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EVALUATION OF SHEAR STRENGTH OF CONCRETE BEAMS REINFORCED WITH FRP

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The shear strength of concrete beams reinforced with FRP was studied through experiment and analysis. The effect of longitudinal tensile reinforcement stiffness on the diagonal tension failure strength was calculated using the extended modified compression fields theory, and an evaluation method was proposed based on the analytical results. Furthermore, shear strength of beams failing due to FRP stirrup rupture was well evaluated by considering the lower stiffness and the strength reduction of FRP at bent corners.

Key Words : FRP, shear strength, bent bar strength, stiffness of reinforcement, modified compression fields theory

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1. INTRODUCTION

In recent years, the use of FRP(Fiber Reinforced Plastic) rods in place of steel bars or prestressing bars as reinforcement for concrete structures has been actively studied. As a result, many useful reports were presented[1], and both deformation and strength were evaluated for the flexure behavior of the concrete beams reinforced with FRP[2]. However, the shear behavior of concrete beams using FRP has not ever been established.

Many reports have made clean that the shear strength of concrete beams without shear reinforcement reinforced with FRP as the longitudinal tensile reinforcement is less than those with steel bars[3][4]. Tsuji et al.[3] proposed a method to evaluate the shear strength for such beams by considering a ratio of Young's modulus of FRP to that of steel bar in a term of the reinforcement ratio of shear strength equation of JSCE. Since the method was, however, obtained on the basis of a small number of experimental results, its applicability is not always clear. One report[5] show that the shear strength can be calculated closely or safely in comparison with the experimental results using the method proposed by Tsuji et al. On the contrary, another report[6] shows that the correction considering a ratio of Young's modulus does not necessarily evaluate the actual strength.

At the same time, it is reported that the shear strength of concrete beams using FRP for shear reinforcement is small, as compared with the calculated value by a modified truss theory (Vc+Vs)[2]. Although several evaluation methods of the strength of such beams are proposed, it seems that, at present, no method gives satisfactory solutions. The main reason why the shear strength of such beams has never actually been evaluated appears to be that the rupture of FRP stirrups corresponding to yielding of the steel bars occurs at a crack intersection or bent corner of a beam due to local stress intensity before the uniaxial strength is reached[7]-[12]. Therefore, it is important to consider such phenomena when evaluating the shear strength.

The purpose of this paper is to evaluate the shear strength of concrete beams reinforced with FRP rods as the tensile and shear reinforcement. First, the shear resistance behavior of the concrete beams reinforced with FRP rod is confirmed by experimental observation. Second, the bent corner strength and the diagonal tension strength of FRP rods, which are typical mechanical properties of FRP in concrete beams, are investigated theoretically, and the effect of Young's modulus of the reinforcements on shear behavior is analyzed using an extended modified compression fields theory. Finally, the shear strength of the beams is evaluated using the results of the above investigations .

2. OUTLINE OF EXPERIMENTS

(1)Materials

FRP used consists of continuous glass fibers impregnated with resin (GFRP). GFRP was used so that the effect of Young's modulus of the reinforcement on the shear strength could be investigated, since the Young's modulus of GFRP is very small compared with the steel bars. Main reinforcements (G16) was formed into grids with spacing of 10cm and the area of cross section of 2.01cm ^{2} . Stirrups(G6) with a cross section of 0.35cm ^{2} were used, and the radius of the bent corner was 5mm or 10mm. The material properties of GFRP and steel bars are shown in Table-1. The tensile strength and Young's modulus of GFRP were obtained by the test proposed by Idemitsu[14]. The compressive strength of the concrete(f'c) is shown in Table-2.

(2) Specimens and Experimental Method

An outline of the beam specimens tested in this study is shown in Table-2. The cross section, without stirrups, was 30cm wide, 20cm high and with an effective beam depth of 15cm. The beams were tested on a two-point loading system with monotonically increasing loads.

The cross section with stirrups was a width of 20cm, height of 30cm, and an effective beam depth of 25cm. Parameters of the experiment are (1) material of longitudinal reinforcement(GFRP, steel bar), (2) material of stirrup(GFRP, steel bar), (3) spacing of the stirrups and (4) the radius of the bent corner of GFRP stirrups. The beams were tested on a one-point loading system with monotonically increasing loads. Fig.-1 gives details and its loading condition of specimens of GG05-20 and GG10-20. This experiments are characterized by the radius of the bent corner of GFRP stirrups and allow investigation of the effect of the bent bar strength on beam shear strength.

3. EXPERIMENT RESULTS

The experimental results are summarized in Table-3. All test specimens failed in shear mode and the longitudinal reinforcements did not yield or break at failure in any specimen.

(1) Beams without Stirrups

For beams without stirrups, the flexural crack propagated to the upper part of the cross section as soon as the crack initiated. The ultimate failure was determined by the diagonal tension crack which occurred from the flexural crack at a distance of about 1.5d from the loading point. The failure mode of the specimens was diagonal tension failure.

(2) Beams with Stirrups

Crack formation in the specimens with stirrups was, before failure, almost the same as in the specimens without stirrups. The diagonal crack occurred from the flexural crack at a distance of 1.5d ~ 2.0d from the loading point. The crack formation of a specimen of GG05-20 at the ultimate is shown in Fig.-2. The load carrying capacity increased by the effect of the stirrups after the diagonal tension crack occurred. The specimens reinforced with steel bars as stirrups failed after the stirrups yielded, while the failure of specimens reinforced with GFRP stirrups, was caused by the breaking of the stirrups. GFRP breaking occurred at the upper or lower bent corner near the diagonal tension crack. One purpose of this experiments is investigation of the effect of the bent bar strength on shear strength. It is understood from the experimental results that the shear strength of the specimens with a bent corner radius of 5mm is obviously lower than that of 10mm; the strength of the bent corner is an important factor for evaluating the shear strength of the beam reinforced with FRP.

Table-1 Material Properties

	Area of cross section cm^2	Tensile strength MPa	Young's modulus MPa	Strain at breaking %
G16	2.01	751.4	2.94×10^4	2.55
G6	0.35	828.0	3.14×10^4	2.64
D19	2.87	371.6	1.80×10^5	———
D10	0.71	324.6	1.90×10^5	———
D6	0.32	370.1	1.80×10^5	———

Table-2 Outline of Specimens

Specimen	Longitudinal reinforcement	spacing of stirrup cm	r mm	Pt %	a/d	f'c MPa
G01	3-G16	———	———	1.34	4.0	22.7
G02	4-G16	———	———	1.79	4.0	27.8
GG05-10	4-G16	G6 (10)	5	1.61	3.0	35.4
GG10-10	4-G16	G6 (10)	10	1.61	3.0	33.4
GG05-20	4-G16	G6 (20)	5	1.61	3.0	35.2
GG10-20	4-G16	G6 (20)	10	1.61	3.0	35.2
DG05-15	3-D19	G6 (15)	5	1.72	3.0	34.7
DG10-15	3-D19	G6 (15)	10	1.72	3.0	34.4
DG05-25	3-D19	G6 (25)	5	1.72	3.0	35.6
DG10-25	3-D19	G6 (25)	10	1.72	3.0	35.8
GD-15	4-G16	D6 (15)	———	1.61	3.0	38.6
GD-25	4-G16	D6 (25)	———	1.61	3.0	37.1

Table-3 Experimental Results

Specimen	Diagonal cracking load				Shear strength			
	$V_{c,exp}$ (tf)	$V_{c,1}$ (tf)	$V_{c,2}$ (tf)	$V_{c,cal}/V_{c,exp}$	V_{exp} (tf)	$V_{c2}+V_{s1}$ (tf)	$V_{c2}+V_{s2}$ (tf)	V_{cal}/V_{exp}
G01	6.75	9.91	6.13	0.91	—	—	—	—
G02	7.41	11.67	7.22	0.97	—	—	—	—
GG05-10	10.00	13.21	8.18	0.82	17.00	33.38	16.08	0.95
GG10-10	9.50	13.04	8.07	0.85	20.40	33.27	17.48	0.91
GG05-20	7.50	13.20	8.17	1.09	11.46	20.77	12.01	1.05
GG10-20	8.50	13.20	8.17	0.96	13.50	20.77	12.88	0.95
DG05-15	16.50	13.61	—	0.82	19.66	30.41	18.87	0.96
DG10-15	14.90	13.57	—	0.91	21.68	30.37	19.84	0.92
DG05-25	13.90	13.73	—	0.99	16.27	23.81	16.72	1.03
DG10-25	14.00	13.76	—	0.98	16.27	23.84	17.52	1.08
GD-15	10.00	14.04	8.69	0.87	15.73	14.71	—	0.94
GD-25	8.50	13.86	8.58	1.01	11.16	12.19	—	1.09

$V_{c,1}$:Niwa's equation, $V_{c,2}$:Eq.(8)
$V_{c,cal}$:$V_{c,1}$ for steel bars, $V_{c,2}$ for GFRP
$V_{s,1}$:Truss Theory($f_yA_s(d/1.15)/s$), $V_{s,2}$:Eq.(11)
V_{cal}:$V_{s,1}$ for steel bar, $V_{s,2}$ for stirrup

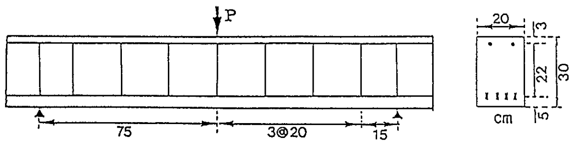

Figure-1 Detail and Loading Condition of Specimen(GG05-20,GG10-20)

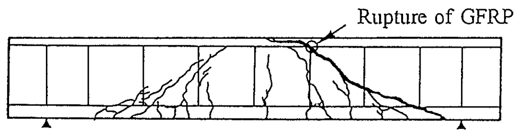

Figure-2 Crack Formation at Ultimate State(GG05-20)

4. EVALUATION OF MECHANICAL PROPERTIES OF FRP IN CONCRETE BEAMS

(1) Bent Bar Strength

As shown in experimental observations, bent bar strength is a major factor for shear strength. Therefore, we must first clarify the bent bar strength in order to evaluate the shear strength of the concrete beams reinforced with FRP. The evaluation equation of the bent bar strength is theoretically derived as follows.

We assume an bent corner FRP cast in concrete as shown in Fig.-3. We consider the deformation at the cross section varied from straight part to bend, since it is experimentally observed that breaking at the bent corner always occur at that location[8]. When a tensile force is applied to the bar as shown in Fig.-3, and there is no bond between concrete and FRP, the FRP stretch δx uniformly at the cross section in the straight part subjected to uniform axial force. It is assumed that the cross section deform rotation angle of ϕ(rad) maintaining a radius of the bent corner of r. Then, if Bernoulli assumption is used, the strain distribution in the cross section is represented by a following hyperbolic curve at any point y in the cross section since FRP should deform δx at the bent part.

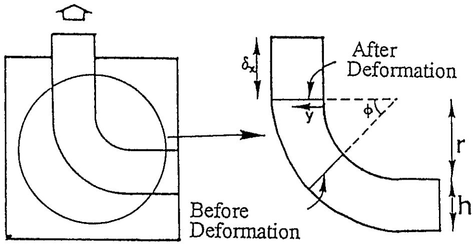

Figure-3 Model of Bent Bar in Concrete

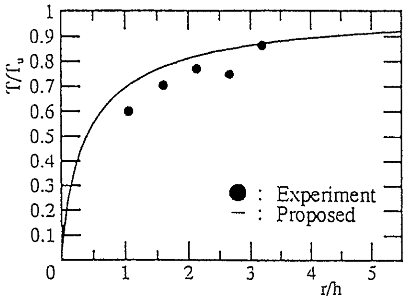

Figure-4 Evaluation of Bent Bar Strength

$$\varepsilon(y) = \delta_x / ((r + y)\phi) \tag{1}$$

Stress distribution is obtained by multiplying Young's modulus E to strain, when FRP behavior is strictly elastic. Then, the stress distribution at breaking is represented by the following equation. Here it is assumed that the breaking of FRP occur when a stress in cross section reach the breaking stress of σy.

$$\sigma = \sigma_y \cdot r / (r + y) \tag{2}$$

Therefore, the evaluation equation of bent bar strength can be derived by integrating the stress distribution of Eq.(2) in the cross section.

$$T = \int_A \sigma_y \cdot r / (r + y) dA \tag{3}$$

The equation is rewritten to Eq.(4) as the function of the radius of the bent corner, r, and the height of the cross section, h, when the cross section is rectangular with the height of h.

$$T = T_u \cdot r / h \ln(1 + h / r) \tag{4}$$

in which Tu is the uniaxial strength, r is the radius of the bent corner, and h is the height of the cross section.

Fig.-4 shows the comparison between the evaluated values using the proposed equation and the experimental results obtained by Miyata et al.[9]. In the figure, the lateral axis shows a ratio of r to h and the vertical axis shows the reduction ratio of the strength of the bent corner from the uniaxial strength. A solid line shows the Eq.(4) and a ● indicates an experimentally obtained value. It is understood that the proposed equation can satisfactorily evaluate the bent bar strength.

(2) Diagonal Tensile Strength

The effect of the diagonal tensile force due to a diagonal crack, as shown in Fig.-5, is considered a factor of shear strength. Next, we investigate the evaluation method for diagonal tensile strength of FRP as anisotropic material.

Consider that FRP with a length L is applied to diagonal tensile force T with angle θ. Then, the stress resultant acted in the cross section is the axial force is T·cosθ and the bending moment is T·L·sinθ. A strain in the cross section reach breaking strain from a smaller tensile force than with uniaxial state because of the effect of the strain gradient. Therefore, the diagonal tensile strength is less than the uniaxial tensile strength since the cross section of FRP without plastic region, breaks as soon as one part of the cross section is broken. Then, the diagonal tensile strength of FRP is represented by the following equations for a rectangular cross section with height h.

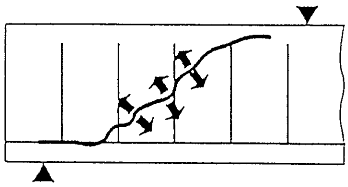

Figure-5 Diagonal Tension due to Diagonal Crack

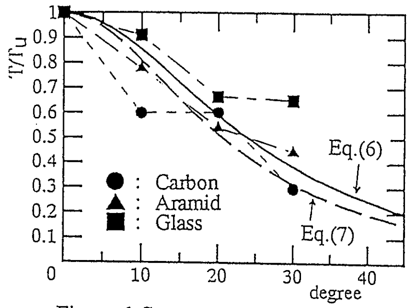

Figure-6 Comparison with Experiment

$$T = T_u / (\cos\theta + 6L \cdot \sin\theta / h) \tag{5}$$

in which Tu is uniaxial tensile strength, θ is the direction of the tensile force, and h is the cross section height.

In the above equation, L is the unknown value. Now, we assume that L is the length of intersection between FRP and the diagonal crack with angle of θ. Then, L=h·tanθ and the diagonal tensile strength is derived as,

$$T = T_u / (\cos\theta + 6\tan\theta \cdot \sin\theta) \tag{6}$$

For circular sections,

$$T = T_u / (\cos\theta + 8\tan\theta \cdot \sin\theta) \tag{7}$$

Figure-6 compares the experimental results from Maruyama et al.[10] and the proposed equations. The experimental results were obtained for carbon(CFRP), aramid(AFRP), and glass(GFRP) with the direction of diagonal tensile force varied from 0 to 30 degree. In this figure, a solid or broken line correspond respectively to Eq.(6) and Eq.(7), while the symbols ● , ▲ and ■ show the experimental results for carbon, aramid, and glass fibers. It is understood that strength reduction due to diagonal tensile force is reasonably evaluated by the proposed equations. However, the equations can not consider the effect of fiber type, even though the strength reduction is different for different fibers in experiment.

5. EVALUATION OF SHEAR STRENGTH OF CONCRETE BEAMS REINFORCED WITH FRP

(1) Shear Strength of Beams without Stirrups

It is known from many experimental observations that the shear strength of a beam reinforced with FRP, which has a lower Young's modulus than steel bar, is smaller than those reinforced with the steel bar[3][4]. This fact is confirmed by this experiment(G01 and G02 specimens in Table-3) and the strength is about 30 ~ 40% smaller than the value predicted by the equation proposed by Niwa et al.[15]. It is reported that the reason is mainly the difference in Young's modulus of reinforcement. Therefore, we will analytically investigate the effect of Young's modulus of the main reinforcement on the shear strength.

Analysis is a method based on the extended modified compression fields theory which can accurately evaluate the shear strength of the concrete beams[13]. The model used in the analysis has a cross section of 20 × 20cm, a beam depth of 16cm, and a longitudinal reinforcement ratio of 2.68%, as shown in Fig.-7. The material properties are that the compressive strength of the concrete is 28MPa and the tensile strength is 2.8MPa. Analysis is performed in which Young's

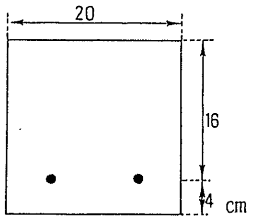

Figure-7 Analytical Model

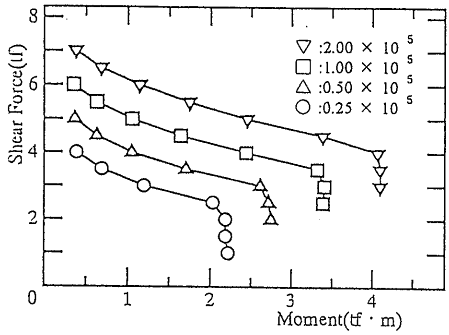

Figure-8 Effect of Young's Modulus of Main Reinforcement

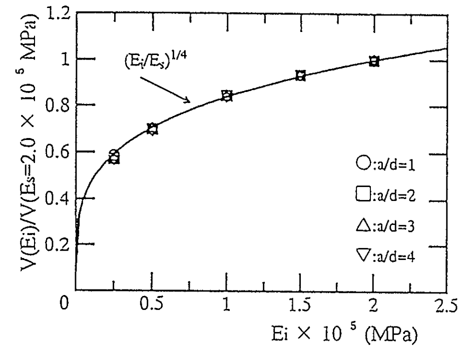

Figure-9 Relationship between Shear Span Ration and Young's Modulus of Main Reinforcement

modulus of the reinforcement is varied from 0.25×10^5 to 2.0×10^5 MPa. The relationships between the shear force and the moment in failure state obtained from the analysis are illustrated in Fig.-8. The results for Young's modulus values of 2.0×10^5, 1.0×10^5, 0.5×10^5 and 0.25×10^5 MPa are respectively marked with ▽, □, △ and ○. The range in which the shear force gradually decreases as the moment increases, corresponds to diagonal tension failure in the analysis. On the other hand, the range in which the moment is constant and the shear force decrease rapidly corresponds to the flexural failure region. The load carrying capacity for two-point loading and simply supported beams can be evaluated for both shear and flexural failure from this figure using the relation of a/d=M/(V·d). However, this analysis can not be applied to shear compression failure, since the analysis does not consider the effect of transverse local compressive stress.

As shown in Fig.-8, the shear strength decrease along with Young's modulus. It is understood that the Young's modulus of the main reinforcement is an important factor influencing shear strength and it can be seen that the Young's modulus curves are almost parallel. This implies that the effect of Young's modulus on shear strength is independent to a ratio of the shear force to the moment and is a function of Young's modulus only. Figure-9 shows the reduction ratio of shear strength from strength in the case of Young's modulus of 2.0×10^5 MPa. The symbols ○, □, △, and ▽ correspond to a/d=1, a/d=2, a/d=3, and a/d=4. The difference of a/d does not appear. It was confirmed and similarly concluded the effects of longitudinal reinforcement ratio, compressive strength of the concrete, and effective depth of the cross section in the analysis. Therefore, the effect of Young's modulus on the shear strength can be introduced by simply multiplying the previously introduced shear strength equations by the effect of the Young's modulus of the longitudinal reinforcement.

The analytical results indicate that the shear strength is proportional to the 1/4 power of a ratio of Young's modulus (Ei/Es). A solid line in Fig.-9 represents the curve of $(Ei/Es)^{1/4}$. Finally, the shear strength of the concrete beam reinforced with FRP can be evaluated using Eq.(8).

$$V = V_c (E_i / E_s)^{1/4} \tag{8}$$

in which Vc is the shear strength of the concrete beam reinforced with steel bar, Ei is the Young's modulus of FRP, and Es is the Young's modulus of the steel bars.

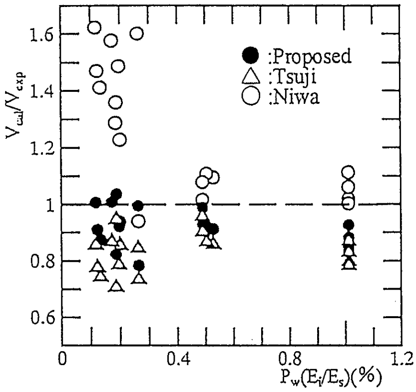

Figure-10 Comparison between Estimated and Experimental Values

Table-4 Comparison between Estimated and Experimental Values

	Vcal/Vexp			Coefficient of variation %		
	Niwa	Tsuji	Proposed	Niwa	Tsuji	Proposed
G, G+C (7)	1.50	0.80	0.94	7.4	7.2	6.9
C (11)	1.09	0.86	0.91	10.1	7.3	8.0
All specimens	1.25	0.84	0.92	18.0	8.0	7.8

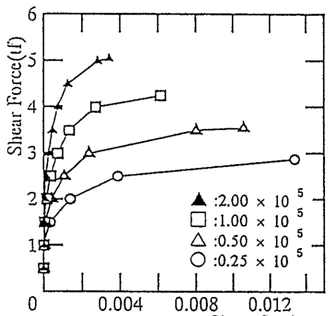

Figure-11(a) Relationship between Shear Force and Shear Strain

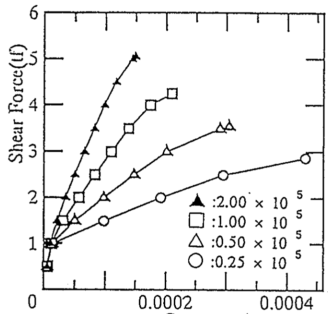

Figure-11(b) Relationship between Shear Force and Curvature

Tsuji et al. proposed a method of evaluating the shear strength of concrete beams reinforced with FRP using the transformed area of reinforcement As(Ei/Es), considering a ratio of Young's modulus of FRP to that of the steel bar[3]. The results in our analysis are identical with the method proposed by Tsuji et al., since the effect of the longitudinal reinforcement ratio on the shear strength is also proportional to the 1/4 power in the analysis[13]. However, the effect of Young's modulus as proposed by Tsuji et al. differs from this analysis. The effect on the shear strength is represented by a 1/3 power, since they use the JSCE equation.

Figure-10 shows the ratio of estimated and experimental values[3][4][6][7] of shear strength. The estimated values are obtained from the equation proposed by Niwa et al.[15], which is proposed for the concrete beams reinforced with steel bars(indicated by ○), the method of Tsuji et al.(indicated by △), and the proposed method of Eq.(8)(indicated by ●). It is assumed that Vc is evaluated by Niwa's equation and that the Young's modulus of the steel bars in Eq.(8) is 2.0×10^5 MPa. The number of experimental results 18 for FRP reinforced with glass(G), while 11 specimens were hybrids of glass and carbon(G+C) as fiber (Ei=$2.9 \times 10^4 \sim 3.34 \times 10^4$ MPa) and 18 used carbon(C)(Ei=$6.73 \times 10^4 \sim 1.4 \times 10^4$ MPa). Table-4 shows the average and a coefficient of variation for the ratio of estimated and the experimental values for each method. The equation for concrete beams reinforced with steel bar overestimated the experimental results and the difference appears remarkably for lower Young's modulus. Tsuji's method underestimated the test results. On the other hand, the proposed method can satisfactorily estimate test results, regardless of the difference of Young's modulus. The coefficient of variation is also satisfactory.

Figure-11 show the relationships between shear force and shear strain and between shear force and curvature for M/(V·d)=3 when the Young's modulus of the main reinforcement is varied in

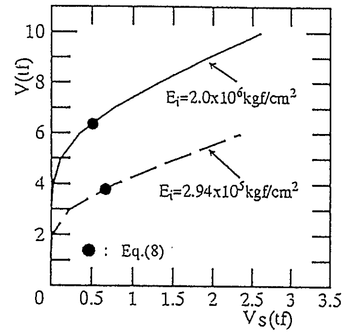

Figure-12 Effect of Young's Modulus of Main Reinforcement

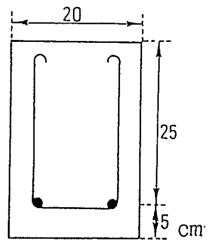

Figure-13 Analytical Model

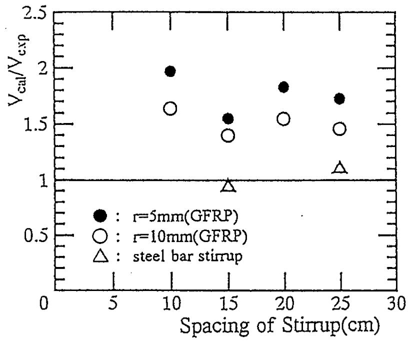

Figure-14 Estimated Result of Experiment Data (ignoring bent bar strength)

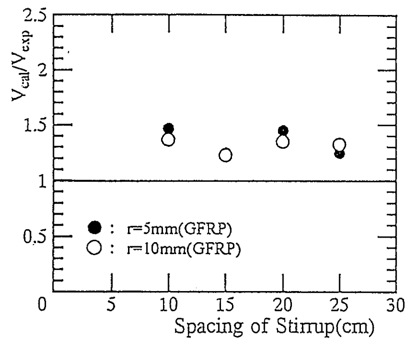

Figure-15 Estimated Result of Experiment Data (considering bent bar strength)

the analysis. The ▲, □, △, and ○ symbols correspond to the analysis for Young's modulus of 2.0×10^5, 1.0×10^5, 0.5×10^5,and 0.25×10^5 MPa, respectively. Though Eq.(8) satisfactorily estimates the effect of Young's modulus on shear strength, the mechanical meaning was not clear. However, as shown in Fig.-11, the shear strain which is identical with shear deformation increases rapidly in proportion as Young's modulus becomes small and the effect of Young's modulus on shear strain is great compared with that on the curvature. Therefore, it seems that the increase of the shear strain due to the lower Young's modulus probably influences shear strength.

(2) Shear Strength of Beams with Stirrups

The shear failure modes of the concrete beams are distinguished into failure caused by yielding or breaking of stirrups and compressive failure of the concrete near the loading point, even if diagonal cracks predominate. Both failure modes are basically different and the shear strength for both modes can not be evaluated with the same estimating method. In this paper, only shear failure caused by the breaking of FRP stirrups or the yielding of steel bar is considered.

Figure-12 shows the analytical result of M/(V·d)=3 obtained from analysis based on the extended modified compression fields theory. The model used in the analysis had rectangular cross section of 20 × 30cm, beam depth of 25cm, longitudinal reinforcement ratio of 1.61%, stirrup area of 2 × 0.35cm^2, and stirrup spacing of 10cm, as illustrated in Fig.-13. Analysis was performed in which the Young's modulus of the longitudinal reinforcement was either 2.0×10^5 or 2.94×10^4 MPa, while that of stirrup was 2.5×10^4 MPa. The vertical axis shows the applied shear force and the lateral axis shows the shear resisting force of the stirrups. The symbol ● in the figure represents the shear strength without stirrup as calculated by Eq.(8). The shear resisting force of the stirrups is calculated by the following equation.

$$V_s = B_w h / n \sum^n \sigma_{yi} / \tan\theta_i \tag{9}$$

in which σ_{yi} is the stirrup stress obtained from the analysis, θ_i is the direction of principal stress, n is the number of subdivided layers in cross section, and Bw and h are the width and the height of the cross section.

As shown in Fig.-12, the shear resisting force of the stirrup yields before the diagonal tension crack occurs and increases gradually after that. Also, it increases in linear proportion after the applied force reaches the shear strength of Eq.(8). This tendency is the same, independent of the Young's modulus of the longitudinal reinforcement and/or the stirrup. This result implies that the shear resisting force of the concrete beam with stirrup is approximately evaluated by the modified truss theory. In this paper, we tried to evaluate the shear strength of such beams by using the modified truss theory in which yielding or breaking of the stirrup at the ultimate state is assumed.

$$V = V_c + V_s \tag{10}$$

in which Vc is the shear strength considering the effect of Young's modulus of the longitudinal reinforcement, Eq.(8), and Vs is the shear force carried by the stirrups calculated by a truss theory.

Figure-14 show the estimated results of the experiment data by Eq.(10). Here, Vs is calculated by assuming that the compressive stress declines to 45 degree and the stirrup stress is the uniaxial yielding stress or the breaking stress. The △ symbols in the figure show the results with steel bar stirrups. The shear strength can now be accurately evaluated by the truss theory. Therefore, the shear strength of the concrete beam with FRP as longitudinal reinforcement and steel bar as stirrup can be calculate considering only the effect of the Young's modulus of the longitudinal reinforcement on Vc. On the contrary, the shear strength of that with FRP as the stirrup is overestimated by Eq.(10). Therefore, in this case, Vs can not be evaluated using uniaxial strength of FRP in the same way as with the steel bar stirrup. One reason is that the strength of FRP in the concrete deteriorates from the uniaxial strength due to the local stress intensity or the combined effect of the tension and the shear at the bent and a crack intersection part. Therefore, we attempted to modify Vs considering the influencing factors.

a) Effect of Bent Corner

In Fig.-14, the shear strength of the specimens with differing radius of bent corner is different. It is understood that the strength of the bent corner must be considered correctly to evaluate Vs. Therefore, we re-evaluate Vs considering the strength of the bent corner using Eq.(4). Figure-15 compares estimated values using Eq.(4) with the experimental results. The differing shear strengths for specimens having different radius of bent corner then disappears. By using Eq.(4) in Vs, the effect of the bent corner on the shear strength is satisfactorily evaluated. However, since the estimated values are larger than experiment values, we show further modification of Vs.

b) Effect of Young's Modulus of Stirrup

The factor related to the shear strength, besides the strength of the bent corner, is the effect of the diagonal tensile force. In this case, Eq.(6) and Eq.(7) proposed for diagonal tensile strength may be applicable. However, the equations require the crack angle at crack intersection and the angle is not always uniquely determined. Therefore, the evaluation of the shear strength using Eq.(6) and Eq.(7) is not considered in this study.

The other effect on the shear strength is Young's modulus of the stirrup. Satoh et al.[17] report from the analytical result of FEM that the shear strength becomes small when Young's modulus of the stirrup is low. We therefore investigate the effect of Young's modulus of the stirrup analytically and evaluate the shear strength considering the effect in this section.

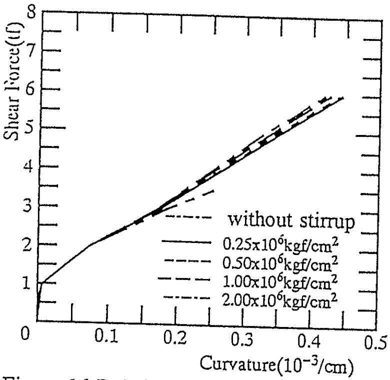

Figure-16 Relationship between Curvature and Shear Force
(Young's modulus of main reinfoecement of 29400MPa)

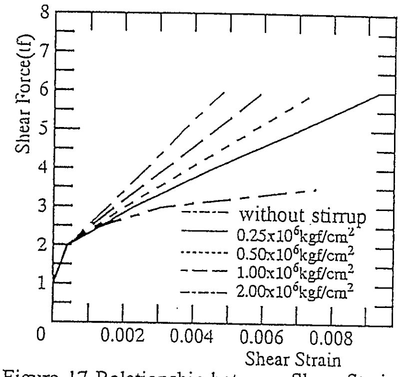

Figure-17 Relationship between Shear Strain and Shear Force
(Young's modulus of main reinfoecement of 29400MPa)

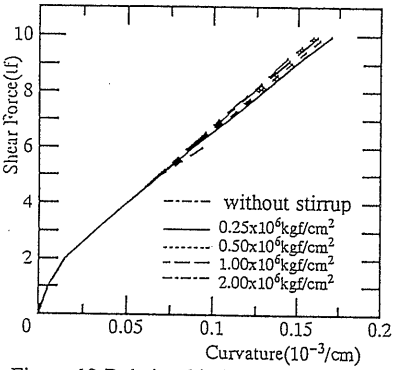

Figure-18 Relationship between Curvature and Shear Force
(Young's modulus of main reinfoecement of 200000MPa)

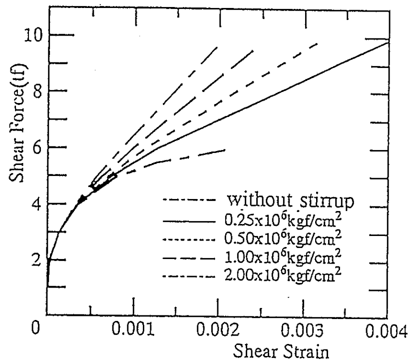

Figure-19 Relationship between Shear Strain and Shear Force
(Young's modulus of main reinfoecement of 200000MPa)

The analytical results based on the extended modified compression fields theory are shown in Fig.-16 ~ Fig.-19, for the RC cross section shown in Fig.-13. Figure-16 shows the relationship between the curvature and the shear force, while Fig.-17 shows the relationship between the shear strain and the shear force for M/(V·d)=3 when the Young's modulus of the stirrup is varied for 4 cases and that of the main reinforcement is 2.94×10^{4} MPa. Fig.-18 and Fig.-19 show the results when the Young's modulus of the main reinforcement is 2.0×10^{5} MPa. The effect of the Young's modulus of the stirrup does not appear for the curvature and shear force relationships as shown in Fig.-16 and Fig.-18. This means that the Young's modulus of the stirrup is not a predominant factor for flexural deformation. On the other hand, the shear strain and the shear force relationships as shown in Fig.-17 and Fig.-19 are influenced by the Young's modulus of the stirrup. The shear strain increases remarkably for lower Young's modulus of the stirrup. If the shear strain is identical with the shear deformation, the deformation of the beam is dominated by the shear deformation and directly depends on the Young's modulus of the stirrups. Therefore, assuming that the shear strength is closely related to the shear deformation, Vs may be evaluated by considering the effect of Young's modulus of the stirrup on the shear deformation. The method considering the effect of the shear deformation in Vs is already proposed by Kobayashi et al.[6]. Considering the shear deformation indirectly represents the strength reduction of FRP due to the combined effect of the tension and the shear caused by the local shear strain in beam.

Since the shear strain and the shear force relationships are almost linear for every case, the effect of Young's modulus of the stirrup is investigated by the shear stiffness. Figure-20 show the reduction ratios of the shear stiffness from the results of Young's modulus of 2.0×10^{5} MPa.

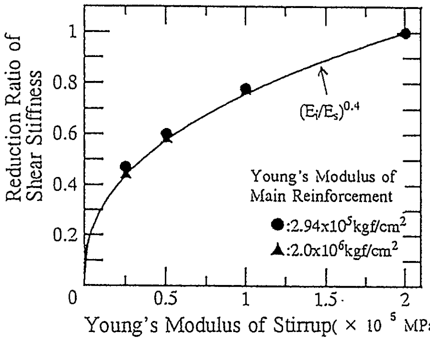

Figure-20 Effect of Young's Modulus of Stirrup on Shear Stiffness

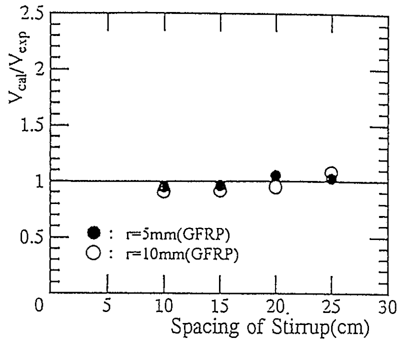

Figure-21 Estimated Result of Expeeiment Data

The symbols with ● and ▲ show the results when the Young's modulus of the main reinforcement is 2.94 × 10 ^{4} MPa and 2.0 × 10 ^{5} MPa, respectively. The reduction of the shear stiffness is independent of Young's modulus of the main reinforcement and depends on that of the stirrup. The effect is proportional to the 0.4 power of the Young's modulus of the stirrup. Assuming that the effect of the stirrup is related to the shear deformation directly, Vs is modified as

Table-5 Comparison with Experiment[11]

Specimen	f_y MPa	E_i MPa	V_{exp} tf	V_{cal} tf	V_{cal}/V_{exp}
C-A2-0	1540.0	7.00×10^4	8.65	9.59	1.11
C-C2-0	1440.0	1.12×10^5	8.50	9.83	1.16
C-CS2-0	2080.0	1.47×10^5	10.0	9.69	0.97

$$V_s^* = V_s(E_i / E_s)^{0.4} \tag{11}$$

in which Ei is the Young's modulus of the stirrup and Es is the Young's modulus of the steel bar(2.0 × 10 ^{5} MPa). Figure-21 show the comparison between the experimental and the estimated values using Eq.(11). The applicability of the proposed method is found satisfactory.

Since the above investigations are obtained from concrete beams reinforced with GFRP, the applicability of this method was then checked by comparing with test results carried out by Wakui et al.(a/d=3, s=25cm) using other FRP[11]. The mechanical properties of these FRP stirrups are greatly different from those of our experiment, as shown in Table-5. The estimated results are summarized in Table-5. The accuracy of estimation by the proposed method is satisfactory.

6. CONCLUSIONS

(1) Experimental investigation shows that the radius of the bent corner is an important factor for the shear strength of a concrete beam with FRP as the stirrup.

(2) Equations to evaluate the bent bar strength and the diagonal tensile strength of FRP are proposed from the theoretical investigation. They are useful for evaluating the shear strength of concrete beams reinforced with FRP.

(3) For the shear strength of concrete beams reinforced with FRP as longitudinal reinforcement, the Young's modulus of the longitudinal reinforcement is a factor influenced to the shear strength and the effect can be evaluated accurately by the term of 1/4 power of the ratio of Young's modulus. The effect of Young's modulus ratio on the shear strength is similarly represented to the effect of the longitudinal reinforcement ratio.

(4) The shear strength of a concrete beam reinforced with FRP as longitudinal reinforcement and with steel bar stirrup can be accurately evaluated by the modified truss theory.

(5) The strength of FRP in concrete deteriorates from the uniaxial strength due to the local stress intensity or the combined effect of the tension and the shear at a bend and a crack intersection. The shear strength of the concrete beam reinforced with FRP stirrups, which fails by breaking of FRP, is satisfactorily calculated by the proposed method when the bent corner strength and the effect of shear deformation is considered in Vs.

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