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ESTIMATION OF THE SHEAR STRENGTH OF RC BEAMS WITHOUT WEB REINFORCEMENT BASED ON THE ZONE SHEAR STRENGTH METHOD

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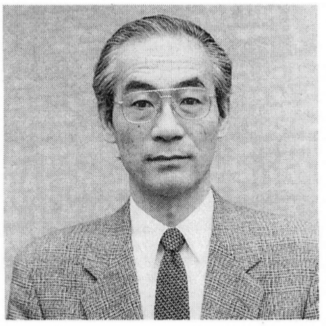

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SYNOPSIS

To calculate shear strength of RC beams subjected to the ordinary loading conditions, such as distributed loading and/or multiple concentrated loading, it is essential to know the cross-sectional shear strength throughout the span. In this study, the shear strength of a zone including a diagonal crack is first studied experimentally as a practical alternative for determining the cross-sectional shear strength. A method for calculating shear strength of RC beams (the zone shear strength method) is proposed and shown to be satisfactory.

Key Words: Reinforced concrete beams, Shear strength, Uniformly distributed load, Multiple concentrated load, Zone shear strength

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1. INTRODUCTION

In simple reinforced concrete beams without web reinforcement and subjected to a single load or a pair of symmetrical concentrated loads, the shear strength can be calculated with good accuracy. For example, Niwa [1] has proposed Eq. (1) to estimate the shear strength of such RC beams.

$$V_u=0.94f_c'^{1/3}p_w^{1/3}(100/d)^{1/4}(0.75+1.4d/a)b_wd \qquad (1)$$

Where, f_c' : compressive strength of concrete [kgf/cm^2]
p_w : reinforcement ratio = $100A_s/(b_wd)$ [%]
a : shear span length [cm]
d : effective depth [cm]
b_w : width of web [cm]
A_s : cross-sectional area of tensile reinforcement [cm^2]

Equation (1) gives no information about the location of the shear failure. Thus, it appears that this type of equation estimates the shear strength of a beam as a whole rather than estimating the shear strength of the failed section. Considering this, it is clearly difficult to calculate the shear strength of beams subjected to uniformly distributed loading and/or multiple concentrated loading; in such cases, the shear force is not constant along the span and the location of section in which failure occurs should be determined first in calculating the working shear force.

In the case of bending moment, design is carried out in consideration of both the external moment and flexural strength at every cross section. It might be convenient to use a similar method also in the design against shear force. To do so, however, it will be necessary to know the shear strength of each cross section throughout the span.

When a simple beam is subjected to a single concentrated load, it is well known that the shear span to depth ratio (a/d) very much affects the shear strength of the beam, and that a beam with a smaller a/d has higher shear strength. This is usually explained as follows: concentrated load applied to the beam and the resulting reaction force at the support will produce local compressive stress σ_y in the web concrete, reducing the principal tensile stress σ_1 that causes the development of diagonal cracks. Considering this, it can be expected that the shear strength of a section would be larger at locations nearer to the loading point and/or the support.

In this study, the relationship between the shear strength of a section and its distance from the load and support was examined experimentally. A method of calculating the shear strength of a beam under uniformly distributed loading or multiple concentrated loading was then proposed. The accuracy of estimates made using the new method was verified by comparing results with published experimental data.

Because shear failure always results from the development of diagonal cracks, it might be argued that the sections in which shear strength is examined should be inclined. In this work, a zone including a diagonal crack and with a finite length is considered instead. When the length of this finite zone is reduced, it converges to a normal vertical cross section. The method may prove convenient in the design of the ordinary flexural members subjected to both flexural moment and shear force.

2. PRELIMINARY LOADING TEST

It would be very difficult to examine the shear strength of every zone throughout a span, if ordinary beams without web reinforcement were used in loading tests. In such cases, the location of the critical diagonal crack would be almost completely determined by the given conditions, such as a/d ratio, and we would only find the strength of the weakest zone in the beam.

Here, specially reinforced beam specimens are used. Only small amounts of stirrups are used in the zone under consideration (the test zone), while heavy stirrups are placed in the remaining parts of the beam to force a shear failure within the test zone.

At first, as shown in Fig. 1, an ordinary beam without web reinforcement (Specimen A) was tested to find the location of the critical diagonal crack and the shear failure load in advance. Specimen B reinforced with stirrups except in the zone equivalent to the critical diagonal cracking zone in specimen A, was then tested. As shown in Table 1, though there was little difference in the location of the critical diagonal cracks in the two specimens, shear failure load of specimen B was 1.34 times larger than that of specimen A. The effect of stirrups placed outside the test zone was clearly recognized in specimen B.

One way to eliminate the effect of the stirrups would be to extend the length of the test zone. However, if this were done, it would be difficult to assign a particular location to the critical diagonal crack. We therefore proceeded to think about another possibility. As shown in Fig.1, specimens C and D were made and loaded. In these beams, the test zone was slightly reinforced using ϕ3.2 stirrups (dotted lines) while the remainder was reinforced with D10 stirrups (solid lines). The spacing of all stirrups was 10 cm. The test zone was 30 cm (1.43d) and 40 cm (1.90d) long in specimens C and D, respectively. To compare the failure load and determine the location of the test zone in C and D, a loading test was carried out on specimen E in advance: in this specimen, ϕ3.2 stirrups were arranged at 10 cm pitch throughout the shear span.

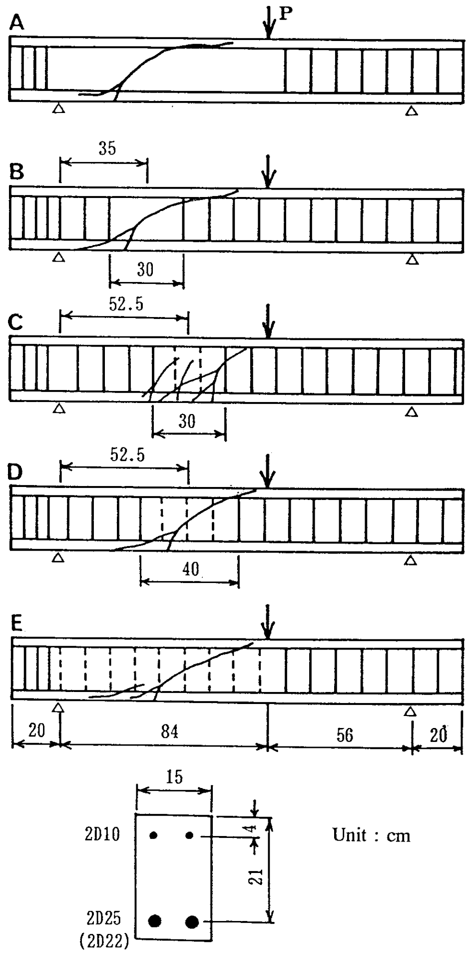

Fig. 1 Preliminary Specimens

A summary of the loading test results is given in Table 1. Specimen C, with a zone length of 30 cm, failed due to yielding of the axial reinforcement before diagonal cracks fully developed. Also the shear strength was clearly seen to increase due to the effects of the stirrups outside the test zone. On the other hand, in specimen D, where the zone length was 40 cm, both the critical diagonal crack location and the shear failure load corresponded to that of the uniformly reinforced beam specimen E, showing that the stirrups outside the test zone had negligible effect. On the basis of these experimental results, the length of the test zone was fixed at 40 cm, with ϕ3.2 (10 cm pitch) stirrups in the test zone and D10 (10 cm pitch) stirrups outside the test zone, for subsequent test specimens.

Table 1 Summary of preliminary test

No.	f_c' kgf/cm^2	Failure load P_u (tf)	Location x (cm)	Failure mode
A	218	11.5 (1.00)	35.0	Shear
B	248	15.4 (1.34)	38.2	Shear
C	288	21.7 [1.42]	–	Flexure
D	277	16.2 [1.06]	56.3	Shear
E	252	15.3 [1.00]	52.5	Shear

In these experiments, the cross section of the beam was 15 cm in width and 21 cm in effective depth. The longitudinal reinforcement ratio (p_w) was 2.46% (2D22) or 3.22% (2D25). The ratio of shear span to effective depth was 4.00 in all specimens. Steel bearing plates 3 cm thick and 8 cm wide were used at both the loading point and supports.

The physical properties of the reinforcing bars used are shown in Table 2. In Fig. 1, the critical diagonal cracks that caused shear failure are also sketched. In Table 1, the location given for the critical diagonal crack (x) denotes the distance from the support to the cross section where the diagonal crack intersects the mid–height of the effective depth.

3. OUTLINE OF LOADING TESTS TO DETERMINE ZONE SHEAR STRENGTH

In a beam with shear span length a, the shear strength v_u of a zone is assumed to be determined by the values (x_n) and (a–x_n), which represent the distances from the nearest support and from the loading point to the zone under consideration, respectively. Namely, the zone shear strength is assumed to be expressed by Eq. (2). Here, the location of a zone is conveniently represented by the abscissa at the center of the zone.

$$v_u = f\,(x_n/d,\ (a-x_n)/d)\,v_0 \qquad (2)$$

Where, v_0 denotes the basic shear strength of the section under ideal conditions; that is, it is located at infinite distance from both the load and reactions and there is no influence from them at all. The basic strength, v_0, is determined by the shape and dimensions of the cross section, the quality of the materials, and the arrangement of shear reinforcement.

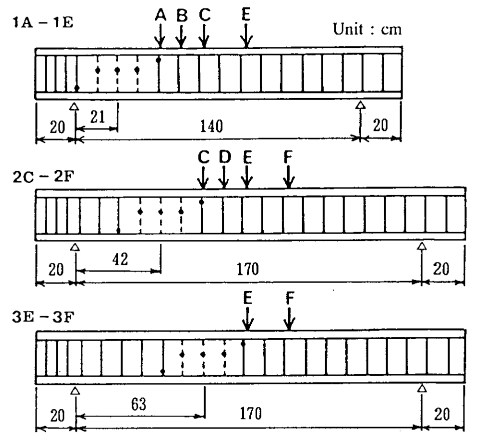

Fig. 2 Specimens for the study of zone strength

Table 2 Properties of bars

Bars	Yield point kgf/cm^2	E_s 10^6kgf/cm^2
ϕ3.2	3,838	2.07
D10	3,820	1.86
D22	3,455	1.77
D25	3,602	1.94

The beam specimens used to examine the zone shear strength are shown in Fig. 2. These beams have identical cross sections to the preliminary specimen D. The nominal distance, x_n, from the support to the center of the test zone is varied between 1.0d (21 cm), 2.0d (42 cm), and 3.0d (36 cm). Two to four identical beams with the same value of x_n were made. A concentrated load was applied to each beam at different position (a/d) to find the effect of x_n/d and (a–x_n)/d on the shear strength of the test zone. The value of a/d ranged from 2.0 to 5.0.

Wire strain gages were attached to three ϕ3.2 stirrups in the test zone and to two D10 stirrups adjacent to the test zone. If the position of the critical diagonal crack deviated very much from its assumed location, stirrups adjacent to the test zone would intersect the diagonal crack, and the correct shear strength of the test zone could not be found. The purpose of the latter strain gages is to detect such a possible situation.

During the loading tests, the applied load and the strains of stirrups were measured. The development of cracks was observed with magnifier lenses. The same steel bearing plates as described in Section 2 were also used here. The compressive strength of concrete at the time of the tests is shown in Table 3, and the physical properties of the reinforcing bars are the same as given in Table 2.

4. ZONE SHEAR STRENGTH EQUATION

A summary of the loading test results is shown in Table 3. All specimens except 2F, failed in shear due to the development of a critical diagonal crack within the test zone. At shear failure, it was confirmed that all stirrups in the test zone yielded. In specimen 1C, a diagonal crack intersected a stirrup outside the test zone, and the effect of this on the stirrup was clearly seen in the strain measurements. Thus, the data for specimen 1C was omitted from the analysis. In Table 3, the test results for the preliminary

Table 3 Summary of loading test to examine zone strength

No.	a/d	x_n/d	f_c' kgf/cm^2	V_u tf	v_u kgf/cm^2	x/d	v_{ucal} kgf/cm^2
1A	2.0	1.0	283	14.68	46.61	1.21	41.03
1B	2.5	1.0	284	7.62	24.19	1.18	27.72
1C	3.0	1.0	264	9.41	29.87	1.68	-
1E	4.0	1.0	260	6.25	19.87	1.51	19.75
2C	3.0	2.0	292	9.06	28.77	2.14	31.02
2D	3.5	2.0	279	7.37	23.39	2.14	21.79
2E	4.0	2.0	278	6.63	21.05	1.98	19.12
2F	5.0	2.0	260	6.83	-	-	-
3E	4.0	3.0	255	7.66	24.32	3.01	25.63
3F	5.0	3.0	272	6.83	21.69	3.62	20.72
D	4.0	2.5	277	6.43	20.42	2.68	21.56
E	4.0	-	252	6.11	19.39	2.50	19.95

specimens, D and E, are also shown.

The purpose of the experiments was to determine the zone shear strength of beams without web reinforcement; that is, to more accurately fix the function f in Eq. (2). As described in Section 2, a small amounts of stirrups were arranged in the test zone and their effect should be isolated from the test results. Scince the effect of stirrups on shear strength has not been fully clarified yet, v_0 is assumed to be expressed by Eq. (3) simply through the ordinary truss analogy.

$$v_0 = v_c + rf_{vy} \tag{3}$$

Where, r : web reinforcement ratio $= A_v/(b_w s)$

f_{vy}: yield point of web reinforcement

A_v : cross–sectional area of a stirrup

s : stirrup spacing

In the experiment, the value of rf_{vy} was 4.04 kgf/cm^2. Strength v_c denotes the basic shear strength of a section without web reinforcement, and it is assumed to be expressed by Eq. (4) [2].

$$v_c = 0.94f_c'^{1/3}(1+\beta_p+\beta_d) \tag{4}$$

$$\beta_p = \sqrt{p_w} - 1.0 \leq 0.732$$

$$\beta_d = (100/d)^{1/4} - 1.0$$

As clearly seen from the results in Table 3 and Fig. 3, the shear strength of a zone is almost constant, at about 20 kgf/cm^2 whatever the value of x/d, when the distance (a–x) from the load point to the crack is greater than about 1.5d. On the other hand, the shear strength increases as the value (a–x)/d falls below 1.0. This tendency is more notable when the value of x/d is smaller.

Several types of function, which might adequately express this behavior, were selected and their adaptability to the experimental results was examined using the non–linear least squares method [3]. The results of using Eq. (5), the hyperbolic function, were found to be most suitable [4]. In this analysis, the actual position of the critical diagonal crack (x) was used instead of the nominal position of the test zone (x_n).

$$f = 0.958[\coth(x/d)]^{1.360}[\coth\{(a-x)/d\}]^{1.484} \tag{5}$$

Where, $\coth(t) = (e^t + e^{-t})/(e^t - e^{-t})$

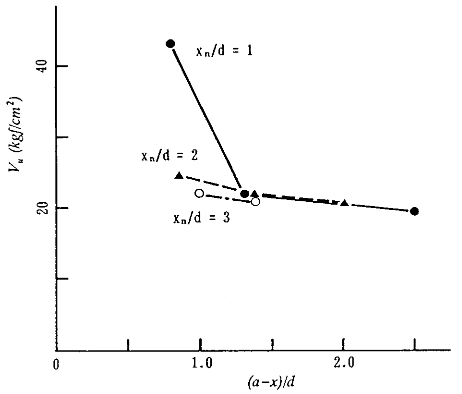

Fig. 3 Effect of x_n and a on V_u

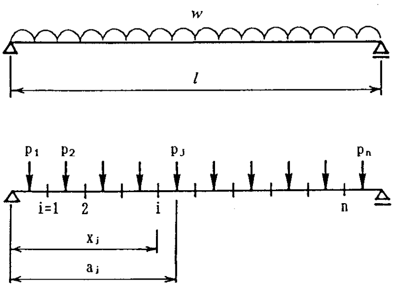

Fig. 4 Transformation of uniformly distributed load into concentrated loads

The ratio of calculated shear strength v_{ucal} as determined by Eq. (5) to the experimental value, v_u, was 1.00 on average with a coefficient of variation of 8.0% for the 10 specimens shown in Table 3. The accuracy of Eq. (5) is satisfactory, and this equation will from now on be used as the "Zone shear strength equation".

5. APPLICATION TO SIMPLE BEAMS SUBJECTED TO UNIFORMLY DISTRIBUTED LOADING

When we use the zone shear strength equation to calculate the shear strength of a simple beam subjected to uniformly distributed loading, the load must first be transformed into a series of concentrated loads. Furthermore, the interaction between these loads should be considered.

First, the uniformly distributed load is transformed into a group of concentrated loads P_1 to P_n, as shown in Fig. 4. Here, w and l denote the intensity of the uniformly distributed load and span length, respectively.

$$P_j = wl/n \tag{6}$$

The shear strength of section i against each concentrated load P_j is expressed by Eq. (2) and Eq. (5). That is, the shear strength increases to f times the basic shear strength v_0. Here a substitution is used: shear strength is left unchanged but the effect of applied shear force is decreased to $1/f$ times the actual value. This imaginary shear force is defined here as the "Effective shear force", (V_e). The effective shear force at section i due to force P_j is expressed by Eq. (7).

$$V_{ij} = P_j\,(l - a_j)/l \qquad (j > i)$$

$$= -P_j a_j/l \qquad (j \leq i)$$

$$f_{ij} = 0.958[\coth(x_i/d)]^{1.360}[\coth\{(a_j - x_i)/d\}]^{1.484} \qquad (j > i)$$

$$= 0.958[\coth\{(l - x_i)/d\}]^{1.360}[\coth\{(-a_j + x_i)/d\}]^{1.484} \qquad (j \leq i)$$

$$V_{eij} = V_{ij}/f_{ij} \tag{7}$$

Then, as the shear failure criterion for the section under the combined action of the concentrated loads, a linear accumulation of damage is assumed. That is, as shown in Eq. (8), shear failure is assumed to occur when the sum of effective shear forces due to n concentrated loads reaches the basic shear strength V_0 of the section.

$$\sum_{j=1}^{n} V_{eij} = V_0 \tag{8}$$

The shear failure load of a section is calculated using the procedure and assumptions described above. Repeating the calculation for every cross section throughout the span, the failure load of a beam is found as the minimum of all the values. The position of the critical section is also given by the x_i that results in the minimum shear failure load [4].

Iguro [5], Ishibashi [6], and Niwa [1] have independently proposed methods for calculating the shear strength of simple beams subjected to uniformly distributed loading. Iguro et al. replace a uniformly distributed load with an equivalent pair of concentrated loads acting at the one–fourth points along the span, following the idea by Kani. Then using the a/d ratio for the substituted concentrated load, the shear capacity of the beam is calculated using published equations for shear strength under concentrated loading.

Ishibashi et al. assume that the shear capacity of a section V_u is expressed by the sum of V_{ur} and V_{us}, as shown in Eq. (9). Here, V_{ur} and V_{us} represent the favorable effects of local compressive stress σ_y due to the load and support reaction, respectively.

$$\begin{aligned} V_u &= V_{ur} + V_{us} \qquad (9) \\ V_{ur} &= V_{u(a_1'=2(a_1-x))}/2.0 \\ V_{us} &= V_{u(a_1'=2x)}/2.0 \\ V_u &= \alpha f_c'^{1/3}(1 + \beta_p + \beta_d) b_w d \\ \alpha &= g(a_1'/d) \end{aligned}$$

Where, a_1 is the clear distance between the bearing plates at load point and support and roughly corresponds to the shear span length. Also, x is the distance between the inner edge of the bearing plate at the support and the section under consideration. That is, V_{ur} and V_{us} are equal to half the shear capacity of beams with shear span lengths $a'_1=2(a_1-x)$ and $a'_1=2x$, respectively.

The assumed failure criterion, under the combined action of n concentrated loads, is that the sum of the ratio of V_j to V_{uj} is unity, as shown in Eq. (10). Here, V_j is the shear force due to P_j, and V_{uj} is the shear strength of the section against P_j.

Table 4 Analysis of Leonhardt's experimental results

No.	Experiment				Zone strength method			Ishibashi's method			Niwa's method		
	l/d	f_c'	P_u tf	x cm	P_{ucal} tf	P_{ucal}/P_u	x_{cal} cm	P_{ucal} tf	P_{ucal}/P_u	x_{cal} cm	P_{ucal} tf	P_{ucal}/P_u	x_{cal} cm
11/1	5.17	355	55.10	29	42.61	0.77	30.0	36.28	0.66	30.0	34.69	0.63	33.0
11/2	5.07	355	59.63	-	44.47	0.75	30.0	37.66	0.63	30.0	36.24	0.61	33.0
12/1	7.33	343	40.50	29	29.41	0.73	34.3	26.14	0.65	34.3	24.23	0.60	31.4
12/2	7.35	343	32.10	-	29.19	0.91	34.3	25.99	0.81	34.3	24.08	0.75	31.4
13/1	9.16	348	27.80	44	24.94	0.90	38.9	22.71	0.82	33.3	21.54	0.77	33.3
13/2	9.19	348	27.80	-	24.77	0.89	38.9	22.54	0.81	33.3	21.41	0.77	33.3
14/1	10.99	337	21.39	38	22.02	1.03	40.9	20.38	0.95	35.5	19.51	0.91	32.7
14/2	10.99	337	21.48	-	22.02	1.03	40.9	20.38	0.95	35.5	19.51	0.91	32.7
15/1	14.71	357	19.05	43	19.37	1.02	45.3	18.39	0.97	37.3	17.98	0.94	32.0
15/2	14.65	357	20.32	-	19.36	0.95	48.0	18.37	0.90	37.3	17.95	0.88	32.0
16/1	18.32	352	19.25	46	17.69	0.92	50.0	16.99	0.88	41.7	16.89	0.88	38.9
16/2	18.25	352	19.15	-	17.71	0.92	50.0	17.00	0.89	41.7	16.89	0.88	38.9
	Number of beams				12			12			12		
	Average P_{ucal}/P_u				0.90			0.83			0.79		
	C.V. of P_{ucal}/P_u				11.1%			14.0%			15.0%		

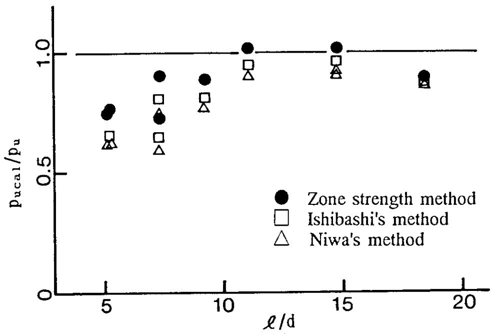

Fig. 5 Calculated shear failure load

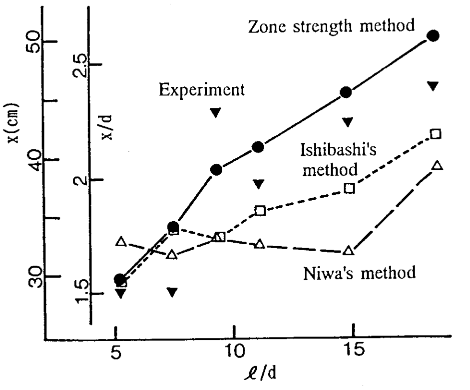

Fig. 6 Position of failed section

$$\sum_{j=1}^{n} V_j/V_{uj} = 1 \qquad (10)$$

In the calculation for simple beams under uniformly distributed loading, the load is represented by a number of concentrated loads, and Eq. (9) and Eq. (10) are used.

In Niwa's method, the idea is similar to that in Eq. (9) but his original equations of shear strength are used. The failure criterion is fundamentaly the same as Eq. (8). The significant difference between the author's method and Niwa's or Ishibashi's is that the distribution of shear strength of the section is formulated on the basis of the experimental results, while it is assumed almost intuitively in the latter two methods.

Leonhardt [7] carried out loading tests to study the shear strength of simple beams subjected to uniformly distributed loading. The cross section of his specimens was 19 cm (width) by 27.3 cm (effective depth) and with an axial reinforcement ratio of 2.05%. No web reinforcement was included in the shear span. A uniform load was applied using water pressure. A summary of his experimental results is given in Table 4. Test data for specimens 17/1 and 17/2 are omitted from the analysis, because one of these two identical specimens failed in flexure. The shear failure loads given in the table include the effect of the beams' own weight.

In Table 4, the shear strength and location of failure as calculated by the author's method, Ishibashi's method, and Niwa's method are shown. Though the calculated value changes according to the number of divisions of the span and load, it is confirmed that the calculation can be considered almost convergent if the number of divisions is greater than 50 for these specimens. In the calculations shown in Table 4, span and load are equally divided with a pitch of about 0.1d to precisely determine the location of failure. As a result, the number of divisions is between 50 and 180. The term l/d in the table denotes the ratio of span length to effective depth, and P_u and P_{ucal} denote experimental and calculated failure loads, respectively. Also, x and x_{cal} are the experimental and calculated distances from the support to the critical diagonal crack. The cubic strength, β_w, of the concrete is transformed into a cylindrical strength, f_c', by multiplying β_w by 0.85.

For all 12 specimens, the average and coefficient of variation (C.V.) of the ratio between calculated and experimental shear failure load (P_{ucal}/P_u) is, respectively, 0.90 and 11.1% by the zone strength method, 0.83 and 14.0% by Ishibashi's method, and 0.79 and 15.0% by Niwa's method. Although, the differences are not large, the accuracy of the estimate made by the zone strength method is the best.

The relationship between l/d and P_{ucal}/P_u is shown in Fig. 5. In the case of the zone strength method, P_{ucal}/P_u is greater than about 90% in the range of l/d higher than about 10, showing the good agreement with the experimental results. On the other hand, when l/d is about 5 and 7, the accuracy is not very good. In these cases, the location of the critical diagonal crack (x/d) is near to 1.0. This problem might be caused by the larger variation in shear strength itself and by the insufficient experimental data

Table 5 Analysis of beams under multiple concentrated loading

No.	Number of loads	a_i/d	Ratio of loads	a'/d	P_u (tf)	P_{ucal} (tf)	P_{ucal}/P_u	x_{cal}/d
K71	4	1.50, 4.50	0.28, 0.22, 0.23, 0.27	3.0	21.4	19.4	0.91	1.90
K72	4	1.00, 3.00	0.29, 0.21, 0.23, 0.26	2.0	28.4	24.1	0.85	1.50
K73	4	0.75, 2.25	0.28, 0.22, 0.25, 0.25	1.5	38.0	30.1	0.79	1.10
K74	4	0.58, 1.72	0.29, 0.22, 0.24, 0.26	1.15	64.6	46.5	0.72	0.90
L75	8	0.75, 2.25 3.75, 5.25	0.18, 0.10, 0.13, 0.11 0.12, 0.11, 0.10, 0.16	3.0	18.6	17.5	0.94	1.50
L76	8	0.50, 1.50 2.50, 3.50	0.20, 0.10, 0.14, 0.07 0.09, 0.13, 0.11, 0.17	2.0	30.6	26.4	0.96	1.20
L77	8	0.38, 1.13 1.88, 2.63	0.16, 0.13, 0.15, 0.06 0.08, 0.12, 0.12, 0.17	1.5	50.2	37.5	0.75	0.90
L78	8	0.29, 0.86 1.44, 2.01	0.17, 0.12, 0.11, 0.09 0.09, 0.12, 0.11, 0.19	1.15	69.6	55.9	0.80	0.90
M79	8	0.75, 2.25 3.75, 5.25	0.27, 0.14, 0.08, 0.02 0.01, 0.07, 0.14, 0.27	2.0	36.6	32.0	0.87	1.30
M80	8	0.56, 1.69 2.81, 3.94	0.27, 0.14, 0.08, 0.02 0.01, 0.07, 0.14, 0.27	1.5	49.8	38.0	0.76	1.10
M81	8	0.43, 1.29 2.16, 3.02	0.29, 0.13, 0.08, 0.02 0.02, 0.06, 0.13, 0.28	1.15	70.8	58.6	0.83	0.90
N82	8	0.56, 1.69 2.81, 3.94	0.07, 0.10, 0.17, 0.17 0.19, 0.17, 0.07, 0.07	3.0	16.2	15.4	0.95	1.50
N83	8	0.38, 1.13 1.88, 2.63	0.10, 0.10, 0.21, 0.10 0.13, 0.21, 0.07, 0.07	2.0	33.4	24.7	0.74	1.10
N84	8	0.28, 0.84 1.41, 1.97	0.10, 0.12, 0.18, 0.11 0.15, 0.18, 0.09, 0.07	1.5	55.0	38.1	0.69	0.90
N85	8	0.22, 0.65 1.08, 1.51	0.07, 0.08, 0.18, 0.16 0.13, 0.18, 0.11, 0.09	1.15	65.0	59.6	0.92	0.70

available in determining the zone strength equation for x/d smaller than 1.0.

Figure 6 shows the relationship between l/d and x. Though there is some scatter in the experimental results, x can be considered to increase monotonously with increasing l/d. In the zone strength method, calculated values of x behave similarly to the experimental results, and there is little difference between the calculated and experimental values. On the other hand, calculated values of x by Ishibashi's method have irregular variations in the vicinity of l/d equal to 8. Those calculated x by Niwa's method are almost constant when l/d is smaller than 15. These results verify that for simple beams without web reinforcement and subjected to uniformly distributed loading, we can well estimate the shear strength and the location of failure by the proposed method.

6. APPLICATION TO SIMPLE BEAMS SUBJECTED TO MULTIPLE CONCENTRATED LOADS

Kobayashi et al. [8][9] carried out a systematic series of experiments concerning the shear strength of simple beams under multiple concentrated loading. The specimens used had a rectangular cross section with width 20 cm, effective depth 24 cm, and a tensile reinforcement ratio of 1.61% (2D19 + D16). No web reinforcement was used in the shear span. Compressive strength of the concrete (f_c') at the time of loading was 239 kgf/cm^2 for all specimens.

Two groups of two or four concentrated loads were loaded symmetrically on the simple 288 cm spans.

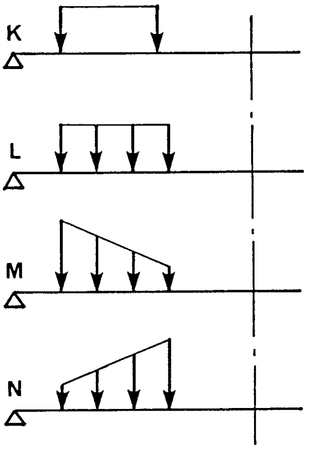

Fig. 7 Pattern of loading

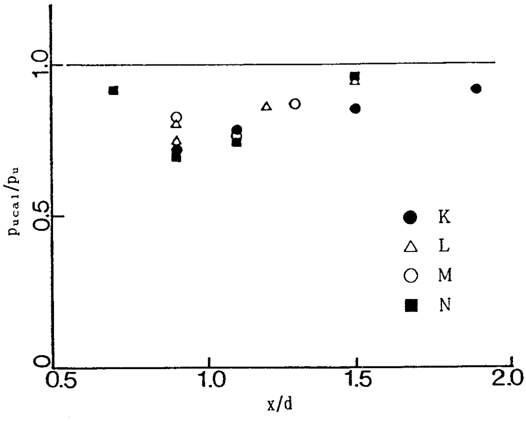

Fig. 8 Relationship between failure load and position of failure

The loading pattern is shown schematically in Fig. 7. All concentrated loads were equal in series K and L. On the other hand, in the M and N series, loads were increased or decreased toward the span center, respectively. The position of each concentrated load on one side (a_i/d) and the measured ratio of the loads at failure are shown in Table 5. Here, P_u denotes the shear failure load, which is the sum of all four or eight concentrated loads on the beam. As summarized in Table 6, for two or four loads on one side, the location of each load (a_i/d) and that of the resultant (a'/d) ranged from 0.22 to 5.25 and from 1.15 to 3.00, respectively. The ratio of maximum load to minimum load in the K and N series ranges from 13.50 to 27.00 and from 1.80 to 3.00, respectively. These load conditions can be considered covering very wide range.

Table 6 Summary of loads

Series	Load position a_i/d	Resultant nominal a'/d	Load ratio max./min.
K	0.58 - 4.50	1.15 - 3.00	1.27 - 1.38
L	0.29 - 5.25	1.15 - 3.00	1.33 - 2.86
M	0.43 - 5.25	1.15 - 2.00	13.50 - 27.00
N	0.22 - 3.94	1.15 - 3.00	1.80 - 3.00

For the 15 specimens above, the shear failure load (P_{ucal}) and the location of failure (x_{cal}/d) were calculated using the zone shear strength equation as in Section 5 (see Table 5). Calculations were carried out for all the 120 cross sections that equally divid the span. The pitch of the sections was 0.1d.

As shown in Table 5, the average ratio of calculated to experimental failure load (P_{ucal}/P_u) for the three or four specimens in each series was between 0.82 and 0.84. No significant difference was observed in the ratio with different loading patterns. For all specimens, the average value and C.V. of the failure load ratio (P_{ucal}/P_u) was 0.83 and 9.8%, respectively. It can be said that the zone shear strength method well estimates the shear strength of beams subjected to multiple concentrated loading, too, though it does on average give a slightly lower failure load.

The relationship between failure load ratio (P_{ucal}/P_u) and the calculated failure position (x_{cal}/d) is shown in Fig. 8. As seen in this figure, the failure load ratio is lower in the range of x_{cal}/d from about 0.9 to 1.1, as in the case of uniformly distributed loading. It would be necessary to accumulate more experimental data on zone shear strength in the range of small x/d to improve the accuracy of the zone shear strength equation.

7. CONCLUSIONS

The conclusions of this study are summarized as below:
(1) In order to calculate the shear strength of an RC beam, the shear strength of all cross sections throughout the span need to be known. Scince a diagonal crack develops over a zone with a finite length, it is more practical to consider the shear strength of a finite zone than that of a simple cross section.

(2) In rectanguler beams without web reinforcement, the shear strength of a zone is determined by the distances from the section to the loading point and to the support, as well as by the basic shear strength that is dictated by the shape and dimensions of the cross section and the physical properties of the materials used. The shear strength of a zone is expressed by Eq. (5).

(3) The shear strength of simple beams subjected to uniformly distributed loading can be calculated with good accuracy by the proposed zone shear strength method, in which the uniformly distributed load is transformed into a series of small concentrated loads, and the zone shear strength equation (Eq. (5)) is applied to the transformed loads. A linear accumulation of damage (Eq. (8)) is assumed as the failure criterion.

(4) The shear strength of simple beams under multiple concentrated loading can also be well estimated using the zone shear strength method, irrespective of the location and the distribution of the concentrated loads.

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