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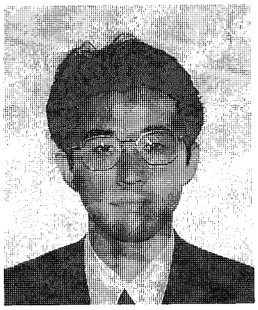
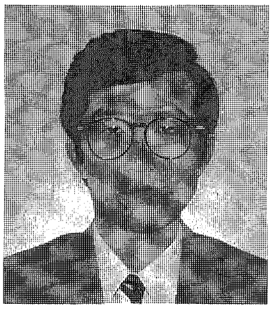
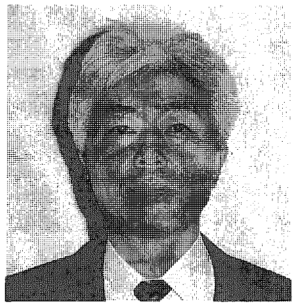
SHEAR STRENGTH OF REINFORCED AND PRESTRESSED CONCRETE BEAMS WITH SHEAR REINFORCEMENT

(Translation from Proceedings of JSCE. No.544/V-32, August 1996)

Yasuhiko SATO Tamon UEDA Yoshio KAKUTA

A shear resisting model for reinforced and prestressed concrete beams using steel bars is proposed. This model is based on the shear resisting model for non-prestressed and prestressed concrete beams reinforced with FRP rods as proposed previously by the authors. In the proposed model the shear strength of beams in which yielding of the shear reinforcement takes place can be calculated with equivalent stiffness of the shear reinforcement taken into account. It has been confirmed that this shear resisting model can predict experimental shear strengths of reinforced and prestressed concrete beams with good accuracy.

Keywords **:** *Steel reinforcement, RC beams, PC beams, Shear strengths, Shear reinforcement, Non-linear finite element analysis*

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1. INTRODUCTION

At present there is ever more demand to construct concrete structures with improved durability and performance. Various studies on improvements to durability, such as by implementing cathodic protection [1], have been conducted.

One response has been an increase in the use of fiber reinforced plastic (FRP) rods, which have good chemical resistance, as reinforcement for concrete structures. Work on the application of FRP rods to concrete structures is in progress [2], and the basic characteristics of concrete members reinforced with FRP rods have been clarified. The need now is to come up with specific design methods for concrete structural members using FRP rods. The aim is to create an environment in which engineers can choose to use not only steel bars but also FRP rods as the reinforcement for their concrete structures. This requires a unified design method which does not depend on the type of reinforcement.

The authors have developed shear strength equations [5][6] for non-prestressed and prestressed concrete beams using FRP rods based on shear resisting behavior which were verified by finite element analysis [3][4]. In these equations, the influence of reinforcement on shear strength is considered as a stiffness obtained by multiplying Young's modulus by the reinforcement ratio. Even if steel reinforcement is used in a concrete beam as the main and/or shear reinforcement, the equations can be applied in the case that the reinforcement does not yield.

Generally, the yield strength of tensile reinforcement is not considered in the design equation because it is assumed that tensile reinforcement does not yield before shear failure [7]. Further, the difference between shear strength with yielding of main reinforcement and that without yielding is not likely to be significant under conditions in which shear failure occurs in a beam.

There is, however, a big difference between the shear strength with yielding of the shear reinforcement and that without yielding. In a case where the shear reinforcement yields, the shear strength is less than when there is no yielding of the shear reinforcement. Therefore, yield strength must be considered in shear strength equations, such as that used in truss theory. The aim of this study is to evaluate quantitatively the shear strength in the case where the shear reinforcement yields. The shear strength equation for reinforced and prestressed concrete beams when shear reinforcement yielding does not occur, as proposed by the authors for concrete beams with FRP rods [5][6], will be applied, considering yielding of shear reinforcement as a reduction of Young's modulus. The result of this study indicates that it is possible to use the same equation to calculate the shear strength for cases where the shear reinforcement does yield (steel bars) and does not yield (FRP rods) .

2. SHEAR STRENGTH EQUATIONS FOR NON-PRESTRESSED AND PRESTRESSED CONCRETE BEAMS USING FRP RODS

2.1 Shear Resisting Model [5][6]

The authors have earlier developed a shear strength equation for non-prestressed and prestressed concrete beams on the bases of numerical experiments using non-linear finite element analysis [8]. The equation is based on the following shear resisting model [5][6] (see Fig.1):

$$V = V_{cpz} + V_{str} + V_{web} - V_{com} \tag{1}$$

where,

V_{cpz} : shear force carried by concrete in compression zone above the neutral axis

V_{str} : shear force carried by other than shear reinforcement in shear cracking zone

V_{web} : shear force carried by shear reinforcement in shear cracking zone

V_{com} : shear force transferred by concrete in horizontal zone

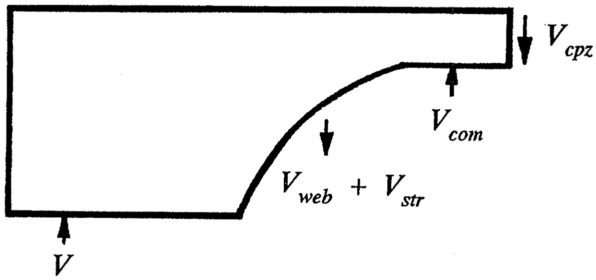


Fig.1 Shear Resisting Model

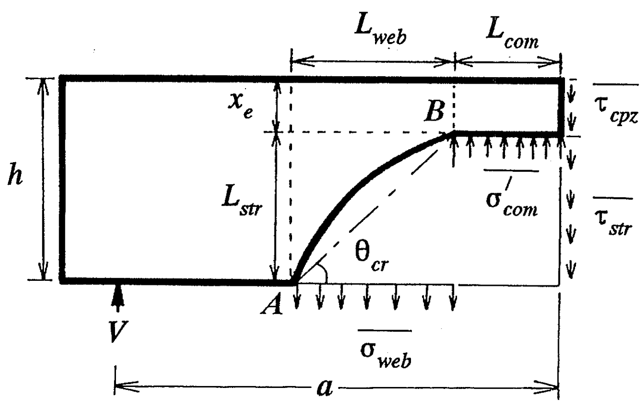


Fig.2 Distribution of Shear Resisting Stresses

2.2 Shear Strength Equation [5][6]

In Eq.(1), the shear forces are calculated from each average stress multiplied by area of its resisting zone. They are defined as functions of shear span to effective depth ratio (a/d; a :shear span, d :effective depth), concrete strength(f_c'), stiffness of main reinforcement ($p_s E_s$; p_s:main reinforcement ratio, E_s:Young's modulus of main reinforcement), stiffness of shear reinforcement ($p_w E_w$; p_w:shear reinforcement ratio, E_w:Young's modulus of shear reinforcement), and prestressing force divided by the cross-sectional area of the beam ($\overline{\sigma_P'}$) (see Fig.2). Equations for the shear resisting stresses and each resisting zone are defined as follows.

$$V = \overline{\tau_{cpz}}\ b\ x_e + \overline{\tau_{str}}\ b\ L_{str} + \overline{\sigma_{web}}\ b\ L_{web} - \overline{\sigma'_{com}}\ b\ L_{com} \tag{2}$$

where,

$\overline{\tau_{cpz}}$: average stress at compression zone

$$\frac{\overline{\tau_{CPZ}}}{f'_c} = 0.65\ \sin\alpha\ \cos\alpha \tag{3}$$

$$\alpha = \tan^{-1}\left(\frac{d}{a}\right) \tag{4}$$

$\overline{\tau_{str}}$: average shear stress at shear cracking zone

$$\frac{\overline{\tau_{str}}}{f'^{1/3}_c} = \frac{1.28}{\sqrt{a/d}+1}\ e^{-11.2\frac{\sigma'_P}{f'_c}} \tag{5}$$

$\overline{\sigma_{web}}$: average tensile stress of shear reinforcement at shear cracking zone

$$\overline{\sigma_{web}} = E_{web}\ \overline{\varepsilon_{web}} \tag{6}$$

$$\overline{\varepsilon_{web}} = 0.0053\frac{\sqrt{f'_c}}{\sqrt{a/d}+1}\ e^{-\frac{1000}{p_s E_s}-0.05\sqrt{p_w E_w}}\left[1+\left(\frac{\sigma'_P}{f'_c}\right)^{0.2}\right] \tag{7}$$

$\overline{\sigma'_{com}}$: average compressive stress at horizontal zone

$$\frac{\overline{\sigma'_{com}}}{f'_c} = 0.64\ \left(\frac{a}{d}\right)^{-1}\ \sin^2\beta \tag{8}$$

$$\beta = 32\left[1-\left(\frac{\sigma'_P}{f'_c}\right)^{0.5}\right] \tag{9}$$

x_e : depth of compression zone

$$\frac{x_e}{x} = \frac{1-e^{-\left(\frac{a}{d}\right)}}{1-3.2^{-0.12(p_w E_w)^{0.4}}}\left[1+\left(\frac{\overline{\sigma'_P}}{f'_c}\right)^{0.7}\right] \tag{10}$$

$$x = -np_s d + \sqrt{(np_{sd})^2 + 2np_s d^2} \tag{11}$$

L_{str} : vertical projected length of shear cracking zone

$$L_{str} = h - x_e \tag{12}$$

L_{web} : horizontal projected length of shear cracking zone

$$L_{web} = \frac{L_{str}}{\tan\theta_{cr}} \tag{13}$$

$$\theta_{cr} = 45\left[1-\left(\frac{\sigma'_P}{f'_c}\right)^{0.7}\right] \tag{14}$$

L_{com} : length of horizontal zone

$$L_{com} = \frac{a}{h}x_e \qquad (a > h) \tag{15}$$

h : beam height

b : beam width

2.3 Failure Mode in Model

In the analysis, softening of the concrete around the loading point was observed at the peak load. It can therefore be said that the failure mode is shear compression. This means that the shear strength equation developed based on numerical experiments can be applied to concrete beams in which shear compression failure occurs. The failure criterion used in the shear strength equation is defined by the principal stresses at compression zone. Equation (3) indicates the shear stress component of the principal stresses .

3. DEVELOPMENT OF SHEAR STRENGTH EQUATION FOR REINFORCED AND PRESTRESSED CONCRETE BEAMS USING STEEL BARS

3.1 Yielding of Shear Reinforcement and Shear Strength

Figure 3 shows relationships between Young's modulus of shear reinforcement and shear strength based on analytical and experimental studies. The vertical axis indicates shear strength (V_u) and the horizontal axis is Young's modulus of the shear reinforcement (E_w). If the yield strength of the shear reinforcement is high enough, shear strength rises as Young's modulus increases ($V_{uPP} > V_{uCP} > V_{uAP}$). When the shear reinforcement yields, however, shear strength drops below that of a specimen in which shear reinforcement does not yield ($V_{uPP} > V_{uPS}$) [4]. In this case, yielding can be treated as reduction in Young's modulus. The shear strength equation developed earlier by the authors is able to predict the shear strength of concrete beams in which shear reinforcement does not yield (V_{uPP},V_{uPC},V_{uPA} in Fig.3). This paper extends it to predict the shear strength of concrete beams in which the shear reinforcement does yield (V_{uPS} in Fig.3).

The ratios of shear resisting forces to total shear resisting force carried by each resisting zone before yielding of the shear reinforcement differ once the shear reinforcement yields. It has been confirmed that the same failure criteria can be applied to shear compression failure both when the shear reinforcement does yield and when it does not. Even when the yielding of the shear reinforcement takes place, it is possible to predict the shear strength of concrete beams which fail by shear compression failure. In other words, the ultimate shear resisting force may be calculated using the shear strength equation developed under the condition in which the shear reinforcement does not yield. The applicability of this concept is investigated below.

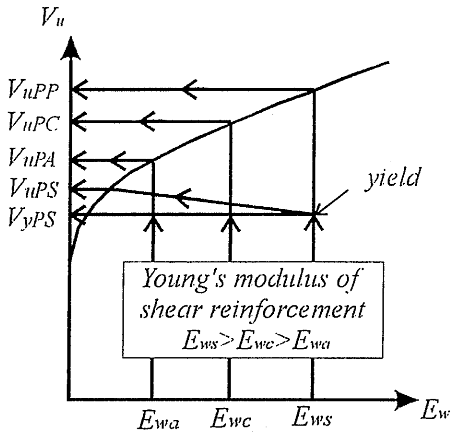


Fig.3 Relationships between Young's Modulus of Shear Reinforcement and Shear Strength

3.2 Evaluation of Equivalent Stiffness of Shear Reinforcement

As mentioned in the previous section, the shear strength of a concrete beam in a case where the shear reinforcement yields is less than that when shear reinforcement does not yield. This is because after yielding the depth of the compression zone is small, as in a concrete beam with shear reinforcement of low Young's modulus [3].

The reduction of Young's modulus due to yielding in the shear strength equation (Eq.(2)), for concrete beams with FRP rods shown in Section 2, is discussed below.

In the shear strength equation for concrete beams with FRP rods, the stiffness of the shear reinforcement affects only the depth of the compression zone and the average stirrup strain in the shear cracking zone [5][6]. It is assumed, therefore, that there is no influence of yielding (reduction of Young's modulus) on the average stress at the compression zone ($\overline{\tau_{cpz}}$), the average compressive stress at the horizontal zone ($\overline{\sigma'_{com}}$), and the average shear stress at the shear cracking zone ($\overline{\tau_{str}}$).

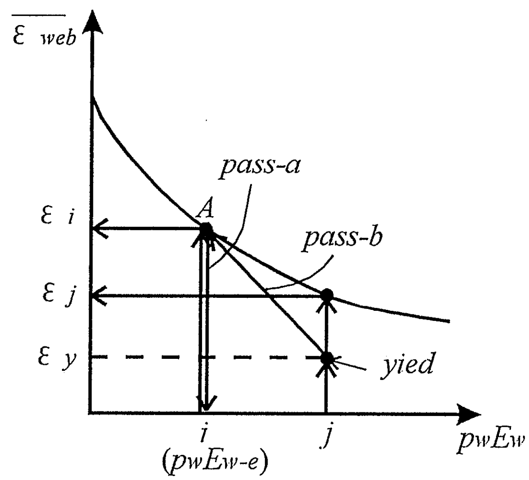


Fig.4 Relationships between Average Strain and Stiffness of Shear Reinforcement

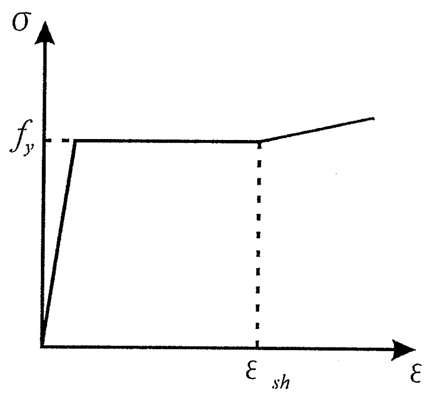


Fig.5 Stress - Strain Relationship of Bare Bar

Figure 4 shows the relationships between average stirrup strain and stiffness of the shear reinforcement in the shear cracking zone as calculated by Eq.(4). In this figure, the average shear reinforcement strain of a beam with a shear reinforcement stiffness of $p_w E_j$ is defined as ε_j.
The dotted line indicates the yield strain, ε_y, of the shear reinforcement. If the yield strain is greater than ε_j, the beam fails before the shear reinforcement yields. On the other hand, if the yield strain is less than ε_j, the shear reinforcement yields and strain increases to point A (ε_i) as if a reduction of Young's modulus had occurred.

The shear force carried by the shear reinforcement is a tensile force sustained by the reinforcement at a crack, so it can be defined as a summation of the average stress of the shear reinforcement and the average stress of the concrete. Therefore, the stress - strain relationship for a steel bar must be treated as the stress- strain relationship of the bare bar, not the average stress - strain relationship. This study uses the following equation (see Fig.5):

$$\sigma_s = f_y + (1.01 f_u - f_y)(1 - e^{(\varepsilon_{sh} - \varepsilon_s)/k}) \tag{16}$$

$$k = 0.032(400 / f_y)^{1/3} \tag{17}$$

where,

f_y : yield stress

f_u : tensile strength

ε_{sh} : hardening strain

If the shear reinforcement is elastic material such as FRP rods, the ultimate strain in the shear cracking zone can easily be obtained by dividing the stress, which is the sum of shear reinforcement stress and concrete tensile stress, by the Young's modulus of the reinforcement. It is difficult, however, to find the ultimate strain in a case where the shear reinforcement yields.

Figure 6 shows the relationships between shear reinforcement stress and strain at the shear cracking zone for both steel bars and FRP rods at the ultimate stage. When the steel strain and stress at the ultimate stage are known, the equivalent Young's modulus giving the same ultimate stress and strain can be calculated by the following equation:

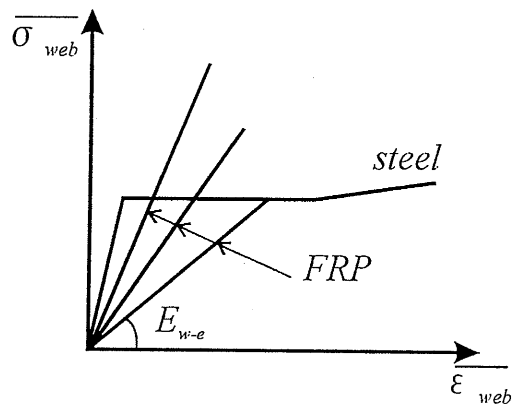


Fig.6 Relationships between Stress and Strain of Shear Reinforcement at Diagonal Cracking Zone

$$\overline{\sigma_s} = E_{w-e}\,\overline{\varepsilon_{web}} \tag{18}$$

where,

E_{w-e} : equivalent Young's modulus (see Fig.6).

Path-a in Fig.4 indicates the strain change in a case where the shear reinforcement has an equivalent Young' modulus and does not yield. On the other hand, the strain follows path-b and reaches point A in a case where the shear reinforcement yields. If this difference in strain path does not affect the shear strength, it can be said that the shear strength of a beam in which the stiffness of the shear reinforcement does not change (path-a) is equal to that of a beam in which the stiffness of the shear reinforcement decreases because of yielding (path-b).

As an example, the equivalent stiffness is calculated for a beam with concrete strength of 44MPa, a shear span to effective depth ratio of 2.4, a main reinforcement stiffness of 4944 MPa, a shear reinforcement stiffness of 824 MPa (p_w=0.4%, E_w=206GPa), and a yield strength of 294 MPa (Specimen RC2 in Table 1). The equivalent stiffness (shear reinforcement ratio × equivalent Young's modulus) is 216 MPa, because the equivalent Young's modulus, $E_{w\text{-}e}$, derived from Eqs.(7), (16), and (18) by assuming that stress calculated by Eq.(16) is the same as that calculated by Eq.(18), is 54GPa. To confirm the applicability of the equivalent stiffness, finite element analysis is carried out for two beam specimens: a concrete beam with a shear reinforcement stiffness of 824 MPa and a yield strength of 294 MPa (yielding case) and another with a shear reinforcement stiffness of 216 MPa and a high yield strength (no yielding). The shear strength in the yielding case and the no yielding case is 188 kN and 186 kN, respectively. Figure.7 shows the shear force - deflection curves of the two cases analyzed. Although there is a small difference before the shear reinforcement yield point, the shear force - deflection curve for the yielding case agrees well with that for the no-yielding case at the ultimate stage. The same behavior was observed for other specimens listed in Table 1. It can be said, therefore, that the shear strengths of beams in the case where the shear reinforcement yields can be calculated from the equivalent stiffness.

3.3 Evaluation of Average Shear Resisting Stresses

Table 1 shows the applicability of the shear strength equation for FRP-reinforced concrete beams to the prediction of the average shear resisting stresses in beams in which the shear reinforcement yields. Six reinforced concrete beams and two prestressed concrete beams are used to prove applicability. Compression zone depths and all resisting stresses are shown in Table 2. Column A indicates the results of FEM analysis while column B indicates the predictions of the shear strength equation. The depth of the compression zone, x_e, and the average stress at the shear reinforcement, $\overline{\sigma_{web}}$, are calculated using the equivalent stiffness. Although there are some cases

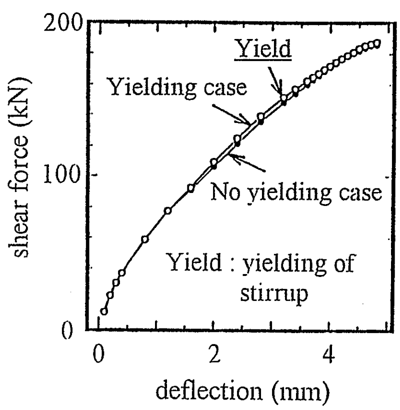


Fig.7 Shear Force - Deflection Curves

Table 1 Analyzed Specimens

Analyzed specimen	P_{eff} (kN)	a/d	p_s (%)	p_w (%)	f_{wy} (MPa)
RC1	0	2.4	2.4	0.4	196
RC2	0	2.4	2.4	0.4	294
RC3	0	2.4	2.4	0.2	294
RC4	0	2.4	1.2	0.4	294
RC5	0	2.4	2.4	0.4	294
RC6	0	1.6	2.4	0.4	294
PC1	100	3.2	2.4	0.4	294
PC2	200	3.2	2.4	0.4	294

Table 2 Analytical Results

Analyzed specimen	x_e (cm)			$\overline{\tau_{cpz}}$ (MPa)			$\overline{\sigma'_{com}}$ (MPa)			$\overline{\tau_{str}}$ (MPa)			$\overline{\sigma_{web}}$ (MPa)			V (kN)		
	A	B	A/B	A	B	A/B	A	B	A/B	A	B	A/B	A	B	A/B	A	B	A/B
RC1	8.0	7.2	1.11	9.5	10.2	0.93	2.2	3.3	0.67	1.7	1.8	0.94	183	196	0.93	176	168	1.05
RC2	8.6	7.6	1.13	9.8	10.2	0.96	3.2	3.3	0.97	1.8	1.8	1.00	282	294	0.96	188	186	1.01
RC3	7.5	7.0	1.07	9.1	10.2	0.89	2.7	3.3	0.82	1.6	1.8	0.89	278	294	0.95	170	158	1.08
RC4	6.5	6.0	1.08	9.3	10.2	0.91	2.8	3.3	0.85	1.6	1.8	0.89	267	294	0.91	168	166	1.01
RC5	9.7	9.2	1.05	4.8	4.5	1.07	1.2	1.5	0.80	1.3	1.4	0.93	274	294	0.93	135	135	1.00
RC6	6.1	6.5	0.94	14.8	12.9	1.15	4.6	4.9	0.94	1.8	2.0	0.90	288	294	0.98	261	232	1.13
PC1	10.7	9.5	1.13	8.6	9.4	0.91	2.4	2.0	1.20	1.3	1.1	1.18	272	294	0.93	122	137	0.89
PC2	11.3	10.0	1.13	10.7	9.4	1.14	1.2	1.6	0.75	0.5	0.7	0.71	285	294	0.97	164	154	1.06

of poor agreement between the compressive stress in the horizontal zone, $\overline{\sigma'_{com}}$, as predicted by FEM and by the shear strength equation, in general the average shear resisting stress in beams where the shear reinforcement yields can be predicted by the shear strength equation for beams where the shear reinforcement whose stiffness equal to the equivalent stiffness does not yield. The values of V in column B in Table 2 show the shear capacity obtained by the shear strength equation using the average shear resisting stresses shown in column B. It is clear that they agree with the values in column A, which are the values predicted by FEM.

4. EVALUATION OF PROPOSED SHEAR STRENGTH EQUATION AGAINST PREVIOUS EXPERIMENTAL RESULTS

The predicted results are compared with previous experimental results to confirm the applicability of the proposed equation. Experimental results for twenty-two reinforced concrete beams and fifteen prestressed concrete beams as obtained by Frantz [10], Saito [11][12], and Cederwall [13] are used for the verification. These data are chosen because they allow us to confirm applicability to various concrete strengths, shear span to effective depth ratios, main reinforcement ratios, shear reinforcement ratios, and prestresses. Details of these specimens are given in Tables 3, 4, and 5, where V_{test} indicates the experimental results and V_{cal} indicates the predicted results. Failure mode in the predictions is shear compression failure for all specimens.

Table 3 Details of Test Specimens and Shear Strengths (FRANTZ) [10]

Specimen	a/d	f_c' (MPa)	p_s (%)	p_w (%)	f_{wy} (MPa)	V_{test} (kN)	V_{cal} (kN)	V_{test}/V_{cal}	Yielding of Stirrup 1) Test	Cal.
B50-3-3	3.6	24	3.36	0.11	323	76	69	1.10	–	Y
B50-7-3	3.6	43	3.36	0.11	323	93	88	1.06	–	Y
B50-11-3	3.6	65	3.36	0.11	323	97	104	0.93	–	Y
B50-15-3	3.6	90	3.36	0.11	323	111	121	0.92	–	Y
B100-3-3	3.6	30	3.36	0.26	269	95	85	1.12	Y	Y
B100-7-3	3.6	51	3.36	0.26	269	120	105	1.14	–	Y
B100-11-3	3.6	75	3.36	0.26	269	150	124	1.21	Y	Y
B100-15-3	3.6	89	3.36	0.26	269	115	133	0.86	–	Y
B150-3-3	3.6	31	3.36	0.36	286	138	97	1.42	–	Y
B150-7-3	3.6	51	3.36	0.36	286	133	117	1.14	–	Y
B150-11-3	3.6	76	3.36	0.36	286	161	136	1.18	–	Y
B150-15-3	3.6	90	3.36	0.36	286	149	158	0.94	–	Y

1) Y : yielding of stirrup NY : no yielding of stirrup – : not reported

Table 4 Details of Test Specimens and Shear Strengths (SAITO) [11][12]

Specimen	a/d	f_c' (MPa)	p_s (%)	p_w (%)	f_{wy} (MPa)	V_{test} (kN)	V_{cal} (kN)	$\frac{V_{test}}{V_{cal}}$	Yielding of Stirrup[1)]	
									Test	Cal.
SV53-1	2.0	21	2.46	0.43	341	121	125	0.97	Y	Y
SV53-2	2.0	25	2.46	0.69	400	173	150	1.15	NY	NY
SV44-1	1.7	19	2.46	0.43	341	141	131	1.08	Y	Y
SV40-2	1.5	21	2.46	0.69	400	176	153	1.15	NY	NY
SV44-3	1.7	19	2.46	1.04	400	180	138	1.30	NY	NY
SV26.5-1	1.0	25	2.46	0.29	341	213	125	1.70	Y	Y
SV26.5-2	1.0	21	2.46	0.34	400	214	149	1.44	NY	Y
SV26.5-3	1.0	21	2.46	0.69	400	229	174	1.32	Y	NY
SV26.5-4	1.0	17	2.46	1.20	373	180	152	1.18	NY	NY
SV13.3-2	0.5	23	2.46	0.29	341	308	125	2.46	NY	Y

1) Y : yielding of stirrup NY : no yielding of stirrup — : not reported

Table 5 Details of Test Specimens and Shear Strengths (CEDERWALL) [13]

Specimen	σ_p' (MPa)	a/d	f_c' (MPa)	p_s (%)	p_w (%)	f_{wy} (MPa)	V_{test} (kN)	V_{cal} (kN)	$\frac{V_{test}}{V_{cal}}$	Yielding of Stirrup[1)]	
										Test	Cal.
734-45	3.1	2.5	33	1.05	0.21	495	90	93	0.97	—	Y
824-1B	2.0	2.5	52	1.45	0.21	495	116	117	0.99	—	Y
824-2B	3.6	2.5	32	1.46	0.22	520	116	99	1.17	—	Y
824-1C	2.4	2.5	20	1.46	0.43	488	88	112	0.79	—	NY
803-2S	3.1	2.5	36	1.12	0.22	235	76	88	0.86	—	Y
803-1S	2.7	2.5	31	1.12	0.27	235	85	83	1.02	—	Y
842-6	3.5	2.5	58	2.19	0.22	529	131	132	0.99	Y	Y
842-7B	3.7	2.5	45	2.20	0.22	529	129	117	1.10	—	Y
842-8	3.9	2.5	70	2.17	0.29	529	160	159	1.01	—	Y
842-10	3.4	2.5	63	2.20	0.22	353	107	122	0.88	—	Y
842-11	3.5	4.2	63	2.19	0.22	353	91	79	1.15	—	Y
842-12	3.1	1.7	67	2.19	0.22	353	191	154	1.24	—	Y
842-13	3.1	3.4	67	2.16	0.21	353	122	100	1.22	—	Y
842-14	1.2	2.5	51	2.19	0.22	529	109	120	0.91	—	Y
842-16	2.1	2.5	65	2.22	0.22	529	157	136	1.15	—	Y

1) Y : yielding of stirrup NY : no yielding of stirrup — : not reported

Figure 8 shows the relationships between experimental shear strength divided by the predicted value (the shear strength ratio) and concrete strength. The applicability of constitutive laws used in the FEM program adapted for this study has been confirmed for the range 20 MPa to 50 MPa in concrete strength [9]. It is clear, however, that the proposed equation can estimate the shear strength of concrete beams in the range 10 MPa to 90 MPa with good accuracy. At higher concrete strengths, it is generally considered that the shear force carried by aggregate interlocking is less than that in concrete beams with normal concrete since because the crack surface of high strength concrete is relatively flatter than that of the normal concrete. The proposed equation should be carefully applied to beams with concrete strengths greater than that used in this verification.

It seems that the shear strength ratio is large in the case of concrete strength between 10 MPa and 20 MPa. This is the case in deep beams where the shear span to effective depth ratio, a/d, is less than 1.5.

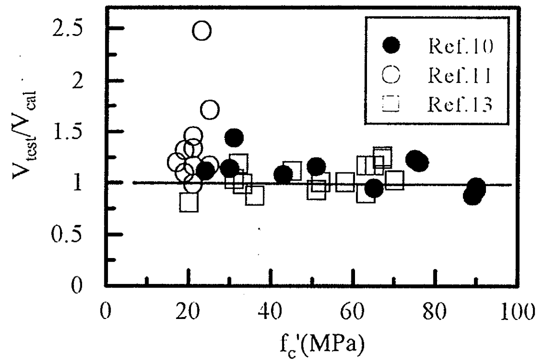


Fig.8 Ratio of Shear Strength - Concrete Strength Relationships

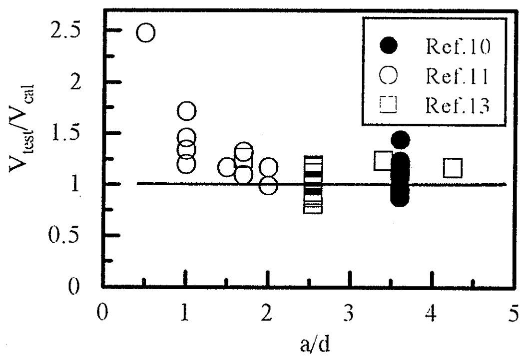


Fig.9 Ratio of Shear Strength - *a/d* Relationships

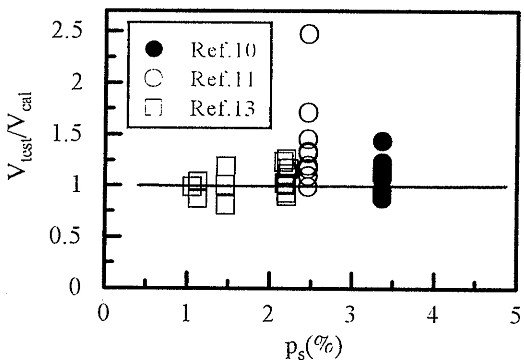


Fig.10 Ratio of Shear Strengths - Main Reinforcement Ratio Relationships

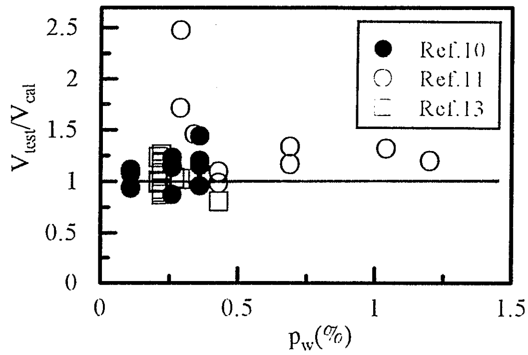


Fig.11 Ratio of Shear Strengths - Shear Reinforcement Ratio Relationships

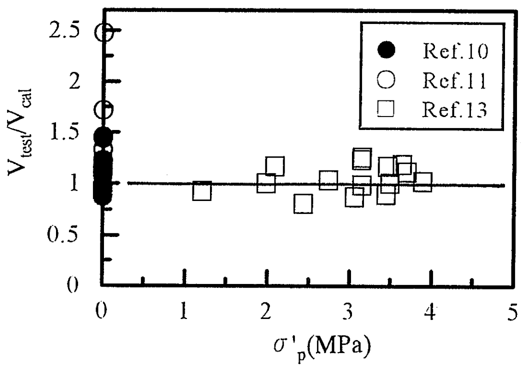


Fig.12 Ratio of Shear Strengths - Prestress Relationships

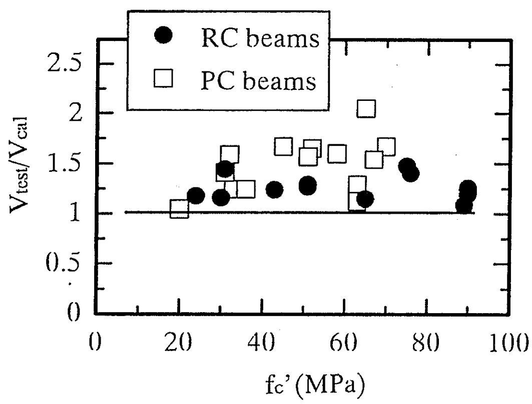


Fig.13 Ratio of Shear Strengths - Concrete Strength Relationships

Figure 9 shows the relationships between shear strength ratio and a/d. The shear strength ratio is approximately 1.0 in the range greater than 1.5 in a/d ratio. The proposed equation clearly underestimates the shear strength of a beam in which a/d is 1.0 and 1.5.
The proposed model is based on the assumption that the shear span (a) is greater than the height (h), that is, $a/h>1$ [5]. In Fig.9, beams in which a/d is greater than 1.5 satisfy this condition. The average shear strength ratio in the case where a/d is smaller than 1.5 is 1.62 with a coefficient of variation of 28%. On the other hand, the average shear strength ratio in the case where a/d is greater than 1.5 is 1.09 with a coefficient of variation of 12%. Thus the proposed equation suitably estimates the shear strengths of beams that satisfy this condition. The discrepancy in the case of deep beams may result from the effects of loading plate width and the three-dimensional confinement by the plate. The effects should be considered in for the revising the proposed model in the future.

Figure 10 shows the relationship between shear strength ratio and stiffness of the main reinforcement, and Fig.11 shows the relationship between shear strength ratio and stiffness of the shear reinforcement. No influence of a/d is observed in these figures.

Figure 12 shows the relationship between shear strength ratio and prestress. The shear strength of concrete beams with a prestressing force is calculated by a modified equation taking into account the equivalent stiffness of the shear reinforcement based on the shear strength equation for prestressed concrete beams with FRP rods proposed by the authors [6]. The equation proposed in this study can estimate the shear strengths of prestressed concrete beams with good accuracy. The average shear strength ratio in the case of prestressed concrete beams is 1.03, with a coefficient of variation of 13%.

Tables 3, 4, and 5 show the results of a comparison to determine whether yielding occurs or not in the predictions and experiments. Predicted results agree with the experimental ones for beams which satisfy the model condition ($a>h$).

Figure 13 shows the relationship between shear strength calculated by the JSCE code [7] and concrete strength. The comparison of experimental results and predicted results is for the beams in which a/d is greater than 2.5 because the JSCE code does not consider the effects of shear reinforcement in deep beams. In the calculation, all the safety factors are 1.0.

The JSCE code underestimate all the experimental results. It can be said, therefore, that the JCSE code can be applied to concrete beams with high concrete strength, such as 90 MPa. The average shear strength ratio is 1.26 with a coefficient of variation of 9.4% in the case of reinforced concrete beams, and 1.48 with a coefficient of variation of 17.6% in the case of prestressed concrete beams.

5. CONCLUSION

(1)A shear resisting model for reinforced and prestressed concrete beams using steel bars was proposed based on the shear strength equation for non-prestressed and prestressed concrete beams reinforced with FRP rods previously proposed by the authors. In a case where the shear reinforcement does not yield, the equation for non-prestressed and prestressed concrete beams with FRP rods can be used directly. On the other hand, where the shear reinforcement yields, shear strengths are calculated by taking into account the equivalent stiffness of the shear reinforcement.
(2)The applicability of the proposed shear strength equation for reinforced and prestressed concrete beams was confirmed by comparing predictions with previous experimental results.

This paper shows that it is possible to use the same equation to calculate the shear strength of concrete beams with FRP rods as well as with steel reinforcement as the main and shear reinforcement.

The proposed equation takes no account of the size effect on shear strength. If the failure mode is shear compression, however, size effect is likely to have little influence because in this failure mode the compressive strength affects the shear strength.

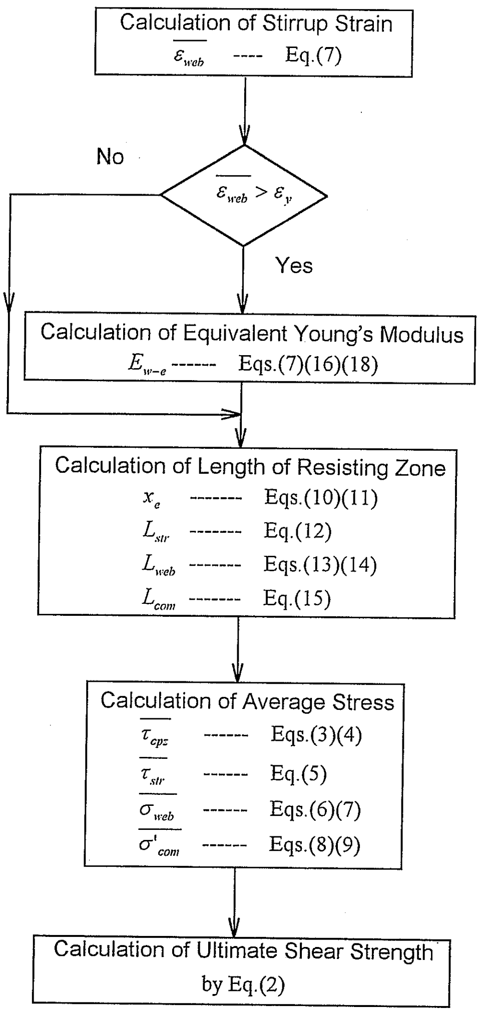
In beams which have low main and/or shear reinforcement stiffness, such as slender beams without shear reinforcement, diagonal tension failure is caused by single shear cracking. However, the finite element program used in this study cannot accurately predict the diagonal tension failure. Since the proposed equation is based on numerical experiments using this program, it is applicable only to shear compression failure and may overestimate the shear strength of beams with low main and/or shear reinforcement stiffness.
This may be due to the reduction of transferrd shear force at a shear crack. A study on the application of the proposed shear strength equation to diagonal tension failure will be reported later.

ACKNOWLEDGMENTS

The authors would like to thank Dr. Nares Pantaratorn who developed the finite element program used in this study.

APPENDIX

The calculation procedure for the proposed model is shown below.



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