circles and source spheres

Considering Eq.(7) and Eq.(9), the number of source spheres with radius $r \sim r+dr$ can be expressed by

$$N_s\{r \sim r+dx\} = 2r\mathrm{AN}_0 f(r)dr \tag{16}$$

Then, the total number of source spheres N_s becomes

$$N_s = 2\mathrm{AN}_0\int_0^\infty rf(r)dr = 2\mathrm{AN}_0\bar{r} \tag{17}$$

Since the right side of Eq.(17) is the same as Eq.(12),

$$N_s = n \tag{18}$$

This result verifies analytically that the total number of apparent circles is equal to that of the source spheres.

3.3 Relation between the distribution of radius frequency of apparent circles and that of air voids

a) Expression of radius by discrete variable

In practice, data on the apparent circles will be ordered by the frequency of circles with a certain range of radius. In that case, the radii x and r are expressed by

$$x = i\Delta x \tag{19}$$

$$r = j\Delta x \tag{20}$$

where Δx is the interval of the class in a histogram. When the number of the classes is m, r_m, the maximum value of r is

$$r_m = m\Delta x \tag{21}$$

b) Notation of numbers

To represent various kinds of numbers, the following symbols will be used.

- n : the total number of apparent circles found in examined area A.
- $n(i)$: frequency of the apparent circles included in class i.
- $n(j)$: number of the apparent circles originating from the source spheres in class j.
- $n(i,j)$: frequency of apparent circles which at once belong to class i and have originated from source spheres in class j.
- N_s : total number of source spheres.
- $N_s(j)$: number of source spheres in class j.
- N_0 : number of air voids per unit volume of concrete (/mm^3).
- $N_0(j)$: number of air voids per unit volume of concrete in class j.
- N : number of air voids contained in 1cm^3 of concrete (/cm^3), that is $$\mathrm{N} = 1000\mathrm{N}_0 \tag{22}$$
- $N(j)$: frequency of the air voids in class j (/cm^3), that is $$N(j) = 1000N_0(j) \tag{23}$$

c) Relation among the radius distributions of apparent circles, source spheres, and air voids

When the density function of a radius distribution for air voids is given by $f(r)$, the frequency of the air voids in class j is expressed by

$$N_0(j) = \mathrm{N}_0 \int_{(j-1)\Delta x}^{j\Delta x} f(r)dr$$

Representing the primitive function of $f(r)$ by $F(r)$, the previous equation becomes

$$N_0(j) = \mathrm{N}_0\left[F(j\Delta x) - F\{(j-1)\Delta x\}\right] \tag{24}$$

Hereafter, $F(j\Delta x)$ will be abbreviated to $F(j)$. From Eq.(7), the frequency of the source spheres in class j is

$$\begin{aligned} N_s(j) &= 2\mathrm{AN}_0 j\Delta x \\ &= 2\mathrm{AN}_0 j\Delta x\{F(j) - F(j-1)\} \end{aligned} \tag{25}$$

By analogy from Eq.(18), the total number of apparent circles originating from source spheres in class j is equal to the number of source spheres in class j. Then

$$n(j) = N_s(j)$$

By referring to Eq.(4), $n(i,j)$ becomes

$$n(i,j) = n(j)\int_{i-1}^{i} g(x)dx$$

$$= \frac{N_s(j)}{j}\left\{\sqrt{j^2-(i-1)^2}-\sqrt{j^2-i^2}\right\} \quad (26)$$

$$= 2AN_0\Delta x\left\{\sqrt{j^2-(i-1)^2}-\sqrt{j^2-i^2}\right\}\times\{F(j)-F(j-1)\} \quad (27)$$

Though the apparent circles in class i have originated from different source spheres, the radius of an apparent circle can not exceed the radius of its source sphere and a circle can not originate from a sphere whose radius is smaller than that of the circle. Thus, the number of apparent circles in class i, the total number of apparent circles, and the probability density of class i are written as follows.

$$n(i) = \sum_{j=1}^{m} n(i,j)$$

$$= 2AN_0\Delta x\sum_{j=1}^{m}\left\{\sqrt{j^2-(i-1)^2}-\sqrt{j^2-i^2}\right\}\times\{F(j)-F(j-1)\} \quad (28)$$

$$n = \sum_{i=1}^{m} n(i) = \sum_{i=1}^{m}\sum_{j=i}^{m} n(i,j) \quad (29)$$

$$p(i) = \frac{n(i)}{n} \quad (30)$$

$F(j)-F(j-1)$ in the previous formulas is not necessarily given by a known function, but it will be sufficient if its numerical value is given.

d) Selection of the number of classes m

If the class interval Δx is fixed, the number of classes m will be decided as a necessary consequence from the minimum and the maximum values of the radius. However, in such a case as the aim of the analysis is to reveal the law of distribution which the radii of air voids obey, the upper limit of x or r is not clear. In that case, it is considered adequate to determine m as follows.

Let $\varepsilon(j)$ be the ratio of the total volume of air voids in class j to the sum total volume of those in class 1~class j, and m be a value of j which satisfies $\varepsilon(j)<\varepsilon$, in which ε is a constant to be selected according to the desired accuracy. Then, m can be determined as the smallest integer which satisfies the condition

$$\varepsilon(m) = (m-1/2)^3\{F(m)-F(m-1)\} / \sum_{j=1}^{m}(j-1/2)^3\{F(j)-F(j-1)\} < \varepsilon \quad (31)$$

Considering the effect on the calculated air content, condition (32) is sufficient to confine the error to within 0.1 percent as a value of air content.

$$\varepsilon = 0.01 \quad (32)$$

4. DEDUCING THE RADIUS DISTRIBUTION AND OTHER PARAMETERS OF AIR-VOID COMPONENT

4.1 Analysis of histogram components

Consider a histogram of apparent circles with m classes and an interval Δx. When x is on the boundary of two classes, it is included in the smaller class, that is,

$$(i-1)\Delta x < x \leq i\Delta x \tag{33}$$

From the first expression of Eq.(28), $n(j)$ for $i = 1 \sim m$ can be written as

$$\left.\begin{array}{l} n(1) = n(1,1) + n(1,2) + \cdots\cdots + n(1,m) \\ n(2) = \qquad\quad n(2,2) + \cdots\cdots + n(2,m) \\ \qquad\cdots\cdots\cdots\cdots\cdots\cdots \\ \qquad\cdots\cdots\cdots\cdots\cdots\cdots \\ n(m) = \qquad\qquad\qquad\quad n(m,m) \end{array}\right] \tag{34}$$

These formulas mean that a histogram of observed circles is composed of m individual histograms of apparent circles originating from source spheres in class m~class 1. This is schematically illustrated by Fig. 3. For this reason, the original histogram for the observed circles will be called an m-dimensional histogram.

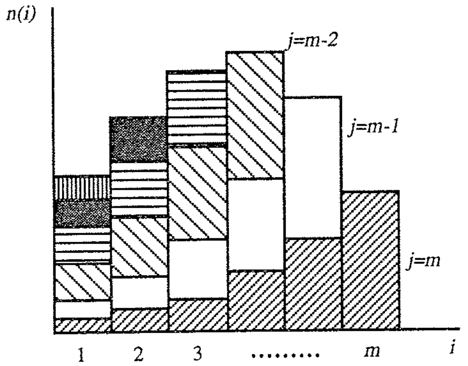

Fig. 3 Composition of the histogram of apparent circles

4.2 Procedure for determining the radius distribution of air-voids from a histogram of observed circles

a) Calculation of the radius distribution of source spheres

As shown in Fig. 3, $n(1,m) \sim n(m,m)$ are the numbers of apparent circles originating from the source spheres in class m, and the total will amount to $N_s(m)$, the number of source spheres in class m. In addition, the apparent circles belonging to the largest class m should all have originated from the source spheres in class m. Then, by substituting m for i and j in Eq.(26), the relationship equation (35) can be obtaind.

$$n(m,m) = \frac{\sqrt{2m-1}}{m} N_s(m) \tag{35}$$

Rewriting Eq.(35), the formula to compute $N_s(m)$ from $n(m,m)$ becomes

$$N_s(m) = \frac{m}{\sqrt{2m-1}} n(m,m) \tag{36}$$

It should be noted that $n(m,m)$ is a known number given by the frequency of apparent circles in class m.

When $N_s(m)$ becomes known, the frequencies of apparent circles originating from source spheres in class m can all be computed by formula (37) for class 1~class $m-1$.

$$n(i,m) = \frac{N_s(m)}{m}\left\{\sqrt{m^2-(i-1)^2} - \sqrt{m^2-i^2}\right\} \quad (i = 1 \sim m-1) \tag{37}$$

By subtracting these results from an m-dimensional histogram, an (m-1)-dimensional histogram is obtained, and the frequency of its uppermost class m-1 becomes a known number, that is

$$n(m-1,m-1) = n(m-1) - n(m-1,m) \quad (38)$$

Consequently, formulas to compute the number of source spheres in class m-1 and the frequency of apparent circle sizes originating from source spheres in class m-1 are obtained by substituting m-1 for m in formulas (36) and (37) respectively. Thus, $N_s(m-1)$ and $n(i,m-1)$ (i=1~m-1) become known numbers. If this process is repeated down to the 1-dimensional histogram, the radius distribution of the entire set of source spheres will be revealed.

In practicing these calculations, it may occur that the number of apparent circles in some class of some dimensional histogram will be negative. In that case, the frequency of such a class should be zeroed by adjusting the value to be subtracted from the preceding histogram. However, this adjustment will cause the calculated number of source spheres to exceed the observed number of apparent circles by the absolute value of the negative number. To compensate for this discrepancy, the number of source spheres obtained by formula (36) is used as a temporary value to carry out the operation of formula (37), and a virtual value for the number of source spheres $N_q(j)$ will be determined by summing the subtracted values $n_c(i,j)$ (i=1~j), that is

$$N_q(j) = \sum_{i=1}^{j} n_c(i,j) \quad (39)$$

b) Relating the radius distribution of air-voids to that of source spheres. Since the relation between the number of source spheres N_{sr} and air voids per unit volume of specimen N_{or} was given by Eq.(7), the number of air voids in class j per unit volume of concrete $N_0(j)$ is

$$N_0(j) = \frac{N_q(j)}{2\mathrm{A}j\Delta x} \quad (40)$$

and the frequency density $f(j)$ is

$$f(j) = \frac{N_q(j)}{2\mathrm{AN}_0 j\Delta x} \quad (41)$$

c) Air content, average spacing of air voids, and modified spacing factor
When $N_0(j)$ is known, the volume of the air voids in class j per unit volume of concrete $V(j)$ is

$$V(j) = 4\pi\{r(j)\}^3 N_0(j)/3 \quad (\mathrm{mm}^3/\mathrm{mm}^3) \quad (42)$$

in which, $r(j)$ is the median of class j computed by

$$r(j) = (j-1/2)\Delta x \quad (43)$$

If the air content of concrete as a percentage is denoted by A, then

$$A = \mathrm{V}_0 \times 100 \quad (\%) \quad (44)$$

where V_0 is the total volume of air included in a unit volume of concrete, that is

$$\mathrm{V}_0 = \sum_{j=1}^{m} V_0(j) \quad (\mathrm{mm}^3/\mathrm{mm}^3) \quad (45)$$

As for the spacing of air voids, an average spacing C_L will be defined as

$$C_L = \sqrt{3}\sqrt[3]{\frac{P_0}{N_0} + \overline{V}_r} - 2\bar{r} \tag{46}$$

where, P_0 is the volume of cement paste exclusive of air voids per unit volume of concrete (mm^3/mm^3), and $\overline{V}_r$ represents the volume of an air void with the average radius $\bar{r}$ (mm), that is

$\overline{V}_r = 4\pi\bar{r}^3/3$ (mm^3)

Since one half of C_L is equivalent to the formula which will be obtained by replacing both the radius in the so called spacing factor formula with the average radius, and the volume of an air void with the volume of an air void of average radius, it can be called the modified spacing factor L'. Then

$$L' = \frac{\sqrt{3}}{2}\sqrt[3]{\frac{P_0}{N_0} + \overline{V}_r} - \bar{r} \quad \text{(mm)} \tag{47}$$

5. VERIFICATION OF THE VALIDITY OF THE PROPOSED METHOD

5.1 Verification by numerical calculations

a) Procedure

In order to verify that the radius distribution of air-voids can be deduced from the histogram of apparent circles, the following steps were carried out.

1: The total number of air voids per unit volume of concrete and the examined area was assumed to be $N_0=10^4$ ($/cm^3$) and $A=1000mm^2$.

2: A density function of air void radius distribution $f(r)$ was selected.

3: An interval of classes as Δx was set, and the frequency distribution of air void radii was tabulatated by multiplying $F(j)-F(j-1)$ by N_0.

4: A table was made containing the frequencies of radius distribution of the apparent circles corresponding to that of the air void radii made in the previous step.

5: By treating the table obtained at step *4* as measured data, a radius distribution of air voids was derived from it by the method discussed in Section 4.2.

6: The processed result was judged to agree with the given distribution of air voids at step *3* by a χ^2-test.

Deducing the radius distribution in step *5* is performed independently of the function $f(r)$, so if the result of step *5* agrees with the tabulated frequency distribution from step *3*, it proves the deducing method to be valid regardless of the radius distribution exhibited by the air-voids.

b) An example of uniform distribution

Let the radii of the air-voids be distributed uniformly in the range of $0 \sim r_m$. Then

$$f(r)=1/r_m,\quad F(r)=r/r_m,\quad F(j)-F(j-1)=1/m$$
$$p(i)=\sum_i/\sum_{ij} \tag{48}$$
$$\sum_i=\sum_{j=i}^{m}\left\{\sqrt{j^2-(i-1)^2}-\sqrt{j^2-i^2}\right\}$$
$$\sum_{ij}=\sum_{i=1}^{m}\sum_{j=i}^{m}\left\{\sqrt{j^2-(i-1)^2}-\sqrt{j^2-i^2}\right\}$$

Assuming r_m=0.5 mm, the radius distribution can be shown as in Table 1. The values of $p(i)$ in Table 2 are the densities of apparent circles computed by formula (30), and the values shown for $n(i)$ are the expected frequencies of apparent circles in each class.

Table 1 Histogram for air voids with a uniform distribution

class	radius	frequency	
j	r (mm)	$N_0(j)$	$N(j)$
1	0.00~0.05	1	1000
2	0.05~0.10	1	1000
....	......	.....	.....
10	0.45~0.50	1	1000
total	-	N_0=10	N=10000

Table 2 Deduced results for uniform distribution

class	-	density	expected frequency	estimated radius distribution
i	Σ_i	$p(i)$	$n(i)$	of air voids
1	1.99188	0.036216	199.19	998
2	4.04143	0.073481	404.15	999
3	5.35936	0.097443	535.94	1000
4	6.23502	0.113364	623.50	1002
5	6.76814	0.123057	676.81	1001
6	6.99892	0.127253	699.89	1001
7	6.93339	0.126073	693.40	999
8	6.54816	0.119057	654.81	1001
9	5.76421	0.104804	579.42	999
10	4.35890	0.079253	435.89	1000
Σ_{ij} = 55.00001		1.000001	5500.00	N=10000

If the integers of $n(i)$ are regarded as observed data, the number of apparent circles in class 10 is 436, and the temporary number of source spheres in class 10 will be computed by formula (36) as

$$N_s(10)=\left[\frac{m}{\sqrt{2m-1}}n(m,m)\right]_{m=10}=1000.25$$

Consequently, the number of apparent circles derived from the source spheres of class 10 can be obtained by formula (37) as follows.

$$n(1,10)=\frac{N_s(m)}{m}\left\{\sqrt{m^2-(i-1)^2}-\sqrt{m^2-i^2}\right\}$$
$$=5.014$$

$n(2,10) = 15.195$

$\cdots\cdots\cdots\cdots$

$n(10,10) = 435.999$

Thus, the total number of source spheres with a radius belonging in class 10 will be estimated as

$N_q(10) = 5.014 + 15.195 + \cdots\cdots + 435.999$

$= 1000.25$

This result verifies that $N_s(10)$ is equal to $N_q(10)$.

By using formulas (40) and (23), the number of air voids with a radius belonging in class 10 can be determined.

$$N_0(10) = \left[\frac{N_q(j)}{2\mathrm{A}j\Delta x}\right]_{j=10} = 1000 \quad (/\mathrm{mm}^3)$$

$N(10) = 1.000 \quad (/\mathrm{cm}^3)$

Then, a 9-dimensional histogram can be obtained by subtracting $n(1,10) \sim n(10,10)$ from the 10-dimensional histogram. The values of the rightmost column of Table 2 are the estimated results from repeating similar calculations down to a 1-dimensional histogram.

Here, the χ^2 value of the estimated results was calculated to be 0.014 when compared with Table 1. Since the value of χ^2-distribution, expressed by χ_0^2 hereafter, with 10 degrees of freedom and 5 percent probability is 18.31, the χ^2 value is far greater than the χ_0^2 value. This test result indicates that the estimated radius distribution corresponds exactly to the assumed distribution.

c) Results for a normal distribution

Table 3 shows the results for a normal distribution with an average radius $\bar{r}$=0.25mm and a standard deviation σ=0.05mm. In this case, the χ^2 value was 1.341 and the χ_0^2 value was 21.03.

Table 3 Deduced results for normal distribution

class i,j	radius r (mm)	theoretical frequency of air voids	expected frequency of circles	estimated radius distribution of air voids
1	0.100~0.125	49	46	56
2	0.125~0.150	165	143	171
3	0.150~0.175	441	257	436
4	0.175~0.200	919	389	918
5	0.200~0.225	1499	510	1499
6	0.225~0.250	1915	569	1919
7	0.250~0.275	1915	526	1912
8	0.275~0.300	1499	396	1501
9	0.300~0.325	919	238	917
10	0.325~0.350	441	114	443
11	0.350~0.375	165	42	163
12	0.375~0.400	49	12	50
total		10000	3242	9981

d) Results for an exponential distribution

The assumed distribution is

$f(r)=\{\exp(-r/\rho)\}/\rho$

and the number of classes m is determined to be 22 by formulas (31) and (32). When the examined area A is 1000mm^2 and the number of air voids is 10^4/cm^3, the calculated number of apparent circles will total 6059. The χ^2 value for the estimated results will be 0.235 and the χ_0^2 value will be 33.92.

5.2 Verification by experiment

a) Procedure

In the previous verification, function $G(x)$ was fundamentally related both to the process of computing the frequencies of apparent circles from the given distribution of air-void radius and to that of deducing the radius distribution of air voids from the given frequencies of apparent circles. Hence, to verify the validity of the method under the condition that the function $G(x)$ is not related to the former, the frequencies of apparent circles were experimentally determined. For this purpose, a two-component mixture of a matrix and spherical particles was prepared and analysed. The matrix material was agar and the particles were made of starch.

The specimen was a rectangular prism 160mm long, 112mm wide and 55mm high, in which 429 particles were contained with a volume ratio of 0.448. The size distribution of the particles is shown in Table 5. To examine the cut surfaces, the specimen was sliced into plates, each about 10mm thick. The total area of the examined surfaces was 92400mm^2 and the total number of apparent circles was 500. All of their diameters were measured and consecutively numbered. The frequencies of the diameters are shown in Table 4.

Table 4 Histogram for observed circles

class	diameter	ID number of circles				
i	x (mm)	1~100	1~200	1~300	1~400	1~500
1	0~5	4	12	18	27	33
2	5~10	45	95	130	166	197
3	10~15	26	53	94	122	15
4	15~20	25	40	58	85	113
examined area (mm^2)		18480	36960	55440	73920	92400

b) An estimate of diameter distribution and its accuracy

In Table 5, the number of particles estimated to be in the total volume of the specimen is shown in comparison with the number actually used in the experiment. The value of χ^2-distribution with 3 degrees of freedom and 5 percent probability is 7.815. So if a value of χ^2 calculated by comparing the result with the actual data is smaller than 7.815, the result can be considered to have a sufficient accuracy. In Table 5, ratios of χ^2 values to 7.815 are shown as χ^2/χ_0^2, which indicates that about 300 apparent circles are required for accuracy.

c) Computing the volume ratio of particles

Table 6 shows the computing process and resulting volume ratio for n=500 by formula (45). As can be seen, the obtained value of V_0 is 0.441--close to the actual value of the specimen.

Table 5 Reduced results of diameter distribution snd accuracy of estimation

class	diameter of	particles	estimated number of particles			
	particles	included in	number of circles used for the estimation			
i,j	(mm)	specimen	100	200	300	500
1	0~5	0	0	0	0	0
2	5~10	202	217	238	206	185
3	10~15	131	92	102	127	122
4	15~20	96	106	85	82	96
total	-	429	409	425	415	403
	χ^2	-	13.1	14.1	2.24	2.05
	χ^2/χ_0^2	-	2.18	2.35	0.37	0.34

Table 6 Calculated result of the volume ratio of the particles

class	$r(j)$	$N_0(j)$	$r(j)N_0(j)/N_0$	$V_0(j)$
j	mm	10^{-3}/mm^3	mm	mm^3/mm^3
1	2.5	0	0	0
2	7.5	0.1877	0.999	0.0415
3	12.5	1.1238	1.098	0.1266
4	17.5	0.0974	1.210	0.2733
N_0=1.409×10^{-3}/mm^3			r=3.31mm	V_0=0.441

6. REQUIRED NUMBER OF APPARENT CIRCLES AND CUT SURFACE AREA TO BE EXAMINED

6.1 Requirements for estimating the radius distribution of air-voids

For practical reasons, it is desirable to observe and measure as few apparent circles as are required to ensure a precise result. So, the number of apparent circles required to keep the value of χ^2/χ_0^2 smaller than unity will be discussed. The number of apparent circles is controlled by choosing to examine an appropriate amount of surface area. The distribution functions and other parameters are the same as those used in Section 5.1.

Figure 4 illustrates the relations between the number of apparent circles n and the χ^2-test result. As can be seen in the figure, the values of n corresponding to χ^2/χ_0^2=1 are about 150 for uniform, 350 for exponential, and 600 for normal distribution. Therefore, even in the case where the radius distribution of air-void component is not known, it can be deduced from approximately 600 appeared circles with an acceptable accuracy. If the number of apparent circles is fixed, the area to be examined will increase with a decrease in the number of air voids per unit volume of concrete. Denoting by A_1 the area required to be examined in terms of air content, A_1 should be larger for concrete with less air content.

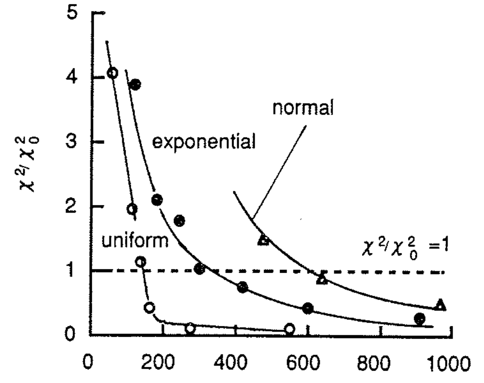

Fig. 4 Relation between number of observed circles and accuracy of estimation

6.2 Area to be examined for determining the air content of concrete and the spacing factor of air-voids

The accuracy of the determined radius distribution of the air-void component is mainly dependent on the number of observed apparent circles. However, such parameters as the air content of concrete, the average spacing of air voids, and the spacing factor of those air-voids are related not only to the air-void component but also to the other constituents of concrete. Therefore, the area to be examined must be large enough to determine the concrete's composition, especially the coarse aggregate content, because the coarse aggregate particles are the fewest in number.

If the symbol A_2 is used to represent this area, A_2 will increase with an increase in the maximum size of the aggregate. So, the value of A_2 will be discussed for three maximum sizes of coarse aggregates:20,25, and 40mm. Their size distributions are shown in Table 7 both by weight ratio and by particle ratio. The properties and mix proportions of concrete are assumed to be as shown in Table 8.

Table 7 Assumed size distributions of coarse aggregates

Max.size	weight ratio			particle ratio		
mm	20mm	25mm	40mm	20mm	25mm	40mm
5~10	0.30	0.18	0.14	0.732	0.659	0.683
10~15	0.40	0.27	0.19	0.211	0.213	0.200
15~20	0.30	0.35	0.17	0.058	0.101	0.065
20~25	-	0.20	0.15	-	0.027	0.027
25~30	-	-	0.14	-	-	0.014
30~35	-	-	0.12	-	-	0.007
35~40	-	-	0.09	-	-	0.004

Table 8 Assumed mix proportions of concrete

Max. size	air content	*W/C*	*s/a*	unit content (kg/m^3)			
(mm)	(%)	(%)	(%)	*W*	*C*	*S*	*G*
20	6.0	50	45	170	340	777	946
25,40	5.0	50	40	158	316	720	1079

The total number of coarse aggregate particles which were calculated from the mix proportions of concrete and the size distribution of aggregates are 96.76×10^5 for 20mm, 5.13×10^5 for 25mm, and 3.85×10^5 for 40mm. The particle ratio in Table 7 is identical to the distribution density, so if the area of the cut surface to be examined is given, the frequency distribution of the radii can be calculated. By assuming this result as observed data, the size distribution of the coarse aggregate can be determined with the procedure described in Section 4.2.

In order to judge the accuracy of estimation, the value of χ^2/χ_0^2 in comparison with the assumed distribution from Table 7 was calculated. Figure 5 illustrates the relation between the examined area and χ^2/χ_0^2 for three sizes of coarse aggregate. As shown, the examined areas corresponding to χ^2/χ_0^2 =1 are about 900cm^2 for aggregate with a maximum size of 20mm, 1300cm^2 for 25mm, and 1800cm^2 for 40mm.

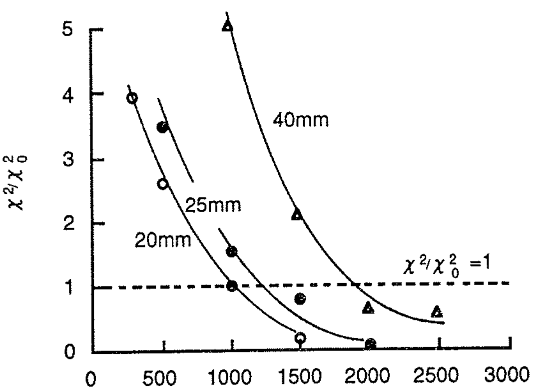

Fig. 5 Effect of examined area on accuracy of estimation

7. CONCLUSIONS

From the theoretical analyses and the experiments described above, the following conclusions may be reached.

(1) When a large set of spheres with a uniform radius are intersected by a plane, the radii of the cut sections of the spheres will distribute in accordance with the relation $G(x)$ given in Section 3.1. According to this function, the radius distribution of air void spheres in hardened concrete is statistically connected with that of the air void circles on the cut surface of the concrete.

(2) To bring the theory into practice, the variables representing the radii of the air void spheres and cut circles in the theoretical formulas need to be expressed as discrete variables. In this way, the observed air void circles can be ordered and represented in the form of a histogram. By analysing this histogram with the procedure developed in Section 4.2, the radius distribution of the entire air-voids will be revealed.

(3) The precision of the estimated results is dependent on the examined area. When the measurement is planned to ascertain the radius distribution of the air-voids, the area is required to include about 600 circles. If the intent of measurement is to determine the air content of concrete or the spacing factor of the air-void component, the area to be examined will be about $1000cm^2$ for concrete containing coarse aggregate with a maximum size of coase aggregate 20mm, $1500cm^2$ for 25mm, and $2000cm^2$ for 40mm.

Reference

[1] ASTM C 457:Standard Recommended Practice for Microscopical Determination of Air-Void Content and Parameters of the Air-Void System in Hardened Concrete