THREE-DIMENSIONAL ANALYSIS OF STRENGTH AND DEFORMATION OF CONFINED CONCRETE COLUMNS

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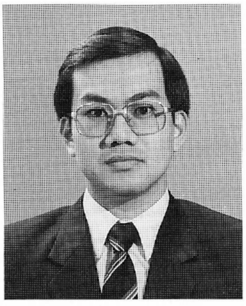

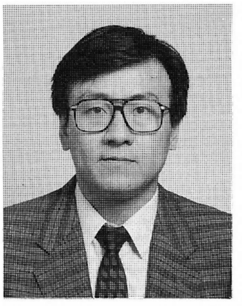

Paulus IRAWAN Koichi MAEKAWA

SYNOPSIS

Material-nonlinear three-dimensional finite element analyses were conducted for laterally confined reinforced concrete columns under axial compression. An elasto-plastic and continuum fracture model served to investigate the mechanism of confinement by steel casing and steel ties or hoops on strength gain and the whole inelasticity of the members. This paper describes two failure modes. One is where the whole lateral steel yields when the reinforced concrete section reaches its ultimate capacity. The other corresponds to the case where some part of the lateral steel remains elastic in the ultimate condition of reinforced concrete columns. A circular casing was found to exhibit the former mode of failure in any case, and the strength gain of the confined concrete is proportional to the volume of steel. On the other hand, square casing with a larger amount of steel was proved to come up with the latter mode of failure, and lateral stress arising in confined concrete was found not to be proportional to the amount of lateral steel. Uniformity of confinement stress and the induced damage were elucidated in consideration of confinement efficiency by discretely distributed lateral steel ties and hoops. The sectional averaged lateral stress in concrete, the minimum of which along the axis of columns governs the capacity of the entire confined columns, was found to be affected by the volumetric averaged lateral stress of the concrete as well as the spacing of the ties associated with the uniformity of stress states. The spacing of lateral ties also influences the volumetric averaged confinement of the concrete, which mathematically corresponds to the axial mean value of sectional averaged confinement stress in each section.

Keywords : *Confinement, finite element analysis, constitutive law, ductility*

P. Irawan is a graduate student in the Department of Civil Engineering at the University of Tokyo. His research is in the mechanical behaviors of concrete under triaxial confinement and non-linear analysis of reinforced concrete shells.

K. Maekawa is Associate Professor of the Department of Civil Engineering at the University of Tokyo. His research interests cover reinforced concrete mechanics, computational approach to seismic behavior of reinforced concrete-soil systems, thermodynamics of cement-concrete composites, and durability design of concrete structures.

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1. INTRODUCTION

Numerous studies of the compressive strength and deformation of core concrete confined by lateral reinforcement have already been conducted with a view to improving the seismic performance of reinforced concrete members [1-7,12-17,20]. In general, design codes state minimum requirements for lateral reinforcement to ensure a member has sufficient ductility and to prevent longitudinal reinforcement from buckling. However, it is not quantified how the lateral steel could enhance the strength and ductility indicated by stress-strain relationships of confined structural concrete of arbitrary shapes, dimensions, amount of steel and spacing. It can also be said that the mechanism and confinement efficiency of lateral steel have not been rationally deduced from a vast pool of experiments. As is widely known, even if the same amount of steel would be placed as lateral confinement, the strength of reinforced concrete section in compression differs according to the shape of the section and the spatial arrangement, as well as the spacing of the lateral reinforcement.

There is no lack of design equations named "constitutive equations for confined concrete". In these equations, the relationships between the confinement efficiency and the geometry of the lateral steel is dealt with as an empirical shape factor [1,2,4,12,20] or geometrically calculated effective sectional area [7,16]. The relationship has never been explained from the microscopic viewpoint of material inelasticity. Thus, many empirical stress-strain relationships for confined concrete do not deserve to be named "versatile constitutive law", but should be regarded as design-oriented empirical equations with few mechanical background. Confinement efficiency has to be theoretically explained as being rooted in the stress field and corresponding local inelasticity induced by the lateral steel indeed.

The objective of this research is to investigate computationally how the strength and ductility of confined concrete are affected by the cross-sectional shape of the concrete, the arrangement of the lateral steel, the amount of steel used, and the spacing of the steel. To do this, considerable technical attention is paid to the stress-induced damage indicated by the fracture parameter [9] as well as to the plasticity, which cannot be directly measured by tests. At first, the authors deal with a steel casing that represents the steel tubes or plates surrounding the concrete. Densely arranged reinforcing bars (the spacing to referential dimension of section less than 0.1) along the axis of columns may be covered in the category of steel casing such as large scale reinforced concrete bridge piers. Stress-induced damage over the section was the point of discussion, and the stress distribution along the axis of the member was assumed to be uniform in this case. Subsequently, core concrete confined by steel ties arranged discretely will be discussed. In this case, the distribution of stress and damage along the axes of members is non-uniform and the effect of spacing of lateral ties and spirals will be a key parameter. Computer simulation was carried out using an elasto-plastic and continuum fracture constitutive model with three-dimensional finite element analysis [9]. The results of three-dimensional finite element analysis are compared with existing proposed design equations. Discussion of analytical results is expected to elucidate the mechanism of confinement subjected to axial compression. Analytical simulation and discussions are somewhat fundamental so that they could serve in future development of a rational design equation of confined reinforced concrete under axial compression.

2. FINITE ELEMENT APPROACH

The strength gain and ductility of confined concrete should not be treated as material properties, but as macroscopic member characteristics. Then, confinement which differs according to the arrangement of the lateral ties, the amount of lateral reinforcement, and the spacing has to be explained by microscopic aspect such as non-uniform stress field developing over the volume and associated local inelasticity in each dimension and shape of members.

This paper attempts to approach this mechanical aspect with the aid of finite element analysis. An enhanced constitutive model that can simulate the inelasticity of concrete mechanics is indispensable in an analytical investigation. Here, an elasto-plastic and continuum fracture model [9] was adopted since the internal damage and plasticity occurring in concrete under three-dimensional stress can be quantified. To understand the effect of the geometrical arrangement of lateral steel on confinement efficiency, a steel casing filled with concrete is selected as the extreme case where the spacing of lateral bars is zero. Spacing of discretely arranged reinforcing bars will be discussed as one of the important factors after clarifying the confinement efficiency related to the shape of section and steel arrangement.

2.1 Full Three-Dimensional Elasto-Plastic and Continuum Fracture Model

Nonlinear behavior of concrete is indicated by plasticity, which denotes residual deformation, and internal damage, which represents the loss of elastic shear stiffness induced by the occurrence of micro defects and the internal stress intensity. Micro defects were found not to affect the volumetric elastic energy capacity of concrete but elastic energy capacity in shear mode. Maekawa et al. [9] introduced fracture parameter K to indicate continuum damage occurring in the shear elasticity of concrete, and proposed the second invariant of elastic strains J_{2e} as a primary indicator of internal stress intensity, which evolves the continuum damage associated with the assembly of micro-defects. The damage is formulated of being suppressed by the three-dimensional confinement denoted by I_{1e} as well. Thus, we have the following indicators of concrete nonlinearity [9]:

Indicator of fracturing damage,

$$K = \frac{J_2}{2\, G_0\, J_{2e}} \tag{1a}$$

Indicator of plasticity,

$$J_{2p} = \int \frac{\partial J_{2e}}{\partial \varepsilon_{eij}}\, d\varepsilon_{pij} \tag{1b}$$

Indicator of internal shear stress,

$$J_{2e} = \sqrt{\frac{1}{2}\, e_{eij}\ e_{eij}} \tag{1c}$$

Indicator of internal confinement,

$$I_{1e} = \frac{\varepsilon_{ekk}}{3} \tag{1d}$$

Indicator of total shear stress,

$$J_2 = \sqrt{\frac{1}{2}\, s_{ij}\ s_{ij}}\ . \tag{1e}$$

Fracture parameter K represents continuum damage of concrete in terms of elastic shear strain energy. The smaller the value of K, the smaller the capacity of the concrete to absorb and release the elastic shear strain energy due to the induced micro defects. G_o is defined as the initial elastic shear modulus. Plastic indicator J_{2p} represents induced permanent deformation in shear mode and isotropic plastic indicator I_{1p} (= $\varepsilon_{pkk}/3$) represents inelastic dilatancy. Notations ε_{eij} and ε_{pij} mean elastic and plastic strain tensors, respectively. Shear elastic intensity J_{2e} represents the internal stress applied to the effective non-damaged volume of concrete. Notations e_{eij} and s_{ij} mean elastic deviatoric strain and stress tensors, respectively. The values of K and J_{2e} are related to the strength gain of confined columns and J_{2p} to the ductility. Constitutive laws were proposed incorporating the above nonlinear indicators. Details are discussed in reference [9]. The constitutive equations concerned were installed in program COM3 in the COncrete Model series (COM2 [8], WCOMR [10], WCOMD [18]).

2.2 Finite Element Model

Two types of element were used in the finite element model. A three-dimensional solid isoparametric element and three-dimensional truss element were adopted for representing concrete and lateral reinforcement, respectively. The use of truss element to model steel casing can be justified, since in this study the steel casing is not expected to resist any axial load, but only in-plane lateral membrane stress to study the amount of confinement produced by the steel casing. Moreover, it is also used to model densely arranged reinforcing bars without any stiffness in the longitudinal action. The contribution of out-of-plane flexural stiffness, which is quite significant for heavily lateral reinforced concrete, is neglected on purpose to model the so called "corner confined" concrete. In this case, confinement is mainly introduced through the corner action. Thus, only the effect of axial stiffness of lateral steel, which dominates the behavior of lightly laterally reinforced concrete is considered. The coupled effect of axial stiffness and out-of-plane flexural stiffness will be discussed in the next stage of this research.

Symmetry was taken into account in the discretization of finite element model as shown in Fig. 1. One-fourth of section was represented by finite elements in lateral direction. To consider uniform lateral stress, a single iso-parametric element consisting of 20 nodes was used to represent concrete for circular section. Since confinement is not uniform within the section, several elements are needed for square section. Thus, four elements were used to obtain more information about fracture parameter of local points, internal stresses and strains around the square section. In the longitudinal direction, no division of element was assigned for concrete and steel in the case of a steel-encased confined column, and two elements were allocated to represent non-uniform distribution of stress along the member axis for a discretely confined column. Figure 1 shows sectional and longitudinal discretization of the model. Second order of Gaussian integration was used in analyzing the model. Elasto-plastic hardening model for steel casing is shown in Fig. 2 with standard material properties used in the analysis. Strain hardening is modeled to avoid numerical instability and divergence of iterative solution.

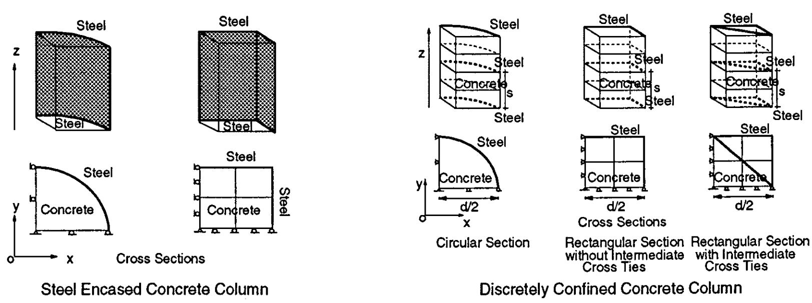


Fig. 1 Discretization of 1/4 Model

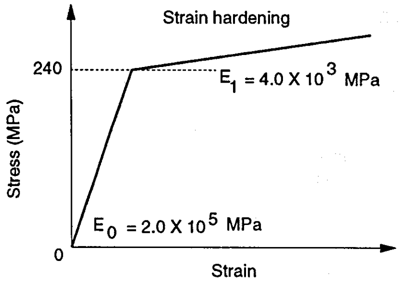


Fig. 2 Elasto-Plastic Hardening Model for Steel Casing (Standard)

Loading was applied to the top surface of the concrete in the direction of the *Z*-axis (See Fig. 1) as an axial enforced displacement with fixed boundary condition at bottom surface. Displacement increment was taken considerably small, being one-ten-thousandth of the height of model, i.e., 100 micro strain as average axial strain increment. It should be noted that reinforcement which was represented by truss element does not resist any direct axial load. Only the confined concrete bears the axial load, and confinement being produced by the steel reinforcement as it resists the lateral expansion of the concrete.

Flexural rigidity through the truss element used in the analysis was not taken into account. It implies that the interacting force from the steel is transferred to the confined concrete only through the corners of the square section. This mode of confinement, called "corner action", may hold in the case where thin steel casing or smaller diameter bars are used. However, in arranging thick steel casing and larger diameter bars around comparatively smaller sections of concrete, there is an additional mode of confinement besides corner action. Actually, the line load directly applied to the confined concrete is also created by a steel casing or steel bars having flexural rigidity. The effect of flexural rigidity on the confinement efficiency will be discussed later.

Longitudinal reinforcement can serve to confine the concrete as well as to bear the axial load together with the core concrete. In this study, longitudinal reinforcement was intentionally eliminated in the analysis to ascertain solely the role of lateral reinforcement. Concrete covering also was not considered because it easily falls down in the ultimate condition, and normally it is not expected to improve the strength and ductility.

The constitutive model of concrete used in this analysis [9] is applicable to non-localized fracturing, where defects in the concrete can be assumed to be dispersed uniformly. Although a descending branch of the mean stress-strain relationship of members is obtained in this research (see Fig. 3), the authors accepted results only up to the ultimate capacity as being reliable and independent on the finite element discretization. For analyzing softening behaviors in compression, localized deformation has to be consistently predicted with view points of fracture mechanics and/or interaction field of micro defects [11].

3. CONFINEMENT INDEX

The strength of concrete columns depends on axial concrete stress at each location of member section. It can be said that member axial strength with lateral confinement is spatial averaged stress of confined concrete over the whole domain of a critical section. As for circular section, local stress over a section can be assumed to be uniform due to its axial symmetry. However, for square section, non-uniform stresses appear over a section. This is why finite element analysis is needed in evaluating cross section based strength and ductility of confined columns. At each Gauss integral point, constitutive equations are applied.

Confinement efficiency by lateral steel depends on the amount of steel, strength of constituent materials, shape of section, and spacing. Since some of these factors are mutually coupled, parametric study has to be done very carefully. In general, one factor must be kept constant while the influence of the other is studied. The effect of shape dependent factors is studied first, since it is important to ascertain this influence in order to investigate the effect of the spacing of lateral ties.

The following stress-based parameters, called sectional averaging of the lateral mean stress of concrete $\overline{\sigma_c}$ and volumetric averaging of lateral mean stress $\overline{\sigma_v}$, which can not be obtained in the experimental approach are introduced as,

$$\overline{\sigma_c} = \overline{\sigma_c}(z) = \frac{1}{A_c}\int_{A_c} \sigma_c(X,Y,Z)\, dA \ , \tag{2a}$$

$$\sigma_c \equiv \frac{\sigma_{c,xx} + \sigma_{c,yy}}{2} \tag{2b}$$

$$\overline{\sigma_v} = \frac{1}{V_c}\int_{V_c} \sigma_c(X,Y,Z)\, dV \tag{3}$$

where $\sigma_{c,ij}$ is defined as concrete stress tensor of two dimension $[ij]$, A_c is the cross-sectional area of concrete confined by lateral steel perpendicular to the Z-axis (see Fig.1), and V_c is the domain of concrete. V_c is being equal to $A_c\, H$, where H is the height of member.

The sectional averaging of the lateral mean stress of concrete $\overline{\sigma_c}$ is supposed to be the lateral local confinement degree by the lateral steel and the volumetric averaging of lateral mean stress $\overline{\sigma_v}$ expresses the entire confinement supplied by the whole lateral steel and is regarded as mean confinement index.

For a continuous steel casing, $\overline{\sigma_c}$ is not dependent on coordinate-Z along the axis of members. Hence, $\overline{\sigma_v}$ is equal to $\overline{\sigma_c}$ at any coordinate-Z.

According to mathematical compatibility, we have,

$$\overline{\sigma_v} = \frac{1}{H}\int_{H} \overline{\sigma_c}(z)\, dz \tag{4}$$

The equilibrium condition of lateral concrete stress with lateral uniaxial steel stress σ_s (see Appendix) yields,

$$\overline{\sigma_v} = -\frac{1}{2\,V_c}\int_{V_s} \sigma_s\, dV \ , \tag{5}$$

where, V_s is the whole volume of the laterally arranged steel.

A negative sign in equations represents compression; a positive sign, tension. When lateral steel would come up to the yield strength of steel f_y over the whole volume of steel, according to the Appendix, Eq. (5) is equal to,

$$\bar{\sigma}_v = -\frac{1}{2}\rho_s f_y , \tag{6}$$

where ρ_s is the volumetric ratio of steel.

The value of $\frac{1}{2}\rho_s f_y$ has been often used as a parameter to exhibit the amount of confinement in design equations proposed by Sargin [12], Sheikh and Uzumeri [16], Mander et al. [7], and others.

The value of $\frac{1}{2}\rho_s f_y$ has a clear physical meaning to represent the level of confinement if steel would reach the full plastic yield condition in the ultimate of the columns. However, provided the elastic zone remains in steel, $\frac{1}{2}\rho_s f_y$ might lose its mechanical background. The authors would like to insist that the sectional shape factor on the confinement efficiency should be discussed under the same magnitude of lateral confinement indicated by Eq. (3), not by $\frac{1}{2}\rho_s f_y$. Finite element analysis allows us to compute the mean confinement index $\bar{\sigma}_v$ directly and rationally. This is the advantage of the microscopic approach to the macroscopic aspects of structural concrete.

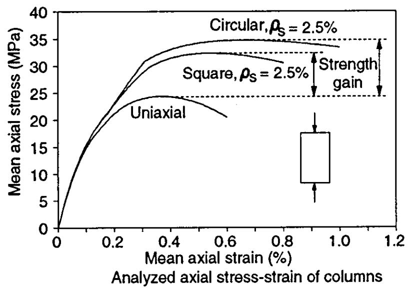
To deduce the effect of the longitudinal arrangement of lateral ties associated with the non-uniform confinement, the potential $\frac{1}{2}\rho_s f_y$ of lateral confinement is kept constant in order to discuss the effect of spacing of lateral ties. The cross section of each tie was varied depending on the distance between ties so that the potential lateral stress denoted by Eq. (6) would be constant. This procedure is possible since, as mentioned before, the contribution of flexural stiffness of ties was not taken into account. Therefore, there is no coupling effect of axial and flexural stiffness from lateral ties to the potential lateral confinement. If the spacing alone were to be changed and the size of the steel section were constant, the volumetric ratio of steel, which is related to the confinement magnitude, subsequently varies, too. This parametric arrangement does not make any sense for the purpose of evaluating confinement efficiency. After this section, compression will be defined positive unless special note is given.

4. STEEL ENCASED CONFINED CONCRETE COLUMNS

4.1 Confinement Efficiency - Geometrical Arrangement of Steel -

The effect of cross-sectional shape is discussed first. In this section, circular and square casings are concerned. The strength of confined concrete, which is equated with axial capacity per unit area, is quantified by the absolute strength gain, not the normalized value of uniaxial compressive strength. This approach has been experimentally and analytically verified and adopted for design equations proposed by others [1,2,3]. The reason is lateral stress level, which enhances axial strength, is also associated with uniaxial strength, and the sensitivity of uniaxial strength to both the strength gain and corresponding lateral stress is nearly the same. The computed strength gain of confined columns of circular and square sections is shown in Fig.3.

The yield strength of steel f_y is 240 MPa with modulus of elasticity before yielding E_0 is 200,000 MPa and after yielding E_1 is 4,000 MPa (see Fig. 2). Concrete uniaxial strength f_c' is 25 MPa and the modulus of elasticity is 20,000 MPa. Volume ratio of steel "ρ_s" was controlled by changing the thickness of the casing.



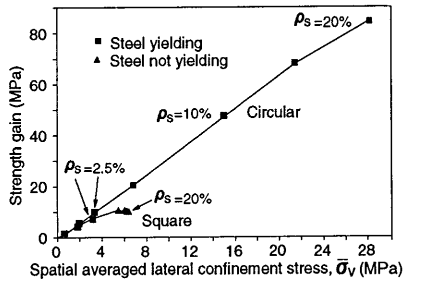


Fig. 3 Computed Strength Gain and Laterally Induced Stress of Steel Casing Confined Columns

Let us compare sectional shape dependency on strength gain. Figure 3 shows that for the same lateral mean confinement $\bar{\sigma}_v$, which was evaluated by numerical integral in analysis, the strength gain of circular section based on f_c' coincides with that of square one for small spatial averaged lateral confinement. Here the level of stresses and confinement at each location around the square section is still almost the same; but as $\bar{\sigma}_v$ increases, the strength gain of circular section is gradually greater than that of the square one until the peak when the confined concrete loses the capability to induce plasticity in steel in square sections. Roughly speaking, the strength gain appears to be proportional to the mean confinement $\bar{\sigma}_v$ defined by Eq. (3) for both circular and square sections if the steel yields. However, the sensitivity is different according to the geometrical placing of lateral steel. Lateral steel does not yield when larger volumetric ratio is allocated in square sections (see Fig. 3). In this case, spatial averaged lateral confinement stress of confined concrete is almost constant, and so is the strength gain.

Lateral stress distribution in concrete is crucial and results in shape dependency. Distribution of axial and lateral local confinement stresses ($\sigma_{c,zz}$ and σ_c) over square shaped cross sections are shown in Figs. 4 and 5. As circular section creates perfect uniformity of stress, values of axial and lateral stresses of circular sectional concrete having the same amount of lateral steel are shown for comparison in the same figures. In this case, the whole lateral steel yields and the same axial mean strain of members is realized. On the other hand, the stress field developed in a square section appears complex. Spatial averaging of local axial stress in Fig. 4 is equal to the strength of the confined structural concrete in the ultimate condition. The sectional average of local lateral stresses for both square and circular sections (see Fig. 5) balances with the common value of $\frac{1}{2}\rho_s f_y$ according to Eqs. (2), (3), and (6).

Around the four corners (Figs. 4 and 5), higher lateral stresses are induced, which may cause higher axial local stresses to be carried by the concrete. Concrete at intermediate zone between corners contributes less to axial load-carrying capacity, owing to lower local confinement in the lateral direction. Around the central zone far from corners, isotropic confinement is attained.

Compared with the uniform stress in circular section, some parts of the square section have higher confinement and greater axial stresses and others, smaller. It can be said the average of those local stresses exhibits lower performance on confinement compared with circular section as a whole. For a deeper understanding, the spatial distribution of fracture parameter as a damage indicator of concrete is useful in elucidating the confinement mechanism. The contour line of fracture parameter and its magnitudes in both circular and square sections are shown in Fig. 6. At the four corners of the square section, value of K is the greatest; and at points mid-way between the corners on each side, the value is the smallest. This means that much damage is concentrated on the concrete close to the steel casing but far from the corners of square section. Mechanically, confinement state at the center halts the further progress of fracturing.

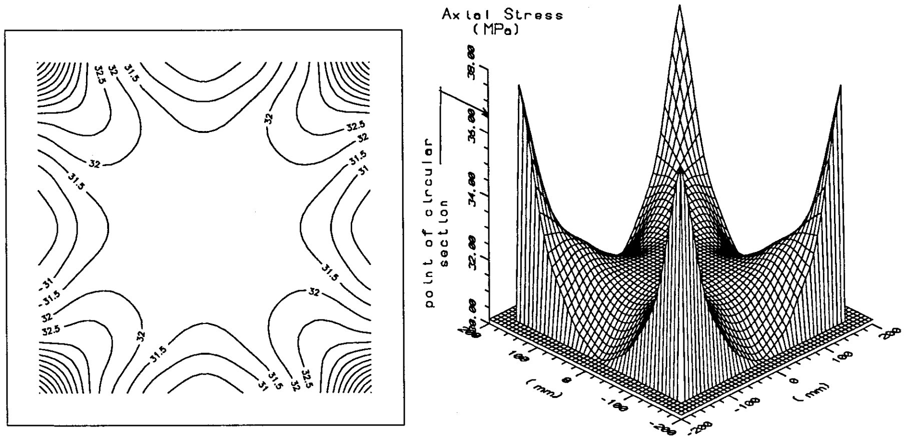


Fig. 4 Distribution of Axial Stress, $\sigma_{c,zz}$ (compression positive)

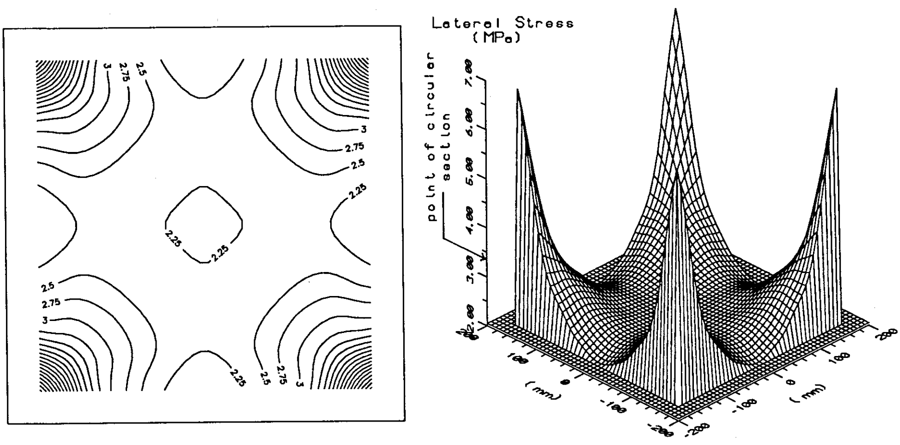


Fig. 5 Distribution of Lateral Stress, σ_c (compression positive)

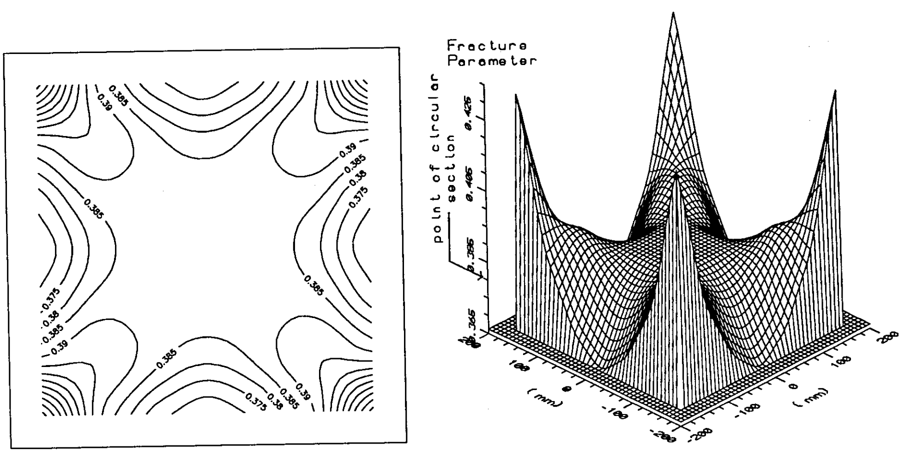


Fig. 6 Contour Line of Fracture Parameter, K

4.2 Confinement Efficiency - Amount of Steel -

To study the confinement efficiency of concrete, the amount of steel was varied as a parameter. As far as mechanical aspects are concerned, the material parameters specified in the previous section remained unchanged. The relationship between the amount of steel "ρ_s" and the induced mean confinement $\overline{\sigma}_v$ over a section is shown in Fig. 7. Also, relationship of strength gain versus amount of steel is obtained as shown in Fig. 8, incorporating Figs. 7 and 3.

Through these figures, it can be found that non-uniformity in stress distribution and confinement action in square section induces two conditions of steel when confined concrete reached ultimate strength. One is that the whole steel has undergone plastic deformation, and the other is that steel is still in the elastic zone. It should be also mentioned that the latter condition never happens with a circular section in the ultimate. For a small amount of steel, the first condition will also occur with a square section, where the steel will yield when the confined concrete reaches the peak.

However, when the amount of steel is increased, a condition occurs in which the steel is still in the elastic zone although the peak stress of the concrete has been reached, as shown in Fig. 7. The magnitude of the lateral stress becomes smaller than $\frac{1}{2}\rho_s f_y$ in Eq. (6) as shown in Fig. 9 and a slight increase in the induced lateral stress level is observed even though a larger amount of steel would be added.

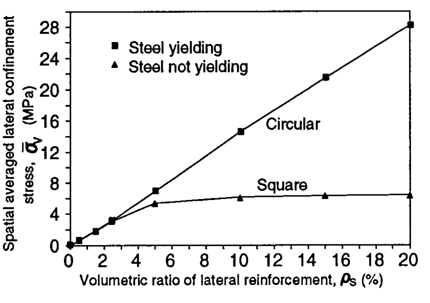


Fig. 7 Effect of Amount of Steel to Mean Confinement

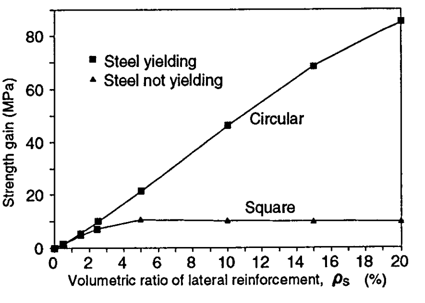


Fig. 8 Effect of Amount of Steel to Strength Gain

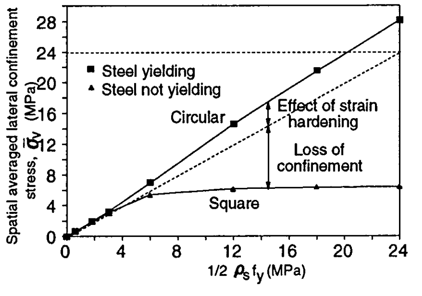


Fig. 9 Relationship between Averaged Lateral Confinement Stress and $\frac{1}{2}\rho_s f_y$

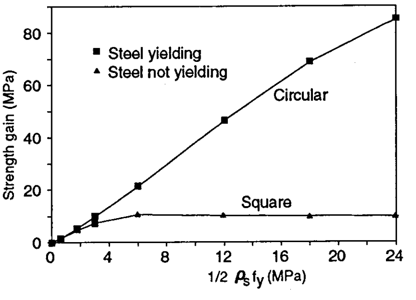


Fig. 10 Sensitivity of $\frac{1}{2}\rho_s f_y$ to Strength Gain

This is because a larger amount of steel gives rise to higher concrete stress around the corners, but at the same time, much internal damage occurs at the points mid-way between the corners on each side, as shown in Figs. 4 to 6. These weak points with inefficient local confinement may trigger the overall failure of the confined structural concrete even though other parts are strongly confined. Thus, confined square-shaped concrete loses the ability to trigger plastic deformation of the steel before failure of concrete itself. Conversely, the uniformity of stress distribution in circular sections produces relatively no weak points in any section. Uniform expansion of confined concrete is always counteracted by confinement action from steel encasing until it would lose its stiffness after yielding. As a result, there exists nonlinear sensitivity of the value of $\frac{1}{2}\,\rho_s f_y$ to the strength gain as shown in Fig. 10 as far as square cross section is concerned.

It is clear that the conventional parameter $\rho_s f_y$, which has been used in design equations, loses its mechanical meaning when a larger amount of steel would be used in the case of square shape. Accordingly, the sensitivity of steel volume to confinement efficiency appears to be complex for square arrangement of steel. Sheikh and Uzumeri [16] empirically adopted the square root of the volume ratio of steel in their design equation proposal. The square root sensitivity of steel ratio is supposed to correspond to the nonlinearity appearing in Fig. 7.

On the other hand, the strength gain of circular columns seems to be directly proportional to the amount of steel, since a perfect proportion between $\overline{\sigma}_v$ and amount of steel is certified. So far, some design equations for square sections are based on linear sensitivity of volume ratio of steel to the confinement efficiency of circular section [12,20].

4.3 Confinement Efficiency - Material Properties -

In order to study the sensitivity of material properties of concrete and steel casing to confinement, it is necessary to change one material property at a time. Three cases were studied for this purpose. A concrete compressive strength of 25 MPa was used in case 1 and 3, while case 2 used a compressive strength of 100 MPa. Case 1 and 2 used similar yield strength of steel of 240 MPa, but case 3 used yield strength of 960 MPa. By comparing cases 1 and 2, the effect of compressive strength of concrete to strength gain and level of confinement can be studied. Cases 1 and 3 can be used to study how the yield strength of steel casing influences strength gain and confinement stress.

Strength gain was calculated based on the peak capacity of confined concrete. As the plastic plateau of steel is not idealized for the requirement of stability of computation (see Fig. 2), the capacity may keep on increasing owing to assumed strain-hardening of steel casing especially for some circular sections. In this case, strength gain was defined as the capacity at the point where the strain reaches the actual hardening strain (ten times the yield strain) of the steel casing.

4.3.1 Strength of Concrete

The relationship of strength gain versus amount of steel for both circular and square sections is shown in Fig. 11 (cases 1 and 2). In case of circular sections, where concrete always has the capability to induce the yielding in the steel, the relationship of strength gain versus confinement stress, which has direct proportion to $\frac{1}{2}\,\rho_s f_y$, is found to be almost unaffected by the strength of the concrete. In the case of square sections, the capability of concrete to cause steel yielding depends on the strength of both materials. In general, a relatively low concrete strength will not be able to induce plasticity in steel and therefore, the confinement efficiency is

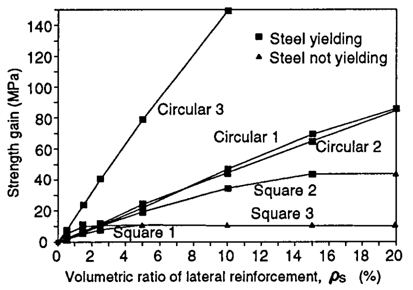


Fig. 11 Effect of Material Properties to Strength Gain
Case 1: fc' = 25 MPa, fy = 240 MPa
Case 2: fc' = 100 MPa, fy = 240 MPa
Case 3: fc' = 25 MPa, fy = 960 MPa

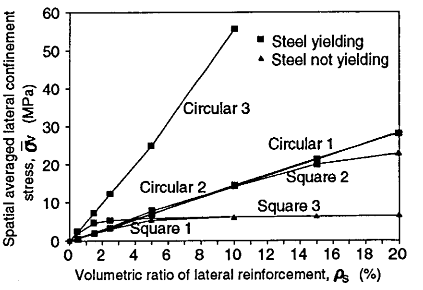


Fig. 12 Effect of Material Properties to Confined Mean Stress
Case 1 : fc' = 25 MPa, fy = 240 MPa
Case 2 : fc' =100 MPa, fy = 240 MPa
Case 3 : fc' = 25 MPa, fy = 960 MPa

lower. To increase confinement efficiency, higher strength of concrete should be used, since it will extend the effective range of steel volume where the steel yields (Fig. 11, cases 1 and 2).

Relationship between confined mean stress and amount of steel can be seen in Fig. 12 for different sets of material properties. As for circular sections, the relationship concerned remains proportional. For square sections, we can see in Fig. 12 (cases 1 and 2) that the range of plastic mode of steel failure expands with the increase of compressive strength of concrete. Higher strength of concrete serves to induce yielding in greater amount of steel surrounding concrete for square section.

It can be summarized that confinement efficiency related to the amount of steel is not affected by the strength of concrete when a circular section is assumed, but is greatly influenced by the concrete strength on the strength gain in the case of square confinement. This interaction has not been rationally linked with design equations to specify the stress-strain relationship of confined concrete for design purposes.

4.3.2 Yield strength of steel

For circular sections, the increment of the yield strength of steel, as predicted, increases strength gain proportionally, as shown in cases 1 and 3 of Fig. 11. It is easily understood that by increasing the yield strength, the level of confinement provided by the steel casing also increases according to Eq. (6), since a circular section always has the capability to induce steel into the plasticity range.

However, with square sections, there is almost no increment in strength gain due to the increment of yield strength for a larger amount of steel. In other words, maximum level of confinement provided by steel casing is similar regardless of the value of yield strength when we use a large amount of steel, as shown in Fig. 12 (cases 1 and 3). Because of non-uniformity distribution of stresses and local damage in square sections, the capacity of confined concrete to induce yielding in steel is not affected by the yield strength of the steel since the ultimate mode is decided by the mode of concrete failure. Only fewer steel and/or higher compressive strength of concrete is needed to reach the most efficient usage of steel when a higher yield strength is employed.

5. DISCRETELY CONFINED CONCRETE COLUMNS

5.1 Lateral Hoop for Circular Section

For a circular section with continuous encasing, concrete of any strength always has the capability to induce steel casing to yield when the core concrete reaches its peak strength, because of the perfect uniformity of stress distribution and confinement actions, not depending on the amount of steel, as discussed in the previous chapter. It means from Eq. (6) that laterally induced concrete stress in XY plane in Fig. 1 always reaches full capacity, denoted by $\frac{1}{2}\rho_s f_y$, which represents volumetric lateral average stress in the case of entire steel yielding.

However, when we have discrete lateral hoops, lateral confinement in concrete is not uniform along the column axis. Hereafter, let s/d denote the ratio of bar spacing to sectional size. For circular sections, diameter is assigned referential size 'd' of the section, and for square sections, the span length of the square is assigned for 'd'. Figure 13 shows sectional averaged lateral stress computed along the axis of columns with different bar spacing when reinforced concrete columns reach the ultimate capacity. Since the section at the position of lateral tie locally undergoes the greatest lateral confinement, it exhibits the maximum $\bar{\sigma}_{c,\max}$ of sectional averaged confinement stress $\bar{\sigma}_c(z)$. Conversely, the section in between lateral ties undergoes the minimum confinement, indicated by $\bar{\sigma}_{c,\min}$. The average sectional confinement with respect to the axial direction of columns coincides with $\bar{\sigma}_v$ in accordance with Eq. (4).

Even though different spacing would be assigned, the value of $\frac{1}{2}\rho_s f_y$ was kept constant. This means that the induced lateral confinement becomes equal if entire steel yields. Smaller spacing gives rise to smaller deviation from the mean value of $\bar{\sigma}_c$, which is mathematically

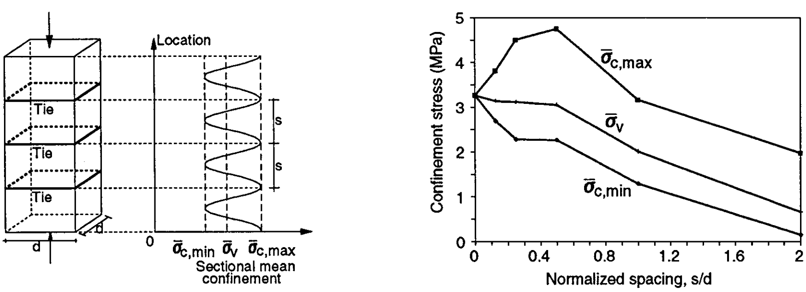


Fig. 13 Sectional Averaged Lateral Confinement

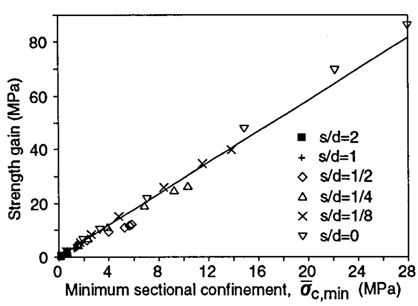


Fig. 14 Relationship between Strength Gain and Minimum Sectional Confinement

equal to $\overline{\sigma}_v$ (see Fig. 13). Here, the value of $\overline{\sigma}_v$ is computed close to $\frac{1}{2}\rho_s f_y$. It means that the entire steel hoop reaches plasticity. The capacity of the confined circular concrete is governed by the weakest section having the minimum value of $\overline{\sigma}_c$ between adjacent ties.

With the increase of spacing, the minimum value of $\overline{\sigma}_c$ between ties deviates from mean value defined as $\overline{\sigma}_v$, where $\overline{\sigma}_v$ ($= \frac{1}{2}\rho_s f_y$) is kept constant since the same volume of steel was arranged and the whole steel also yields. However, a larger distance between adjacent hoop ties is found to lose the capability to induce yielding to entire steel. As shown in Fig. 13, the value of $\overline{\sigma}_v$ gets reduced even though the value of $\frac{1}{2}\rho_s f_y$ intentionally remained unchanged. Minimum lateral confinement at critical section between adjacent ties decreases with the increase of spacing. This is owing to the less lateral stresses arising in steel as well as the far location from the tie. Strength gain, which is defined as the increment of the ultimate capacity of confined concrete in comparison with uniaxial condition, is supposed to be governed by the minimum sectional averaging of lateral stress $\overline{\sigma}_{c,\min}$ at a critical section between ties. A relation of strength gain and $\overline{\sigma}_{c,\min}$ looks linear as shown in Fig. 14. By combining Figs. 13 and 14, we have relationship between strength gain and the spacing of hoops as shown in Fig. 15.

The effect of spacing on strength gain and ductility is associated with the amount of steel, as shown in Fig. 15. Steel volumetric ratio ranges from 0.5% to 15%. A value of 15% is regarded as the extreme case, which is however advisable for comprehending overall behavior and tendency on strength gain. Peak strain in the ultimate (ε_c) normalized by the one for unconfined concrete ($\varepsilon_c^{'}$) is taken to represent ductility. The confinement efficiency of lateral steel, which is obtained by comparing volumetric average confinement $\overline{\sigma}_v$ actually induced with the potential lateral stress $\frac{1}{2}\rho_s f_y$, can be seen in Fig. 16. If $\overline{\sigma}_v$ is less than $\frac{1}{2}\rho_s f_y$,

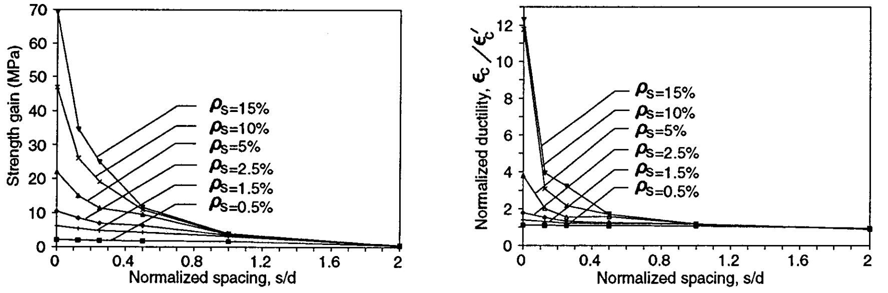


Fig. 15 Effect of Spacing on Strength Gain and Ductility of Circular Section

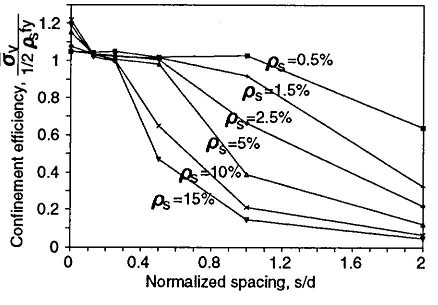


Fig. 16 Effect of Spacing on Confinement Efficiency of Circular Section

lateral steel still remains elastic even though confined concrete reaches its capacity. This means that the level of confinement that can be supplied by the steel is not fully achieved.

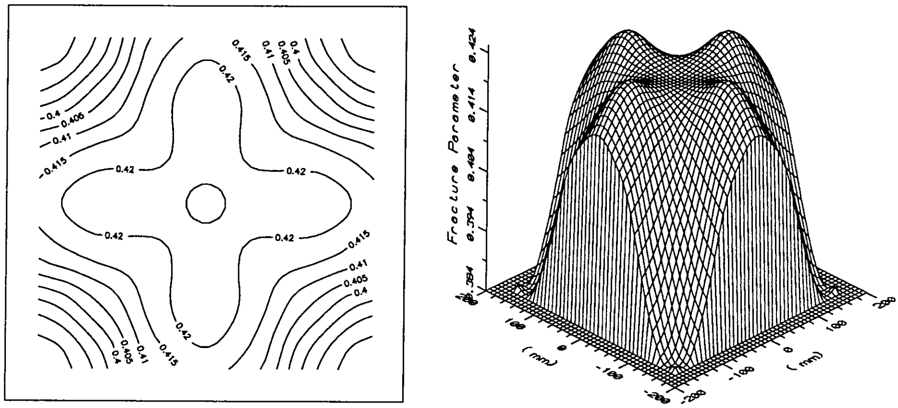
When we use reinforcement with less than 1.5% of ρ_s by volume ratio and less than 1.0 of *s/d* by spacing to diameter ratio, confinement efficiency becomes close to unity. Thus, concrete with a compressive strength of 25 MPa can induce plasticity in steel within the conditions stated above. Conversely, if bulk of the steel were to be discretely arranged (e.g. 15% of ρ_s), a smaller hoop spacing (e.g. $s/d < 0.3$) is required to induce complete plasticity in the steel. However, regardless of the amount of steel, lateral steel undergoes plasticity when the spacing converges to zero, that is, when a continuous casing is formed. To avoid instability in the calculation, an elasto-plastic model with strain hardening immediately after yielding was applied for lateral steel (Fig. 2). As a result, after the steel yields, the lateral steel stress can still keep increasing. Therefore, confinement efficiency based on $\frac{1}{2}\rho_s f_y$ can be slightly greater than 1.0, as appears in Fig. 16. Confinement efficiency greater than 1.0 should be interpreted as full utilization of the lateral steel.

5.2 Hoop and Tie of Square Section

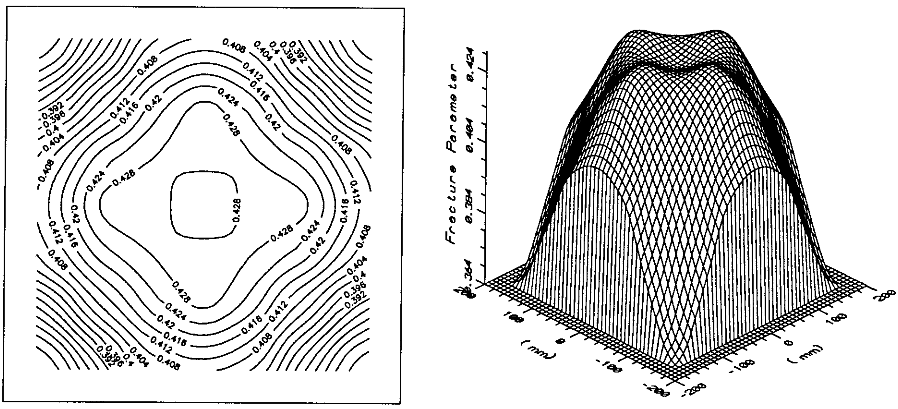
Two types of lateral reinforcement were considered: square sections with and without intermediate cross ties, where the stress distribution and confinement actions are not uniform in space. The cross-section of these shapes can be seen in Fig. 1. The effect of the shape, especially for a circular and square section steel-encased core concrete, has already been discussed in the previous chapter. Analysis revealed that square lateral encasing does not undergo plastic yielding provided the larger amount of steel or relatively lower strength of concrete. In other words, the concrete fails prior to the square casing yielding. This is not the case for circular encasing.

It was reported that, in general, cross ties with square hoops gives rise to higher strength gain and ductility than lateral steel without cross ties. Figure 17 illustrates spatial distribution of local damage indicated by non-dimensional variable K in Eq. (1) within critical section in both types of bar arrangement (see Fig. 1). In the ultimate capacity, sectional averaged lateral stresses $\overline{\sigma}_c$ are 1.33 MPa and 0.96 MPa in square hoop with and without intermediate cross ties, respectively. In both cases steel is still in elastic zone. It is obvious that intermediate cross ties enhance the confinement efficiency. It can be seen from Fig. 17 that the presence of cross ties creates different damage distribution with a slightly higher value of K from the case of square section without cross ties.

The relationship of strength gain and ductility to the spacing of lateral ties are shown in Fig. 18. The efficiency of lateral confinement is seen in Fig. 19. This value which indicates $\overline{\sigma}_v$ induced by steel stress in terms of $\frac{1}{2}\rho_s f_y$ is useful for clarifying the nonlinearity condition of steel in the ultimate capacity of columns. In general, the sensitivity of spacing to strength gain depends on the amount of lateral steel as in the case of circular section. As for square columns, there exists an upper limit of strength gain of confined concrete even if the volume of lateral ties would be amplified so much (see Fig. 18). As shown in Fig. 19, zero spacing as an extreme case does not always exhibit full plasticity but elastic zone remains in steel if greater amount of lateral steel is placed. It can be clarified that the reduced confinement efficiency by a greater amount of lateral steel offsets the benefit granted by the larger volume of lateral steel. This is also rooted in the geometrical aspect of sectional shapes already discussed analytically by the authors in the previous chapter and experimentally by others [5,19].



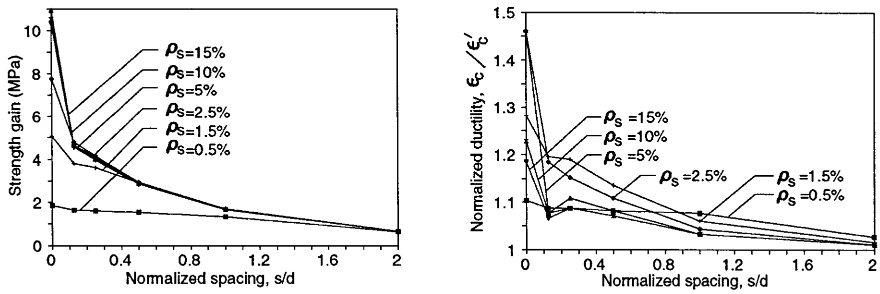
(a) Square Section without Intermediate Cross Ties



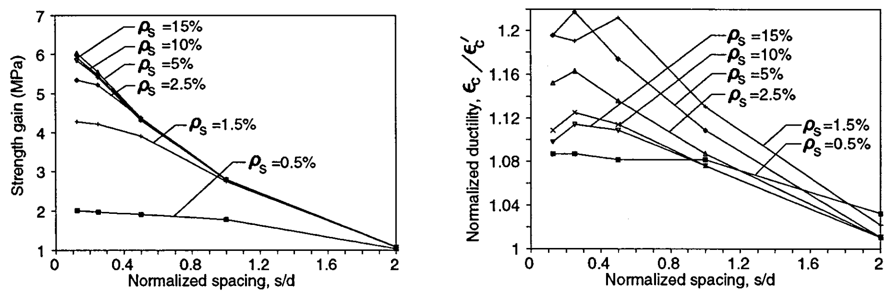
(b) Square Section with Intermediate Cross Ties

Fig. 17 Spatial Distribution of Damage within the Critical Section

The above sensitivity of volume ratio of steel could be fairly pointed out from Fig. 20. In the case that steel is still in the elastic zone at the peak of the stress-strain diagram (confinement efficiency less than 1.0), almost no increment of strength was observed. In the elastic zone, the increment of volumetric ratio only affects the mean steel stress being reduced in lateral reinforcement without increasing the strength of the confined concrete totally. This is because the level of confinement induced by the whole lateral reinforcement is not improved. Internal damage to the concrete nullifies the capability of the confined concrete to expand and forces the lateral reinforcement to produce greater confinement action. In other words, the use of excessive lateral reinforcement does not evolve the performance of confined concrete efficiently. Through parametric studies by the finite element method, it was found that the spacing of ties affects the whole confinement stress level represented by $\overline{\sigma}_v$ as well as the distribution of local lateral confinement stress denoted by $\overline{\sigma}_c$.

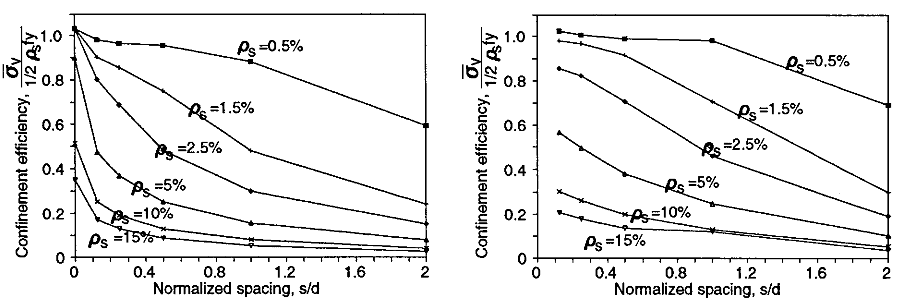


(a) Square Section without Intermediate Cross Ties



(b) Square Section with Intermediate Cross Ties

Fig. 18 Effect of Spacing on Strength Gain and Ductility of Square Section



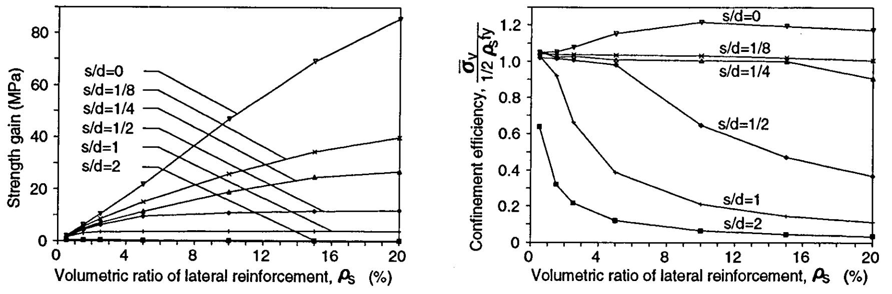
(a) Square Section without Intermediate Cross Ties

(b) Square Section with Intermediate Cross Ties

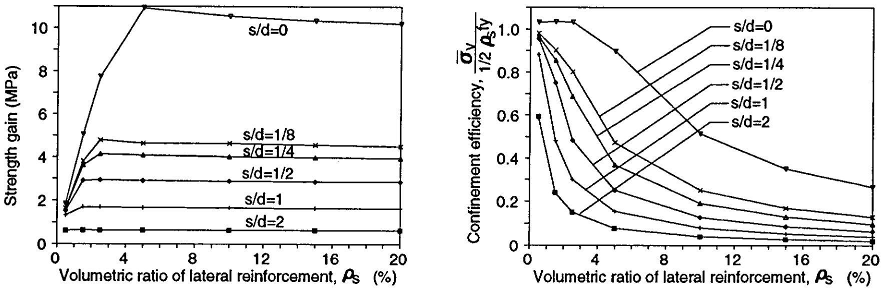
Fig. 19 Effect of Spacing on Confinement Efficiency of Square Section

5.3 Comparison with Existing Design Equations

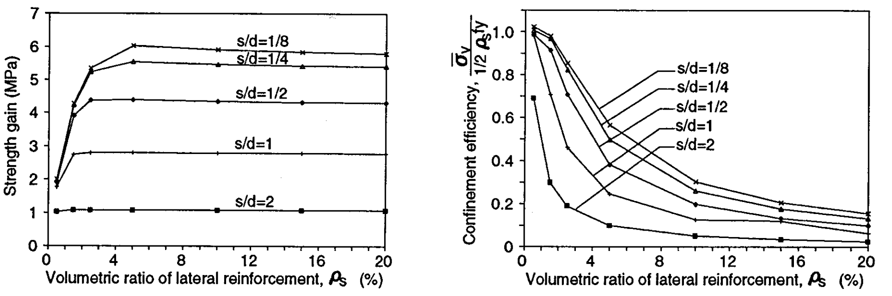
As mentioned earlier, considerable work has been done regarding the strength and deformation of confined concrete. Among previous researches on rectangular sections, the influence of volumetric ratio of lateral reinforcement has received the most attention. Burdette et al. [3] observed that only this parameter and the yield stress of the steel reinforcement influence the performance of a confined concrete core. Iyengar et al. [4] included the compressive cylinder strength of concrete in their proposal for predicting the strength gain of confined concrete with rectangular shapes. The influence of spacing ratio appeared in the proposal by Sargin [12],



(a) Circular Section



(b) Square Section without Intermediate Cross Ties



(c) Square Section with Intermediate Cross Ties

Fig. 20 Relationship between Amount of Lateral Reinforcement and Strength Gain and Confinement Efficiency

which was agreed by Vallenas et al. [20]. Finally, Sheikh and Uzumeri [16] included the effect of lateral reinforcement configuration to predict the performance of confined concrete.

Results by finite element analysis are presented with the existing design equations which empirically derived from previous researches (see Figs. 21-23). Two parameters, the lateral reinforcement ratio and the ratio of spacing to sectional size denoted by *s/d*, are varied systematically. It must be stated that each empirical equation might be invalid in some range conducted in parametric study no matter what applicability is specified. Observed is discrepancy among the empirical equations whatever experimental database is assumed. For circular sections, as far as smaller spacing of circular hoop ties is concerned, the predictions by authors' analyses seem close to the proposal by Mander and Iyengar. As for the range of larger

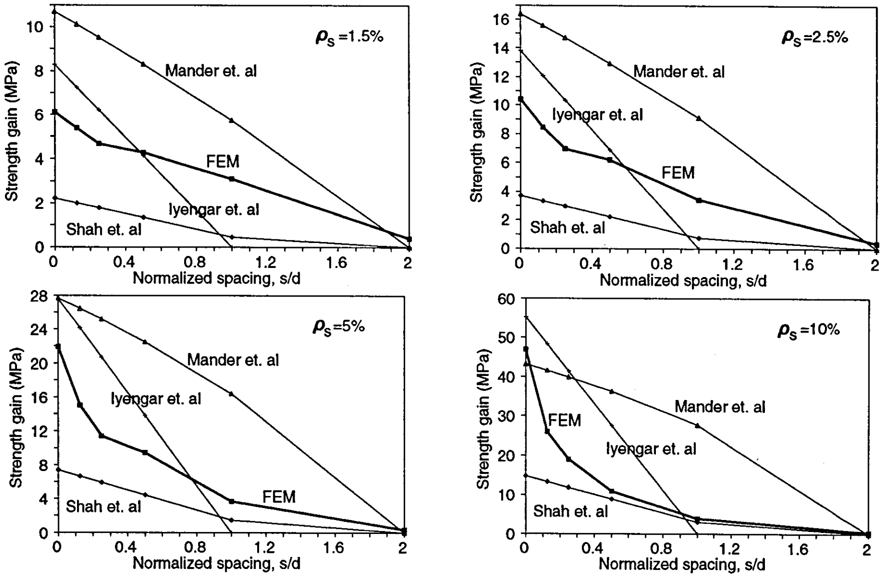


Fig. 21 Strength Gain of Confined Concrete Columns of Circular Section

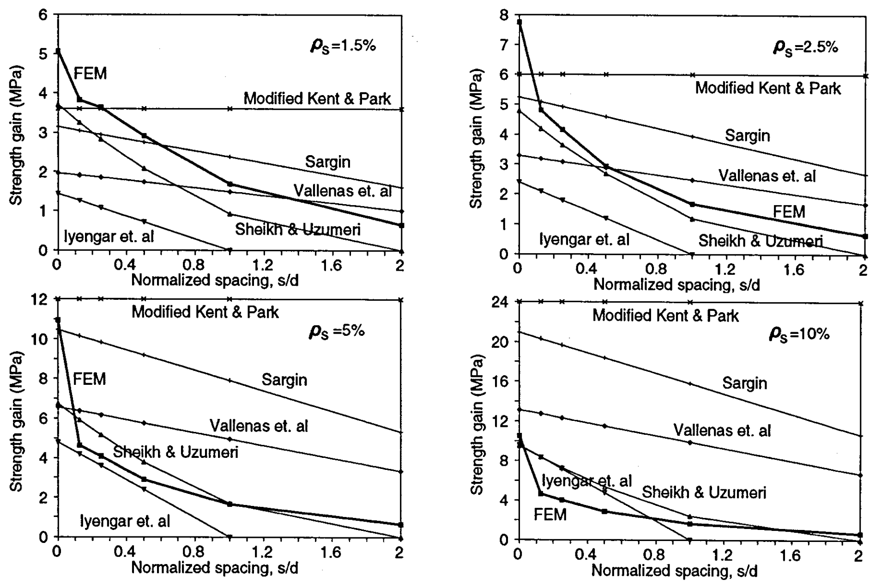


Fig. 22 Strength Gain of Confined Concrete Columns of Square Section
without Intermediate Cross Ties

spacing, however, the strength gain computed by constitutive model appears to be close to the prediction by Shah et al. [13].

Figure 22 shows strength gain of square sections without intermediate cross ties. Analytical results are comparatively close to the prediction by Sheikh and Uzumeri [16] except for very small spacing less than 0.1 by *s/d*. It should be remembered that these design equations were not derived from circular casing. A large amount of steel was analytically proved to be inefficient on confinement (see Section 4.2). The models except for Sheikh and Uzumeri's one give higher predictions because the sensitivity of volume ratio on strength gain is assumed linear. Sheikh and Uzumeri's model alone assumes the square root sensitivity of volume ratio.

According to analytical and experimental investigations, volumetric ratio should not be regarded as being proportional to the induced sectional confinement due to the remaining elasticity in steel, which means inefficient usage of lateral reinforcement. The accuracy of analysis was verified in the case of steel lateral encasing in the previous chapter.

Intermediate cross ties change stress distribution over a cross section. Contrary to the simple square hoop, the analytically obtained compressive capacity of cross tied members is found to be smaller than the one predicted by Sheikh and Uzumeri (see Fig. 23). Other models are avoided in the discussion since the presence of intermediate cross ties cannot be taken into account in their proposals. The tendency is that smaller spacing shows bigger discrepancy between the predictions of authors and those of Sheikh and Uzumeri. Actually, in the analysis, there exists no greater difference between the strength gain of cross tie reinforcement and that of having no ties. In reality, design equations including Sheikh and Uzumeri's were derived from the loading experiments having longitudinal reinforcing bars which can serve to confine the lateral expansion of the core concrete concerned as well as to carry some of the axial load. Cross tie arrangement implies effective lateral confinement by longitudinal reinforcement.

The finite element analysis proposed in this paper does not take into account the presence of longitudinal reinforcement as lateral confinement agent. In other words, it can be understood that design equations do not separate confinement efficiency into those originating in the lateral and longitudinal reinforcement. This will be analytically clarified in the next stage of investigation as well as the effect of flexural rigidity of discretely arranged bars.

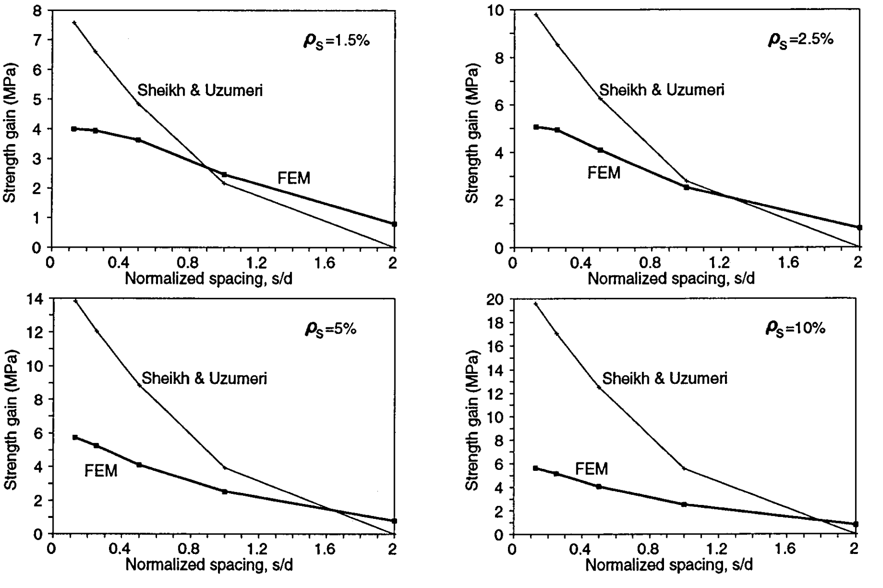


Fig. 23 Strength Gain of Confined Concrete Columns of Square Section with Intermediate Cross Ties

6. CONCLUSIONS

The compressive load bearing capacity of laterally reinforced concrete results from complex three-dimensional stress distribution. To elucidate the confinement efficiency by lateral steel, a three-dimensional stress field is analyzed using finite element method and elasto-plastic and fracture models for concrete. Sectional averaged lateral stress and volumetric averaging were quantitatively evaluated. This stress-based information cannot be obtained experimentally. The following are analytically investigated.

6.1 Steel Encased Confined Concrete Columns

(1) It was clarified that the strength gain due to confinement is approximately proportional to sectional averaging of lateral mean stress at each particular point in a section, regardless of the geometry of steel arrangement, while the proportional sensitivity of strength gain versus spatially averaged lateral mean stress is dependent on the type of steel laterally placed. Three-dimensional finite element analysis was verified to quantify confinement efficiency related to the geometry of lateral steel.

(2) Sectional averaging of lateral mean stress was found to be exactly proportional to the amount and yield strength of steel for circular sections. But as for square sections, it was numerically shown that the linearity between the mean confinement stress in concrete and the amount of steel does not hold. It means that the strength gain cannot be rationally predicted by the conventional factor "$\rho_s f_y$" where ρ_s is the volume ratio and f_y is the yield strength of steel.

(3) The confinement efficiency for a square section is not improved when the amount of lateral steel becomes large. The nonlinear relationship between the strength gain and the volume ratio of steel casing is rooted in the premature failure of concrete before the yielding of the lateral steel. It was numerically investigated that confined concrete with a square section will lose the ability to induce plasticity to greater amount of lateral steel. Even if more steel were be added to the confinement system, the stress arising in the steel is simultaneously reduced. As a whole, the confinement efficiency is not improved effectively by increasing the volume of steel.

(4) It was verified that compressive strength of confined concrete and yield strength of steel affect the relationship between lateral averaged stress of concrete and the amount of steel. Higher strength of concrete, which can act to induce greater plasticity to steel, extends the effective range of steel volume where the whole steel yields and the confinement efficiency is the maximum under the given material. Higher yield strength of steel reserves more confinement stress that can be mobilized, as long as concrete can induce plasticity to the steel.

6.2 Discretely Confined Concrete Columns

(1) For circular columns with discrete lateral hoops, larger spacing between adjacent hoops results in less lateral confinement to the critical section of core concrete. It is computationally shown that lateral confinement stress which is transferred along an axis is reduced as the distance increases. In addition, absolute hoop stress itself is reduced with greater spacing. Spacing has duplicated effects on the whole confinement efficiency.

(2) For square columns with or without cross ties, spacing has a similar influence on confinement to the core concrete. It is analytically pointed out that square hoops lose confinement efficiency when a larger spacing and/or greater amount of steel would be assumed.

(3) The presence of cross tie is taken into account in the analysis. The tie improves confinement efficiency since the intermediate steel reinforcement makes internal stress and damage distribution smoother even when the same amount of steel would be allocated in concrete. The mean value of sectional non-uniform damage, which is indicated by the fracture parameter, is proved to be greater than the one of lateral square ties without any cross reinforcement. This comparison is made under the same amount of steel which means the same volumetric averaging of lateral confinement stress, which can be attained provided the entire plasticity of lateral steel reinforcement.

(4) Analytical results are compared with the prediction by empirical formula oriented to the design for confined members. The computed strength gain is examined to be close to some empirical equations. Through verification, the sensitivity of each factor to strength gain is studied for a new proposal of design equations having the mechanical background in future.

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APPENDIX

Let us consider laterally confined concrete, as shown in Fig. A1. Applying the virtual work principle, we obtain,

$$\int_V \sigma_{ij}\, \delta\varepsilon_{ij}\, dV = \int F \delta u \, dS \,, \tag{A-1}$$

where F is the external force applied on the boundary of the domain and V denotes the domain of the whole volume. When we apply the following virtual displacement and associated field of perfect isotropic virtual strain,

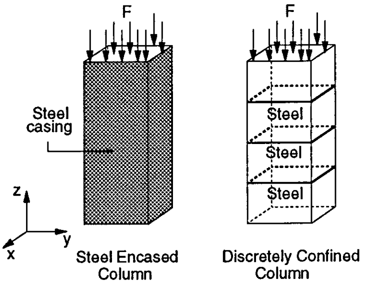

Fig. A1 Confined Concrete

$$\delta\varepsilon_{xx} = \delta\varepsilon_{yy} \neq 0 \tag{A-2a}$$
$$\delta\varepsilon_{zz} = 0 \tag{A-2b}$$
$$\delta\varepsilon_{ij} = 0 \quad \text{for } i \neq j\ , \tag{A-2c}$$

the right term of Eq. (A-1) becomes zero because the X-Y component of F as a surface force applied to the confined concrete is not present.

By substituting the above equations into Eq. (A-1) and assuming uniaxial stress state σ_s in the steel, we obtain the following integrals in the concrete and steel domains, V_c and V_s:

$$\int_V \sigma_{ij}\,\delta\varepsilon_{ij}\,dV = \int_{V_c} \sigma_{c,ij}\,\delta\varepsilon_{ij}\,dV + \int_{V_s} \sigma_{s,ij}\,\delta\varepsilon_{ij}\,dV = 0 \tag{A-3}$$

$$\delta\varepsilon_{xx} \int_{V_c} (\sigma_{c,xx} + \sigma_{c,yy})\,dV + \delta\varepsilon_s \int_{V_s} \sigma_s\,dV = 0 \tag{A-4}$$

According to the compatibility condition of the prescribed field, virtual strain along lateral reinforcement coincides with the value designated by Eq. (A-2) regardless of the way of steel arrangement. Then, we have,

$$\int_{V_c} (\sigma_{c,xx} + \sigma_{c,yy})\,dV + \int_{V_s} \sigma_s\,dV = 0 \tag{A-5}$$

Here, let us define the local lateral confinement stress, which is the first invariant under a two-dimensional stress field, by taking,

$$\sigma_c \equiv \frac{\sigma_{c,xx} + \sigma_{c,yy}}{2} \tag{A-6}$$

By substituting Eq. (A-6) into Eq. (A-5), we have,

$$2\int_{V_c} \sigma_c(X,Y,Z)\,dV + \int_{V_s} \sigma_s\,dV = 0 \tag{A-7}$$

Using the above equation, we can compute the spatial averaging of local lateral confinement induced in concrete with respect to the steel stress as follows;

$$\bar{\sigma}_v = \frac{1}{V_c} \int_{V_c} \sigma_c\,(X,Y,Z)\,dV \tag{A-8}$$

$$\bar{\sigma}_v = -\frac{1}{2V_c} \int_{V_s} \sigma_s\,dV \tag{A-9}$$

Provided that the whole domain of lateral steel would be in yield condition where the steel reaches yield strength f_y, spatial averaging of lateral confinement becomes,

$$\bar{\sigma}_v = -\frac{1}{2V_c} \int_{V_s} f_y\,dV = -\frac{1}{2V_c} f_y\,V_s = -\frac{1}{2}\left(\frac{V_s}{V_c}\right) f_y = -\frac{1}{2}\,\rho_s\,f_y\,. \tag{A-10}$$

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