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A STUDY OF DESIGN EQUATION FOR THE SHEAR STRENGTH OF RC BEAMS SUBJECTED TO AXIAL TENSION

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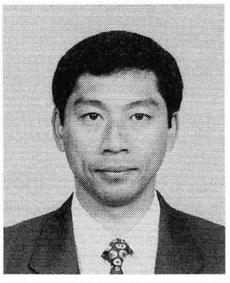
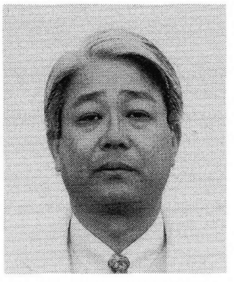

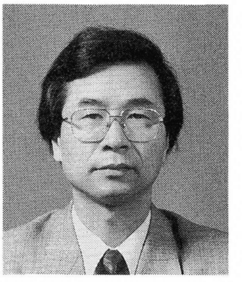

Takahiro TAMURA   Tsunemi SHIGEMATSU   Takashi HARA   Kyuichi MARUYAMA

This paper looks into the influence of reinforcing ratio on shear strength of the reinforced concrete beams subjected to axial tension and bending. Using both experimental results and numerical analysis, it is clearly shown that the reinforcing ratio influences the shear capacity of a beam. To take account of this dependence on reinforcing ratio, suitable regressive equations are defined based on the experimental and numerical results. Earlier experimental results obtained by the authors for rectangular beams and T-shaped beams, as well as the results of Mattock et al., verify the accuracy of the proposed equations.

Keywords : reinforced concrete beam, shear strength, axial tension, reinforcing ratio

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1. INTRODUCTION

Few studies have been published on the shear strength of reinforced concrete (RC) beams subjected to an initial axial tension because it is quite difficult to examine this problem. However, engineers frequently encounter stress conditions including axial tensile stress due to restraints imposed by shrinkage; such stress conditions are an inevitable issue in the statically indeterminate structures such as rigid frames. In the current JSCE design equation, the shear strength of a RC beam without shear reinforcement is determined from a combination of concrete compressive strength, effective depth of cross section, reinforcement ratio, and applied axial force(Eq.(1))[1].

$$V_{yd} = V_{cd} + V_{sd} + V_{ped} \tag{1}$$

$$V_{cd} = f_{vcd} \cdot b_w \cdot d/\gamma_b \tag{2}$$

$$f_{vcd} = 0.9\beta_d \cdot \beta_p \cdot \beta_n \cdot \sqrt[3]{f'_{cd}}(kgf/cm^2) \tag{3}$$

Where:

$\beta_d = \sqrt[4]{100/d}$ (d : cm) β_d is 1.5 when $\beta_d > 1.5$
$\beta_p = \sqrt[3]{100p_w}$ β_p is 1.5 when $\beta_p > 1.5$
β_n: the term related to the axial force, $2M_0/M_u = M_0/M_d$, as defined in Eq.(4)
M_d: designed bending moment
γ_b: shape factor, which is generally 1.3
b_w: web width
d: effective depth (m)
$p_w : A_s/(b_w \cdot d)$
f'_{cd}: compressive strength of concrete(kgf/cm^2)
V_{sd}: term on shear reinforcement
V_{ped}: term on prestressed reinforcement

All parameters in this equation are reliable and suitable for actual design practice since they are derived from the results of many experimental studies. Regarding the parameter for axial force, the JSCE code adopts the ideas of decompression moment as shown in Fig.1, as also adopted in the CEB/FIP model code and examined experimentally by Haddadin et al.[2]. However, if the structural member is subjected to an axial tension, structural safety considerations, mean that the coefficient of decompression moment, β_n should be twice as large as when under the axial compression, as shown Eq.(4), because there are few studies on the shear strength of RC members under an initial axial tensile force[3].

$$\begin{aligned} \beta_n &= 1 + 2\frac{M_0}{M_u} \quad (N'_d > 0) \\ \beta_n &= 1 + 4\frac{M_0}{M_u} \quad (N'_d < 0) \end{aligned} \tag{4}$$

where N'_d is the design axial load, which is taken as positive in compression and negative in tension. M_u is the ultimate moment and M_0 is the decompression moment. If M_0 has the same sign as M_u, it is taken as positive.

The authors have studied the shear strength of RC beams subjected to axial tension and also the effects of axial force in the recommended equation both experimentally and numerically. The results of experimental studies on rectangular beams (reinforcement ratio $p_w = 0.011$; shear span to effective depth ratio a/d =1.75

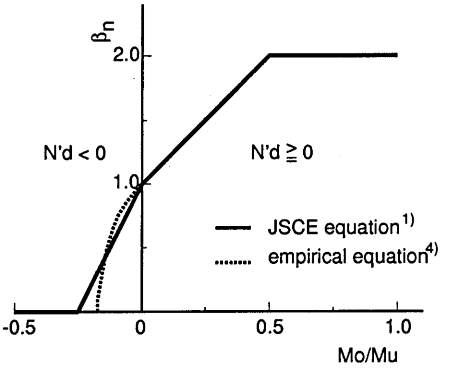


Fig.1 Influence of Axial Tension on Shear Strength of RC Member

$\sim$4.0) confirm that the specified equation is safe for certain applications in which the shear span to effective depth ratio is greater than 3.0. However, when the axial tensile stress exceeds the $30kgf/cm^2$ the safety of design values calculated by this equation deteriorate if the shear span to effective depth ratio is less than 3.0(a/d=1.75$\sim$3.0)[4].

An empirical formula for evaluating the effects of axial force within the experimental region has been given in previous reports by the authers:

$$\beta_n = \frac{0.0143}{M_0/M_u - 0.18} + 1.08(N'_d < 0) \tag{5}$$

This equation is a hyperbolic function (as shown by the broken line in Fig.1) based on the concept of the decompression moment. However, recent experimental studies indicate that for beams with comparatively poor longitudinal reinforcement and small shear span to effective depth ratio, this proposed equation requires further investigation. Furthermore, the results of a finite element approach to this problem, using isoparametric degenerated shell elements [5][6], show that the axial tensile force governs the fracture process of in such beams and their shear capacity [7]. On the other hand, in a recent paper, Collins et al. report that the shear capacity of RC member subjected to axial tension is affected by the total amount of longitudinal reinforcement[8]. Against this background, the purpose of this paper is to improve the accuracy of the design equation by introducing a modified formula for the effects of axial tension in the design equation using experimental and numerical techniques. Our previous experimental results for rectangular beams[2] and T-shaped beams[4] as well as the results of Mattock et al.[9] are used to verify the accuracy of the new equation considering the relation of shear span to effective depth and the axial tensile force, simultaneously.

2. EXPERIMENTAL PROGRAM

In the previous experimental study, rectangular beams shown in Fig.3 and the T-shaped beams shown in Fig.4, were tested. The test apparatus comprised the longitudinal and lateral actuators shown in Fig.2. On the basis of the results, we discussed the relation between applied axial force and shear span to effective depth ratio, as well as parameters influencing the shear strength of the beam. The reinforcement ratio p_w of the beam was held constant in every test. In this new study, on the other hand, the applied axial force and shear span to effective depth ratio are fixed, and the reinforcement ratio is changed in parametrically (as shown in Fig.3, Type C). The dimensions of the beam tested by Mattock et al., verifying the new equation, are shown Type-B in Fig.3 and Type D in Fig.4, respectively. The experimental procedure of this paper, Type C in Fig.3, is outlined as follows.

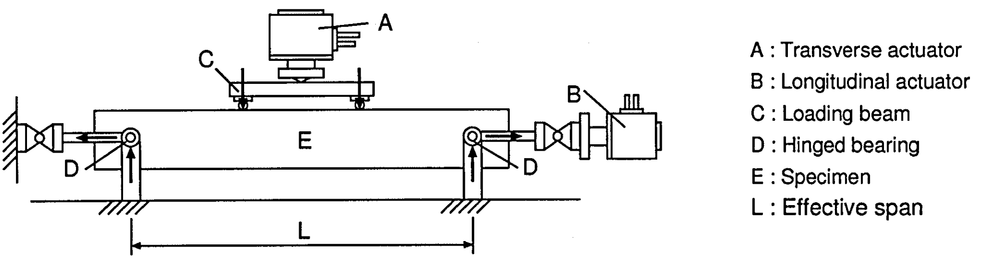


Fig.2 Test Apparatus

Table 1 Details of Specimens used in Tests

Beam No.	p_w	N	f'c	Pb	Mb	Mu	Mb/Mu	Vb	Vyd0	Vb/Vyd0	Remarks
1	0.0075	2.0	398	4.50	121500	104551	1.16	2.25	2.27	0.99	M
2	0.0100	2.0	429	4.90	132300	134262	0.99	2.45	2.53	0.97	S
3	0.0125	2.0	424	5.75	155250	165222	0.94	2.88	2.77	1.04	S
4	0.0150	2.0	373	5.75	155250	183125	0.85	2.88	2.72	1.06	S
5	0.0175	2.0	378	6.25	168750	241570	0.70	3.13	3.01	1.04	S
6	0.0200	2.0	377	6.00	162000	267333	0.61	3.00	3.03	0.99	S
7	0.0075	4.0	414	3.75	101250	101615	1.00	1.88	2.19	0.86	M
8	0.0100	4.0	465	3.25	87750	134873	0.65	1.63	2.51	0.65	S
9	0.0125	4.0	339	4.10	110700	162216	0.68	2.05	2.44	0.84	S
10	0.0150	4.0	443	4.88	131760	206708	0.63	2.44	2.87	0.85	S
11	0.0175	4.0	390	5.38	145260	237946	0.61	2.69	3.06	0.88	S
12	0.0200	4.0	377	4.88	131760	252040	0.52	2.44	2.93	0.83	S

N : Axial tension (tf) f'c : compressive strength of concrete (kgf/cm²)

Mb : Ultimate moment [experiment] (kgfcm) Mu : Ultimate moment [calculation] (kgfcm)

Vb : Shear at ultimate load [experiment] (tf) Pb : Ultimate load [experimental result] (tf)

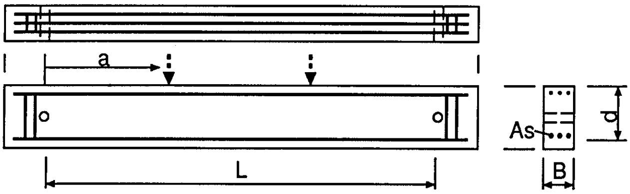
M : Bending failure S : Shear failure Vyd0 : Shear at ultimate load [calculated result by eq.(5)] (tf)

2.1 Experimental Procedure

In this experimental study, twelve specimens were tested under several parameters. The shear span to effective depth ratio was a/d=3.0 since the decrease in shear capacity was large. The applied axial force was 2tf or 4tf, and reinforcement ratio ranged within the range from 0.0075 to 0.02. The loading capacity and the behavior of the specimens was examined.

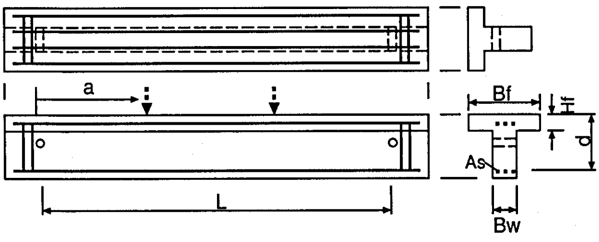
2.1.1 Test beams

The dimensions of the beams and the reinforcement arrangements are shown in Fig.3. These are rectangular beams. All specimens are 180cm long and the effective span is L=150cm. The width and height are B=10cm and H=20cm, respectively. At the ends the specimens have holes ($\phi = 24mm$) for the tag through which axial force is introduced. The general structural steel rods ($\phi 8mm$) are placed as necessary to achieve tensile reinforcement and the half of them are placed for the compressive reinforcement. The cover depth is 1.5cm. In the previous study, deformed bars were used; however, in this analysis slim round bars were used because the reinforcement ratio must be changed parametrically. Consequently, some differences arise in the results of maximum strength for each member. However, the influence of axial force on the shear strength takes the same form. No stirrups are placed to give shearing reinforcement



Type	a/d	p_w	B (cm)	d (cm)
A	1.75 ~4.00	0.011	10.0	18.0
B	2.74 ~5.14	0.010 ~0.031	15.2	30.5
C	3.00	0.005 ~0.020	10.0	18.0

Fig.3 Specimens (rectangular beams)



Type	a/d	p_w	Bw (cm)	Bf (cm)	d (cm)	Hf
A	2.00 ~2.50	0.025	8.0	24.0	21.0	6.0
B	1.50 ~2.50	0.025	8.0	24.0	21.0	4.0
C	1.50 ~2.50	0.012	8.0	24.0	21.0	6.0
D	2.50 ~6.00	0.034	17.8	61.0	38.1	10.1

Fig.4 Specimens (T-beams)

so as to clarify the shear behaviour of the model. The concrete was from 6 to 8 weeks of age at the time of the bending tests. Also, the mechanical properties of the concrete are shown in Table 1.

2.1.2 Test apparatus and procedure

The test apparatus was the same as that used in the previous experiment. Axial tension was introduced into the beams through the hinged bearing at the beam ends via a longitudinal actuator to avoid eccentricity. Once the axial tension reached the desired tensile force, it was held constant. Then the transverse load was applied by a transverse actuator. The load was distributed onto two points by the loading beam, and was increased under in a load controled way. The transverse load increased under a displacement controll system. The transverse load was increased until the beam failed. The load and bending strain of the reinforcement were measured and new cracks were marked on the beam faces at every displacement stage.

2.2 Experimental Results and Considerations

The ultimate load P_b and ultimate moment M_b of all specimens are shown in Table 1 along with the theoretical values. Failures are classified into two types corresponding to the failure state of the beam. Shear failures occurred due to the growth of shear cracks before the flexural

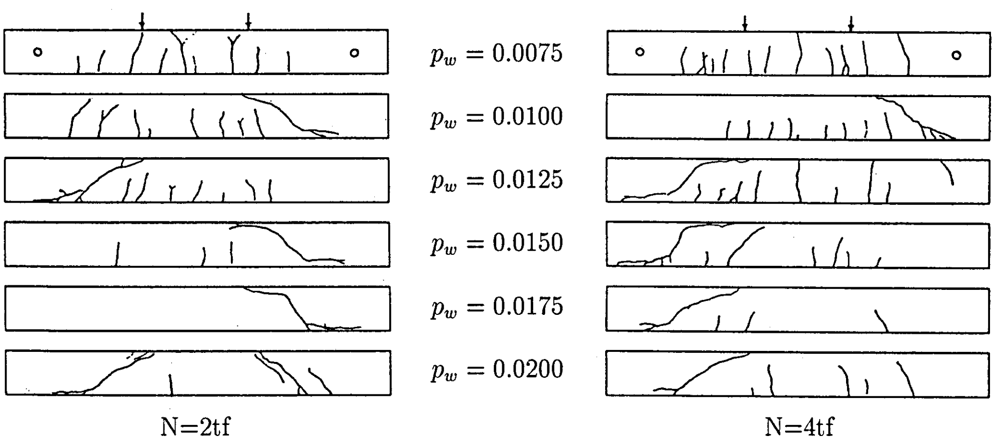


Fig.5 Failure Modes of Beams

reinforcement yielded, while flexural failures occured due to yielding of the flexural reinforcement. These failure types are denoted by (S) and (M) in Table 1, respectively.

2.2.1 Modes of failure

Figure 5 illustrates the cracking patterns on beams at the final loading stage. It is evident from this figure that when the reinforcement ratio of the beam is 0.0075, flexural failure occurs in the beam at both levels of axial force. This confirms that the flexural reinforcement yields before the beam fails. When the reinforcement ratio of the beam is greater than 0.01, the shear cracks appeared. However, the failure of the beam is then caused not by shear failure but by bending-shear failure, and the beam does not fail unexpectedly; the smaller the reinforcement ratio, the greater the number of flexural cracks. In other words, the greater the reinforcement ratio, there are the more unexpected occurrence of shear failure. The higher axial tension, the more flexural cracks appear and the narrower their spacing.

2.2.2 Shear capacity

Figure 6 shows the load-carrying capacity of the beams. The ordinate is the ratio of breaking moment M_b to ultimate moment M_u, as calculated by ultimate theory. The abscissa represents the reinforcement ratio. (The data for bending failure is not shown.) This figure indicates that the larger the reinforcement ratio is, the smaller the shear capacity. However, the smaller the reinforcement ratio is, the greater the loss in shear capacity under axial tensile loading.

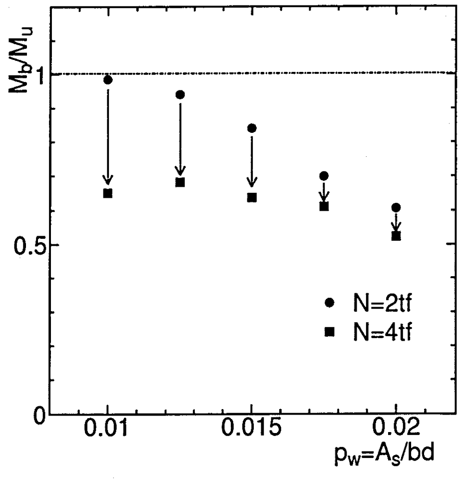


Fig.6 Relationships between M_b/M_u and Reinforcement Ratio (Experimental Results)

3. INFLUENCE OF REINFORCEMENT RATIO ON THE SHEAR CAPACITY OF RC MEMBERS SUBJECTED TO AXIAL TENSION

3.1 The Effects of Reinforcement Ratio

The design equation for shear failure of RC beam under axial tension the JSCE's Specifications for Concrete is based on Zsutty's curve[10] for the reinforcement ratio term from the typical study[9]-[12] as shown in Fig.7. The bold line in Fig.7 shows the result of finite element analysis by the authors. Figure 8 shows numerical results by the finite element method which clarify the effects of axial tension on shear failure. It is clear that the axial tension affects the relationship between reinforcement ratio and shear capacity. Furthermore, when the reinforcement ratio is less than 0.015, the value of shear capacity rapidly falls for every axial tension. The relationship between the function β_n expressing the influence of axial tension, as calculated by finite element analysis, and M_0/M_u are shown in Fig.9 with various reinforcement ratios. From these lines, the influence of axial tension on the shear capacity of the member for any reinforcement ratio is approximately expressed by following function:

$$\beta_{p_w-n} = 1 + \frac{M_0}{M_u}(2.5 - 100p_w) \quad (6)$$

When the reinforcement ratio $p_w > 0.015$, M_0/M_u does not exceed 0.1, even if the member

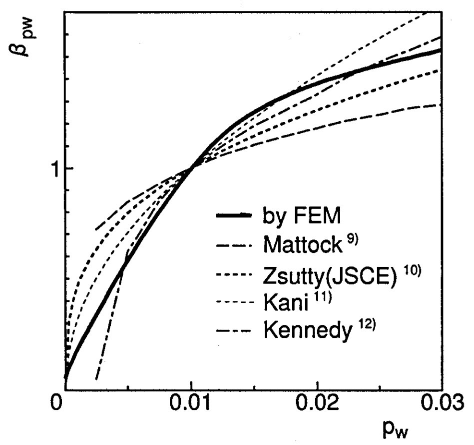


Fig.7 Influence of Reinforcement Ratio on Shear Capacity of RC Member

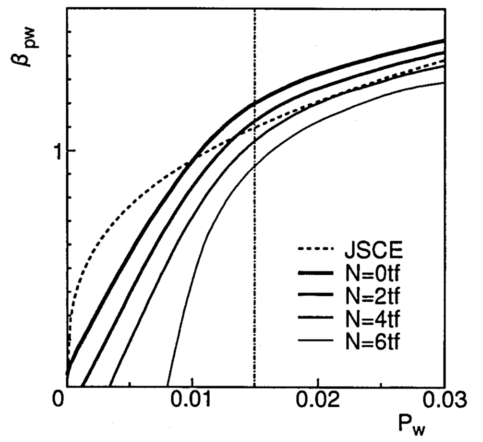


Fig.8 Relationships between Shear Capacity and Reinforcement Ratio

is subjected to a large axial tension. (Though in particular cases, some cracking occurs as a result of the axial tensile force.) The tangents of the lines in Fig.9 are approximately the same. From this viewpoint, it can be concluded that the influence of reinforcement ratio is negligible in this region and the function β_{p_w-n} can be written as Eq.(7).

$$\beta_{p_w-n} = 1 + \frac{M_0}{M_u} \quad (p_w \geq 0.015) \tag{7}$$

3.2 Verification of the Proposed Equation

The results of our own experiments and those of Mattock are used to confirm the proposed equation. Figure 10 shows the relation between M_0/M_u and the shear capacity V_b/V_{yd0}. The shear capacity V_b/V_{yd0} is obtained from the experimental data divided by the shear capacity

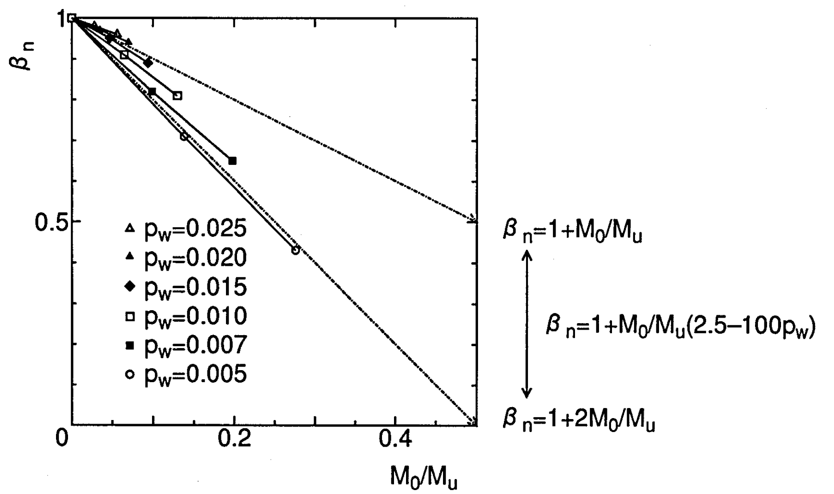


Fig.9 Influence of Axial Tension for Shear Capacity of RC Member at each Reinforcement Ratio (by FEM)

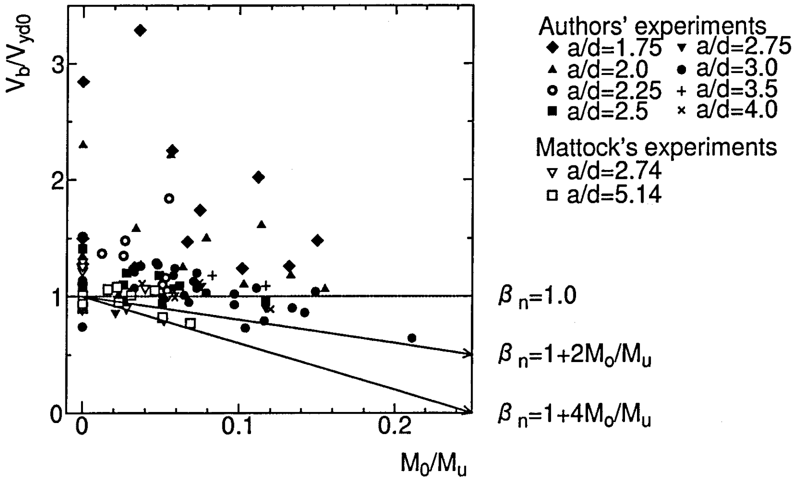


Fig.10 Relationships between V_b/V_{yd0} and M_0/M_u (for Experimental Results an Rectangular Beams)

proposed by Niwa et al.[13] for each shear span to effective depth ratio. (Where a/d is defined as 5.6 in calculating Eq.(8).)

$$V_{yd0} = 0.94\sqrt[3]{f'_{ck}}\sqrt[3]{100p_w}\sqrt[4]{100/d}(0.75 + \frac{1.4}{a/d}) \cdot b_w d \qquad (8)$$

The function expressing the influence of axial tension by using the concept of decompression moment, as adopted in Specification for Concrete of JSCE, is shown by arrows in Fig.10. This shows that the JSCE equation overestimates the shear capacity of almost all members. However, the smaller the shear span to effective depth, the more the experimental result/calculated

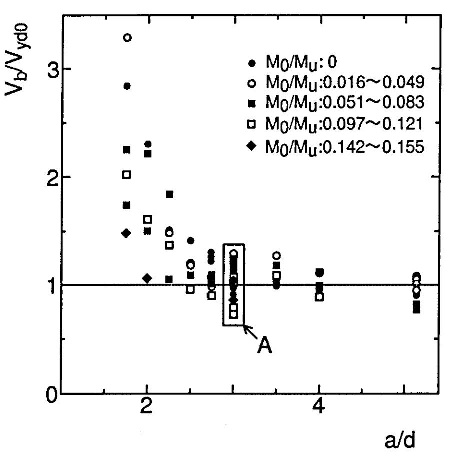


Fig.11 Relationships between V_b/V_{yd0} and a/d (for Rectangular Beams)

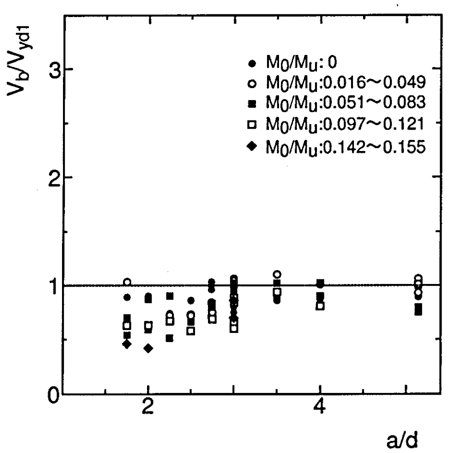


Fig.12 Relationships between V_b/V_{yd1} and a/d (for Rectangular Beams)

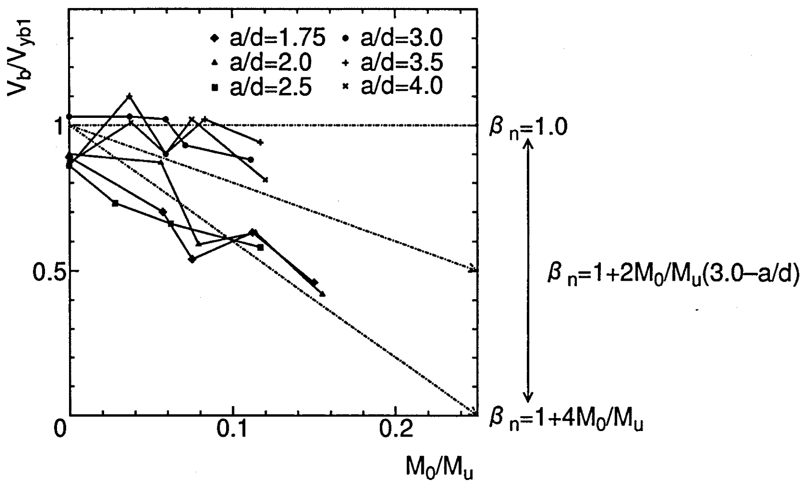


Fig.13 Influence of Axial Tension on Shear Capacity of RC Members for each a/d (Experimental Results)

value (V_b/V_yd0) diverges from β_n, because the term of shear span to effective depth ratio is rearranged as $a/d = 5.6$. Figure 11 shows the Fig.10 in another form. The abscissa is the shear span to effective depth ratio. Each data point is plotted as a dimensionless factor M_0/M_u expressing the magnitude of the applied axial tension. From this figure, it is evident that the shear capacity of a member rapidly falls in the region where the shear span to effective depth ratio is less than 3.0. These observed values also differ considerably from the shear capacity (Eq.(8)) at the point where the shear span to effective depth is a/d=3.0.

Before analyzing of these data using the previously proposed equation (6), the data are first analyzed by the function on the shear span to effective depth ratio. Subsequently, the data are analyzed for the influence of axial tension using a derivative function obtained from the results of experiments, as follows.

Firstly, the data are analyzed for shear span to effective depth ratio. The data between $2.75 < a/d < 6.0$ are analyzed using the reliable function given in Eq.(9) as proposed by Okamura and Higai[14], and those at $a/d \leq 2.75$ are analyzed using Eq.(10) as proposed by authors from an experimental study[4].

$$\beta_{a/d} = 0.75 + \frac{1.4}{a/d} \qquad (2.75 < a/d \leq 6.0) \tag{9}$$

$$\beta_{a/d} = \frac{9.1}{a/d} - 2.0 \qquad (1.75 \leq a/d \leq 2.75) \tag{10}$$

Figure 12 compares these shear capacities V_{yd1} with the experimental results V_b. It can be seen that the influence of axial tension changes in accordance with changes in the shear span to effective depth ratio when the ratio (a/d) is less than 3.0. Considering this, the relations V_b/V_{yd1} and M_0/M_u are then examined for each shear span to effective depth ratio, as shown in Fig.13. From this figure, the difference between V_{yd1} and V_b at each shear span to depth ratio as M_0/M_u changes can be written as Eq.(11).

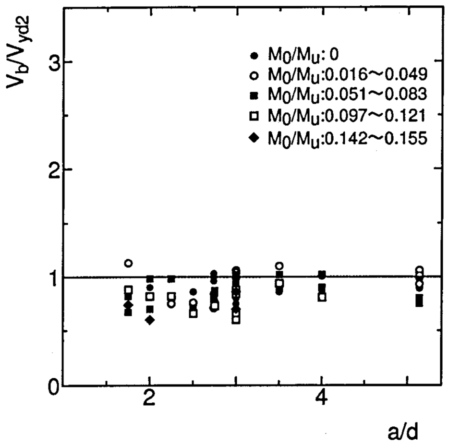


Fig.14 Relationships between V_b/V_{yd2} and a/d (Rectangular Beams)

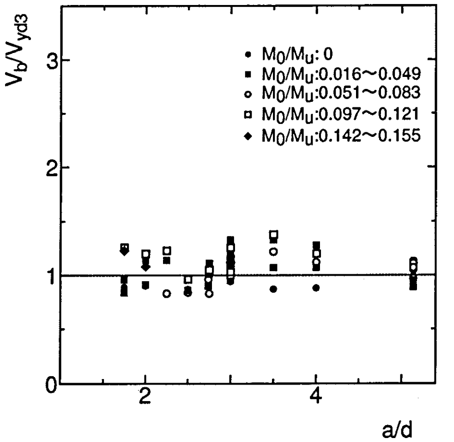


Fig.15 Relationships between V_b/V_{yd3} and a/d (Rectangular Beams)

$$\beta_{a/d-n} = 1 + 2\frac{M_0}{M_u}(3.0 - a/d) \tag{11}$$

If a/d=1.0, the Eq.(11) becomes equal to the previous JSCE equation (4). However, when $a/d > 3.0$, the influence of axial tension based on the shear span to effective depth ratio becomes constant. Therefore,

$$\beta_{a/d-n} = 1.0 \qquad (a/d > 3.0). \tag{12}$$

The comparison of V_{yd2}, obtained for V_{yd1} in considering Eq.(11) or Eq.(12), with the experimental results V_b is shown as Fig.14.

Finally, the results in Fig.14 are multiplied by the previous function β_{pw-n} of Eq.(6) or Eq.(7) for reinforcement ratio. The deviation of the average of $V_b/V_{yd3'}$ from 1.0 is found to be 0.15. Considering the differences for experimental results, Eq.(6) and Eq.(7) are replaced to Eq.(13) and Eq.(14) respectively, because Eq.(6) and Eq.(7) are obtained by finite element analysis. If Eq.(13) or Eq.(14) is used to calculate the shear capacity of the beam, then the deviation of the average of V_b/V_{yd3} from 1.0 becomes 0.06 (as shown in Fig.15).

$$\beta_{p_w-n} = 1 + 2\frac{M_0}{M_u}(2.5 - 100p_w) \tag{13}$$

$$\beta_{p_w-n} = 1 + 2\frac{M_0}{M_u} \quad (p_w \geq 0.015) \tag{14}$$

If $p_w = 0.005$, Eq.(13) is equivalent to the previous JSCE equation (4) if the coefficient of the axial tension factor is doubled for safety,

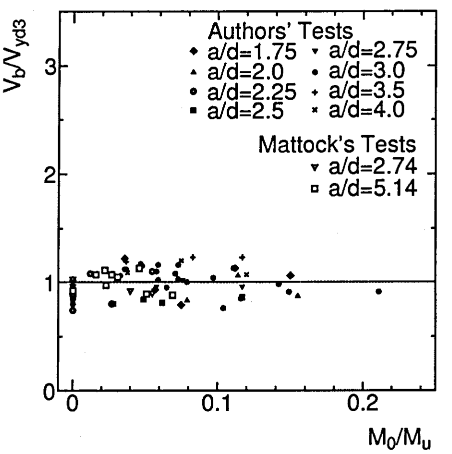


Fig.16 Relationships between V_b/V_{yd3} and M_0/M_u (Rectangular Beams)

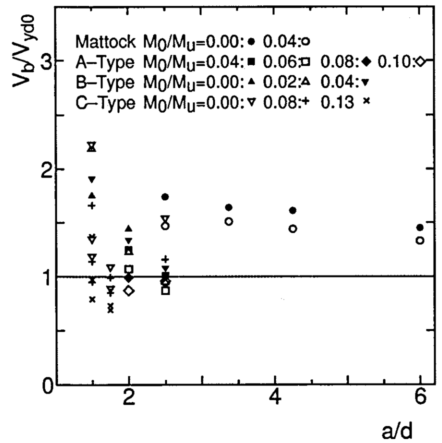


Fig.17 Relationships between V_b/V_{yd0} and a/d (for T-Beams)

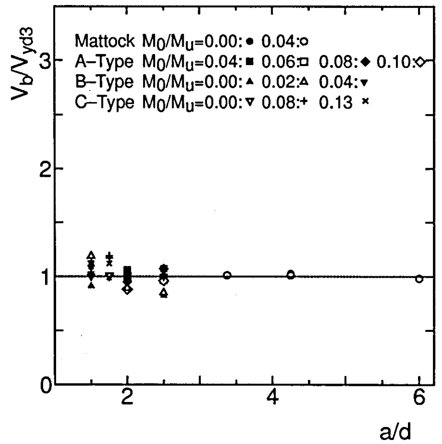


Fig.18 Relationships between V_b/V_{yd3} and a/d (for T-Beams)

and Eq.(14) becomes equal to the previous JSCE equation for members subject to axial compression. It is suggested that Eq.(13) and Eq.(14) reflect the deterioration when the influence of axial tension appears at reinforcement ratios $p_w > 0.015$. The accuracy of these equations is confirmed by the relation of V_b/V_{yd3} and M_0/M_u in Fig.16. These are in good agreement for each shear span to effective depth ratio. Figure 17 and 18 show the results of applying the above approach to a T-beam. (The analysis is omitted.) The equation is quite accurate as regards the results for T-beams obtained in our experiments and by Mattock. The function proposed by the authors and the JSCE function are shown in Fig.19. In the proposed equation, if the shear span to effective depth ratio is greater than 3.0, the influence of axial tension is equal to that of axial compression when the reinforcement ratio is greater than 0.015. If the shear span to effective depth ratio is less than 3.0, the smaller the reinforcement ratio is, the greater the influence of axial tension is. The functions match the present JSCE equation when the reinforcement ratio $p_w = 0.005$.

4. CONCLUSION

This work confirms that the present JSCE equation overestimates the experimental results, because the equation includes a safety factor for the shear span to effective depth ratio and the coefficient for axial tension is doubled to account for its disadvantageous influence. However, the JSCE equation is safe in some cases, because there are not discussed on the disadvantageous influence of the axial tension for the shear capacity of the member. The authors confirm that the deviation appears when the shear span to effective depth ratio is less than 3.0 and the reinforcement ratio is less than 0.015. In this paper, terms expressing the influence of axial tension as a function of reinforcement ratio are proposed as a modification of the JSCE equation. The proposed equation is shown to give an effective and accurate evaluation of shear capacity. The proposed equation also represents the shear capacity of a T-beam accurately.

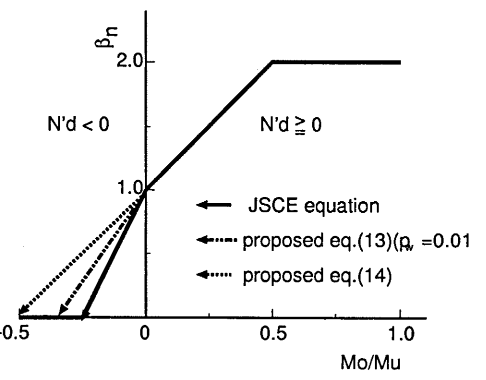


Fig.19 Influence of Axial Tension on Shear Capacity of RC Members considering Reinforcement Ratio

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