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ANALYTICAL STUDY FOR SHEAR RESISTING MECHANISM USING LATTICE MODEL

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The authors have developed a lattice model, which can be considered a simplified analytical model, to clarify the changes in the shear resisting mechanism of reinforced concrete beams during, for example, the initiation of diagonal cracking, yielding of the shear reinforcement, and crushing of the web concrete. This is a practical and macroscopic analytical model to explain the shear resisting mechanism. The applicability of this lattice model is examined by shear strength equations proposed in the past and available experimental data. After verification of the lattice model, the validity of the modified truss analogy, which forms the basis of current Japanese shear design specifications, is examined thoroughly using the lattice model.

Key Words; *shear resisting mechanism, shear strength, lattice model, modified truss analogy*

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1. INTRODUCTION

In the current JSCE (Japan Society of Civil Engineers) Standard Specifications for Design and Construction of Concrete Structures, the stipulated method of calculating shear carrying capacity of reinforced concrete beams subjected to shear is based on the modified truss analogy. This means that the shear carrying capacity, V_y, should be the sum of the contribution made by the concrete, V_c, and the resistance of the truss mechanism resulting from web reinforcement, V_s.

If only the resistance of a truss mechanism with diagonal struts at 45 degrees is taken into account, experiments have shown that the actual shear carrying capacity will be underestimated. Based on this experimental evidence, the contribution made by the concrete, V_c, was added to that of the truss mechanism. In the ACI Building Code Requirements for Reinforced Concrete (ACI 318-89), the same fundamental design procedure is stipulated. On the other hand, the CEB-FIP Model Code 90 [1] gives a different method, in which the inclination angle of the diagonal struts is not fixed at 45 degrees. With this method, a design engineer can choose an arbitrary inclination angle for the diagonal struts within the limitations and calculate the resistance of each mechanical component forming the truss mechanism. By comparing the resistance of each component with the applied force, the safety of reinforced concrete beams subjected to shear is examined. This method is clearly not a direct way to calculate the actual shear carrying capacity.

Design procedures based on the modified truss analogy have been widely accepted in Japan because of abundant existing practice. However, it is also true that the modified truss analogy still includes several problems. For example, although V_s is called the shear carrying capacity, it actually represents a shear resistance corresponding to yielding of the web reinforcement. It is known experimentally that concrete beams may exhibit an increase in shear carrying capacity once the web reinforcement yields depending on the web reinforcement ratio. In estimating V_y, this increase in shear carrying capacity is completely neglected. Moreover, since yielding of the web reinforcement is assumed in this method, V_y cannot be calculated for *FRP* rods, because they do not exhibit yielding behavior.

The contribution due to the concrete, V_c, is assumed to be equal to the shear carrying capacity of concrete beams without web reinforcement. After the initiation of diagonal cracking, it is quite natural to assume that the contribution made by the concrete will fall as the diagonal crack extends, the crack width increases, and the concrete beam deforms. In design, however, V_c is assumed to remain constant from the initiation of diagonal cracking to the ultimate state.

Fixing these various problems with the modified truss analogy and presenting a unified analytical method applicable to concrete beams reinforced with any kind of reinforcement have become a matter of urgency in shear design.

Sato, Ueda, and Kakuta have reported some numerical research work in detail [2]. In their report, the variations in the contribution to shear carrying capacity made by concrete and web reinforcement and the effect of the reinforcement stiffness are investigated based on nonlinear finite element analysis. It is really an elaborate form of numerical study; however, objectivity might be lost during the discrete rearrangement of internal forces within a concrete beam modeled as a continuum in FEM. Aside from such a possible loss of objectivity, it is very difficult to perform the discrete rearrangement itself.
Schlaich has proposed a strut-and-tie model [3]. This is a simple and ingenious way to design

concrete structures, especially in the discontinuous region. Since reinforced concrete structures are modeled into strut and tie components in this model, the paths for internal resisting forces are restricted and the post processing of calculated results becomes easier.

Considering objectivity in the post processing of calculated results and the simple representation of the shear resisting mechanism, this research work takes up the issue of the lattice model [4], in which concrete beams are modeled into an assembly of truss components. Although a lattice model incorporating the compatibility condition, the equilibrium condition, and the constitutive model for concrete beam materials is a more simplified method than FEM, it is able to represent the shear behavior of concrete beams reasonably throughout the changes in the shear resisting mechanism.

In this research, analytical results obtained from the lattice model are used to clarify the variations in contribution to shear strength made by concrete and web reinforcement as deformation of a concrete beam increases after the initiation of diagonal cracking. The validity of the modified truss analogy is also examined.

2. LATTICE MODEL

2.1 Outline of the Lattice Model

Figure 1 shows the schematic diagram of a concrete beam after diagonal cracking initiates. If the shear stress along the crack surface is neglected, biaxial compression-tension stresses exist in an infinitely small element parallel to the diagonal crack direction (Fig. 2).

Considering the existence of this biaxial stress state in the web concrete (Fig. 2), we assume the lattice model shown in Fig. 3. In this lattice model, a reinforced concrete beam that is essentially a continuum is assumed to be an assembly of truss components.

The concrete is modeled into a flexural compression member, a flexural tension member, a diagonal compression member, a diagonal tension member, and an arch member. The reinforcement is modeled into horizontal and vertical members. The modeling of the diagonal tension member of the web concrete is one of the major peculiarities of the lattice model. Once the diagonal tension member of the web concrete is suitably chosen, the shear behavior of concrete beams before and after the initiation of diagonal cracking can be captured properly.

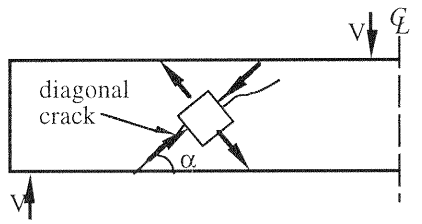


Fig.1 Concrete Beam with Diagonal Crack

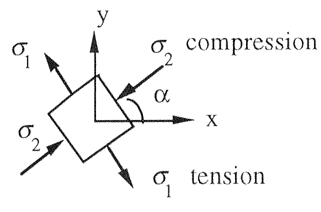


Fig.2 Stress State in Concrete Element

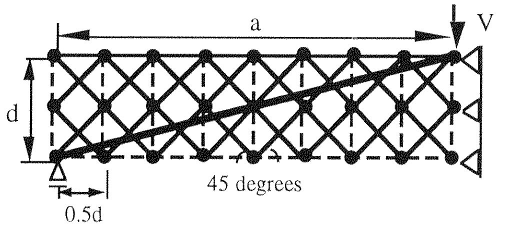


Fig.3 Schematic Diagram of the Lattice Model

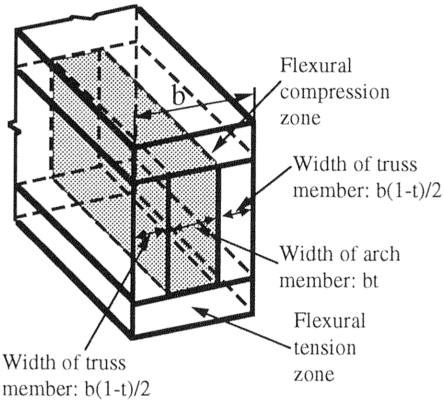


Fig.4 Cross Section of Concrete Beam in the Lattice Model

The thick solid line in Fig. 3 represents the arch member of the web concrete. Although the inclination angle of this diagonal strut is fixed at 45 degrees in the lattice model, the stress redistribution in the concrete beam after yielding of the web reinforcement can be adequately represented using this arch member. The arch member is assumed to be flat and slender and be connected with the nodes at each end.

2.2 Modeling of Each Member

Figure 4 shows a schematic diagram of a cross section through a concrete beam as represented in the lattice model. The web concrete is divided into a truss member and an arch member (the hatched area in Fig. 4). The ratio of the width of the arch member to the beam width is assumed to be "t". The value of t is determined as follows.

Assuming that a unit shear force is acting on a concrete beam with a specified t value ($0<t<1$), the potential energy can be calculated based on elastic analysis; it is obtained from the strain energy in each element and the external work due to a unit shear force. A value of t is found which minimizes the total potential energy for the whole of the structure. Although the calculated potential energy changes with increasing nonlinearity of the concrete beam, this method of determining the t value is adopted as a first approximation. Figure 5 shows an example of the change in this potential energy with t value. In this case, the chosen t value is 0.6.

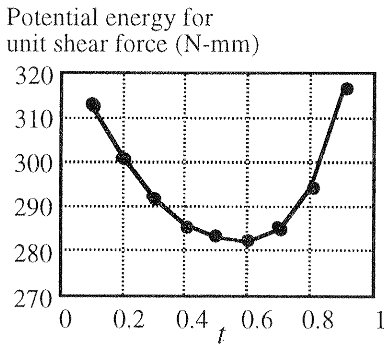


Fig.5 Change in Potential Energy with t value

The displacements of the arch member are coincident with those of the truss member at each end. However, except at the ends, the compatibility of the displacements is not considered, and the displacements of the arch member are completely independent of those of the truss member. This means that in the lattice model the plane stress condition is not assumed because of the concentration of stirrup arrangement in the direction of beam thickness. This is an assumption; however, with wider beams, we can expect that application of the plane stress condition would be impossible.

The depth of the flexural compression member is made equal to the depth of the flexural compression zone at the flexural ultimate state; that is, $x = (A_s \cdot f_y) / (0.68 f_c' \cdot b)$. The depth of the flexural tension member is assumed to be the twice the distance between the centroid of the bars as flexural tensile reinforcement and the bottom of the beam. A trial calculation confirms that assumptions regarding the depth of these horizontal members have less effect on the estimation of shear carrying capacity.

The height of the lattice model is assumed to be coincident with the effective depth of the beam. Thus, diagonal members and the arch member are placed so as to connect the top surface of the beam and the centroid of the bars as flexural tensile reinforcement. The horizontal distance of vertical members is assumed to be half the effective depth. Therefore, the thickness of the truss member and the arch member as seen from the side of the beam are equal to $(d/2)\, sin 45°$ and $d \sin\theta$, respectively, where θ is the inclination angle of the arch member.

2.3 Stress-Strain Relationship for Each Member

a) Diagonal tension member of concrete

The concrete's diagonal tension member resists the principal tensile stress resulting from shear forces. It is elastic before cracking. However, once a crack occurs, the concrete can be assumed to exhibit tension softening behavior. Therefore, after cracking, the tension softening curve for concrete is applied. The curve employed is the one-fourth model shown in Fig. 6. The crack width, w, in Fig. 6 is divided by the length of the diagonal tension member and converted into a strain. The fracture energy of concrete, G_F is assumed to be *100 N/m*. Thus, for the concrete of tensile strength *3.0 MPa*, w_1 and w_2 are equal to *0.025 mm* and *0.167 mm*, respectively.

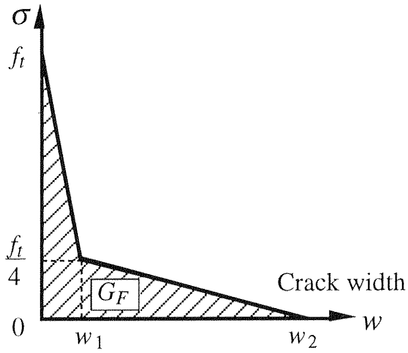


Fig.6 One-Fourth Model of Tension Softening Curve

b) Diagonal compression member of concrete and arch member

The concrete's diagonal compression member and arch member resist the diagonal compression caused by shear. The model of compression softening behavior proposed by Collins et al.[5] is adopted. Equation (1) shows the compressive stress-strain relationship of the concrete used in this research work.

$$\sigma_c = -\eta f_c' \left[2 \left(\frac{\varepsilon_c}{\varepsilon_o}\right) - \left(\frac{\varepsilon_c}{\varepsilon_o}\right)^2 \right] \tag{1}$$

where, $\eta = \dfrac{1}{0.8 - 0.34\,(\varepsilon_t / \varepsilon_o)} \leq 1.0, \qquad \varepsilon_o = -0.002$

c) Vertical and horizontal members

The stress-strain relationship for the reinforcing bars is assumed to be elasto-plastic. As for the horizontal member in the flexural tension zone, Okamura's tension stiffening model is added to take into account the bond behavior. For the horizontal member in flexural compression, the effects of the concrete are considered; however, compression softening is not taken into account.

For the vertical member, the effect of the concrete is not considered because the resistance of concrete to tension is already incorporated into the diagonal tension member. Using the lattice model, nonlinear incremental analysis is performed by displacement control. The convergence technique employed is the Newton-Raphson Method.

3. EXAMINATION OF THE APPLICABILITY OF THE LATTICE MODEL

3.1 For Beams without Web Reinforcement

As the shear strength equation for concrete beams without web reinforcement, Eq.(2) has been proposed [6].

$$v_c\,(MPa) = 0.20\, f_c'^{1/3}\, p_w^{1/3}\, d^{-1/4}\left[0.75 + \frac{1.4}{a/d}\right] \tag{2}$$

where, f_c' is the compressive strength of the concrete (MPa), p_w is the reinforcement ratio ($= 100\,A_s/(b_w\, d)$), d is the effective depth of the concrete beam (m), and a/d is the shear span-effective depth ratio. The validity of Eq.(2) has already been confirmed by numerous experimental data and Eq.(2) is accepted as the basis of the design equation in the JSCE Standard Specifications. To examine the applicability of the lattice model to concrete beams without web reinforcement, the analytical results given by the lattice model are compared with Eq.(2).

Figure 7 shows a comparison of the shear carrying capacity predicted by the lattice model and Eq.(2). For this comparison, a total of 81 concrete beams were calculated. Three levels of concrete strength (f_c'=20, 28, and 35 MPa), reinforcement ratio (p_w =1.36, 2.0, and 3.0 %), effective depth (d=0.3, 0.4, and 0.5 m), and shear span-effective depth ratio (a/d= 2.0, 3.5, and 5.0) were selected and combined. The width of the beams was fixed to b_w = 0.3 m. For the 81 cases, the average and coefficient of variation of the ratio of shear carrying capacities predicted by the lattice model and Eq.(2) are 0.953 and 9.3%, respectively. Although the shear carrying capacity predicted by the lattice model is slightly smaller than Eq.(2), the variation is admissible.

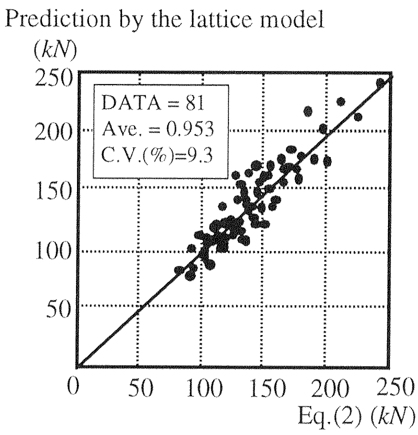


Fig.7 Comparison of the Lattice Model and Eq.(2)

Figure 8 shows the change in the ratio of predicted shear carrying capacity by the lattice model and Eq.(2) as each parameter is varied. Predictions by the lattice model are not exactly same as Eq.(2). However, the ratio of the two is mostly from 0.9 to 1.1. As for the size effect, the predictions of the lattice model are similar to Eq.(2). The shear failure mode predicted by the lattice model is failure of the diagonal tension member. This corresponds to the diagonal tension failure observed in experiments. These results mean that predictions of shear carrying capacity by the lattice model are just about adequate.

3.2 For Beams with Web Reinforcement

In the case of beams with web reinforcement, the validity of the lattice model has been examined based on existing experimental data. An outline of the data used to validate the lattice model is given in Table 1. The lattice model can easily predict changes in the stress state of each member in addition to predicting the applied shear force-displacement relationship.

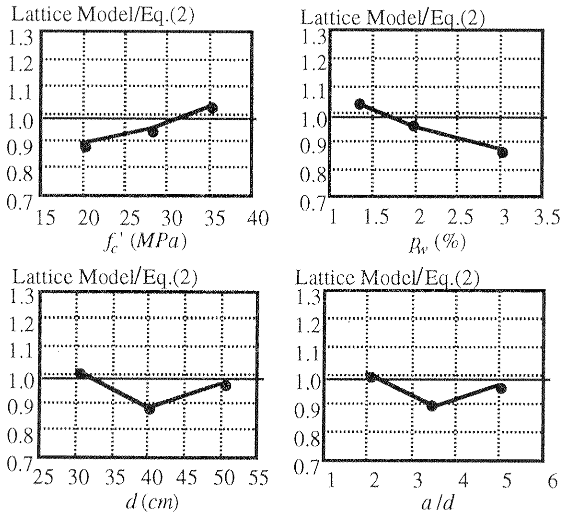


Fig. 8 Change in Shear Carrying Capacity with Variation of Each Parameter

Table 1 Outline of Experimental Data

No	Researcher	Cross Section*	b cm	h cm	d cm	a/d	fc' MPa	As cm^2	fy MPa	Aw cm^2	fwy MPa	s cm	Shear Carrying Capacity Vu (kN) Vtest	Vcal1**	Vcal2**
1	Ramirez	R	20.3	50.8	42.5	2.15	31.0	23.1	530	1.42	530	13.3	386	397(1.03)	341(0.88)
2	Clark	R	20.3	45.7	38.9	2.00	24.6	24.5	320	1.42	320	18.3	222	214(0.96)	207(0.93)
3	Leonhardt	R	30.0	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0	130	132(1.02)	131(1.01)
4	Leonhardt	T	30(15)	35.0	30.0	3.50	23.7	12.2	419	0.56	314	11.0	127	117(0.92)	98(0.77)
5	Ohuchi	R	45.0	60.0	52.5	2.86	43.9	95.7	383	1.43	355	25.0	519	556(1.07)	480(0.92)
6	Ohuchi	R	45.0	60.0	52.5	2.86	66.2	95.7	383	1.43	355	15.0	637	669(1.05)	599(0.94)

*) R: Rectangular section, T: T-shaped section. No.4: flange width=30cm, flange depth=7.5cm, web width=15cm.
**) Vcal1: Calculated value by the lattice model, Vcal2: Calculated value by the modified truss analogy,
( ) means the ratio of calculated value/ experimental value.

Thus, the changes in the shear resisting mechanism within a concrete beam, i.e., the changes in contribution by each member, can be estimated without adding any subjective operation. This is a strong incentive to develop the lattice model.

a) Shear force-displacement relationship

To examine the applicability of the lattice model to the shear resisting mechanism of concrete beams with web reinforcement, several experimental data for the applied shear force-displacement relationship are selected. The predicted applied shear force-displacement relationship by the lattice model is compared with these experimental results.

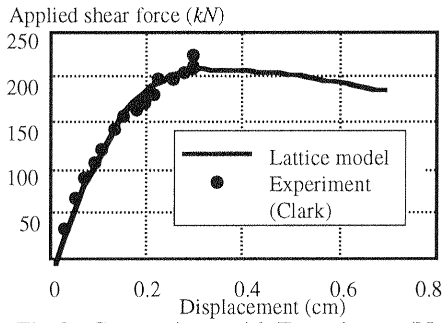


Fig.9 Comparison with Experiment (No.2)

Figure 9 compares the results calculated by the lattice model with Clark's experiment [8] (No. 2 in Table 1). This comparison goes only up to the peak resistance, because post-peak behavior was not reported by Clark. Figure 9 confirms that the lattice model reproduces the displacement behavior adequately. As for shear carrying capacity, the predicted value is very close to the experimental data.

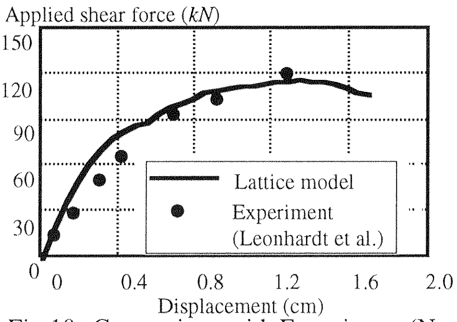


Fig.10 Comparison with Experiment (No.4)

Figures 10 and 11 give the comparisons with Leonhardt's data [9] (No. 4 in Table 1) and Ohuchi's data [10] (No. 5 in Table 1), respectively. Compared with these experimental data, the lattice model has a tendency to slightly overestimate the stiffness. However, the predicted displacement at the peak is almost identical to the experimental values. No. 4 in Fig. 10 is the case of the T-shaped beam. Here, the sum of the width of the arch and concrete diagonal members is assumed to be equal to the web width of the beam.

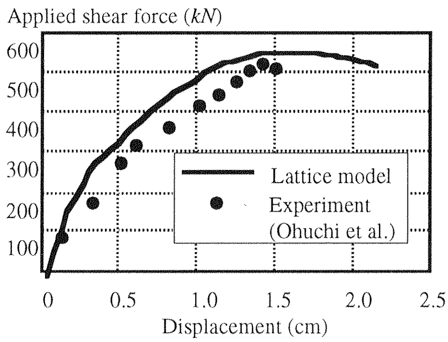


Fig.11 Comparison with Experiment (No.5)

The gradient in the predicted displacement curve changes at around V= 60 kN. This can be treated as the point where diagonal cracking occurred. On the other hand, no corresponding change in the experimental data is visible. As for the peak resistance and displacement at the peak, the predicted values are similar to the experimental ones.

In Ohuchi's experiment (Fig. 11), high strength concrete was used. In this case, the gradient change in the experimental displacement curve at around V= 300 kN is captured by the lattice model. As for the peak resistance, the lattice model gives a slightly high prediction. Although this newly developed lattice model is a fairly simple analytical method as compared with nonlinear finite element analysis, these results confirm that it can predict the shear behavior of concrete beams almost exactly.

b) Change in stress state in each member

Clark's experiment (No. 2) was chosen as the subject of an investigation of stress state changes in each member. The change in average stress on diagonal members of the concrete and stirrups and the stress on the arch member is examined. Figure 12 shows the lattice model for the No. 2 beam. The average stress on members located in the center of the shear span, as represented by solid lines in Fig. 12, is calculated with increasing in displacement of the loading point. The result is shown in Fig. 13.

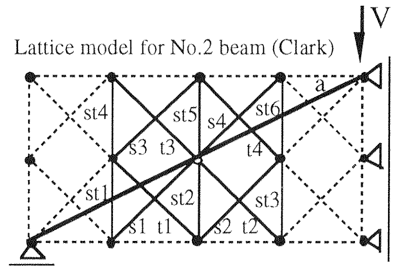


Fig.12 Members in the Shear Span Center used to Calculate the Average Stress

As shown in Fig. 13(a), the average tensile stress on diagonal tension members of the concrete decreases rapidly after the initiation of diagonal cracking. On the other hand, the average compressive stress on diagonal compression members of the concrete and the average tensile stress on stirrups increases significantly (Fig. 13(b),(c)).

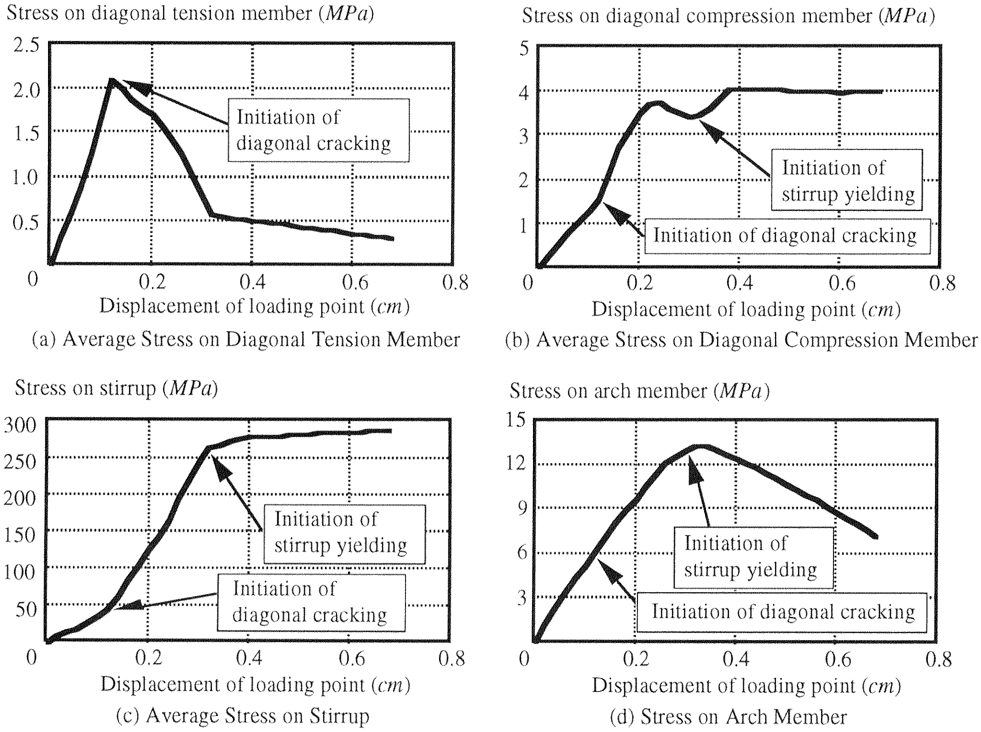


Fig. 13 Change in Average Stress on Each Member with Increasing Displacement

The average stress on diagonal compression members has a tendency to stay almost constant after a certain amount of increase in average stress. The average stress on stirrups increases slightly with increasing displacement after the initiation of yielding. The compressive stress on the arch member shows a significant increase after the initiation of stirrup yielding; however, due to the softening in compression, the arch member reaches the ultimate state (Fig. 13(d)). Thus, the predicted shear failure mode for the No. 2 beam is compression failure of the arch member after the initiation of stirrup yielding. This is coincident with the experimental results.

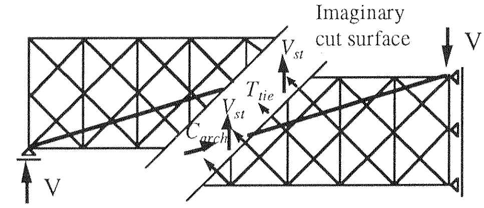
4. CONTRIBUTION OF EACH MEMBER TO SHEAR FORCE

The shear design equation for concrete beams with web reinforcement prescribed in the JSCE Standard Specifications (Eq.(3)) is based on the modified truss analogy. In Eq.(3), the contribution of the concrete, V_c, is assumed to remain constant after the initiation of diagonal cracking.

$$V_y = V_c + V_s \tag{3}$$

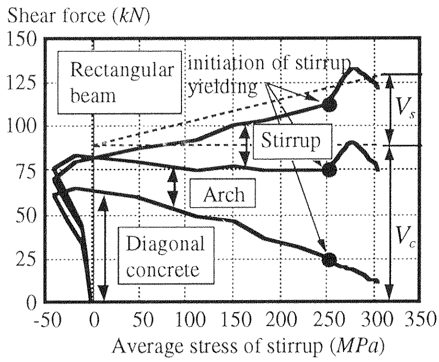
If the concrete contribution, V_c, results from only the tensile resistance of the concrete, it is quite reasonable, as mentioned before, to consider that after diagonal cracking begins the concrete contribution falls as the diagonal crack extends, the crack width increases, and the concrete beam deforms. The lattice model can provide the answer to this question, because the contribution of each member forming the shear resisting mechanism can be evaluated quantitatively by the lattice model. Based on the calculated result, the contribution for shear of each member is investigated.

The experimental data by Leonhardt and Walther (No. 3, No. 4) were chosen as the subject for this calculation. Considering the stress states represented in Fig. 13, the predominate members forming the shear resisting mechanism are the diagonal tension member of the concrete, the arch member, and the stirrup. Therefore, assuming a free body taken from the lattice model at the center of the shear span (Fig. 14), the contribution of each member is quantitatively estimated.

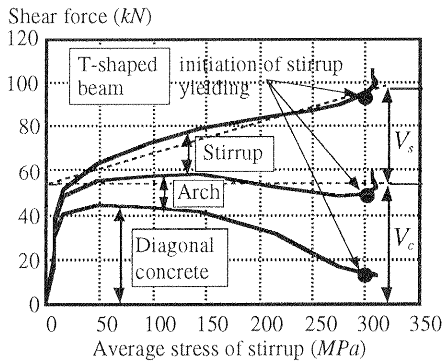


T_{tie} : Tension of diagonal tension member of concrete
C_{arch} : Compression of arch, V_{st} : Tension of stirrup

Fig.14 Free Body Taken from Lattice Model



(a) Contribution to Shear of Each Member (No.3)



(b) Contribution to Shear of Each Member (No.4)

Fig.15 Estimation of the Contribution to Shear by the Lattice Model

Figure 15(a),(b) show the change in contribution to shear of each member obtained from the lattice model. The broken line represents the prediction by the modified truss analogy.

In the modified truss analogy, the contribution of the concrete, V_c, can be obtained from Eq.(2) and the resistance of the stirrup, V_s, is calculated by assuming the 45-degree truss mechanism as follows:

$$V_s = A_w \, \sigma_w \, z / s \tag{4}$$

According to Fig. 15, the contribution of the concrete predicted by the lattice model comprises the resistance of the concrete's diagonal tension and the compression of the arch. In the vicinity of the point where the average stirrup stress is zero, the ratio of the diagonal tension of the concrete is relatively high. As the average stress on the stirrup increases — that is, as the concrete beam deforms —, the resistance of the concrete's diagonal tension decreases monotonically. However, because the increase in arch compression compensates for the decrease in the concrete's diagonal tension, the contribution of concrete, which almost corresponds to V_c, is maintained during loading. If it is assumed that V_c in the modified truss analogy does not result from only the tensile resistance of the concrete, but rather is composed of the tensile resistance and arch compression, the reason for V_c being maintained after the initiation of diagonal cracking is well explained.

According to Fig. 15, the contribution of the stirrup increases monotonically based on the 45-degree truss mechanism until the yielding. After the initiation of stirrup yielding, the resistance of the arch member increases rapidly (● in Fig. 15 corresponds to the initiation of stirrup yielding). The point at which the arch compression rapidly increases corresponds to the initiation of stirrup yielding. Because the tensile resistance of the stirrup is maintained as long as the stirrup does not fail, it is predicted that the final shear carrying capacity is dominated by failure of the arch member. In Fig. 15(a), the softening behavior of the arch member is clearly visible. Since the softening of the arch member occurs relatively early after yielding of the stirrup in this case (No. 3), the shear carrying capacity predicted by the lattice model (132 kN) is very similar to the value calculated by the modified truss analogy (131 kN) (Table 1).

Regarding No. 4 in Fig. 15(b), because of the delay in the initiation of stirrup yielding, the point of rapid increase in arch compression is also delayed. Since the horizontal axis of Fig. 15 is the average stirrup stress, the softening of the arch member cannot be clearly observed in Fig. 15(b). In this case, the resistance of the arch member continues to increase after the initiation of stirrup yielding. The shear carrying capacity predicted by the lattice model (117 *kN*) is considerably larger than the value calculated by the modified truss analogy (98 *kN*) (Table 1).

In Fig. 15(b) (No. 4), the average stirrup stress is always tension. On the other hand, the average stirrup stress in Fig. 15(a) (No. 3) is compression during the early stages of loading. No. 4 is a T-shaped beam and No. 3 is a rectangular beam. Since the width of the diagonal tension member of the concrete is reduced in the No. 4 beam compared with the No. 3 beam, the average stirrup stress is considered to be in tension from the beginning.

5. RELATIONSHIP BETWEEN SHEAR REINFORCEMENT RATIO AND SHEAR CARRYING CAPACITY

According to the lattice model, the shear carrying capacity of concrete beams with web reinforcement is controlled by softening of the arch member. When such softening occurs relatively early after the initiation of stirrup yielding, the shear carrying capacity is close to the value calculated by the modified truss analogy. On the other hand, if softening of the arch member is delayed, the shear carrying capacity predicted by the lattice model is significantly greater than that given by the modified truss analogy.

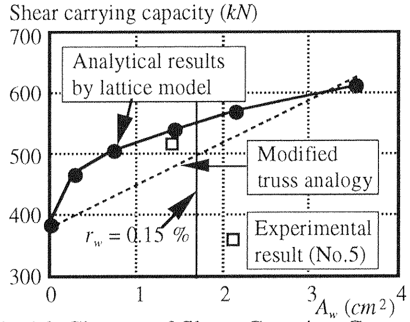


Fig.16 Change of Shear Carrying Capacity with Shear Reinforcement Ratio

In the modified truss analogy, the increase in shear carrying capacity after stirrup yielding is not taken into account. In this sense, it can be considered a conservative prediction. However, the modified truss analogy, which is based on yielding of the web reinforcement, cannot be directly applied to the problem of shear in concrete beams reinforced with *FRP* rods, since *FRP* rods do not exhibit yielding.

When conventional reinforcing bars are used as the web reinforcement, if the shear reinforcement ratio is small, experimental results and qualitative evidence show that the shear carrying capacity is considerably larger than the value calculated by the modified truss analogy. To quantitatively clarify the increase in shear carrying capacity after stirrup yielding, a simulation is performed using Ohuchi's data (No. 5). In this simulation, all factors except the area of the web reinforcement (A_w) are coincident with the original No. 5 data. The result is shown in Fig. 16. As this figure shows, the predicted shear carrying capacity increases significantly more than given by the modified truss analogy as the shear reinforcement ratio increases from 0 to 3.5 cm^2 (corresponding to the shear reinforcement ratio, $r_w = 100\, A_w/(b_w\, s) = 0$ to 0.31 %).

In the specific case, the shear carrying capacity predicted by the lattice model is more than 10% greater than that calculated by the modified truss analogy. In the JSCE Standard Specifications, 0.15 % is required as the minimum shear reinforcement ratio. According to Fig. 16, an almost 10% increase over the modified truss analogy can be expected for r_w = 0.15 %. This increase in shear carrying capacity over that predicted by the modified truss analogy is not constant and varies depending on the amount of shear reinforcement.

6. INSTABILITY OF LATTICE MODEL IN POST-PEAK REGION

To investigate the instability behavior of the lattice model around the peak point, an eigenvalue analysis is carried out. Ohuchi's data (No. 6) was chosen as the subject. Figure 17 shows the shear force-displacement relationship predicted by the lattice model. At point *A* in Fig. 17, just before the peak, all eigenvalues of the tangential stiffness matrix of the lattice model are positive. However, at point *B*, which corresponds to the peak, the minimum eigenvalue of the tangential stiffness matrix turns negative for the first time. At point *C*, just after a sudden drop in shear resistance, several negative eigenvalues are obtained.

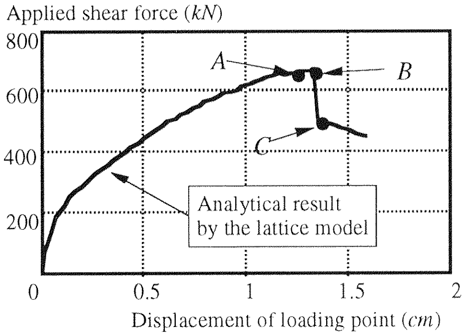
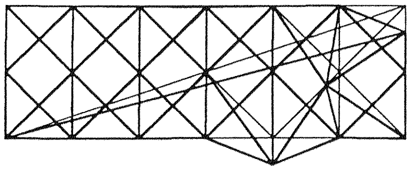
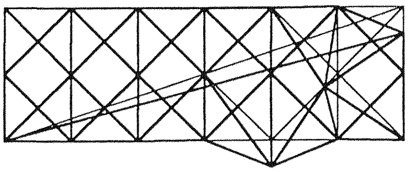


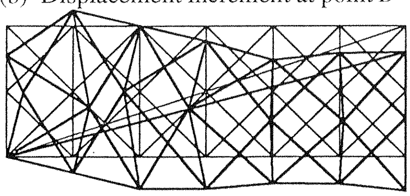
Fig. 17 Analitycal Result of Shear Force-Displacement Relationship (No. 6)



(a) Displacement increment at point A

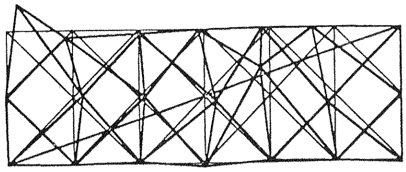


(b) Displacement increment at point B

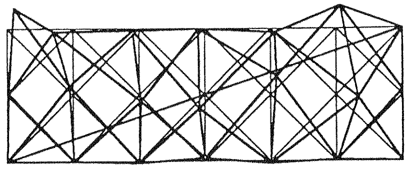


(c) Displacement increment at point C

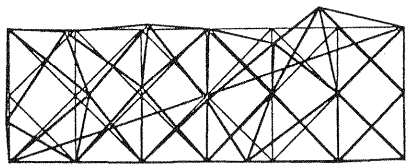
Fig. 18 Predictions of Displacement Increments



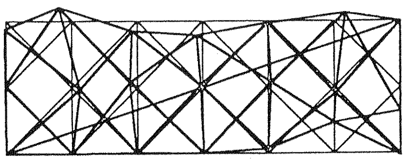
(a) First eigenmode at point A (Eigenvalue = 12525.8)



(b) First eigenmode at point B (Eigenvalue = -5851.4)



(c) First eigenmode at point C (Eigenvalue = -2207551.6)



(d) Second eigenmode at point C (Eigenvalue = -529350.1)

Fig. 19 Predictions of Eigenmodes

Figure 18 shows the displacement increment at points A, B, and C. The thick lines represent the displacement increment obtained by the analysis. The thin lines are the original shape of the lattice model. The shapes of displacement increments at points A and B are almost identical. Because the predicted shear failure is controlled by softening of the arch member, the arch is compressed during loading process toward the peak. Thus, a downward movement of the loading point due to softening of the arch member can be observed. At point C, where several negative eigenvalues exist, a displacement increment appears over the whole of the beam and significant deformation of the beam can be observed.

Figure 19 shows the eigenmode at points A, B, and C. The thick line represents the eigenmode obtained by eigenvalue analysis. The first eigenmodes of points A and B are fairly different, especially in the vicinity of the loading point, in spite of the fact that the displacement increments are similar. This can be considered due to the drastic change of the eigenvalue. At point C, where several negative eigenvalues exist, the change in eigenmode for the whole of the beam can be observed in the second eigenmode. The change in eigenmode of the whole beam is quite similar to the change in displacement increment at this point.

7. CONCLUSIONS

In the newly developed lattice model, a concrete beam subjected to shear force is converted into an assembly of truss and arch members. Using this lattice model, a nonlinear incremental analysis is performed. As well as the conventional truss members, a concrete arch member and diagonal tension members are incorporated into the lattice model. Although this lattice model is a simplified analytical method in which the total degree of freedom is fairly small compared with normal finite element analysis, and despite assumptions in the calculation such as for the ratio of the width of the arch member and the concrete diagonal member, a comparison with experimental data shows that predictions of the shear resisting mechanism of concrete beams are quite adequate. For example, the accuracy of the prediction of shear carrying capacity of a concrete beam without web reinforcement is equivalent to the macroscopic shear strength equation. The shear force-displacement relationship of concrete beams with web reinforcement can be predicted almost exactly by the lattice model.

In particular, since each member forming the shear resisting mechanism is made discrete from the beginning, the lattice model can illustrate the contribution of each member to shear without adding any intentional operation. It can also predict the change in shear carrying capacity with increasing shear reinforcement ratio.

The conclusions reached in this research work are as follows:

(1) In the modified truss analogy, it is assumed that the contribution of concrete, V_c, will be maintained after the initiation of diagonal cracking. This can be explained by considering the compression of the arch in addition to the diagonal tension of the concrete. As the concrete beam continues to deform after the initiation of diagonal cracking, the resistance of the diagonal tension decreases rapidly. However, the compression of the arch compression compensates for the decrease in the diagonal tension of the concrete. Therefore, the contribution of concrete to shear is almost equivalent to V_c even after the diagonal cracking.

(2) The shear resisting action of the stirrups becomes significant after the initiation of diagonal cracking. Their resistance may be estimated based on the 45-degree truss model. After the initiation of stirrup yielding, the resistance of arch compression increases steadily. Finally, because of softening of the arch member, the shear resisting mechanism reaches its peak.

(3) Depending on the shear reinforcement ratio, the shear carrying capacity may show a significantly higher value than predicted by the modified truss analogy. The discrepancy may reach more than 10%. Although the neglect of this discrepancy is essentially conservative in practical design, it

should be noted that the safety margin changes with the amount of shear reinforcement.

(4) According to an eigenvalue analysis, the peak point of the shear force-displacement curve corresponds to the appearance of a negative eigenvalue in the tangential stiffness matrix of the lattice model. After the appearance of several negative eigenvalues, it is possible that the shape of the displacement increment may follow another eigenmode other than the first eigenmode.

The lattice model can be applied to concrete beams regardless of the kind of reinforcement. For concrete beams reinforced with *FRP* rods that do not exhibit yield behavior, the shear carrying capacity corresponding to stirrup yielding does not exist. Thus, application of the modified truss analogy itself is a problem. It is hoped that a parametric study of the lattice model and experimental verification over a wide range will provide useful information for the shear design of concrete beams reinforced with *FRP* rods.

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