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REEVALUATION OF CURRENT EQUATIONS FOR FLEXURAL CRACK WIDTH OF RC BEAMS

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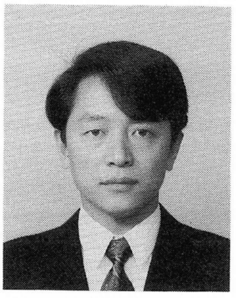

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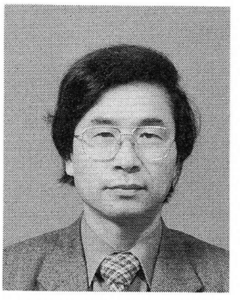

Kyuichi MARUYAMA

Current equations for the flexural crack width are first compared with each other and with the results of experiments on RC beams with multi-layered bar arrangements, and the characteristics of the equations and their problems are pointed out. Based on Kakuta's theory and the concept of local effects in stress distribution, new equations for calculating both crack spacing and crack width are then proposed. The proposed equations feature a definite physical interpretation, a wide range of applicability, simplified calculation, and an accuracy better than the present equations.

Key Words: *crack spacing, crack width, location of stress distribution localization, multi-layers of bars*

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1. INTRODUCRION

In the design of RC beams, it is necessary to not only ensure resistance to bending and shear from a safety viewpoint, but also to take into account the width of cracks that will occur in areas under bending-induced tension in terms of both durability and applicability. Although much research has already been carried out, no definite method of evaluating flexural crack width has been established because cracking is associated with a variety of factors.

The most crucial area of research into cracking-related problems can be considered that of crack spacing. As a result of active studies of flexural crack spacing on the upper and lower faces of RC beams, which have continued ever since the bond propagation theory was proposed by Saliger [1] in 1950, many empirical, semi-theoretical equations and stress distribution models have been proposed for determining the cracking mechanism. Current mainstream thinking, however, is to use empirical or semi-theoretical equations formulated from experimental data as the design equations.

Such design equations can be categorized into three groups according to the primary design parameters in use: Group 1 equations are based on the reinforcing bar diameter or steel-to-concrete ratio; Group 2 on the concrete cover; and Group 3 on both the reinforcing bar diameter/steel-to-concrete ratio and the concrete cover. Group 1 equations - which are also known as equations derived from classical theories based on the assumption that the bond strength of steel reinforcement is proportional to concrete tensile strength - have a simple and clear mechanical meaning. However, it has been shown that they do not agree well with experimental results [2], and that, in addition to tensile strength, bond strength depends on the concrete cover, the reinforcing bar diameter, and the effective concrete sectional area [3].

Group 2 equations, featured by simplified expression, incorporate the relationship between bond strength and these additional factors, but cannot account for the effects of the second or higher layers in multi-layered bar arrangements [4]. Group 3 equations, an improvement over Groups 1 and 2, suffer from no practical problems, but do not have a clear mechanical interpretation. Clearly, all the equations so far formulated for calculating crack width have room for improvement. There are problems that need to be resolved in terms of applicability range, consistency between mechanical models and experimental results, and rationality of the mechanical model.

The first part of this study consists of a comparison of typical equations and methods currently in use for calculating crack width. Further, the results are compared against experiments on RC beams with multi-layered bar arrangements [4], leading to clarification of the problems with the equations. Using the results of past studies as a basis, a new method of evaluating crack spacing is then structured through an approach in which a distinction is drawn between the local and overall effects of the stress distribution. The resulting new equations are able to more accurately calculate crack spacings on RC beams, not only in a single-layered bar arrangement, but also where the bars are multi-layered. Finally, a new method of calculating flexural crack width is proposed based on these equations.

2. CURRENT EQUATIONS FOR CALCULATING CRACK WIDTH

Cracks on RC beams are classified - according to the conditions under which cracking occurs - into two types: cracks in the initial and normal phases. For practical reasons, this paper considers lateral cracking in the normal phase. Furthermore, since flexural crack widths vary with height in the beam cross section, this paper focuses on the crack width on the side of a beam at the height of the first reinforcing bar layer.

A vast majority of the equations proposed in recent years belong to Groups 2 and 3, and five typical examples of these are described below. For easy comparison and description, standard nomenclature is used wherever possible.

2.1 JSCE Concrete Standard Specifications [5]

$$W_{max} = k_1(4c_{min} + 0.7e)(\sigma_s / E_s + \varepsilon'_{cs}) \qquad (1)$$

where W_{max} = maximum crack width on reinforcing bars; k_1 = coefficient representing the effects of reinforcing bar bonding characteristics and generally equal to 1.0 for deformed bars and 1.3 for ordinary round bars and prestressing steel; c_{min} = concrete cover on the side or at the bottom of the RC beam, whichever is smaller; e = clearance between reinforcing bars; σ_s = increase in tensile stress in tension bars in a cross section where cracking occurs; E_s = Young's modulus of reinforcing bars; and ε'_{cs} = strain to take into account an increase in crack width due to drying shrinkage and creeping of concrete, for which 150 x 10-6 may be generally adopted.

2.2 ACI 318-83 [6]

$$W_{max,\,b} = 0.00108\,\beta (c_1 A_e)^{1/3} \cdot \sigma_s \times 10^{-3} \qquad (2)$$

where $W_{max,\,b}$ = maximum crack width at the bottom of the RC beam; c_1 = distance between bottom surface of the beam and the center of the nearest tensile reinforcing bar; A_e = Acel/m = effective concrete sectional area per tension bar, or the sectional area of concrete having the same centroid as that of tensile bars divided by the number of tensile bars, m; and β = ratio of distance between a neutral axis and the bottom surface to distance between the neutral axis and the centroid of tensile reinforcing bars (equal to about 1.2 for beams), and where this equation is derived by changing the unit of σ_s from ksi to kgf/cm^2.

2.3 CEB-FIP Model Code 1978 [7]

$$W_k = 1.7 L_{av}\, \varepsilon_{s,\,av} \qquad (3)$$

$$L_{av} = 2(c_b + 0.1 e_o) + k_2 k_3\, \phi / p_{e2} \qquad (4)$$

$$\varepsilon_{s,\,av} = \{1 - \beta_1 \beta_2 (\sigma_{sr} / \sigma_s)^2\}\sigma_s / E_s > 0.4\,\sigma_s / E_s \qquad (5)$$

where W_k = crack width characteristic value (a width which 95% of cracks do not exceed); L_{av} = average crack spacing; $\varepsilon_{s,\,av}$ = average increase in strain in reinforcing bars; c_b = concrete cover at bottom of the beam (note: since this code does not provide a clear interpretation of this concrete cover, and the adequacy of Eq. (4) worsens over the rage of data collected in this study when c_b is regarded as the concrete cover on the side of the beam, c_b is assumed in this paper to be the concrete cover at the bottom); e_o = spacing between reinforcing bars, equal to 15 ϕ when $e_o > 15\,\phi$; ϕ= diameter of a reinforcing bar; $p_{e2} = A_s/A_{ce2}$ = effective steel-to-concrete ratio; A_s = cross sectional area of steel reinforcement; A_{ce2} = effective sectional area of concrete, i.e. cross sectional area of concrete within a distance of 7.5 ϕ from the center of each reinforcing bar; k_2 = coefficient relating to the bonding characteristics of reinforcing bars, equal to 0.4 for deformed bars and 0.8 for round bars; k_3 = coefficient relating to the shape of the strain distribution, equal to 0.25 for pure tension and 0.125 for bending; σ_{sr} = increase in stress in reinforcing bars at a cross section where cracking is assumed when tensile fiber stress becomes equal to concrete tensile strength; β_1 = coefficient relating to the bonding characteristics of reinforcing bars, equal to $(1/2.5)k_2$; β_2 = coefficient taking into account load characteristics, equal to 1.0 for initial loads and 0.5 for continuous and repeated loads.

2.4 PRC Specifications by the Architectural Institute of Japan [6]

$$W_{av} = L_{av}\, \varepsilon_{s,\,av} \qquad (6)$$

$$W_k = 1.5\, W_{av} \qquad (7)$$

$$L_{av} = 2(c_{av} + 0.1 e_o) + 0.1\, \phi / p_{e1} \qquad (8)$$

$$\varepsilon_{s,\,av} = \{\sigma_s - \beta_3 \beta_4 \sigma_t / p_{e1}\} / E_s > 0.4\,\sigma_s / E_s \qquad (9)$$

$$\beta_3 \beta_4 = 1/(2{,}000\,\varepsilon_{s,av} + 0.8) \tag{10}$$

where W_{av} = average crack width; $c_{av} = (c_b + c_s)/2$ = mean value of concrete cover at the bottom of the beam, c_b and that on the side, c_s; $p_{el} = A_s/A_{cel}$ = effective steel-to-concrete ratio, which should be calculated based on the effective tensile sectional area having the same centroid, A_{cel}; σ_t = tensile strength of concrete; and β_3 and β_4 = coefficients representing the mean value of average concrete tensile stress in the axial direction.

2.5 Kakuta's equation [3]

$$W_{av} = L_{max}\{\sigma_s/E_s - \sigma_{cm}/(E_s \cdot p_{el}) - \varepsilon_\phi\} \tag{11}$$

$$\text{For } e/c_{av} \leqq 2.5 \qquad L_{max} = k_4\, c_{av} \tag{12}$$

$$\text{For } e/c_{av} > 2.5 \qquad L_{max} = k_4\, c_{av}(1 + 0.18\, e/c_{av})/1.45 \tag{13}$$

where L_{max} = maximum crack spacing; k_4 = coefficient taking into account the bonding characteristics of reinforcing bars, equal to 5.4 for lateral node deformed bars; σ_{cm} = reduction in stress in reinforcing bars due to bonding between cracks, which is converted into a mean tensile stress over an effective concrete cross sectional area and is equal to 0.4 σ_t for temporary loads, 0 for continuous loads, 0.2 σ_t for repeated upper limit loads, and -0.2 σ_t for repeated lower limit loads; $p_{el} = A_s/A_{cel}$ = effective steel-to-concrete ratio, which should be calculated based on the effective tensile sectional area having the same centroid, A_{cel}; ε_ϕ = difference in elastic strain between reinforcing bars and concrete due to shrinkage and creep.

Crack widths are determined from the difference in elongation between the steel reinforcement and the concrete between cracks. As the above equations make clear, this is generally given by the product of a term relating to crack spacing and one relating to strain. Furthermore, there are two methods of calculating the maximum crack width, and this becomes significant in the design process; in one, the average crack width, W_{av}, is found based on the average crack spacing, L_{av}, and then the maximum crack width is obtained by multiplying the result by an extra factor (Method 1); the other entails obtaining the maximum crack width, W_{max}, directly from the maximum crack spacing, L_{max}, (Method 2.)

The CEB-FIP Model Code 1978 (abbreviated as the CEB-FIP equation) and the PRC Specifications by the Architectural Institute of Japan (the AIJ equation) use Method 1, while ACI 318-83 (the ACI equation), Kakuta's equation, and the JSCE's Standard Specifications (the JSCE equation) adopt Method 2. Although the factor used in Method 1 generally ranges in value from 1.4 to 1.5, the CEB-FIP's equation uses a value of 1.7 because of the large standard deviation in crack widths, about 0.4, obtained in experiments by Rusch et al [9]. The calculation of crack width is carried out at the bottom of the beam in the ACI's equation, and on the side where longitudinal reinforcing bars lie in the other equations.

Table 1 Summary of equations for calculating average crack spacing

JSCE equation	$L_{av} = 2.76c_{min} + 0.48e$
ACI equation	$L_{av} = 1.88(c_1 A_{cel}/m)^{1/2}$
CEB-FIP equation	$L_{av} = 2(c_b + 0.1e_o) + 0.2A_{ce2}/u$
AIJ equation	$L_{av} = 2(c_{av} + 0.1e_o) + 0.4A_{cel}/u$
Kakuta's equation	$L_{max} = 3.72c_{av}$ (for $e/c_{av} \leqq 2.5$)
	$L_{max} = 2.57c_{av} + 0.48e$ (for $e/c_{av} > 2.5$)

Note: The JSCE, ACI, and Kakuta's equations assume that Lmax/Lav = 1.45.

Generally speaking, the major difference between various researchers' approaches to cracking is how they formulate the equations in the crack spacing problem. This paper also studies the problem of crack spacing. Although the location at which a maximum crack width is calculated and the method of calculating the maximum width vary in these approaches, this seems to have no effect on average crack spacing results. Accordingly, equations representing the average crack spacing are extracted as summarized in Table 1 by assuming $W_{max} / W_{av} = L_{max} / L_{av}$, and $L_{max} / L_{av} = 1.45$ in the JSCE ACI, and Kakuta's equations. In Table 1, the notation ϕ / p_{e1} (or p_{e2}) is changed to A_{ce1} (or A_{ce2}) / u, where u = total circumference of tension reinforcing bars.

3. COMPARING THE EQUATIONS WITH EXPERIMENTS ON BEAMS WITH MUTI-LAYERED BAR ARRANGEMENTS

The differences between the equations, as well as the problems they suffer, stand out more when the bar arrangement is altered. The authors compare the equations with the results of experiments they carried out on beams with multi-layered bar arrangements. Details of the experiments themselves are given in reference [4]. The cross-sectional dimensions of the experimental specimens and their bar arrangement are shown in Fig. 1, and the results of the experiments, for crack spacing and width only, are summarized in Table 2.

Figure 2 shows a comparison between measurements and calculated values: average crack spacings are given in Fig. 2-a) and maximum crack spacings in Fig. 2-b). Figure 2-a) clarifies the overall behavior of the equations, though it is not always appropriate to compare calculated values directly with measurements in the case of a limited number of specimens since the JSCE, ACI, and Tsunoda equations assume Lmax/Lav = 1.45.

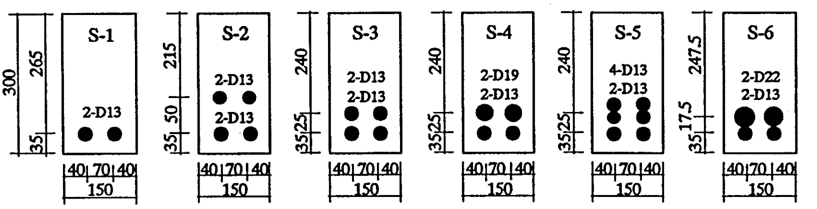

Fig. 1 Cross-sectional dimensions and bar arrangement of specimens

Table 2 Measured crack spacing and width

Specimen	Crack Spacing (mm)			Crack Width* (mm)		
	Ave.	Max.	Max./Ave.	Ave.	Max.	Max./Ave.
S-1	117	167	1.43	0.123	0.186	1.51
S-2	102	140	1.37	0.112	0.159	1.42
S-3	96	148	1.54	0.105	0.184	1.75
S-4	90	143	1.59	0.098	0.150	1.53
S-5	94	127	1.35	0.107	0.160	1.50
S-6	85	130	1.53	0.099	0.147	1.48
Average	97	143	1.47	0.107	0.164	1.53

* Indicates crack width (analytical value) when the reinforcing bar strain in the lowest layer is es = $1{,}200 \times 10^{-6}$

As is clear from Fig. 2, the measurements and values calculated using the ACI, CEB-FIP, and AIJ equations vary as the bar arrangement is altered. In contrast, values calculated using the JSCE and Kakuta equations are almost constant regardless of the bar arrangement; this is because they focus on the first-layer reinforcing bars. In particular, the JSCE equation may underestimate the reinforcing bars in the first layer or overestimate the multi-layered bar arrangement.

On the other hand, the ACI, CEB-FIP, and AIJ results follow the measurements fairly closely as regards the effect of the multi-layered bar arrangement; this is because they adopt an effective concrete cross-sectional area, Ace1 (or Ace2), as a parameter. Furthermore, as shown in Fig. 2-b), although the difference between the JSCE & Kakuta's equations and the CEB-FIP & AIJ equations is small when the steel-to-concrete ratio is large, they diverge - with the former tending to fall short - as the ratio falls or as the bar arrangement approaches a single-layered arrangement.

Figure 3 shows the results of a close look at the effect of effective concrete cross-sectional area, Ace1 (or Ace2), in the ACI, CEB-FIP, and AIJ equations. In this figure, average values with little scatter are plotted as measurements. The bold shaded lines in the figure represent the measured average crack spacings, and the solid lines show the gradient of each equation.

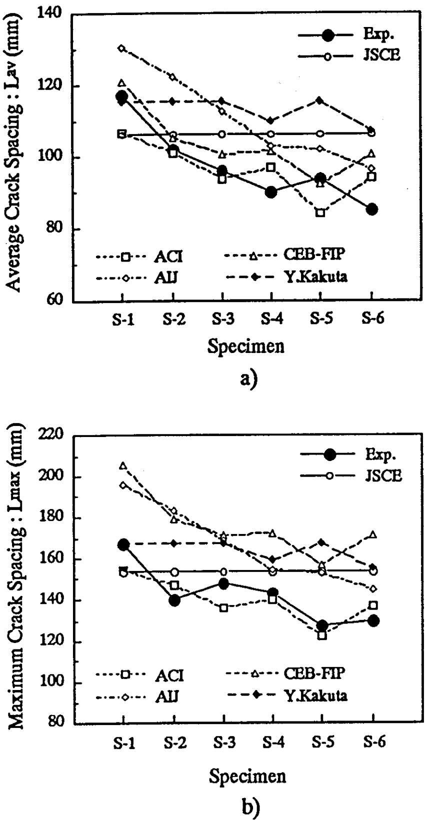

Fig. 2 Comparison between measurements and values calculated using the equations

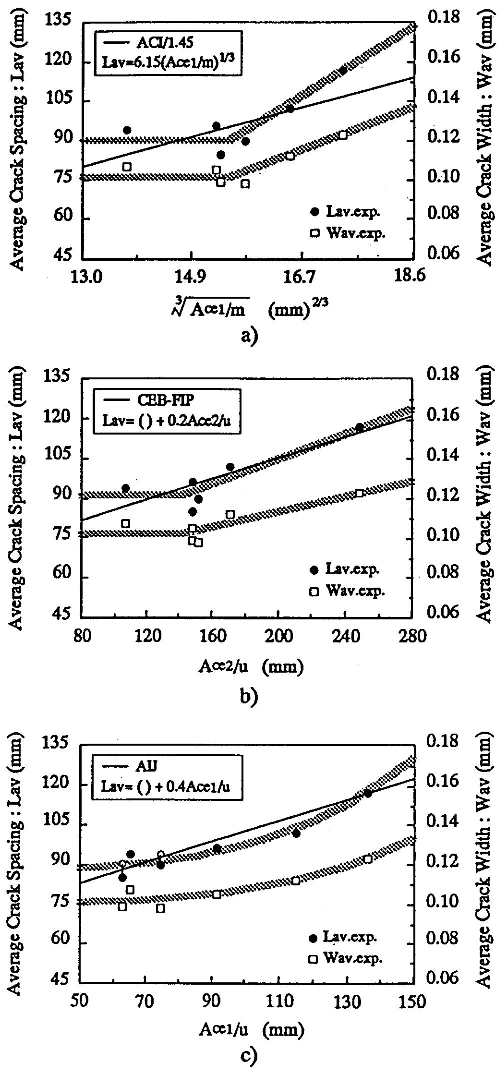

Fig. 3 Effects of effective concrete sectional area on crack spacings and widths

It is apparent from this figure that the CEB-FIP and AIJ equations, in which the effective concrete sectional area is determined by dividing the concrete sectional area by the total circumference, u, are more suitable than the ACI equation, in which the effective area is found by dividing the area by the number of reinforcing bars, m. However, none of these linear equations seems to satisfactorily reflect the experimental trend. This seems to be because the distribution of stress in the concrete section, which varies depending on characteristics of the multi-layered bar arrangement such as the vertical spacing between reinforcing bars and the reinforcing bar diameter, is not well expressed by Ace/u. In general, the changes in crack width are less pronounced than those in crack spacing, which indicates that concrete between the cracks has a confining effect.

4. BASIC EQUATIONS REPRESENTING CRACK SPACING

The form of equations representing crack spacings differs depending on theories adopted. Crack spacings on beams in single-layered bar arrangement are given by Kakuta's theory [3] with satisfactory accuracy. In multi-layered bar arrangement, however, crack spacings are affected by the arrangement of reinforcing bars in the second or higher layers, as shown in Fig. 2. Since both Kakuta's equation and the JSCE equation (which is formulated on the basis of Kakuta's equation) take into account only a single-layered bar arrangement, calculations lose accuracy when they are applied separately to single- and multi-layered bar arrangements, even if cracks in actual structures are taken into account in fixing the coefficients and constants in the equations.

Further, since concrete cover is considered one of the major factors influencing crack spacing, the choice of which cover value to use is significant. The JSCE equation takes the smaller of the concrete cover on the side, cs, and that at the bottom, cb. In relation to this, it has been pointed out in a study by Kakuta et al. [10] that the concrete cover on both the side and the bottom may have an effect on crack spacing. Thus the suitability of an equation becomes very questionable when only the concrete cover at the bottom is considered, according to a study by Suzuki et al. [2]. The Kakuta and JSCE equations take the average of concrete cover depths on the side and at the bottom. In contrast with the case of a bi-directional tensile test, when a beam with multiple reinforcing bars in a horizontal plane is subject to bending, it can be expected that the concrete cover at the side does not always play the same role as that at the bottom; there is thus a need to clarify the role of each.

The effects of the ratio of these two measures of cover is investigated by looking at the results of experiments carried out by Hognestad [11] (specimens 29, 30, and 31). Figure 4 is a comparison of average crack spacings in these experiments with those calculated using the JSCE equation and listed in Table 1.

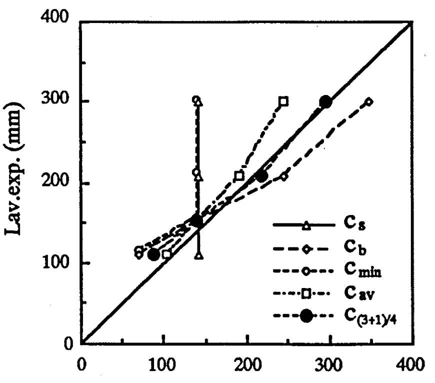

Fig. 4 Comparison of concrete cover values used in calculation

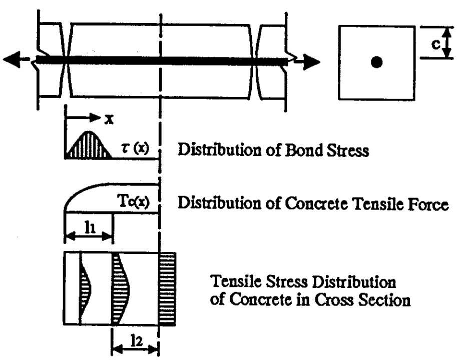

Fig. 5 Cracking mechanism proposed by Suzuki et al.[12]

As is evident from this figure, crack spacings calculated based on $c = (3cb + cs)/4$ match the measured values more closely than those calculated from only one value of concrete cover (at the side or on the bottom) or those based on the average of the two concrete cover values.

On the other hand, the CEB-FIP equation is a modified version obtained by adding the effects of concrete cover and the reinforcement spacing into Saliger's theoretical equation. The AIJ equation is obtained by simplifying and modifying the CEB-FIP equation; such as by replacing Ace2 with Ace1 in the CEB-FIP equation [2]. As shown in Fig. 5, Suzuki et al. [12] interpret Eq. (8) physically as follows: the second term of the equation represents the distance, l_1, along the steel reinforcement from the crack section at which the summed bond stress between concrete and reinforcement reaches the sectional tensile stress, Te(x), corresponding to the concrete strength at that cross section; the first term of the equation represents the distance, l_2, at which the sectional tensile stress, Tc(x), is distributed uniformly within the cross section.

As a result of our review of Eq. (8) using 37 specimens with a single-layered bar arrangement, however, the average ratio of distance l_2 to concrete effective sectional height, $(2c + \phi)$, is 1.91. This indicates that Suzuki et al.'s interpretation of the first term of the equation is contrary to St. Venant's principle, and that there is need for further study. The ACI equation, which is obtained through a statistical analysis of experimental results, is unclear in its mechanical meaning.

It is not always appropriate to apply a cracking mechanism based on a very simple model to a complex bar arrangement - such as a multi-layered bar arrangement - without modification. Local effects of stress distribution are present, and the cracking mechanism seems to be dominated by these local effects.

Figure 6 gives a schematic representation of tensile stress distribution patterns in concrete cross sections between cracks for three different bar arrangements. Stresses (1) and (2) in the figure indicate the stress when tensile forces transmitted by the first- and second-layered reinforcing bars are assumed to be distributed uniformly in the concrete cross section around the first- and second-layered reinforcing bars, respectively. (These are called local effects in this paper.) The mean stress in the figure refers to the stress when the total tensile force is distributed uniformly over the effective concrete cross section with the same centroid as that of all tensile reinforcing bars, Ace. (These are called overall effects in this paper.) The curves in the figure assume the actual stress distribution.

Figure 6 leads us to development of the following discussion: 1) According to the interpretation made by Suzuki et al., the stress distribution curves in Fig. 6 correspond to stress in the cross section near $x = l_1$ in Fig. 5. From the relation, $c_{a)} = c_{b)} < c_{c)}$, the distance (shown by the dotted line) at which the concrete stress becomes distributed uniformly over the area, Ace, is $l_{2a)} = l_{2b)} < l_{2c)}$. However, in consideration of the uneven distribution of stress, b) seems to be of the same order or larger than a) and c). The order of this distance seems to be $l_{2a)} \leqq l_{2b)} \leqq l_{2c)}$, which is difficult to explain using the Suzuki interpretation.

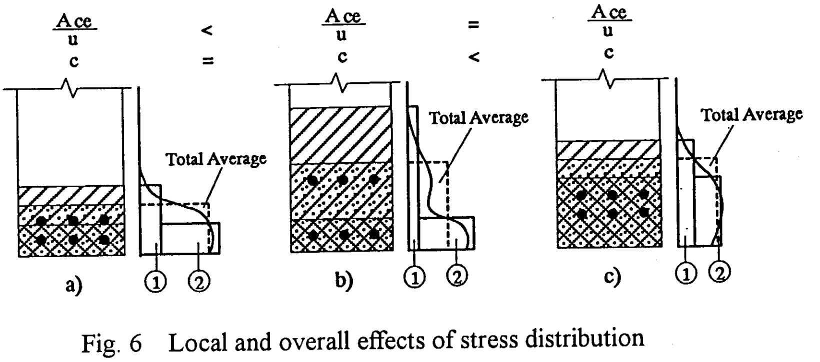

Fig. 6 Local and overall effects of stress distribution

2) Consequently, it can be considered that the concrete cover represents the local effects of stress distribution (the degree of stress concentration near beam ends), rather than distance at which the stress is distributed uniformly. Moreover, the crack spacing in multi-layered bar arrangements is dominated by the local stress due to the first-layered reinforcing bars, ②, and is affected by the stress contributed by the second- and higher-layered reinforcing bars, ①.

3) Reviewing the physical interpretation of the AIJ equation in accordance with discussion 2) above, the first term can be said to represent local effects and the second term the effects of reinforcing bars in the second and higher layers. As is clear from the ratio of stresses ① to ②, the effects of reinforcing bars in the second and higher layers are ordered as c) > a) > b), but as a) > b) = c) in the AIJ equation. Basically, the same is true in the CEB-FIP equation. Therefore, it can be concluded that Ace/u, representing the overall effects, is not a good index of effects of multi-layered bar arrangements.

Another important factor in deriving the basic equation for crack spacing is which of Methods 1 and 2 (described in Section 2) should be used in determining the maximum crack width - which may become an important issue at the design phase. Assuming that crack widths obey a normal distribution, the maximum crack width obtained can be expressed in the form of a probability, such as 5% or less, if the extra factor in Method 1 is determined statistically. On the other hand, in the case of the maximum crack width (a mean value) determined by Method 2, the risk for one specimen is 50%, but the probability of each crack is unclear.

Further, it has been made clear by the authors' in another study [4] that the shorter the relative length of a specimen the smaller the maximum crack spacing measured; that is, shape effects act on the maximum crack spacing. In addition, in contrast to Method 2 by which only one value each of Lmax and Wmax, which vary with the measuring point, is given for each specimen, Method 1 gives data uninfluenced by measuring point yet containing a large amount of useful information because it makes use of all data measured.

Based on the above discussion and on the results of studies so far, a basic equation for calculating crack spacing is derived by means of a new approach in which a distinction is drawn between the local and overall effects of stress distribution. The mathematical model used here achieves a separation between the main and sub terms; that is, Kakuta's theory is applied to reinforcing bars in the first layer (using the representation of the JSCE equation) to give the dominant term, and the effects of reinforcing bars in the second and higher layers are taken into account separately with a correction factor. Method 1 is adopted to determine the maximum crack spacing and width. The basic equation for crack spacing is given in the following forms.

$$L_{av} = 1/a \cdot K_1 K_2 (3c + 0.7e) \tag{14}$$

$$L_{max} = \eta \cdot L_{av} \tag{15}$$

where c = concrete cover = $(3cb + cs)/4$; K_1 = coefficient representing the effects of reinforcing bars' bonding characteristics, equal to k1 in the JSCE equation; K_2 = coefficient representing the effects of a multi-layered bar arrangement; η = ratio of maximum to average crack spacing (or the ratio of maximum to average crack width), as determined statistically; and a = coefficient for correction of the coefficients in parentheses in Eq. (14) based on experimental results.

5. DETERMINATION of K_2, a, and η

The authors' previous study [4] made it clear that the effects of reinforcing bars in the second and higher layers were dependent on three factors - (1) steel bar diameter (2) vertical spacing, and (3) the lack of bonding in bundled bars - and that the mechanism was related to the effects of transmission of bonding around reinforcing bars in the second and higher layers on an increase in concrete tensile stress at the beam edges . Therefore, the effects of reinforcing bars in the second and higher layers on crack spacing are evaluated with a non-dimensional parameter, the ratio of multi-layers of bars, given by the following equation.

$$\text{Ratio of multi-layers of bars} = (u_v/u_l)(c_l/c_v) \quad (16)$$

where u_l = the sum of the effective circumference of reinforcing bars in the first layer; u_v = the sum of the effective circumference of reinforcing bars in the second and higher layers; c_l = distance between the centroid of bonding around reinforcing bars in the first layer and the beam bottom; and c_v = distance between the centroid of bonding around reinforcing bars in the second and higher layers and the beam bottom. The term "effective" means that one fourth of the circumference of a smaller-diameter reinforcing bar should be subtracted as the region where bonding is lacking in bundles of two, tied steel bars. The meaning of these definitions is illustrated in Fig. 7.

In Eq. (16), the term u_v/u_l represents the effects of factors (1) and (3), while c_l/c_v gives the effects of factor (2). Principally, the lack of bonding increases as the vertical spacing between bars falls. From a design viewpoint, however, since longitudinal reinforcing bars are either bundled or arranged in layers separated by 2.5 reinforcing bar diameters or more, there is no need to take into account arrangements in between these two cases. Incidentally, either c_l or c_v should be the distance not from the centroid of the cross section containing the reinforcing bars but rather from the centroid of the resultant bond stresses (called the centroid of bonding in this paper). However, the values defined above may be used as an approximation because the difference between the two distances is very small.

Figure 8 shows the relationship between the effects of reinforcing bars in the second and higher layers and the ratio of multi-layers of bars; that is, the results of an investigation with data given in Table 2. The horizontal axis represents the ratio of multi-layers of bars, and the vertical axis is the ratio of average crack spacing measured to the calculated right-hand term of Eq. (14), with the exception of K2. The undefined value a is provisionally taken to be 1.45 according to the assumptions in Section 2.

As seen in the figure, the ratio of measurement to calculation decreases along a hyperbola as the ratio of multi-layers increases, indicating a good correlation between the two ratios. A regression analysis of these data on the condition that K2 = 1 at u_v =0 yields the following equation, and the good fit of this equation shown in Fig. 8.

$$K2 = 1/\{0.2(u_v/u_l)(c_l/c_v) + 1\} \quad (17)$$

To give the equation greater authority, it is necessary to check and calibrate it against a wide range of experimental data, since the JSCE equation neither clarifies the relationship between maximum and average values nor removes shape effects, and there is a wide range of variations in the results of experiments on cracking as well as differences in experimental methods (such as the criteria for secondary cracks adopted by different researchers).

In addition to the results of our own experiments [13], including the results given above for six specimens, we also make use of the results of experiments on RC beams with deformed bars obtained by other researchers, as follows: Koyama et al. [14], Okuizumi et al. [15], Ikeda et al. [16], Suzuki et al. [17], Matsumoto et al. [18],

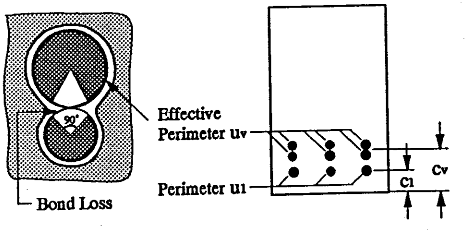

Fig. 7 Illustrated sectional parameters

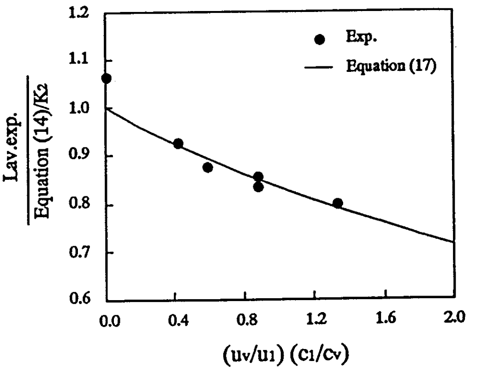

Fig. 8 Evaluation of the effects of multi-layered bar arrangement

Hognested [11], and Kaar and Mattock [19]. Table 3 summarizes the sectional properties of the specimens used by author. The total number of specimens in these studies is 86, of which 37 have a single-layered bar arrangement and 49 are multi-layered. In the case of references that do not give crack spacing data, crack spacings are measured from the results given in figures. As listed in Table 3, a fairly wide spectrum of data is available in this work by other researchers.

By substituting the properties of these various specimens, the results of experiments, and Eq. (17) into Eq. (14), the average correction factor is obtained as $a = 1.300$. The equation of average crack spacing, Eq. (14), is then written as follows.

$$L_{av} = K_1 K_2 (3.1c + 0.54e) \tag{18}$$

The coefficient h in Eq. (15) is then determined. The authors' earlier study [4] revealed that both crack spacing and width obeyed a normal distribution and that the standard deviation of crack spacing and width was 0.288 and 0.305, respectively. This is of the same order as experimental results obtained by other groups [2, 20]. Since the variation in crack spacing is of the same order as that in crack width, it can be considered that the assumption $W_{max}/W_{av} = L_{max}/L_{av}$ holds in statistical terms.

When the risk probability of the maximum crack spacing and width is set, as a control target in the design phase, to 5%, the extra factor for the average crack spacing and width, η, is calculated as follows using the results above.

$$\eta_L = 1 + 1.645 \times \sigma_L = 1 + 1.645 \times 0.288 = 1.474 \tag{19}$$

$$\eta_W = 1 + 1.645 \times \sigma_W = 1 + 1.645 \times 0.305 = 1.502 \tag{20}$$

These values agree well with the mean of the measured maximum-to-average ratios listed in Table 2 of Section 3. It can be deduced from this that the average of the maximum crack spacings and widths obtained from the experiments has a risk probability of about 5% for each crack. With the data listed in Table 3 for the 86 specimens, a calculation of the average of the maximum-to-average ratios as derived from the term in the JSCE equation yields the result 1.334.

Table 3 Sectional properties of specimens used for the study on average crack spacing

Researcher(s)	Cross Section		Cover		Main Bar						Number of Specimens
	b (mm)	D (mm)	Cb (mm)	Cs (mm)	pt (%)	Number	Diameter ϕ (mm)	Number of Layers	Spacing		
									e h (mm)	e v (mm)	
Authors	150~200	300~500	29~61	29~54	0.6~2.7	2~6	13~32	1~3	70	13~60	12
S.Koyama,et al.	720	1300	46~98	58~88	1.1~1.5	3~8	29~64	1~2	144~240	105	3
T.Okuizumi,et al.	150	200	19~20	24~25	1.6~1.9	4~6	10~13	2~3	90	10~25	4
S.Ikeda,et al.	300	300	15~29	20~29	1.5~1.7	3~18	10~22	1~3	50~100	20~30	4
K.Suzuki,et al.	120	200	25	20	0.3~1.4	2~3	6~13	1	35~74	—	14
Y.Matsumoto,et al.	250	800	34~64	34~38	0.3~0.7	3~10	25~32	1~2	40~75	60	11
Hognestad	100~300	200~610	10~73	10~73	0.7~3.3	1~15	10~25	1~3	0~162	38	29
Kaar&Mattock	120~810	390~440	35	35	0.4~2.5	8	13	1~4	38~243	38	9
Range	100~810	200~1300	10~98	10~88	0.3~3.3	1~18	6~64	1~4	0~243	10~105	86

This is smaller than the value in Eq. (19) or the previously noted value, or 1.4 to 1.5. This indicates that the risk of the maximum crack spacing as calculated by the JSCE equation exceeds 5% for each crack; that is, the JSCE equation underestimates the single-layered bar arrangement rather than overestimating the multi-layered bar arrangement.

For practical purposes, it seems reasonable to take 1.5 as the extra factor for average crack spacing or width, η. Thus Eq. (15) is given by the following expression.

$$L_{max} = 1.5\ L_{av} \tag{21}$$

6. ACCURACY OF THE PROPOSED EQUATION FOR CRACK SPACING

The suitability of Eq. (18) was compared with that of the equations listed in Table 1 (in Section 2) using the results of experiments on the 86 specimens listed in Table 3. Experimental data points for maximum crack spacing are few, so only the results of a comparison of average crack spacing are given here.

Figure 9 compares measured and calculated average crack spacings. Table 4 lists the average ratio of measured values to calculated values, as well as the coefficient of variation, for three cases: for all the 86 specimens; for a single-layered bar arrangement; and for a multi-layered bar arrangement.

In this figure, data for single-layered and multi-layered bar arrangements based on the JSCE equation and Kakuta's equation tends to deviate slightly from the straight line. However, data based on the CEB-FIP, AIJ, ACI equations as well as the proposed equation all fall approximately on a straight line. The ACI equation gives results that generally tend to fall short of the measurements.

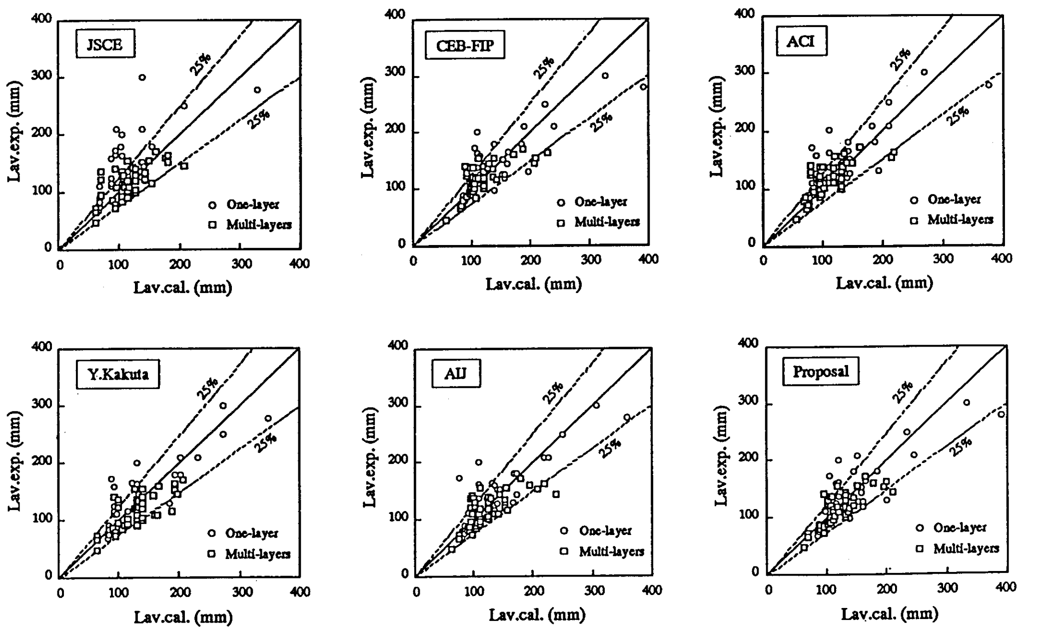

Fig. 9 Suitability of various equations for average crack spacing calculations

Table 4 Average and coefficient of variation of the ratio of measured to calculated values

Equation	Totality		One-layer		Multi-layers	
	Average	Variation	Average	Variation	Average	Variation
JSCE	1.162	0.284	1.312	0.264	1.049	0.254
ACI	1.106	0.229	1.137	0.253	1.083	0.204
CEB-FIP	1.032	0.212	1.028	0.241	1.034	0.187
AIJ	1.013	0.249	1.046	0.286	0.989	0.209
Y.Kakuta	0.990	0.229	1.080	0.230	0.921	0.197
Proposal (18)	1.000	0.198	1.014	0.230	0.985	0.168

These behavior trends can also be seen in Table 4. Let's look first at the overall average and the coefficient of variation. Assuming Lmax/Lav = 1.45 in the JSCE, ACI and Kakuta's equations, then Kakuta's equation is very close to the measurements, but the JSCE and ACI equations underestimate by 16.2% and 10.6%, respectively. On the contrary, the ratio of the CEB-FIP and AIJ equation results to the measured values is just about 1.0, and the average ratio calculated from the proposed equation is 1.0. The overall coefficient of variation of the proposed equation, 19.8%, is the lowest of the six equations. Next is the CEB-FIP equation, then Kakuta's equation, the ACI equation, the AIJ equation, and then the JSCE equation.

Next, let's look at a comparison of the single-layered and multi-layered bar arrangement. The ratio of measured to calculated results in the case of both the CEB-FIP equation and the proposed equation is very close to 1.0, regardless of the number of layers. In the case of the four other equations, however, the ratio differs between single-layered and multi-layered bar arrangements. This difference is particularly noticeable in the case of the JSCE equation and Kakuta's equation, where it is about 25% and 16%, respectively. With the ACI and AIJ equations it is about 5%.

Further, the deviation resulting from the JSCE equation differs in nature from that of Kakuta's equation. The former tends to overestimate the single-layered bar arrangement while accurately modeling with double-layered arrangement, while the latter does the opposite but to approximately the same degree. The proposed equation has the smallest coefficient of variation of the six equations, both in the single-layered and multi-layered cases. Table 4 and Fig. 9 also demonstrate that variations are greater in the case of a single-layered bar arrangement than for a multi-layered arrangement.

Overall, these results indicate that Eq. (18) is able to cope with various bar arrangements and, of the equations considered, gives the closest fit to the experimental crack spacing results. A the same time, it is a less complex calculation.

7. CRACK WIDTH

Another important determining factor for both crack depth and spacing is the differential strain between the concrete and reinforcing bars. This differential plays an important role in three effects: the average elongation of the concrete surface between cracks under loading; the average elongation of reinforcing bars; and the difference in strain between the concrete and reinforcing bars induced by creep and drying. Since the first makes a negligible contribution [3], only the latter two effects need to be taken into account.

The average reinforcing bar strain between cracks is less than the strain in reinforcing bars at a cracked cross section, since the concrete has a confining effect. The effect this lower strain has on crack width are generally small, but it cannot be neglected when the steel-to-concrete ratio is small, or the stress on reinforcing bars is small. The JSCE and ACI equations do in fact neglect this reduced strain, on the grounds that the stress on reinforcing bars under normal cracking conditions is small, and that, anyway, taking this line gives design results that err on the safe side.

On the other hand, Kakuta's equation incorporates a constant ratio of reduced strain for each effective steel-to-concrete ratio. The CFB-FIP and AIJ equations take it into consideration as a function of steel-to-concrete ratio and reinforcing bar stress, so they can be applied to a wide range of steel reinforcement stresses. In the latter case, though, the calculations are very complex.

It has been pointed out by many researchers that the crack width is greatly affected by drying shrinkage and creep [20 to 22]. Past research has also made clear that these effects can be rationally evaluated by introducing a strain increment into the strain term of the equations for crack width [3]. However, the coefficients of drying shrinkage and creep obtained from experiments and surveys of actual beams vary from specimen to specimen and with environmental conditions. Thus, there has as yet been no definitive conclusion as to the extent to which a design coefficient should be used.

In this study, the effects of steel-to-concrete ratio are taken into account indirectly in evaluating crack spacing. In consideration of this, and to enable a wider range of steel-to-concrete ratios to be covered, the confining action of the concrete is taken into account. Further, it is assumed that the design aims to control crack width under normal cracking conditions, and consequently Kakuta's equation is adopted to simplify calculations. That is, the average strain in reinforcing bars, $\varepsilon_{s,av}$, is evaluated using the following equation:

$$\varepsilon_{s,av} = \sigma_s / E_s - \sigma_{cm} / (E_s \cdot p_e) \qquad (22)$$

where Ace = effective tensile cross-sectional area of concrete, where its centroid coincides with the centroid of bonding around all tensile reinforcing bars in calculating the effective steel-to-concrete ratio, pe = As/Ace, and the lack of bonding in bundled bars is taken into account in calculating the centroid of bonding.

Figure 10 demonstrates the adequacy of Eq. (22) against the experimental results given in Table 2 (of Section 3). It is evident from this figure that Eq. (22) is more accurate than the CEB-FIP and AIJ equations, despite the simplicity of the calculations. Of the literature referred to by the authors in comparing crack width calculations, some gives only crack spacing data and, further, each group took results at different crack width measuring points and with different steel reinforcement stress levels under different drying and shrinkage conditions. Thus, it was impossible to systematically compare crack width data. Since a comparison of average steel reinforcement strain is, indirectly, a comparison of crack spacing, it is possible to verify crack width to some degree. The number of specimens is a few, however, at just six.

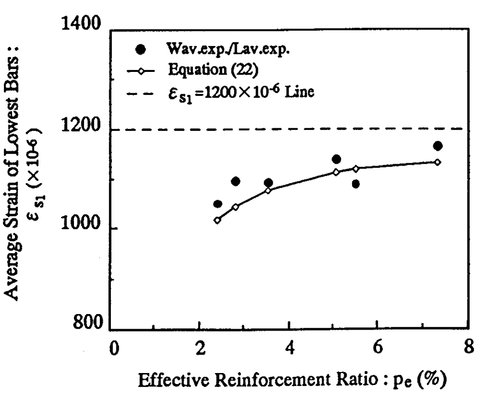

Fig. 10 Confining effects of concrete

Furthermore, taking into account the effects of drying shrinkage and creep, e_ϕ, the average crack width and maximum crack width can be determined to a confidence level of 95% using the following equations:

$$W_{av} = L_{av}\{\sigma_{av}/E_s - \sigma_{cm}/(E_s \cdot p_e) - e_\phi\} \qquad (23)$$

$$W_{max} = 1.5W_{av} \qquad (24)$$

where, for e_{ϕ}, it would be reasonable to use measured values, or about $-(100 \text{ to } 300) \times 10^{-6}$ with reference to past results.

8. CONCLUSIONS

This paper comprises a study of the features and problems of current equations for flexural crack width, as well as a proposal for a new equation. The results of the study can be summarized as follows.

(1) When the results of experiments on RC beams with multi-layered bar arrangements were compared with five typical equations for crack width, none of the equations proved satisfactory. This is primarily because the complicated bar arrangements were taken into account in the equations by using extremely simplified mathematical models.

(2) A study of the effects of concrete cover on the side and at the bottom of a beam, based on the results of experiments conducted by Hognested, indicated that calculated values were most accurate when $c = (3 c_b + c_s)/4$. Thus, the effect of concrete cover on the side of a beam is less than that at the bottom when RC beams with multiple layers of reinforcing bars are subject to bending.

(3) It was also clarified that the effects of the second and higher layers of reinforcing bars on crack spacing can be evaluated by adopting an overall parameter that includes all physical and geometrical factors; i.e. the ratio of the multi-layers of bars. Crack spacing decreases along a hyperbola as this ratio increases.

(4) Based on Kakuta's theory and the concept of the local effects of stress distribution, a new equation for average crack spacing was proposed. This equation gives a better fit than the current equations, and is also able to take into account various bar arrangements, despite a less-complex calculation.

(5) A new equation of average crack width was proposed taking into consideration correspondence with the equation of crack spacing, the assumed normal cracking conditions, and the confining effects of concrete between cracks on reinforcing bar strain. This equation would give practically satisfactory accuracy.

(6) To provide a clearer statistical definition of maximum crack width, which is one of the targets in controlling cracking, a method of calculating maximum crack width is adopted. This maximum crack width is determined by multiplying the average of crack width, Wav, by an extra factor determined from the average crack spacing, Lav. If a risk probability of 5% is applied to both maximum crack spacing and maximum crack width, it is reasonable to take 1.5 as the value of this extra factor, η.

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