FLEXURAL BEHAVIOR AND PROPOSAL OF DESIGN EQUATION FOR FLEXURAL STRENGTH OF EXTERNALLY PRESTRESSED CONCRETE MEMBERS

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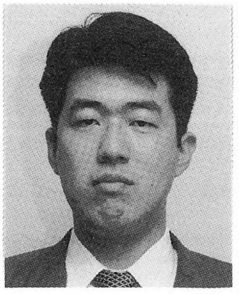
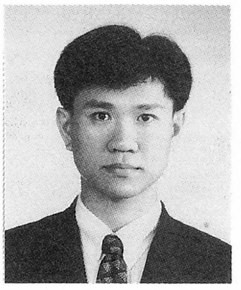
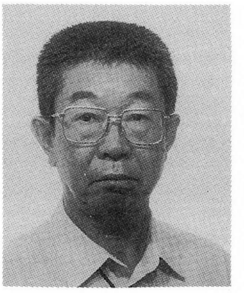




Hiroshi MUTSUYOSHI Kazuteru TSUCHIDA Songkiat MATUPAYONT Atsuhiko MACHIDA

The objectives of this study are to investigate the flexural behavior of externally prestressed concrete beams and to propose a flexural strength design equation for such beams. First, series of tests were conducted on PC beam specimens with external and internal unbonded tendons and with bonded PC beam. It was found that the effect of changes in tendon eccentricity (secondary effect) is dominant in flexural strength of externally prestressed concrete beams. An established analytical approach based on the compatibility of deformation and change of eccentricity effect showed a good agreement with test results. Finally, design equations to predict flexural strength of such externally PC beam were proposed and improved prediction accuracy was obtained with the proposed equations as compared with other equations.

Keywords: *external prestressing, prestressed concrete, flexural analysis of external PC, design equation for external PC*

Hiroshi Mutsuyoshi is a Professor in the Department of Civil Engineering at Saitama University, Urawa, Japan. He received his D.Eng. from the University of Tokyo in 1984. His research interests include seismic response behavior of RC structures, externally PC and application of FRP to concrete structures. He is a member of JSCE.

Kazuteru Tsuchida is attached to the Civil Engineering Technical Division of Shimizu Corporation, Tokyo. He obtained his M.Eng. from Saitama University in 1994. His master thesis was on the flexural behavior of external prestressed concrete. He is a member of JSCE.

Songkiat Matupayont is a Senior Bridge Designer at Jean Muller Int. Inc., Bangkok, Thailand. Formerly, a graduate student of Department of Civil Engineering, Saitama University. He obtained his Ph.D. from Saitama University in 1995.

Atsuhiko Machida is a Professor in the Department of Civil Engineering at Saitama University, Urawa, Japan. He received his D.Eng. from the University of Tokyo in 1976. His research interests include steel-concrete hybrid structures, behavior of RC structures under earthquake loading, and behavior of fresh concrete. He is a member of JSCE.

1. INTRODUCTION

The prestressing technology has been in used over a century in the successful construction of many prestressed concrete (hereinafter referred as PC) structures. The traditional methods of prestressing could be categorized as either pre-tensioning or post-tensioning, in which the tendon is placed within the concrete region. However, structures have recently been successfully constructed, mainly in Europe and North America, where the prestressing tendon is placed outside the concrete region and this is categorized as external prestressing. Some full scale structures of this kind are under construction in Japan. A comparison of external prestressing with internal prestressing shows that 1) reduced thickness of web and slab due to placement of tendons outside these regions, 2) eased construction work due to elimination of sheaths in the web and slabs, 3) added possibility of replacing the external tendons and stressing them in a future date, and 4) applicability for re-strengthening existing structures, are some of the advantages.

Unlike bonded tendons, in external tendons the tendons are not bonded with concrete, thus the increase of the concrete strain at the tendon level is not equal to the increase of strain in the prestressing tendon. Therefore, these types of structures are generally treated as the unbonded type and designed using the methodology for unbonded prestressed structures. However, restraining of the tendons only at the deviators and end anchorages causes position changes of the tendon and greatly influences the ultimate strength of such beams. Although some investigations of the flexural behavior of external prestressed members have been carried out, there is a need to more fully understand the behavior of such beams, especially the changes of prestressing force and eccentricity in external tendons, and a need to accurately predict the ultimate strength. As the advantages of this type structures means more will be built in the future, it is essential to have a better understanding of the behavior of such structures.

In this study, an experimental investigation was carried out to clarify the behavior of externally prestressed beams and the proposed analytical methodology was verified using the experimental results. In addition, a suitable design equation for externally PC members was proposed.

2. EXPERIMENTAL OUTLINE

The experimental setup and the dimension of the specimens are shown in Fig. 1. The length of the specimens was 560 cm having a 'T' shaped cross-section. Specimens No. 1, No. 2, and No. 3 were provided with deviators to keep the external tendons in position. The deviators were monolithically cast with the specimens. For the specimen with unbonded tendon, polyethylene sheaths were provided and grease was applied between the tendon and sheath. Teflon sheets were inserted between the cable and the surface of deviators to reduce the frictional effects in the external tendons.

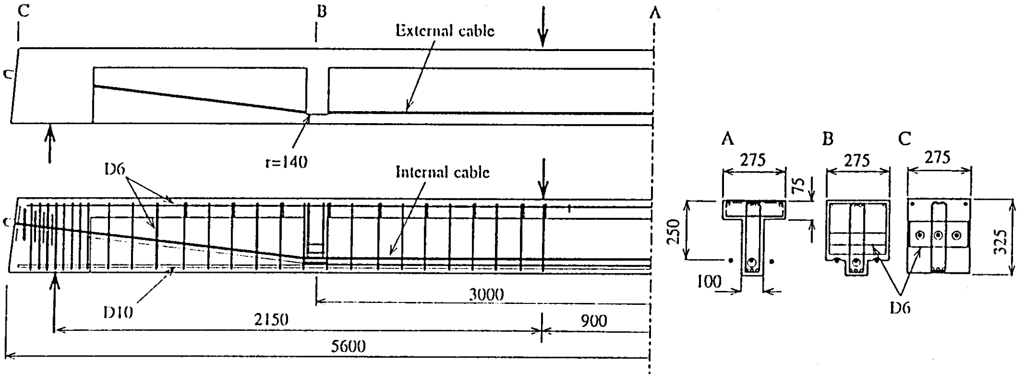


Fig. 1 Dimensions of test specimen (mm)

Table 1 Test variables

No.	Type	Number of deviators	Deviator distance (m)	Type of cable	
				External cables	Internal cables
1	External PC	2	1.8	SWPR7A 15.2*2	–
2	External PC	2	3.0	SWPR7A 15.2*2	–
3	External PC	3	1.5	SWPR7A 15.2*2	–
4	Unbond PC	–	–	–	SWPR19 19.3*1
5	Bond PC	–	–	–	SWPR19 19.3*1
6	External and Internal PC	2	3.0	SWPR7A 12.4*2	SWPR7A 12.4*1

Table 2 Mechanical properties of cables

Type of cable	Area (cm^2)	Yielding load (tf)	Ultimate load (tf)
SWPR7A ϕ 12.4	0.930	13.9	16.3
SWPR7A ϕ 15.2	1.387	20.8	24.5
SWPR19 ϕ 19.3	2.437	39.5	46.0

The test variables are given in Table 1. The distance between the deviators was set to 180 cm and 300 cm for specimens No. 1 and No. 2 respectively. Specimen No. 3 was provided with an additional deviator at the midspan to prevent the variation of eccentricity at midspan. Specimen No. 4 was a unbonded type while No. 5 was a bonded type. Specimen No. 6 was of a combined type with both external and internal bonded tendons. The total tendon area of all the specimens was made approximately equal by providing two tendons of SWPR7A-15.2 mm diameter as external cables in specimens No. 1, No. 2, and No. 3 while providing one tendon of SWPR19-19.3 mm diameter as an internal cable in specimens No. 4 and No. 5. For the combined specimen No. 6 three tendons of SWPR7A-12.4 mm diameter were used, two as external and one as internal tendon. The mechanical properties of the prestressing tendons are given in Table 2. The total initial prestressing was 27 tf for all the specimens, corresponding to 55-59% of the ultimate strength of the cable. Three D10 (SD345) bars were provided as tensile reinforcement and 4 Nos. D6 (SD345) were provided as compressive reinforcement. Shear reinforcement was provided with D6 bars at 10 cm spacing. The nominal compressive strength of the concrete was 400 kgf/cm^2.

Static monotonic two point loading was applied with the span being 520 cm and the loading span being 90 cm. The displacements at the midspan and deviator positions were measured together with the strain of concrete at the midspan region. The variation of prestressing force in external tendons was measured by load cells attached at the anchorage points and the strain in the tendon was also measured.

3. EXPERIMENTAL RESULTS

3.1 Load-Displacement Characteristics

a) Comparison between bonded, unbonded and external tendons
The load-displacement characteristics of specimens of externally PC (No. 2), unbonded type (No. 4), and bonded type (No. 5) are illustrated in Fig. 2(a). All the specimens showed a reduction in stiffness after concrete cracking. In externally prestressed and unbonded specimens the tensile reinforcement yielded when the applied load was 8 tf, resulting in a rapid increase in the deflection. On the other hand, in the bonded specimen the stiffness was not reduced until the yielding of the internal tendon. The load increased till 10 tf when the deflection then rapidly increased. In the unbonded specimen, after tendon yielding the rate of increase in deflection and the applied load was constant and the ultimate strength was 92% of the bonded specimen. In the externally prestressed specimen, due to deformation of the member which changes the relative tendon position, thus resulting in reduction of eccentricity, the ultimate strength was only 79% of the bonded type, corresponding to 86% of the unbonded type. Moreover, the ultimate deformation was 84% of the bonded type.

b) Comparison of deviator distance and the number of deviators

The experimental behavior of externally prestressed specimens Nos. 1, 2, and 3 together with the unbonded specimen No. 4 is presented in Fig. 2(b). Comparing the ultimate strength of specimens No. 1 and No. 2 where the distance between the deviators is different, the strength of No. 2, with a larger deviator distance, was 8% lower than No. 1. This is attributed to the reduction of tendon eccentricity from tendon position changes due to vertical deformation of the beam. It should be noted that specimen No. 3 was provided with 3 deviators in order to prevent the change of eccentricity at the critical section. The behavior of this specimen was almost the same as the unbonded type specimen No. 4, and, compared to specimen No. 2, the ultimate strength and deformation were about 1.2 times greater. From the above results it can be concluded that the ultimate flexural strength of beams with external prestressing is greatly influenced by the distance between the deviators, and it is necessary to prevent the change of eccentricity in order to minimize the reduction in ultimate strength.

c) Load-displacement characteristics of specimen with internal and external prestressing

The behavior of specimens with external prestressing (No. 2), bonded type (No. 5), and combined type (No. 6) are shown in Fig. 2(c). In the case of combined type specimen No. 6, the tensile reinforcement yielded at 8.0 tf and the bonded internal cable yielded at 8.5 tf. After this, the behavior was similar to that of external PC specimens where there was no apparent increase in the load while the deflection increased rapidly. In this specimen, the reduction in the ultimate strength was also similar to that of specimen No. 2 for the same reason as explained above. Eventually, the failure mode of all specimens was due to crushing of concrete within the maximum moment region; all failures were sudden leading to complete specimen collapse.

3.2 Relationship Between Applied Load And Tendon Force

a) Comparison between external and unbonded types

The load versus tendon force characteristics of externally prestressed specimens No. 1, 2, and 3, together with unbonded type (No. 4) is illustrated in Fig. 3. The rate of increase in tendon force was small for all the specimens before cracking. However, the tendon force increased rapidly, especially after yielding of tensile reinforcement in the specimen. The ultimate tendon force of specimens No. 1 and No. 2, which had two deviators, was about 38 tf. Nevertheless, the ultimate flexural strengths differed due to large eccentricity changes. The ultimate tendon force reached 93% and 100% of the yielding force in the unbonded type specimen No. 4 and external type specimen No. 3 respectively. As such, the ultimate flexural strength

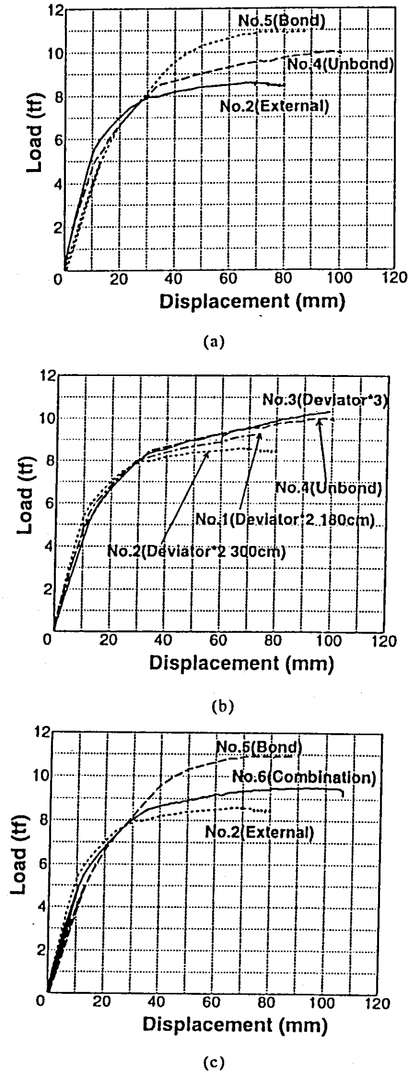


Fig. 2 Load-displacement curve

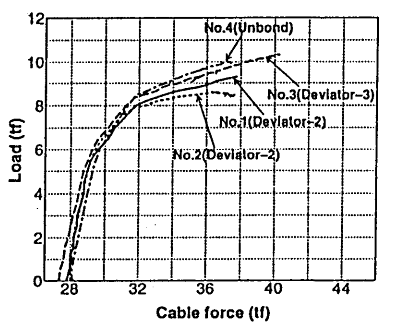


Fig. 3 Load-cable force curve

of specimen No. 3 is nearly equal to that of unbonded type No. 5.

b) Relationship between applied load and tendon Force in combined prestressing
The applied load with increase in tendon stress for specimen No. 6 is given in Fig. 4. Before cracking, the increase in tendon stress of external tendons was small and that of internal tendons was large. After yielding of tensile reinforcement and internal tendon, the increase in external tendon stress grew considerably large. The ultimate tendon stress was 75% of the unbonded type.

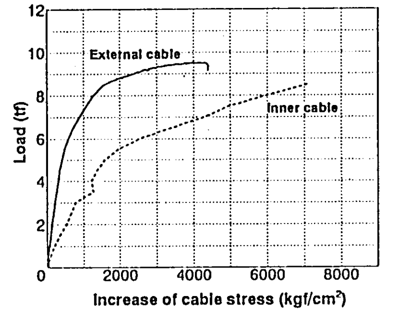
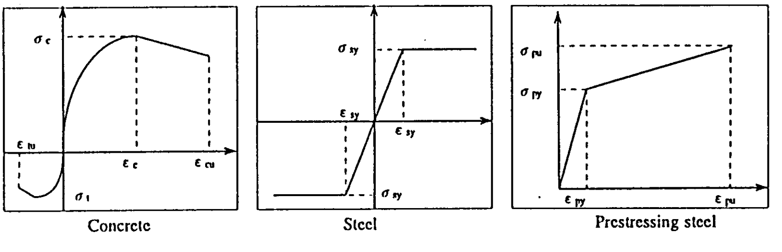


Fig. 4 Load-increase of cable stress curve

4. ANALYTICAL METHODOLOGY OF EXTERNALLY PRESTRESSED MEMBERS FOR FLEXURAL BEHAVIOR

Due to lack of adhesion between concrete and tendon in the externally prestressed and unbonded type PC beams, the strain of tendon is not the same as that of the concrete at the tendon level. As such, the analysis of such beams is member dependent, rather than section dependent. It is necessary to consider compatibility of the overall elongation of the tendon and the expansion of concrete in the whole beam under a particular loading. Moreover, in case of externally PC, the variation of eccentricity caused by deflection of the beam should also be taken into account. Fig. 5 illustrates the stress-strain relationships for prestressing tendon, reinforcing steel, and concrete. These models were based on the experimental results. In this analysis, the member was considered to have reached the ultimate state when either the concrete reaches the ultimate compressive strain (0.35%) or the prestressing steel reaches the ultimate tensile strain. It was assumed that there was no loss in the prestressing stress due to frictional effects.

The beam was divided into small sections (20 elements) along the longitudinal axis and each section was further subdivided into small strips (30 elements) along the depth as shown in Fig. 6. The flowchart of the analytical methodology is shown in Fig. 7. In this analysis, based on the strain of concrete of the topmost fiber at the critical section, the moment distribution, member deformation, and tendon eccentricity along the beam are calculated. For the analytical step under consideration the tendon position of the previous step was used. To minimize resulting error, the concrete strain of the critical section was increased in small increments of 100 μ. The procedure of the analysis is as follows: 1) The strain distribution at each section, deformation of the beam, and the tendon



Concrete				
σ c	σ t	ε c	ε cu	ε t
400kgf/cm^2	27kgf/cm^2	0.25%	0.35%	0.015%

Steel		Prestressing steel			
σ sy	Es	σ py	σ pu	ε pu	Ep
3500kgf/cm^2	2.1$\times10^6$kgf/cm^2	15000kgf/cm^2	17500kgf/cm^2	3.5%	1.9$\times10^6$kgf/cm^2

Fig. 5 Model of stress-strain curve

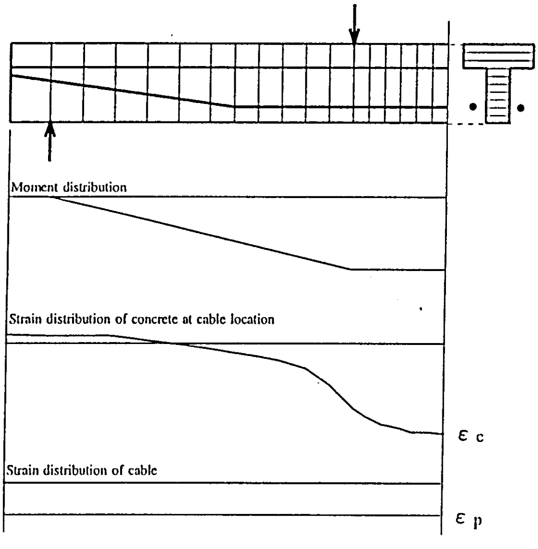


Fig. 6 Division of beam and cross section

position at the initial stage are calculated. 2) Based on the strain of concrete in the topmost fiber at the critical section, the increase in the tendon strain is assumed. Then, the strain distribution at the critical section and the internal moment of resistance is calculated to satisfy the equilibrium of internal forces using the assumed cable strain and the position of tendon in the previous step (For the first step, the condition after full prestressing is used). 3) The strain distribution at all the other sections is calculated to satisfy the force and moment equilibrium. Then the increase in tendon extension is checked with the expansion of the whole beam at the tendon level. When this condition is not satisfied, the tendon strain is reassumed and the above process is repeated until it is within the allowable limits. 4) The member deformation and the tendon position at each section are calculated. Using the above methodology it is possible to predict complete flexural behavior of externally PC beams.

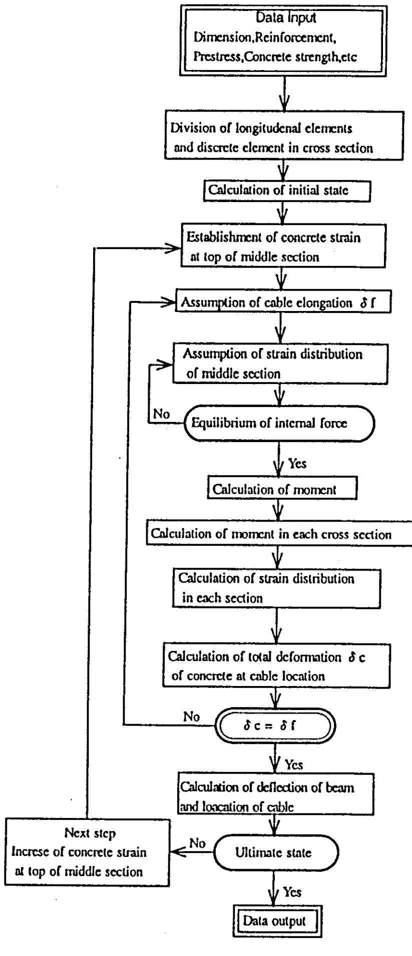


Fig. 7 Flow chart of analysis

5. ANALYTICAL RESULTS

The comparison of experimental and analytical results is summarized in Table 3. The predicted cracking load was nearly the same as the observed value. The flexural strength shows very good agreement, with the ratio of experimental to predicted strength being 94% - 101%. The ratio of the experimental to predicted ultimate tendon stress was in the range of 86% - 101%, thus showing fairly good agreement.

The comparison of analytical and experimental results for the load-displacement characteristics (for specimens No. 1, 2 and 6) is shown in Fig. 8. It can be seen that the results agree well in each stage. In the case of combined specimen No. 6, the results show very good agreement except in the ultimate displacement. Comparison between the experimental and analytical results for the load

Table 3 Comparison of experiment and analysis

No.	Cracking load (tf)			Flexural strength(tf)			Displacement (mm)			Cable force(tf)		
	Exp.	Cal.	Exp./Cal.	Exp.	Cal.	Exp./Cal.	Exp.	Cal.	Exp./Cal.	Exp.	Cal.	Exp./Cal.
1	5.5	5.3	1.03	9.3	9.3	1.00	80.2	69.9	1.15	37.4	38.8	0.96
2	5.5	5.3	1.03	8.6	8.6	1.00	74.2	70.3	1.05	37.9	37.7	1.01
3	5.5	5.3	1.03	10.3	10.6	0.97	99.1	83.5	1.19	41.8	42.3	0.99
4	5.0	5.3	0.94	10.0	10.6	0.94	95.2	83.5	1.14	36.4	42.3	0.86
5	5.0	5.3	0.94	10.9	10.8	1.01	88.2	87.4	1.01	39.5～	43.5	—
6	5.0	5.3	0.94	9.5	9.6	1.01	106.2	76.5	1.39	40.7～	41.3	—

with tendon force is shown in Fig. 9, which shows very good agreement between the results. Fig. 10 shows the increase in tendon stress with load for internal and external tendons in specimen No. 6. Here too there is fairly good agreement. The loss of tendon eccentricity with load is shown in Fig. 11, which shows good agreement. From these it is evident that the proposed analytical methodology could accurately predict the change in tendon position.

From the above comparison, it can be concluded that the flexural behavior of externally prestressed concrete members could be accurately calculated with the proposed analytical methodology considering the change of eccentricity of the external tendon.

6. PROPOSED FLEXURAL STRENGTH DESIGN EQUATION FOR EXTERNALLY PRESTRESSED CONCRETE BEAMS

It is possible to calculate the flexural behavior and ultimate strength for external prestressed beams by the proposed methodology discussed in the previous section. However, in practical situations, this methodology is fairly complex and it is necessary to establish a simplified, accurate design equation. This section proposes an equation to predict the flexural strength of externally prestressed members, based on the existing equation for unbonded type PC beams.

6.1 Outline and Evaluation of Existing Design Equation for Unbonded Tendons

The ultimate strength of unbonded PC beams can be calculated by general flexural analysis if the ultimate tendon stress given by Eq. (1) is known.

$$f_{ps} = f_{pe} + \Delta f_{ps} \qquad \text{Eq. (1)}$$

where f_{ps} : ultimate tendon stress
f_{pe} : effective initial prestress
Δf_{pe} : increase of tendon stress

In the existing prediction equations the estimation of Δf_{ps} varies based on the equations. The accuracy of the existing prediction equations was checked through

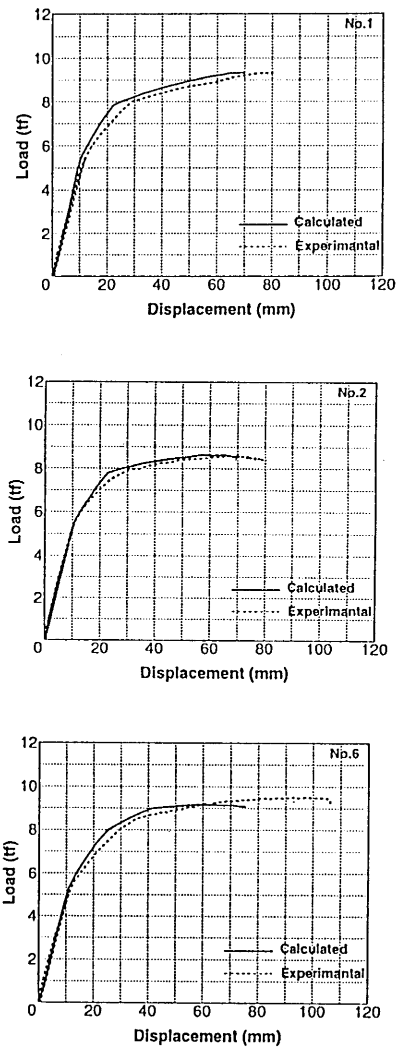


Fig. 8 Load-displacement curve

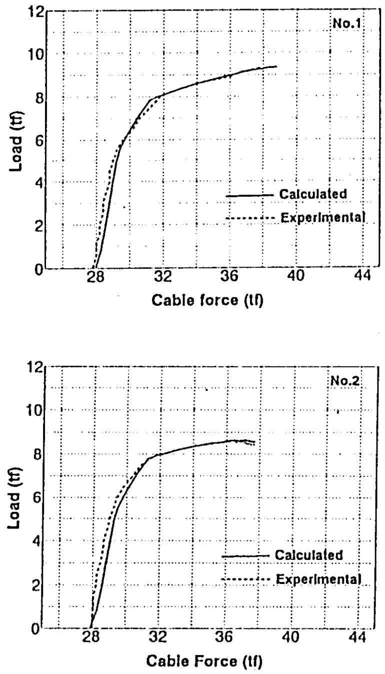


Fig. 9 Load-cable force curve

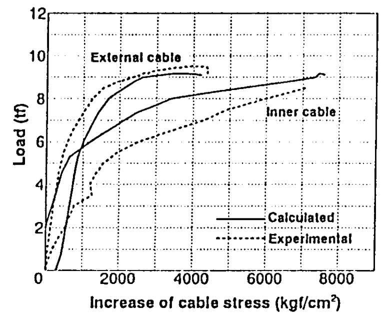


Fig. 10 Load-increase of cable stress curve

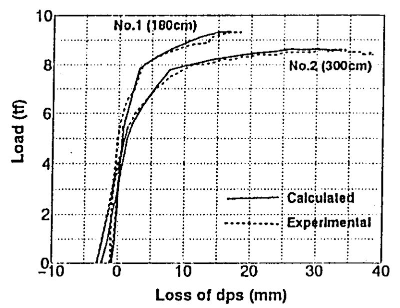


Fig. 11 Load-loss of d_{ps} curve

comparison with the existing experimental data for unbonded PC beams. A total of 67 specimens reported in the literature (1, 8, 13, 20) were used. Table 4 indicates the accuracy of the estimated results of each equation compared with the experimental results. it was found that the equation proposed by Naaman was the most accurate (9). The average correlation for this equation was 1.01 with the standard deviation of 5.96% for the ultimate stress in tendon and it could satisfactorily predict the ultimate stress in the tendon.

The equation proposed by Naaman was evaluated for its applicability to PC beams with external tendons. This was done by comparing predicted values with existing experimental data of 30 specimens found in the literature (17-23). The average correlation of experiment of analysis was 0.97 with a standard deviation of 14.9%, a larger scatter compared to the unbonded case. Fig. 12

Table 4 Comparison of accuracy of different equations

Equations	Cable stress at ultimate		Flexural strength	
	Average of correlation	Avg/STD (%)	Average of correlation	Avg/STD (%)
Warwaruk[2]	1.207	8.46	1.358	14.43
Mattock[3]	1.156	10.49	1.291	14.38
Mojtahedi[4]	1.154	10.92	1.291	14.38
Pannel[5]	1.002	8.49	1.158	11.46
Du and Tao[6]	0.876	9.81	0.868	13.92
Harajli[7]	1.155	10.53	1.291	13.92
Harajli(New)[8]	1.100	7.88	1.238	13.19
Naaman[9]	1.009	5.96	1.140	11.06
Takemoto[10]	1.221	8.63	1.383	15.47
AIJ[11]	1.226	8.14	1.288	14.77
JSCE[12]	–	–	1.523	11.62

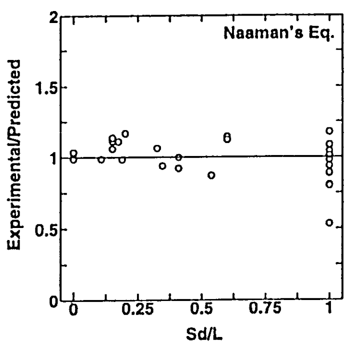


Fig. 12 Influence of S_d/L on accuracy of Naaman's Eq.

gives the ratio of experiment to prediction by Naaman's equation for beams with varying deviator distances. When the deviators numbered two, the distance between the deviators was taken as S_d. When the number of deviators was three or more, or when there was a deviator at the midspan, S_d/L was considered to be zero. When the value of S_d/L grew larger, the proposed equation overestimated predictions. This is attributed to the fact that Naaman's equation does not consider changes in eccentricity. As such, it is evident that using this unbonded tendon prediction equation for external tendon cases will produce inaccurate results proportional to the S_d/L ratio.

6.2 Basis of Flexural Strength Equation for External PC Beams

In external prestressed beams, due to rectilinear behavior of cables between the deviators and anchorages, the elongation of tendon is a slightly less than with unbonded type PC members. Considering this influence, the proposed strain reduction coefficient (Ω_u) is modified for external PC. Moreover, tendon eccentricity is reduced between the deviators. In this section, the concept of effective depth of cable is introduced. To verify the influence of these two factors using the analytical methodology proposed in section 4 a numerical analysis was carried out by varying the parameters.

The strain reduction coefficient, (Ω_u) and the reduction factor for tendon's effective depth at ultimate (Rd) are defined as follows:

$$R_d = d_{ps,u}/d_{ps} \qquad \text{(Eq. 2)}$$

$$\Omega_{u,e} = \Delta\varepsilon_{ps}/\Delta\varepsilon_{cps} \qquad \text{(Eq. 3)}$$

Where d_{ps} : Initial external tendon position at the critical section
$d_{ps,u}$: Ultimate tendon position at the critical section
$\Delta\varepsilon_{ps}$: Increase in tendon strain
$\Delta\varepsilon_{cps}$: Increase in tendon level concrete strain at the critical section

a) Determination of reduction coefficient, R_d
The variation of R_d with S_d/L, L/d_{ps} was obtained by this numerical analysis. The parameter L/d_{ps} was varied from 8 to 24 in steps of 4 in five patterns, and six values were used for the parameter S_d/L for each case of L/d_{ps}. The sectional properties of the beam were the same as those used in the experiment. A third point loading pattern was used. Fig. 13 gives the relationship between R_d and S_d/L based on the analysis. As it is evident from this figure, the variation was linear for each L/d_{ps} and the following equation was obtained.

$$R_d = 1.0 - 0.022 * (L/d_{ps} - 5) * (S_d/L - 0.2) \qquad \text{(Eq. 4)}$$

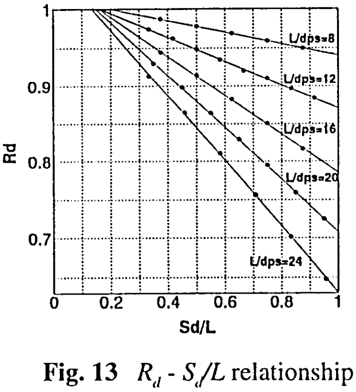


Fig. 13 R_d - S_d/L relationship

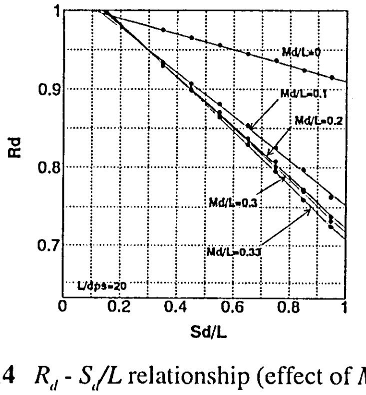


Fig. 14 R_d - S_d/L relationship (effect of M_d/L)

b) Modification of R_d based on moment distribution

The proposed equation given by Eq. 4 was derived for third point loading. However, the relationship among R_d, S_d/L, and L/d_{ps} changes depending on the moment distribution pattern. The effect of M_d/L on R_d was investigated where M_d is the distance between the loading points.

The variation of R_d with S_d/L for different M_d/L is shown in Fig. 14. It was found that for the value of M_d/L from 0.1 to 0.33 there was not much of variation while for single point loading it was considerably high. Considering these results, for one point loading the equation of R_d is expressed as follows.

$$R_{d1} = 0.71 + 0.29R_d \qquad \text{(Eq. 5)}$$

where R_{d1} : Effective depth reduction factor for one point loading
and R_d : effective depth reduction factor for two point loading

c) Effect of non prestressing steel arranged in concrete

Stress increases in external tendons occur after yielding of reinforcement. When the percentage of reinforcement is large, the stress increase in the external tendon after yielding of reinforcement is comparatively small. In this case the change of tendon eccentricity is also reduced. As such the effect of non-prestressing steel on R_d was verified. Fig. 15 gives the variation of R_d with P_m. The percentage of nonprestressing steel, P_m, is defined as follows.

$$P_m = A_s f_{sy} / b d_s f_c' \qquad \text{(Eq. 6)}$$

where A_s: Area of tensile steel
F_{sy} : yield stress of tensile reinforcement
b : width of the section
d_s : effective depth of tensile reinforcement
f_c' : compressive strength of concrete

The variation of R_d with P_m was linear, as seen in Fig. 15, and increased proportionally with the value of P_m. From the above results, R_d is expressed as follows:

$$R_d = 1.0 - 0.022 * (L/d_{ps} - 5) * (S_d/L - 0.2) + 0. - 186 * L/d_{ps} * P_m \qquad \text{(Eq. 7)}$$

d) Determination of strain reduction coefficient Ω_u

Comparing the increase in stress in the unbonded tendons with that of external tendons, the value of Ω_u is smaller than that of unbonded type due to rectilinear shape of the external cable between the deviators. The variation of Ω_u with S_d/L and L/d_{ps} was investigated by numerical analysis and Fig. 16 shows the analytical results. The variation of Ω_u with S_d/L is linear and can be expressed as follows:

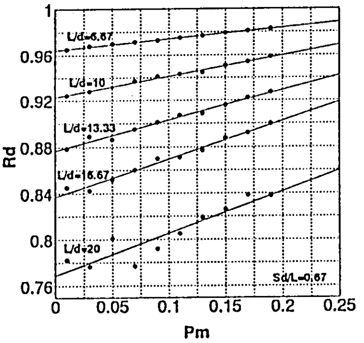


Fig. 15 R_d - P_m relationship

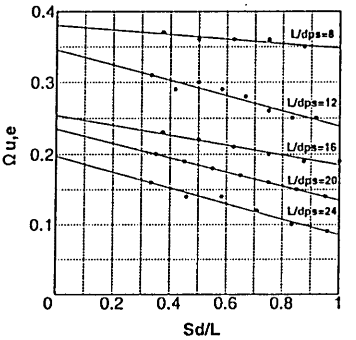


Fig. 16 $\Omega_{u,e}$ - S_d/L relationship

$$\Omega_{u,e} = 4.36/(L/d_{ps}) - 0.084 * S_d/L \quad \text{(Eq. 8)}$$

Since the relationship given by Eq. 8 was obtained for third point loading it is necessary to verify the effect of M_d. Fig. 17 shows the relationship of Ω_u with S_d/L for different values of M_d. It was found that the influence of Md on Ω_u was smaller as the value of M_d decreased. From the above results, Ω_u is expressed as follows.

$$\Omega_{u,e} = (1.47 + 10.3 * M_d/L)/(L/d_{ps}) - 0.29 M_d/L * S_d/L \quad \text{(Eq. 9)}$$

The factors R_d and Ω_u obtained from the above evaluation were applied to the equation proposed by Naaman that gave the best results among the equations for unbonded PC. The application of Ω_u in Naaman's equation can be expressed as follows:

$$f_{ps} = f_{pe} + \Omega_u * E_{ps} * \varepsilon_{cu}(d_{ps}/c - 1) \quad \text{(Eq. 10)}$$

and $c = \dfrac{-B1 + \sqrt{B1^2 - 4 * A1 * C1}}{2A1}$

$$A1 = 0.85 f'_c b_w \beta 1$$
$$B1 = A_{ps}(E_{ps}\varepsilon_{cu}\Omega_u(L1/L2) - f_{pe}) + A'_s f'_y + A_s f_y + 0.85 f'_c (b - b_w) h_f$$
$$C1 = -A_{ps} E_{ps} \varepsilon_{cu} \Omega_u d_{ps}(L1/L2)$$

where h_f : thickness of flange
b_w : Web width

However in case of rectangular section, b_w is taken as b.

When evaluating the flexural strength of external PC members, the value of Ω_u is obtained from Eq. 9 and, by substituting it into Eq. 10, the ultimate tendon stress f_{ps} could be obtained. In addition, the final tendon position, $d_{ps,u}$, given in Eq. 11 is calculated from the effective depth reduction factor R_d obtained from Eq. 7.

$$d_{ps,u} = R_d * d_{ps} \quad \text{(Eq. 11)}$$

With the above evaluation methodology, it is possible to predict the ultimate flexural strength of external PC beams using the general calculation methods for flexure.

6.3 Evaluation of Accuracy of the Proposed Equation

The accuracy of the equation by using Ω_u and R_d was evaluated by predicting the ultimate strength using existing experimental data. Fig. 18 shows the ratio of experimental to predicted results for the same data that were used in Fig. 12. By using Naaman's equation the average correlation was 0.97

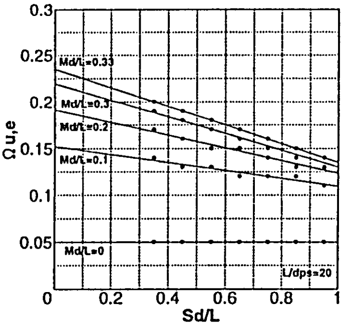


Fig. 17 Effect of M_d/L ($\Omega_{u,e}$)

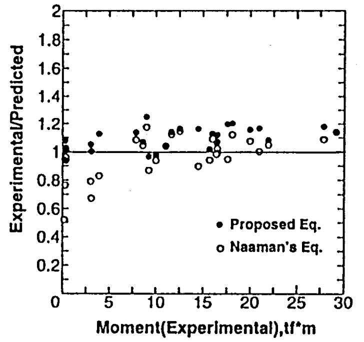


Fig. 18 Comparison of experimental and predicted results

with a standard deviation of 14.9%. However, with the proposed modified equation the corresponding values improved to 1.09 and 7.18%, thus the ultimate flexural strength of external PC members was predicted with a better accuracy.

7. CONCLUSIONS

Based on the experimental investigation, parametric study, and the proposed design equation for external PC members, the following points were concluded.

1) The distance between the deviators has a significant influence in the behavior of external PC members. This affects the increase in tendon stress and the effective tendon position and, as the distance between the deviators grew large, the flexural strength of the member and the deformation ability reduces.
2) By providing a deviator at the midspan, a change of eccentricity in external tendons could be prevented, thus resulting in behavior similar to unbonded PC members.
3) It is possible to predict the flexural behavior of external PC beams by properly incorporating the change of eccentricity and the compatibility of elongation between concrete and tendon.
4) By considering the effective depth reduction factor and strain reduction coefficient for external tendon, it is possible to accurately predict the ultimate flexural strength of external PC members.

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