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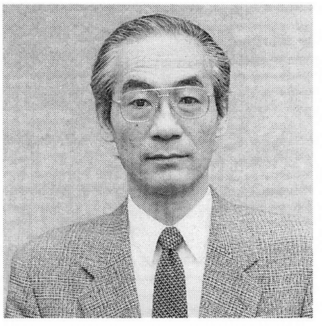
SHEAR STRENGTH OF BEAMS WITH MOMENT INFLECTION UNDER THE EFFECT OF UNIFORMLY DISTRIBUTED LOAD

(Translation from Transaction of JSCE, No. 460, V-17, 1993)



Takeshi HIGAI

SYNOPSIS

Many RC members are subjected to parabolic moment distribution and linear shear force distribution under the combined action of distributed and concentrated loading. There are also usually some inflection points within the span. In this study, the shear resisting behavior and shear strength of this type of beam are examined based on experimental results. Then, a method of estimating the shear strength of the beams is proposed, based on the "zone shear strength equation" as proposed separately by the author.

Keywords: Reinforced concrete beams, Shear strength, Uniformly distributed load, Moment inflection, Zone shear strength method

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T.Higai is a professor in the department of Civil and Environmental Engineering at Yamanashi University, Kofu, Japan. He received his Doctor of Engineering Degree in 1972 from University of Tokyo. His research interests relate to the shear failure of reinforced concrete structures, dynamic behavior of concrete structures and maintenance and strengthening of existing concrete structures. He is a member of JSCE and JCI.

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1. INTRODUCTION

As a result of active research into the shear failure of reinforced concrete members, the shear failure behavior of simple structures, such as simple beams, under simple loading - such as with one or two symmetrical concentrated loads - has to some extent been clarified. The shear strength of such members can be well estimated based on empirical equations. The accumulation of knowledge and experimental data on the shear behavior of simple beams subjected to the uniformly distributed loads is under progress.

On the other hand, the shear failure behavior of even a simple beam under a number of concentrated loads has not been clarified yet, and shear strength cannot be estimated with a good accuracy. For frame or continuous beam members, in which the bending moment changes from positive to negative even under a single concentrated load or the bending moment and shear force vary parabolic and linear, respectively, under a uniformly distributed load, we do not yet have enough knowledge to implement rational design. As a result, such structures are generally designed using underestimates of shear strength.

The effect of the dead load is usually significant in the design of reinforced concrete bridges. In the case of underground structures - such as box culverts, water intake pits and ducts for nuclear power plants - earth pressure is one of the major loadings. Thus, distributed loading is a very important load effect, and a study of shear failure of RC members under distributed loading is also very important.

The purposes of this study are to examine the shear failure behavior of RC members under general loading and to propose a method of calculating shear strength based on the results of loading tests of RC beam specimens. These specimens were subjected to a parabolic bending moment distribution and linear shear force distribution under the effect of a uniformly distributed load, and had a moment inflection point within the span. Loading tests of 26 such beams with different moment distribution were carried out to examine the diagonal cracking behavior and ultimate shear strength. A method of estimating the shear strength based on the "zone shear strength equation" is developed, and its accuracy is examined through a comparison with the experimental results. One of the characteristics of this study is that the importance of accurately determining the position of the critical diagonal crack, or the failure location, becomes very clear.

2. OUTLINE OF THE EXPERIMENT

2.1 Specimens

In examining the shear resisting behavior and shear strength of beams subjected to a parabolic bending moment distribution and a linear shear force distribution and with moment inflection points within the span, simple RC beam specimens with an overhang are used for experiments [1] (see Fig. 1). With this type of specimen, while the bending moment and shear force distribution of the members in statically indeterminate structures can be easily represented, we can precisely determine the moment and shear force at any sectionin the beam, because it is actually

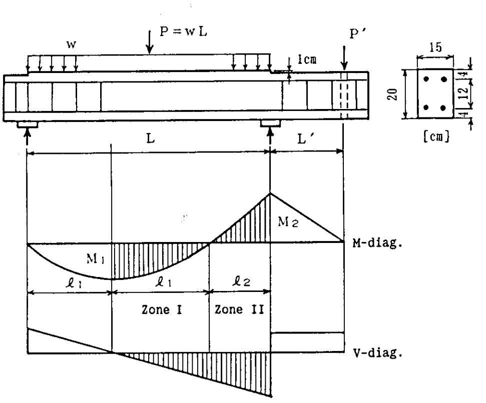

Fig. 1 Dimension of test beams

Table 1 Summary of specimens

Specimen	L (cm)	L' (cm)	M_2/M_1	l_1/d	l_2/d	f'_c (kgf/cm^2)	p_w (%)
II-10	100	37.5	1.0	2.59	1.08	320	3.23
II-20			2.0	2.29	1.68	315	
II-30			3.0	2.08	2.08	320	
IV-00	140	42.5	0.0	4.38	0.00	238	3.23
IV-10			1.0	3.63	1.50	248	
IV-22			2.2	3.14	2.48	193	
IV-30			3.0	2.92	2.92	225	
V-025	160	45.0	0.25	4.72	0.56	330	3.23
V-050			0.50	4.49	1.01	313	
V-075			0.75	4.30	1.39	341	
V-10			1.0	4.14	1.72	299	2.39
V-30			3.0	3.33	3.33	290	
V-40			4.0	3.09	3.86	339	3.23
V-50			5.0	2.90	4.20	319	
V-70			7.0	2.61	4.78	322	
I-10	180	47.5	1.0	4.66	1.93	300	3.23
I-20			2.0	4.12	3.01	435	
I-30			3.0	3.75	3.75	338	
VI-01	220	52.5	0.1	6.71	0.33	418	3.23
VI-02			0.2	6.56	0.63	300	
VI-05			0.5	6.18	1.39	335	
VI-10*			1.0	5.70	2.36	320	
VI-20*			2.0	5.03	3.68	313	
VI-30			3.0	4.58	4.58	336	
VI-50			5.0	3.99	5.78	334	
VI-70			7.0	3.59	6.57	329	

statically determinate. The results of this study can, of course, be applied to statically indeterminate structures, but the changes in moment and shear force distribution due to the change in cross-sectional stiffness accompanying crack development need to be considered separately.

In our loading tests, a uniformly distributed load w (resultant force P) was applied to a simple span and a concentrated load, P', was applied to the top face of the overhang. By changing the ratio of P' and P, the ratio of positive span moment, M_1, and negative support moment, M_2, could easily be adjusted. As shown in Table 1, specimens had a rectangular section 15 cm in width and 20 cm in height, with an effective depth of 16 cm. Equal amounts of compression and tension reinforcement were used. The standard reinforcement ratio, p_w ,was 3.23% (2D22), but in some specimens the ratio was reduced to 2.30% (2D19). The length of the simple span was varied within the range 100 to 220 cm. The ratio of simple span length to effective depth (L/d) was thus between 6.25 and 13.75, and the ratio of M_2/M_1 was varied within the wide range of 0 to 7.0.

Measurements and observations were concentrated in zone I, between the maximum span moment (M_{max}) and the moment inflection point (I.P.), and zone II, between the inflection point and the internal support. To force shear failure in these zones, closed stirrups (2D10, 10 cm pitch) were arranged as shear reinforcement in the remaining parts of the specimen. In specimens VI-10 and VI-20, zone II was also reinforced with closed stirrups (2D10, 10 cm pitch) to force a shear failure to occur in zone I.

2.2 Loading test method

The system for applying the uniformly distributed load is shown in Fig. 2. Two fire-fighting hoses (diameter 75 mm, maximum pressure 32 kgf/cm^2), connected at one end with a U-shaped steel pipe and filled with an appropriate quantity of water, were held to the simple span with an H-profile steel beam. The concentrated load was applied through another H-beam, as shown. The height of upper surface of uniformly loaded part was 1 cm higher than that in the remaining parts, so that the transmission of uniformly distributed load could be worked well.

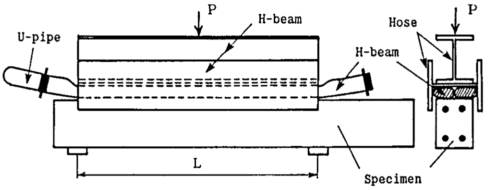

Fig. 2 Method of appliying uniformly distributed load

Table 2 Reinforcement

Series	Bars	f_y kgf/cm^2	E_s kgf/cm^2
I,II	D22 D10	3,660 3,390	1.80*10^6 1.51*10^6
IV	D22 D10	3,750 3,390	1.87*10^6 1.68*10^6
V	D22 D19 D10	3,620 3,760 3,390	1.78*10^6 1.87*10^6 1.68:10^6
VI	D22 D10	3,620 3,390	1.78*10^6 1.68*10^6

Steel bearing plates 3 cm thick and 8 cm wide were used at both supports. A concentrated load, P', was applied to the overhanging part through a vertical hole in the specimen using a PC bar and a center-hole hydraulic jack. Loads P and P' were incremented until failure. The increment in load P' was 0.25 tf or 0.5 tf.

2.3 Observations and measurements

The resultant of uniformly distributed load, P, and concentrated load, P', on the overhang were measured by each load cell. Water pressure in the hoses was monitored using a pressure gauge attached to the U-pipe. During testing, the ratio P'/P was continually monitored and held within certain limits by the fine adjustment of load P'.

The deflection of specimens was measured at both supports, at the concentrated load point, and at the point of maximum span moment. At the same time, the reinforcing bar strain was measured at the points of maximum span moment, internal support, and moment inflection. The development of cracks was observed and recorded manually using magnifier lenses.

2.4 Materials

D22 and D19 were used for axial reinforcement, and D10 for stirrups. The yield point and Young's modulus of these bars are shown in Table 2. High-early-strength Portland cement, crushed stone with maximum size of 20 mm, and river sand were used for the concrete. The W/C ratio of the concrete was 55% to 70%, and the target slump value was 8 cm or 15 cm. Even though the standard age at the time of loading tests was 14 days, the actual age of specimens ranged from 8 to 44 days. The compressive strength of the concrete at the time of loading tests is also shown in Table 1.

3. DIAGONAL CRACKING AND SHEAR FAILURE

The shear failure of all 26 specimens occurred in zone I or zone II. No yielding of the axial reinforcement was noted until the time of failure.

Typical cracking patterns of specimens in the test zone are shown in Fig. 3. These are examples of the V series of specimens with a simple span length of 160 cm and an M_2/M_1 ratio of 0.25 to 7.0. When the M_2/M_1 ratio is small, a critical diagonal crack develops and the shear failure occurs in zone I, where the moment is positive. When the ratio is bigger, the critical diagonal crack and shear failure are observed in zone II, which has a negative moment. For the specimens used in this study, the failure zone moved from zone I to zone II at the limit M_2/M_1 value of about 2.

As shown in Fig. 4, when the position of the first diagonal crack ① happened to be near the point of maximum moment in zone I, or when it was near the internal support in zone II, an additional diagonal crack ② then developed and this was followed by shear failure. There was some difficulty judging which diagonal crack played a more important role in this type of failure. Depending on the ratio of zone I to zone II length (l_1/l_2), there were some cases in which an additional diagonal crack ③ also occurred in the non-failure zone.

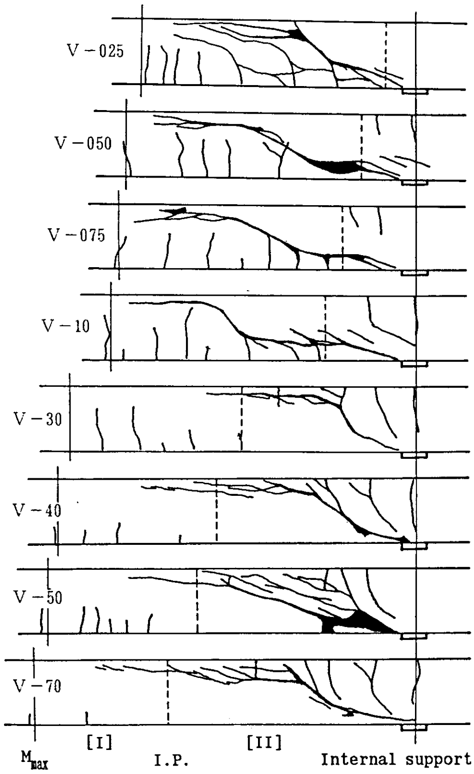

Fig. 3 Diagonal cracking pattern (V Series)

Where a diagonal tension failure occurs in a simple beam subjected to a concentrated load, shear failure usually occurs just after the formation of the critical diagonal crack. In the case of specimens V-075 and V-70 among others, though the crack pattern was very similar to that of for a simple beam under concentrated load, a relatively larger load increment could be sustained after diagonal cracking. As a result, the ratio of failure load (P_u) to diagonal cracking load (P_{cr}) ranged from 1.00 to 2.18. But because the value of P_u/P_{cr} approached 1.0 asymptotically when l_1/d or l_2/d were bigger than 5, as shown in Fig. 5, this might not be a peculiar feature of beams subjected to a uniformly distributed load.

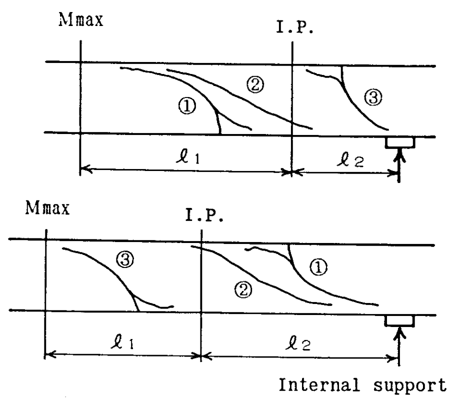

Fig. 4 Schematic diagonal cracking pattern

The test results for shear failure load (P_{cr}) and position of shear failure - or position of the critical diagonal crack - (x_1/d, x_2/d) are shown in Table 3. Here, x_1 denotes the distance from the inflection point to the inclined crack in zone I, and x_2 denotes the distance from the internal support to the inclined crack in zone II. Here the position of the diagonal crack is defined as the point at which the crack intersects the mid-height of effective depth.

Table 3 Summary of loading tests and calculations

Specimen	M_2/M_1	Experiment				Calculation(1)			Calculation(2)				
		Zone	P_u (tf)	Position x_1/d	Position x_2/d	Zone	P_{uc1} (tf)	P_{uc1}/P_u	Zone	P_{uc2} (tf)	Position x_{1c}/d	Position x_{2c}/d	P_{uc2}/P_u
II-10	1.0	I	35.17	0.94		I	29.20	0.830	I	28.18	1.04		0.801
II-20	2.0	I	27.26	0.75		II	23.43	(0.860)	II	23.43		0.75	(0.860)
II-30	3.0	II	25.09		0.93	II	16.74	0.667	II	16.74		0.94	0.667
IV-00	0.0	I	18.70	1.63		I	13.53	0.724	I	13.54	1.36		0.724
IV-10	1.0	I	17.60	1.13		I	19.19	1.090	I	15.35	1.24		0.872
IV-22	2.2	II	14.00		0.94	II	10.81	0.772	II	10.81		1.11	0.772
IV-30	3.0	II	14.70		0.98	II	9.66	0.657	II	9.66		1.26	0.657
V-025	0.25	I	19.19	1.72		I	15.18	0.795	I	15.18	1.47		0.795
V-050	0.50	I	22.90	1.63		I	16.19	0.707	I	16.19	1.40		0.703
V-075	0.75	I	20.50	1.31		I	17.95	0.876	I	15.02	1.34		0.733
V-10	1.0	I	16.00	1.79		I	16.60	1.038	I	12.23	1.35		0.764
V-30	3.0	II	10.40		1.21	II	8.43	0.811	II	8.43		1.44	0.811
V-40	4.0	II	12.40		1.44	II	9.01	0.727	II	9.01		1.53	0.727
V-50	5.0	II	11.40		1.71	II	8.38	0.735	II	8.38		1.58	0.735
V-70	7.0	II	9.50		1.70	II	7.87	0.828	II	7.87		1.62	0.828
I-10	1.0	I	14.21	1.03		I	16.90	1.189	I	11.54	1.45		0.812
I-20	2.0	II	13.45		1.45	II	11.36	0.845	II	11.36		1.31	0.845
I-30	3.0	II	12.52		1.28	II	9.01	0.720	II	9.01		1.59	0.720
VI-01	0.1	I	12.93	1.74		I	13.02	1.007	I	13.02	1.65		1.007
VI-02	0.2	I	13.28	1.49		I	12.04	0.907	I	12.04	1.62		0.907
VI-05	0.5	I	12.42	1.47		I	13.69	1.102	I	11.46	1.65		0.923
VI-10*	1.0	I	12.49	1.34		I	15.28	1.223	I	10.18	1.55		0.815
VI-20*	2.0	I	15.96	1.11		I	18.46	1.157	I	12.31	1.51		0.771
VI-30	3.0	II	8.85		1.81	II	8.05	0.910	II	8.05		1.74	0.910
VI-50	5.0	II	7.91		1.78	II	7.31	0.924	II	7.31		1.83	0.924
VI-70	7.0	II	7.43		1.17	II	6.89	0.927	II	6.89		1.88	0.927

4. METHOD OF CALCULATING SHEAR STRENGTH UTILIZING THE ZONE SHEAR STRENGTH EQUATION

4.1 Outline of zone shear strength equation

It is well known that when a simple beam is subjected to concentrated loading, its shear strength changes greatly depending on the ratio of shear span to effective depth (a/d). This is generally thought to be the effect of local vertical compressive stress σ_y, which is generated by the load acting on the top surface of the beam and the supporting reaction acting on the bottom of the beam. This stress reduces the principal tension stress σ_1 which causes the development of

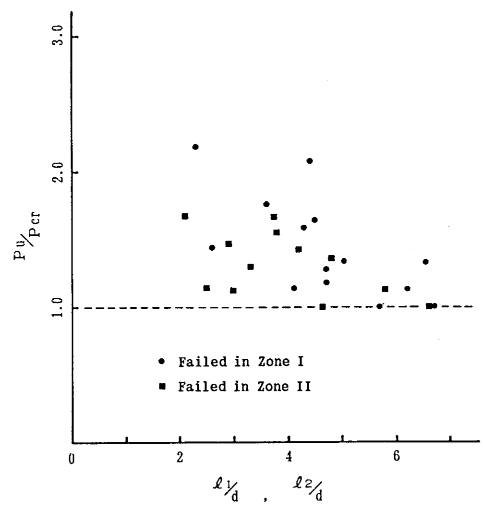

Fig. 5 Relationship between diagonal cracking load and failure load

diagonal cracks in the web concrete. The shear strength in the vicinity of the support and load point is higher than at the other points due to the larger value of σ_y there. In a beam with a short shear span, σ_y is larger due to the short distance between the load and support, resulting in a greater shear strength than in a beam with longer shear span. Generally, load and support conditions under which a vertical compressive stress occurs in the web are called "direct loading" and "direct support," respectively, while "indirect loading" and "indirect support" are the terms for a condition in which no local vertical compression stress occurs.

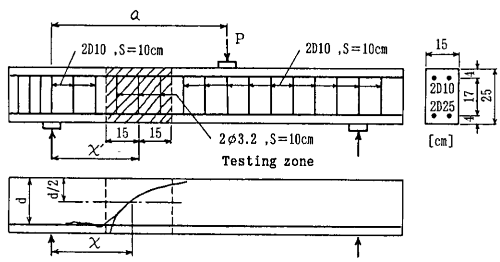

Fig. 6 Specimens for determining zone shear strength

The author has examined experimentally the change in shear strength in a test zone as the distance from a diagonal crack to support (x) and to loading point (a-x) varies. Specially reinforced beam specimens were used in this experiment as shown in Fig.6 : the shear reinforcement in one part (the test zone) of the shear span was substantially reduced in order to force a shear failure within the test zone. Based on the results of this experiment, Eq. 1 was proposed [2].

$$V_u = R V_0 = 0.958[\coth(x/d)]^{1.360}[\coth\{(a-x)/d\}]^{1.484} V_0 \qquad (1)$$

V_0 is the basic shear strength of a cross section when there is no influence from σ_y, and is assumed to be expressed by Eq. 2.

$$V_0 = 0.94 f_c'^{1/3} p_w^{1/3} (100/d)^{1/4} b_w d \qquad (2)$$

Where, f_c' : compressive strength of concrete
p_w : axial reinforcement ratio $= 100A_s/(b_w d)$ [%]
d : effective depth [cm]
b_w : web width [cm]

After calculating V_u for every cross section between the support and loading point, the shear strength of the beam and the location of the critical diagonal crack can be solved by finding the minimum value of V_u and the corresponding value of x, respectively.

4.2 Application of zone shear strength equation to the test specimens

The specimens tested here are simple beams with an overhang. A uniformly distributed load w (w = P/L) acts on the simple span and a concentrated load P' acts on the top of the overhanging part. Based on the ratio of P to P', the ratio of span moment M_1 and support moment M_2 varies and, consequently, the position of the inflection point of bending moment (I.P.) also changes. Here, we denote the distance from the left support to the inflection

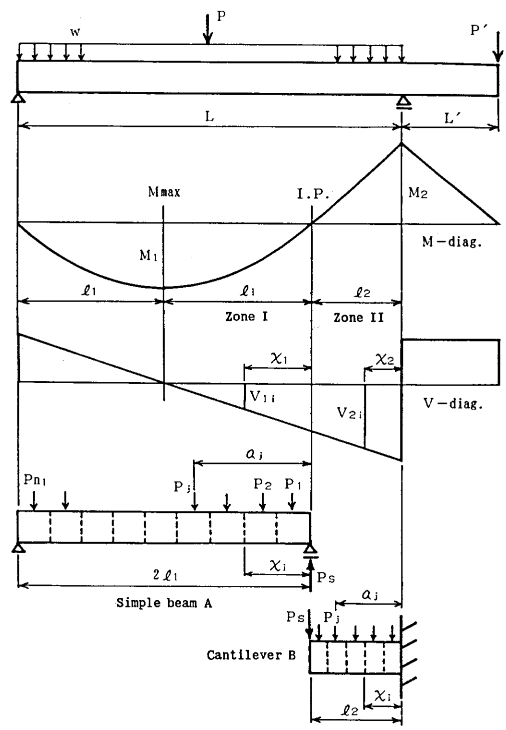

Fig. 7 Simplified model of the specimen

point and that from the inflection point to the internal support as $2l_1$ and l_2, respectively. The shear strength is analyzed only in zone I and zone II (see Fig. 7).

First, the specimen is divided into a simple beam A - a span from the left support to the inflection point - and a cantilever B fixed at the internal support and of length l_2.

(a) Analysis in zone I

The uniformly distributed load w is divided equally into n_1 concentrated loads P_j ($P_j = P/n_1 = wL/n_1$, $j=1,2,\ldots,n_1$). Then the shear strength of each of the n_1 equally divided sections is calculated. We must consider the interactive effect of the large number of concentrated loads, and it is more convenient to assume that the effect of the applied shear force will decrease to 1/R than to consider the shear strength increasing by R times in response to the value of x and a. Based on this way of thinking, the effective shear force V_e is defined as 1/R times the applied shear force. It is also assumed that shear failure occurs when the total effective shear force due to n_1 concentrated loads reaches the basic shear strength V_0 of the section. The shear force V_{ij} and effective shear force V_{eij} produced in x_i section by load P_j can be expressed by Eq. 3.

$$V_{eij}=V_{ij}/R_{ij} \tag{3}$$

Where,

$$V_{ij}=P_j(2l_1-a_j)/(2l_1), \qquad j>i \tag{4}$$

$$V_{ij}=-P_ja_j/(2l_1), \qquad j\le i$$

$$R_{ij}=0.958\,[\coth(x_i/d)]^{1.360}\,[\coth\{(a_j-x_i)/d\}]^{1.484} \tag{5}$$

The total effective shear force produced by P_1 to P_{n1} can be written as Eq. 6, and the intensity of the uniformly distributed load w_{u1i} at shear failure of section x_i can be expressed by Eq. 7.

$$V_{ei}=\sum_{j=1}^{n_1} V_{eij} \tag{6}$$

$$W_{u1i}=\frac{V_0}{\displaystyle\sum_{j=1}^{i}\frac{-a_j}{n_1R_{ij}}+\sum_{j=i+1}^{n_1}\frac{2l_1-a_j}{n_1R_{ij}}} \tag{7}$$

Repeating these calculations from section 1 to $n_1/2$, the shear failure load w_1 and the position of the critical diagonal crack for zone I can be determined be finding the minimum value of w_{u1i} and the corresponding x_i.

In the calculation above, both supports of the simple beam A are treated as direct supports, but in fact the right support is imaginary and there is no actual reaction on the bottom of the beam. Hence, this support should be treated as an indirect support. This discrepancy in the support conditions will be discussed in section 5.

(b) Analysis in zone II

In zone II, the span and uniformly distributed load are divided into n_2 equal parts, and the shear strength is calculated using a method basically the same as that used in zone I. In this case, however, an imaginary concentrated load ($P_s = wl_1$) - equal to the reaction of an imaginary support for simple beam A - is applied to the free end of the cantilever beam.

Taking into account the fact that loads (P_j, $j \le i$) located nearer to the fixed end than the section under consideration x_i exert no shear force on section x_i, the shear force and effective shear force in section x_i due to P_j are expressed by Eq. 8 and Eq. 9. The total effective shear force due to all loads is expressed by Eq. 10. Finally, the value of the uniformly distributed load w_{u2i} upon the shear failure of section x_i can be calculated using Eq. 11.

$$V_{ij}=P_j, \qquad j>i \tag{8}$$

$$V_{eij}=V_{ij}/R_{ij} \tag{9}$$

$$V_{ei}=\sum_{j=i+1}^{n_2} V_{eij}=\sum_{j=i+1}^{n_2} \frac{P_j}{R_{ij}}+\frac{P_s}{R_i} \tag{10}$$

$$W_{u2i}=\frac{V_0}{\sum_{j=i+1}^{n_2} \frac{l_2}{n_2 R_{ij}}+\frac{l_1}{R_i}} \tag{11}$$

Where, R_i denotes the factor R for P_s, as expressed by Eq. 12.

$$R_i=0.958[\coth(x_i/d)]^{1.360}[\coth\{(l_2-x_i)/d\}]^{1.484} \tag{12}$$

By repeating these calculations from section 1 to n_2, the shear failure load w_2 and the position of the critical diagonal crack for zone II can be found as the minimum value of w_{u2i} and the corresponding x_i.

Finally, the failure load w_u of the specimen is determined as the smaller of the two values w_{u1} and w_{u2}, and the value of x corresponding to w_u represents the location of the critical diagonal crack which induces the shear failure.

Some similar methods have been proposed in the past [3],[4], but in these methods the shear strength multiplier or the shear force reduction factor, corresponding to the factor R here, has generally been assumed almost intuitively, with no experimental nor theoretical support. No application of these other methods to RC members with a moment inflection point has been published as yet, so far as the author knows.

5. EXAMINATION OF SHEAR STRENGTH AND FAILURE LOCATION

The shear failure loads P_{uc1} of specimens as calculated according to the method proposed in section 4 are shown in Table 3 (Calculation (1)).

The average ratio of calculated to experimental failure loads (P_{uc1}/P_u) for all specimens is 0.886, and the coefficient of variation (C.V.) is 18.2%. For the 12 specimens which failed in zone II, the average P_{uc1}/P_u and C.V. values are 0.794 and 11.6%, respectively, while for the 13 specimens which failed in zone I, the values are 0.973 and 17.5%, respectively. It is clear that in the latter case, the average P_{uc1}/P_u and C.V. are relatively high compared with the former specimens.

As already mentioned in section 4.(2), this can be considered a result of the assumption that the inflection point is a direct support in calculations of shear strength in zone I. If the length l_2 from inflection point to the internal support is very short - that is, if the real support is very near to the imagined support - then the inflection point might be approximated in this way, but if l_2 is greater than a certain value, such an approximation can be expected to lose applicability, and it should be treated as an indirect support.

The relationship between the load ratio P_{uc1}/P_u and the values of l_2/d, for specimens which failed in zone I, is plotted in Fig. 8. As is clear, l_2/d has no influence over the load ratio if it is less than 1.0. But in the range from 1 to 2, the load ratio clearly increases with rising l_2/d value. Then, if l_2/d is greater than 2, the load ratio does not change any further. This result could be taken as proof that the above explanation concerning the physical condition of the imaginary support in beam A is correct.

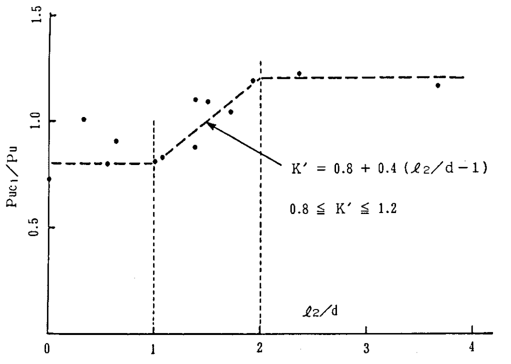

Fig. 8 Correction factor for shear strength in zone I

Thus it is understood that, in analyzing the shear capacity of zone I, the shear strength calculated by the zone strength equation needs to be reduced by dividing it by the correction factor K in Eq. 13. While the ratio of P_{uc1}/P_u can be expected to be 1.0 in the range $l_2/d<1.0$, it is actually around 0.8 as shown in Fig. 8. This discrepancy is regarded as the error caused by the zone shear strength equation, and the relationship representing the broken line (K') in Fig. 8 is transformed into Eq. 13.

$$K=1+0.5(l_2/d-1) \qquad (13)$$

Where, $1.0 \leq K \leq 1.5$

The calculated shear failure load (P_{uc2}) and the location of the shear failure x_{1c}, x_{2c} taking into consideration the correction factor K are shown in Table 3 (Calculation (2)). Here, x_1 and x_2 are the respective distances from the location of the failure to the inflection point in zone I and to the internal support in zone II. The suffix c denotes that these are calculated results. Hereafter, I will describe the shear strength calculation method based on the zone shear strength equation and on this correction factor K the "zone shear strength method".

The average and coefficient of variation of P_{uc2}/P_u for all 26 specimens is 0.808 and 10.9%, respectively. Moreover, the values are 0.817 and 10.3% for the 13 specimens that failed in zone I, and 0.794 and 11.6% for the 12 specimens that failed in zone II. The average calculated shear strength is about 20% lower than the experimental result, but the coefficient of variation is small enough and there is a good balance in the accuracy of the estimates of shear strength in zone I and zone II.

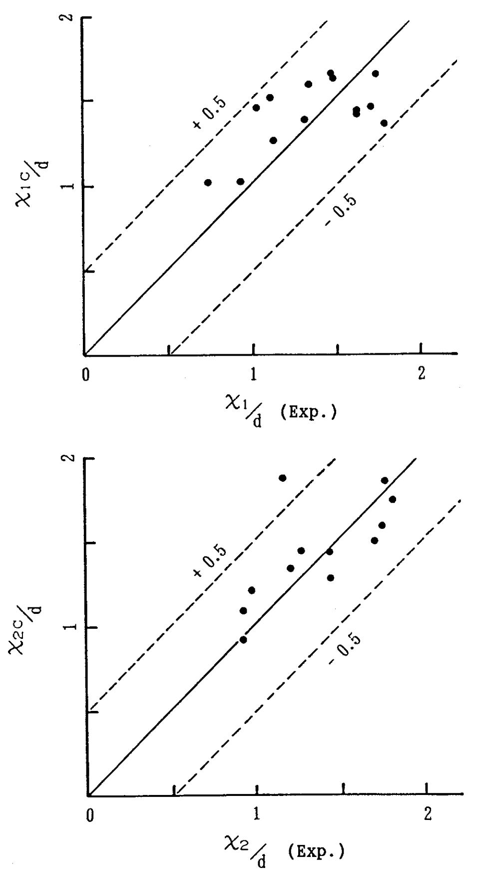

Fig. 9 Position of the failure section

The calculated failure zone agrees with the experimental result in all specimens except for II-20, in which the calculated shear capacity of both zone I and zone II is very close. The relationship between calculated and experimental failure locations is shown in Fig. 9. The average difference between the calculated and experimental value of x/d, whether the failure is in zone I or zone II, is less than 0.1 and the standard deviation is within 0.25. Hence, it could be said that the accuracy of the estimation is satisfactory. Since the error in estimating x/d is directly reflected in the error in the calculation of the shear force to be considered, accuracy in estimating the position of failure is very important in the analysis of the members subjected to a non-uniform shear force distribution.

These results indicate that the zone shear strength method can be used to estimate the shear strength and failure position of specimens with good accuracy. Table 4 gives comparisons of calculated shear strength v_{uc} and basic shear strength v_0. The value of v_0 is basically identical to the shear strength given in the JSCE Standard Specifications for Concrete, except that the safety margins are not included in v_0. As all values of v_{uc}/v_0 are larger than 0.85 and are mostly in the range 1.0 to 3.4, and considering the estimation error of the zone shear strength method, it might be said that the Standard Specification equation frequently underestimates the shear strength.

Table 4 Shear strength of critical section

Specimen	M_2/M_1	Failure Zone	v_0 (kgf/cm^2)	v_{uc} (kgf/cm^2)	v_{uc}/v_0
II-10	1.0	I	15.0	29.2	1.95
II-20	2.0	II	14.9	50.0	3.36
II-30	3.0	II	15.0	36.1	2.41
IV-00	0.0	I	13.6	19.4	1.43
IV-10	1.0	I	13.8	17.4	1.26
IV-22	2.2	II	12.7	23.2	1.83
IV-30	3.0	II	13.4	21.0	1.57
V-025	0.25	I	15.2	20.6	1.36
V-050	0.50	I	14.9	20.8	1.40
V-075	0.75	I	15.3	18.6	1.22
V-10	1.0	I	13.3	14.3	1.08
V-30	3.0	II	13.1	18.3	1.40
V-40	4.0	II	15.3	20.2	1.32
V-50	5.0	II	15.0	19.3	1.29
V-70	7.0	II	15.0	18.9	1.25
I-10	1.0	I	14.7	13.7	0.93
I-20	2.0	II	16.6	24.5	1.48
I-30	3.0	II	15.3	19.7	1.29
VI-01	0.1	I	16.4	20.0	1.22
VI-02	0.2	I	14.7	18.1	1.23
VI-05	0.5	I	15.3	15.8	1.03
VI-10*	1.0	I	15.0	12.8	0.85
VI-20*	2.0	I	14.9	13.1	0.88
VI-30	3.0	II	15.3	18.1	1.18
VI-50	5.0	II	15.2	17.6	1.16
VI-70	7.0	II	15.2	17.3	1.14

The zone shear strength equation was originally derived from a loading test of beams under a concentrated load. It has been shown that it is suitable for application to uniformly distributed loads, too. It now seems that this method is also applicable to general load conditions, such as where there is a large number of concentrated loads, a non-uniformly distributed load, or a combination of these. However, when there is no load acting in zone II, a somewhat special condition, zone II will be subjected to entirely indirect loading, and the calculation of shear strength under such unusual loading might need to be examined separately. It is not suggested that the results given in this paper be applied directly to practical design; studies of the application of this method to practical design is in progress and will be published at another opportunity.

6. CONCLUSIONS

The objective of this study was to develop a calculation method for the shear strength of members in statically indeterminate structures - such as continuous beams, frame structures, and underground box structures - that are subject to parabolic bending moment distributions and linear shear force distributions and that have some moment inflection points. Loading test, under the combined action of a uniformly distributed load and a concentrated load, were carried out using the simple beam specimens with an overhang. Then, based on the experimental results, a new method for calculating the shear strength was proposed. The conclusions of this study can be summarized as follows:

1. Though the applied shear force in zone II (negative moment zone) is always larger than that in zone I (positive span moment zone), the occurrence of shear failure is not limited in zone II only. It can occur even in zone I, if the ratio of support moment and span moment (M_2/M_1) is small.

2. The margin of shear strength after the development of a diagonal crack depends mainly on the span length; a beam with short span will fail under shear compression while a beam with a long span will fail under diagonal tension. This characteristic is very similar to that of a simple beam subjected to one or two symmetric concentrated loads.

3. The zone shear strength method was proposed for calculating the shear failure load of RC members subjected to a parabolic moment distribution and with a moment inflection point. In this method, a member is decomposed into some simpler elements, the uniformly distributed load is replaced by a number of small concentrated loads, and the "zone shear strength equation" - proposed separately by the author - is applied. The accuracy of the method was verified by comparing the results with experimental measurements.

4. From the results of calculations using the zone shear strength method, it was demonstrated that, in many cases, the equation given in the JSCE standard specifications underestimates the shear strength of RC members.

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