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TOWARDS UNIFYING ULTIMATE LIMIT STATE DESIGN OF STRUCTURAL CONCRETE

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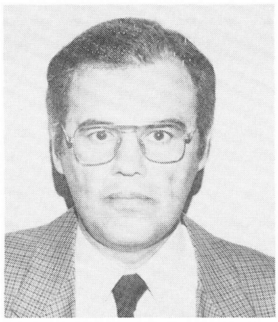

Michael D. KOTSOVOS   Naoki MASUI

SYNOPSIS

The paper contributes towards the development of a methodology capable of simplifying and unifying ultimate limit state design of structural concrete. It is shown that such a methodology could be based on the proposed concept of the compressive force path which, for the case of simply supported reinforced concrete beams, has been found to yield design solutions, not only significantly more economical, but also, safer than those obtained by using the methods recommended by current Code provisions. It is demonstrated that the proposed design method can easily be extended to apply for any type of skeletal structural concrete configuration which represents the structural elements between consecutive points of inflection being modelled as an "internal hinged support" effected by the provision of transverse reinforcement.

Keywords : ultimate limit state, shear strength, flexural strength, triaxial stresses

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TOWARDS UNIFYING ULTIMATE LIMIT STATE DESIGN OF STRUCTURAL CONCRETE

Michael D. KOTSOVOS* and Naoki MASUI**

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Keywords : *ultimate limit state, shear strength, flexural strength, triaxial stresses*

1. INTRODUCTION

The need for an approach that would unify and simplify design procedures has long ago been recognised. The efforts to achieve this aim have been intensified in recent years and resulted in the introduction of new ultimate limit state design methodologies[1)~6)], some of them placing more emphasis on overall load paths and resisting elements[1),6)] and others including JSCE Design Code on sectional load effects and resistances[2)~5)], the latter concepts having dominated design to date.

However, the introduction of new design methodologies has only once[6)] reflected a significant departure from the concepts that have always been used to date to describe the causes of failure of structural concrete. In fact, most new methodologies[1)~5)] essentially involve reformulations of the very same basic concepts which underlie the design approaches that the new methodologies were originally introduced to replace. And yet, there has been published evidence suggesting that these concepts lie at the heart of both the complexity and the inconsistency of current design methods[7),8)].

Clearly, there is a need for an appraisal of the new design methodologies in order to, not only test their validity, but also, identify those of the underlying concepts which are capable of providing a realistic description of the causes of the observed behaviour of structural concrete. Such an appraisal is deemed essential mainly for two reasons. First, although the new methodologies have often been found to yield realistic predictions of structural concrete behaviour, there have also been cases reported for which the predicted response exhibits a deviation from that established by experiment[9)]. And second, many of the concepts which underlie the design methodologies are in conflict with fundamental properties of concrete as established from material tests[10)].

To this end, the present paper presents evidence obtained from experiments designed such as to test the validity of both the new design methodologies and the underlying concepts. Although most of this evidence has already appeared in other publications, it is presented in a unified form and logical sequence that helps not only to elucidate the ultimate limit state behaviour of structural concrete but also to identify a design methodology that is capable of simplifying and unifying design procedures. Such a methodology is concisely described in the following and its application in practical structural design is discussed.

2. STRUT AND TIE MODELS

New design methodologies, such as those based on the use of strut and tie models[1)], place emphasis on structural member rather than on section load effects and resistances. Such methodologies rely on the assumption that, through design and detailing, the designer may dictate the formation of convenient load paths within the body of a structural concrete member which, depending on whether they carry compressive or tensile forces, are considered to form the struts and the ties or various types of truss models that may be used to represent the structural member. A particular type of truss

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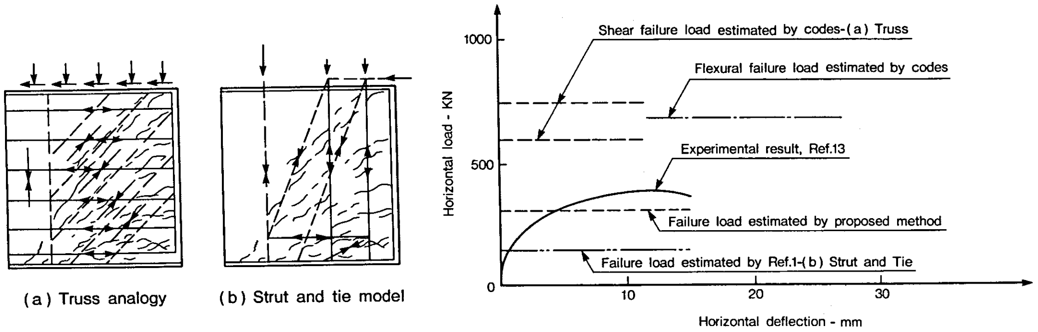


Fig.1 Predicted load carrying capacity of typical structural wall.

(in most cases in the form that was first introduced by Mörsch[11] and Ritter[12]) underlies current design methodologies[1)~5)], most of them relying on sectional load effects and resistances.

Fig.1 shows the values predicted for load-carrying capacity by two different types of truss models used to represent a structural concrete wall under the combined action of vertical and horizontal loading, together with the experimentally[13] established load-deflection relationship for the wall. It is interesting to note in the figure the wide variation of the predicted values as well as their large deviation from the experimental values; while the classical truss model, in **Fig.1** (a), overestimates considerably the load-carrying capacity (in some cases the predicted value being nearly double the experimental value), the strut and tie model shown in **Fig.1** (b) predicts a value less than half the experimental value. It should be noted that the predicted value in the latter case is a maximum value that has been obtained after optimizing the geometric characteristics of the model ; similarly, the values predicted by the classical truss model exhibit a scatter which is dependent on the particular design expressions adopted by the various Codes.

The bad correlation between predicted and experimental values can only be considered to reflect the inadequacy of the strut and tie or truss models and, in particular, to invalidate the view that any structural concrete member could be designed or detailed so as to behave in a prescribed manner (i.e. in a manner compatible with the constraints imposed on the member by adopted model). It should not be expected, therefore, that a load path could be imposed on a structural concrete member through design or detailing. Instead, it is considered that a path forms naturally and that it should be identified before attempting to visualize a physical model that could be used as the basis for the development of a suitable design methodology.

3. IDENTIFICATION OF LOAD PATHS

An indication of load paths that may form within a structural concrete member is often obtained either by inspection of the stress trajectories (established by linear elastic analysis) or by attempting to visualize a statically admissible stress field that does not violate the adopted failure criteria[1]. Such an approach, however, does not account for the significant change in the distribution of stiffness within the member due to the progressive cracking that concrete undergoes under increasing load. As, at the ultimate limit state, only a small part of the member remains uncracked, it should be expected that a knowledge of the distribution of stiffness is essential, not only for identifying the load paths, but also for assessing the magnitude of the load transferred through such paths. In fact, the assessment of the stiffness of the elements that constitute a structural configuration forms a major part of any structural analysis scheme.

(1) Load path in the absence of transverse reinforcement

In the absence of transverse reinforcement, the identification of the load path through the shear span of a RC beam, may be based on an approximate assessment of the stiffness of cracked concrete within the tension zone of this span by considering the experimental information depicted in **Fig.2**. The figure presents the results obtained from tests on beams, similar in all respects but for the arrangement of the transverse reinforcement, subjected to two-point loading. Although a large portion of the shear span of beams C and D is without transverse reinforcement, the results clearly indicate that the presence of transverse reinforcement does not represent the necessary condition, as widely believed, for the beam to attain its flexural capacity when shear capacity is attained at an earlier load stage. Moreover, the

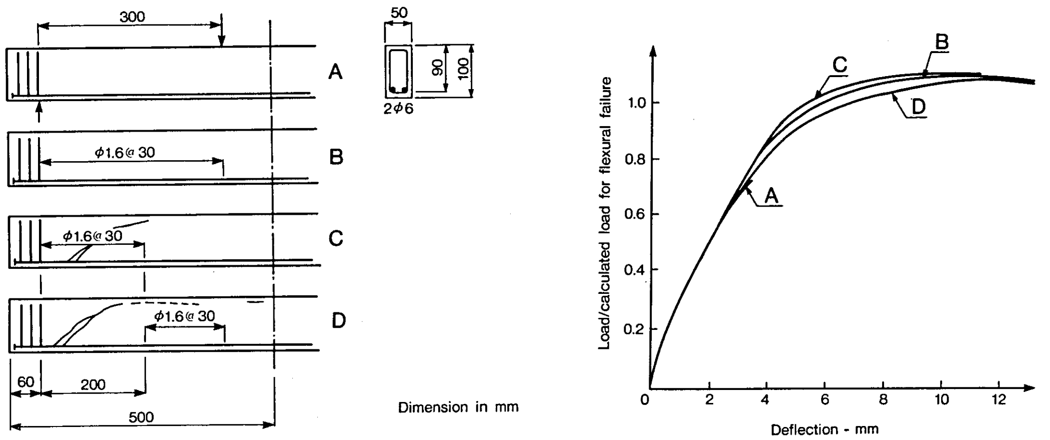


Fig.2 Load-deflection relationships obtained from tests on simply-supported reinforced concrete (RC) beams with different arrangements of transverse reinforcement subjected to two-point loading.

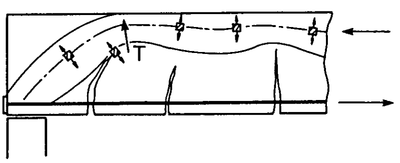


Fig.3 Schematic representation of region of the internal compressive force path.

large width of the inclined crack within the shear span of beams C and D (which failed in flexure) precludes aggregate interlock, and, therefore, the shear modulus of any concrete element intersecting the inclined crack (see **Fig.3**) is zero and, hence, there can be no shear load transfer through such elements. As a result, the external load can only be transferred to the supports through the combined action of the compressive zone and the tension reinforcement and, thus, the path of the internal compressive stress resultant should remain near-horizontal in order to by-pass the inclined crack before changing direction towards the support. A schematic representation of the load path is given in **Fig.3**.

(2) Load path in the presence of transverse reinforcement

An indication of the stiffness of the tensile zone of RC beams with transverse reinforcement may be obtained by considering the experimental information presented in **Fig.4** which has been obtained from tests on RC panels subjected to either pure shear or combined shear and normal forces[14]. The figure indicates that both the stiffness (secant modulus) and the load-carrying capacity of cracked concrete in compression in the direction of cracking decrease as the tensile strain in the orthogonal

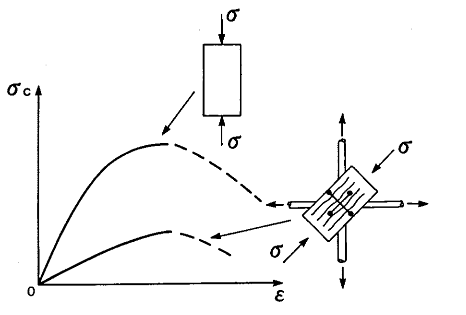


Fig.4 Axial stress-axial strain relationships for uncracked and reinforced cracked-concrete elements in compression.

direction increases. In fact, at the ultimate limit state the results indicate that the stiffness of cracked concrete is much smaller than that of uncracked concrete. As the amount of transverse reinforcement is widely considered to be the main contributor to the shear capacity of a structural concrete member, it should be expected that the shear capacity of the structural concrete walls of **Fig.5**[15], which are identical except for the amount of horizontal reinforcement, should reflect the difference in the amount of such reinforcement. It should be noted that the horizontal reinforcement of wall B is approximately 30% that of wall A which was designed in compliance with the shear design provisions of current Codes. And yet, the figure indicates that the walls exhibited a similar load-carrying capacity in spite of the fact that wall B should have failed in shear at an earlier load stage due to the insufficient amount of transverse reinforcement. Similar results have also been obtained for slender beams and these are discussed

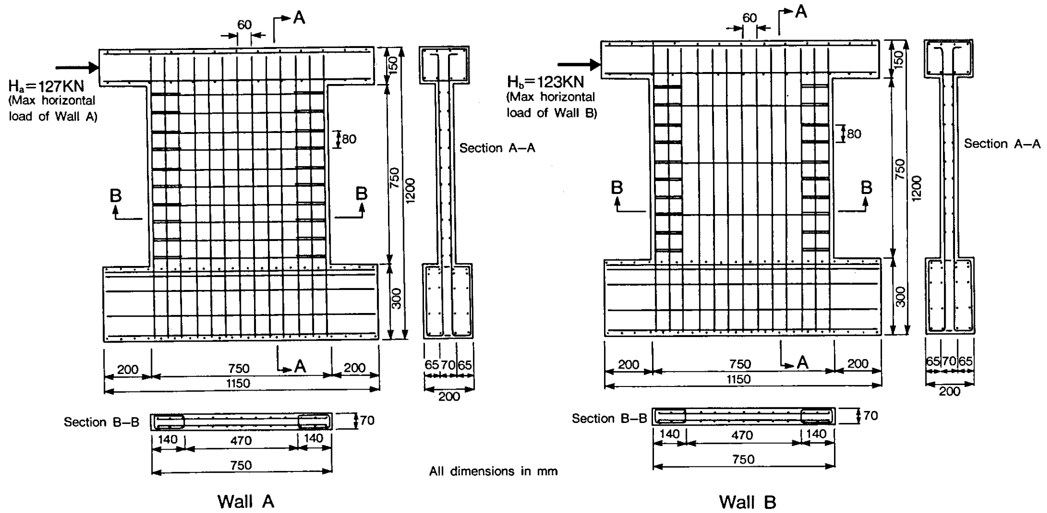


Fig.5 Effect of transverse reinforcement on load-carrying capacity of structural concrete walls subjected to horizontal loading.

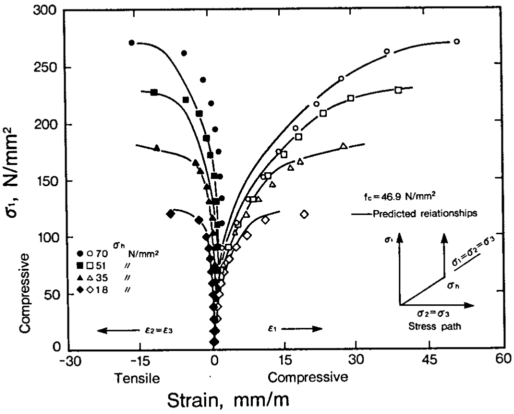


Fig.6 Typical stress-strain relationships for concrete under triaxial stress conditions.

in a subsequent section. This apparent insensitivity of shear capacity to transverse reinforcement is considered to reflect the effect of the difference in stiffness between cracked and uncracked concrete indicated in **Fig.4**. With increasing load, such difference in stiffness should lead to a continuous redistribution of the internal actions from the "soft" cracked concrete in the tensile zone to the "hard" uncracked concrete in the compressive zone and to the tension reinforcement. It would appear, therefore, that, as for the case of the reinforced concrete beams without transverse reinforcement within a significant portion of the shear span (see **Fig.2**), the main contributor to shear capacity is the compressive, and not as widely believed, the tensile zone of the member.

The above conclusion is in compliance with recent experimental information[16] which indicates that, at the ultimate limit state, the stress conditions in the compressive zone are triaxial and not uniaxial, as widely considered. Under such conditions the compressive zone is capable of sustaining the shear force that is widely considered to be sustained by the tensile zone. In fact, it is considered that ignoring the triaxial stress conditions results in an unrealistically small shear capacity of the compressive zone and this has led to the widely accepted view that the main contributor to the shear resistance of a critical section is the tensile zone.

The development of triaxial stress conditions in the compressive zone leads to an increase of the stiffness of the zone as indicated in **Fig.6**[17]. It is further considered that such an increase in the stiffness of the compressive zone when combined with the reduction of the stiffness of the tensile zone leads to a redistribution of the internal forces towards the compressive zone and the tension reinforcement such that, at the ultimate limit state, the cracked concrete in the tension zone will only be subjected to a secondary stress field caused by the restraining effect of the transverse reinforcement to the opening of the inclined cracking. As a result, the external load is transferred to the supports through the combined action of the compressive zone and the tension reinforcement with the cracked concrete in the tension zone essentially remaining ineffective.

4. TIED FRAME MODEL

The main conclusions drawn from the above

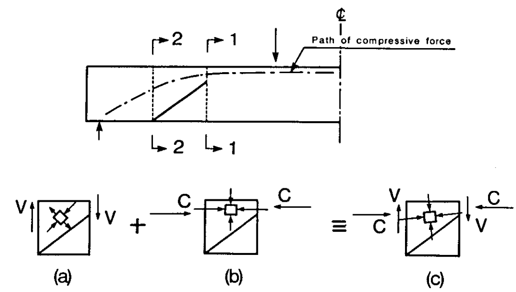


Fig.7 Triaxial stress conditions due to the longitudinal compressive force and the shear force within the compressive zone in the region of the inclined crack between sections 1-1 and 2-2.

experimental information are twofold :

(a) At the ultimate limit state, the region through which the resultant of the internal compressive stresses is transmitted to the supports remains essentially horizontal for as long as it is required for it to by-pass the deep inclined crack forming closest to the support (see **Fig.3**), and

(b) for a simply-supported RC beam, it is this region that provides strength in both flexure and shear, with the tension zone (excluding the tension reinforcement which is essential for flexural capacity) making an insignificant, if any, contribution.

These conclusions have led to the formulation of the concept of the compressive force path which stipulates that the strength of the above region is due to the development of triaxial compressive stress conditions as critical locations within the region, with failure occurring due to the presence of tensile stresses, the most critical of those usually developing at the location where the path changes direction (see **Fig.3**)[18]. It is interesting to note that the existence of triaxial compressive stresses in this region (see **Fig.7** (b)) eliminates the large tensile stresses due to the shear force (see **Fig.7** (a)) and the resulting state of stress essentially remains triaxial compressive (see **Fig.7** (c)) ; hence, the strength of concrete at the location where the path changes direction is significantly larger than that corresponding to uniaxial stress conditions.

The concept of the compressive force path underlies the physical model of a simply-supported RC beam without transverse reinforcement shown in **Fig.8**[19]. The model represents the beam as a frame with inclined legs tied by the tension reinforcement, with the frame providing a simplified representation of the region of the path of the internal compressive stress resultant. The location of the joint between the horizontal and the inclined members of the frame is dependent on the beam

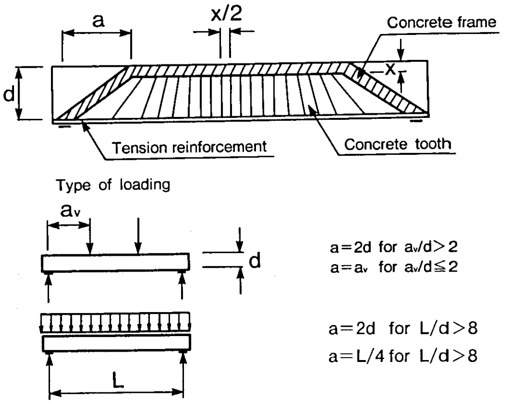


Fig.8 Proposed model for a simply-supported RC beam.

slenderness and the type of loading as indicated also in **Fig.8**. The locations indicated in the figure have been obtained as described in Ref. 18.

The implementation of the above model in design is based on the use of a failure criterion capable of yielding close predictions of the load-carrying capacity of a RC beam. Such a criterion may take the form of the well-established relationship between the strength of a RC beam (expressed as the moment corresponding to the failure load) and the shear span ($a_v = M/V$)-to- depth (d) ratio, as depicted by the curve of **Fig.9**[20]. The figure indicates that the curve is essentially divided into four portions, each one corresponding to a particular type of beam behaviour characterized by a particular mode of failure. While the horizontal portions of the curve describe the ultimate moment of resistance of a section in pure bending, the inclined portions essentially reflect the effect of the shear force on the maximum moment that can be sustained by the section. It should be noted that, although the curve in **Fig.9** implies that for $a_v/d \leqq 1$ the beam exhibits a flexural mode of failure, this, in fact, may not always be true. Failure may occur within the shear span and be brittle in nature under a failure load corresponding to the maximum bending moment which is not usually significantly different from flexural capacity (M_f). In order to prevent such failure, the legs of the frame may be designed as indicated in **Fig.10**[21]. An empirical expression that has always been found to date to yield a close fit to experimental data such as those in **Fig.9** is given in **Appendix 1**.

Collapse of the model may occur due to failure of either the horizontal or inclined members of the frame or their joint (excluding the possibility of snapping of the tie). Failure of the inclined members and failure of the joint are brittle in

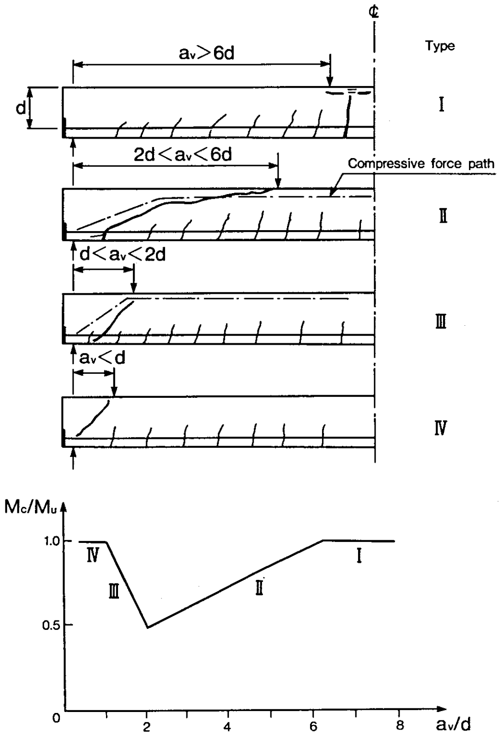


Fig.9 Typical types of behaviour exhibited by RC beams without shear reinforcement.

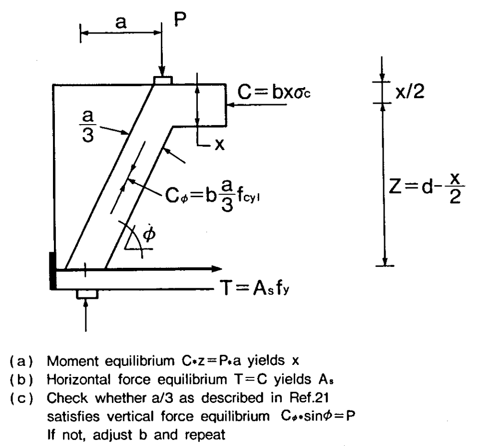
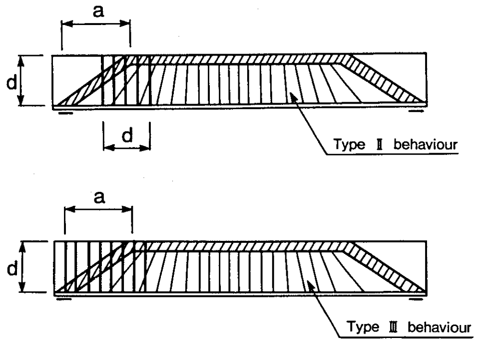


Fig.10 Design procedure for type IV behaviour.

nature, whereas failure of the horizontal member is ductile providing the beam is under-reinforced. For the case of slender beams, brittle failure may be prevented by providing transverse reinforcement as indicated in **Fig.11** (for type II and III behaviour as defined in **Fig.9**), whereas, for the case of deep beams, it may be prevented as discussed in the



Note: Only "calculated" transverse reinforcement is indicated. Throughout remaining of beam, nominal transverse reinforcement is provided in compliance with current design practice.

Fig.11 Schematic representation of transverse reinforcement for RC beams exhibiting types II and III behaviour.

preceding paragraph (see **Fig.10**). The approach followed for the assessment of the transverse reinforcement is briefly discussed in **Appendix 2.**

An indication of the efficiency of the proposed method is given in **Fig.12** which shows the cross-sectional characteristics of a simply-supported RC T-beam under uniformly distributed loading as established by designing the beam in compliance with the provisions of current Codes of Practice as well as the proposed method. JSCE Design Code[2)] and BS Code[4)] give the same results. Full design calculations in accordance with the proposed method are given in **Appendix 3**. The figure clearly demonstrates that the proposed method yields the most economical design solution, since only nominal web reinforcement is required throughout the length of the beam. On the other hand, the current codes specify web reinforcement in larger quantities (see also **Fig.12**), and yet, the resulting design solutions may not always be safer than those obtained by the proposed method since transverse reinforcement may be specified in regions not including the critical section as defined in **Fig.11** (section through the joint of the horizontal and inclined members of the frame).

5. WIDER IMPLICATIONS

The applicability of the proposed design method can be extended not only to structural concrete members other than simply-supported beams, but also to any form of skeletal structural concrete configuration. Examples of the application of the tie-frame model to structural concrete members other than simply-supported beams are illustrated in **Fig.13**, while **Fig.14** shows typical portions of

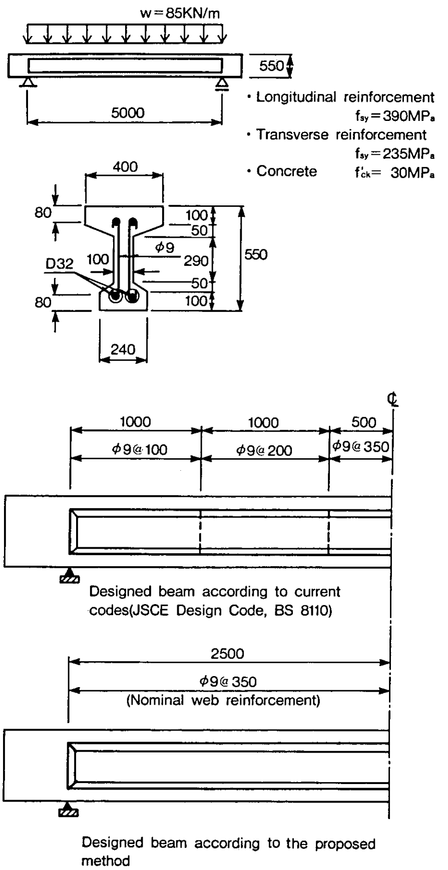


Fig.12 Shear design solutions obtained by using current design practice and proposed methods.

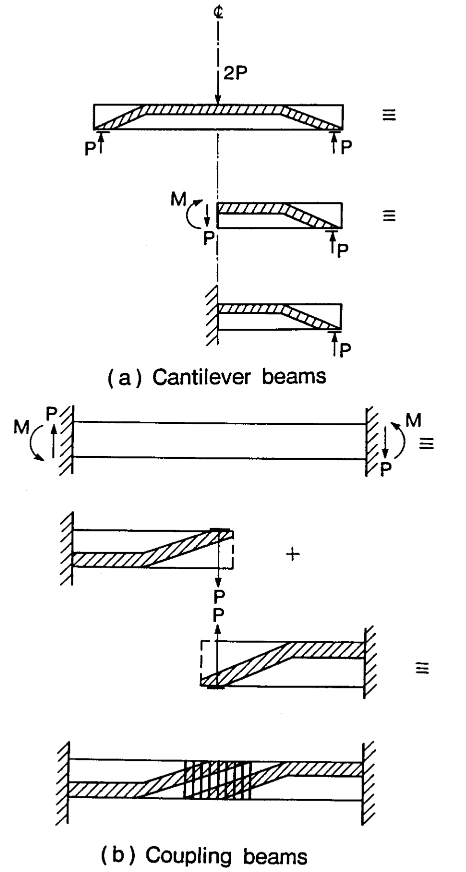


Fig.13 Application of proposed model to structural members other than simply-supported beams.

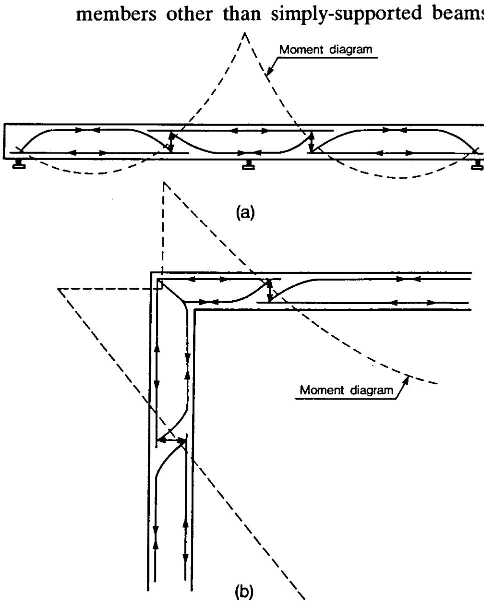


Fig.14 Application of proposed model to assemblages of structural concrete members.

more complex structural concrete configurations modelled in compliance with this model. **Fig.13** shows that a cantilever beam subjected to a point load at its free end can be designed as a simply-supported beam subjected to point-loading at mid-span, since the fixed-end conditions of the cantilever beam are similar to the conditions of the mid-span cross-section of the simply-supported beam. Similarly, a structural concrete wall which, under horizontal loading, is essentially a cantilever beam can also be designed by using the proposed method ; in fact, the application of the proposed method for the design of structural concrete walls has been found to yield safe and efficient design solution[22].

Fig.13 also indicates that a beam with fixed-ends, such as a beam coupling two structural concrete walls, can also be designed in compliance with the proposed method, since it can be divided into two portions between the fixed-end and the section through the point of inflection, each of them being essentially a cantilever beam. In this case, however, the design should be complemented so as to allow for the interaction of the two portions of the coupling beam. The middle region of the beam may be modelled as an internal support where the reaction is equal to the shear force that develops in the section through the point of inflection. The provision of web reinforcement in the form of stirrups in an amount sufficient to sustain the action of the shear force should yield a satisfactory design solution; such reinforcement should extend over a length equal to $2d$ symmetrical about the point of inflection. This method of design can, in fact, be extended to any skeletal structural concrete configuration with the proposed frame model being used to represent elements between consecutive points of inflection and the regions including such points being modelled as internal supports. Typical cases where this design approach may be followed are those of a continuous beam and a frame indicated in **Fig.14**.

6. CONCLUSIONS

The main conclusions drawn from the work are the following :

1. Widely used design models for structural concrete such as the "strut-and-tie" or "truss" models are found to be often incapable of yielding realistic predictions of structural concrete behaviour.

2. It is shown that the cause for such predictions is that the underlying assumption that "a structural concrete member can be forced through design and detailing to transfer the applied load to the supports in a prescribed manner" is not valid.

3. In contrast with the above assumption, the development of the compressive force path has been based on the identification of the "natural" paths along which internal compressive stress resultants are transmitted to the supports.

4. It is shown that, in compliance with the concept of the compressive force path, modelling a simply-supported reinforced concrete beam as a frame with inclined legs tied by the tension reinforcement leads to realistic predictions of load-carrying capacity in all cases investigated to date.

5. By considering the elements of a skeletal structure between points of inflection as simply-supported beams, it is proposed that the "tied frame" model be used as the basis for modelling any such structural concrete configuration with the region of the points of inflection being modelled as an "internal hinged support" effected by the provision of transverse reinforcement.

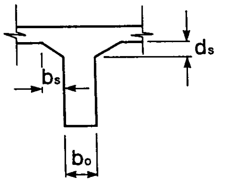


Fig.A1 Definition of section parameters b and d used in eqn. (A-1)

APPENDIX

(A1) Failure criterion

An analytical expression of the inclined portion of the curve of **Fig.9** for $a_v/d \geqq 2$ ($L/d \geqq 4$) has been proposed by Bobrowski & Bardham-Roy[23] and this, in a slightly modified form[24], is as follows :

$$M_c = 0.875sd\left(0.342b_1 + 0.3\frac{M_f}{d^2}\sqrt{\frac{z}{s}}\right)^4\sqrt{\frac{16.66}{\rho_w f_y}} \quad \cdots\cdots \text{(A-1)}$$

where s is the distance of the critical cross-section from the support (mm)

s : is the shear span for two-point loading, $2d$ for uniformly distributed loading

M_c : is the moment corresponding to shear failure load (Nmm)

M_f : is the flexural capacity (Nmm)

d : is the effective depth (mm)

z : is the lever arm of the horizontal internal actions (mm)

$$\rho_w = \frac{\text{area of tension steel}}{\text{web area of concrete to effective depth}}$$

f_y : is the characteristics strength of the tension steel (N/mm^2)

b_1 : is the effective width (mm) given by the lesser of b_0+2b_s, b_0+2d_s ; b_0, b_s, d_s are as shown in **Fig.A1**.

The portion of the curve for $1 < a_v/d < 2$ can be defined by linear interpolation between the values of $M_c = M_f$ for $a_v/d = 1$ and M_c as obtained from eqn. (A-1) for $a_v/d = 2$. In the remaining two regions $M_c = M_f$.

The use of the above failure criterion in design will involve (a) designing the horizontal member of the frame so as to be capable of sustaining a compressive force C such that $C = T = M/z$ (see **Fig.11**) and (b) checking whether the moment (M_a) applied at the critical cross-section (s) is larger or smaller than M_c. If $M_a \leqq M_c$, only nominal web reinforcement would be required similar to that specified by current design practice, whereas if $M_a > M_c$, web reinforcement should be designed in accordance with the requirements described in Appendix 2.

(A2) Assessment of transverse reinforcement

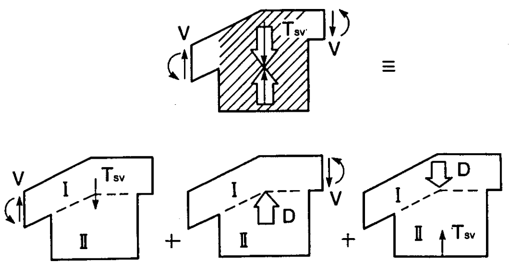


Fig.A2 Additional internal actions developing due to presence of transverse reinforcement in region where path changes direction for type II behaviour.

Such reinforcement is only required to prevent brittle failure associated with types II and III behaviour.

For type II behaviour ($a_v \geqq 2d$), the most likely cause of brittle failure of the model is the tensile force (V_a) that develops in the region of the joint in order to balance the action of the resultant of the compressive forces acting on the joint in the direction of the frame members (see **Fig.8**). A measure of the tensile force that can be resisted by concrete alone in this region is given by

$$V_c = \frac{M_c}{s} \cdots\cdots (A\text{-}2)$$

where M_c and s are as defined by eqn. (A-1).

In order to prevent brittle failure of the joint, stirrups should be provided in order to sustain the portion ($V_a - V_c$) of the force (V_a) in excess of that (V_c) that can be sustained by concrete alone. The presence of such reinforcement will slightly modify the "comb-like" model (shown in **Fig.8**) as indicated in **Fig.A2**. The figure provides a schematic representation of the additional internal actions developing in the region of the joint and indicates that the stirrups, not only sustain the action of the vertical component ($V = V_a - V_c$) of the compressive force resultant, but also subject the shaded concrete block, where it is anchored, to a compressive force (D). This force balances the shear force (V) acting at the right-hand side of the above block. It should be noted that the stirrups will be activated only when the capacity of concrete to sustain alone the action of the internal tensile force is attained. When this occurs, the excess tensile force will be sustained by the stirrups with a cross-sectional area equal to

$$A_{sv} = (V_a - V_c)/f_{yv} \cdots\cdots (A\text{-}3)$$

placed over the length, d, of the shaded portion of the block in **Fig.A2**. It has been found that the inclined and horizontal members of the concrete frame are capable of sustaining the internal actions which develop for equilibrium purposes as indicated in **Fig.A2** and, therefore, it will be sufficient to provide nominal reinforcement outside the shaded block so as to comply with current design practice[2)~5)].

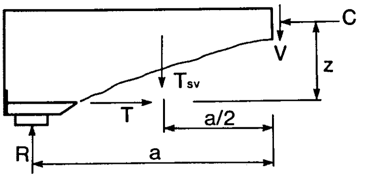


Fig.A3 Contribution of transverse reinforcement to load-carrying capacity for type III behaviour (Note, $M_a = Ra$, $M_{sv} = T_{sv}a/2$; $M_c = M_a - M_{sv}$).

It should be noted that, if the beam is subjected to point loads, stirrups may also be required within the horizontal member of the frame of the model in the region of the point load. The need for such reinforcement arises due to the development of tensile stresses within this region which occurs when the tension reinforcement is debonded from the concrete in the vicinity of the section including the load-point[18)]. A method for the assessment of the additional reinforcement required is described elsewhere[23)].

For type III behaviour ($d < a < 2d$), brittle failure is associated with failure of the horizontal member of the frame in the region of the joint[23)]. Apparently, the penetration of the inclined crack deeply into the compressive zone (see **Fig.9**) reduces the zone cross-sectional area and thus leads to a reduction of both the load-carrying capacity of the zone as well as the moment that can be sustained by the model section. Failure can be prevented by increasing the section moment of resistance to the level of the applied bending moment through the provision of web reinforcement in the form of stirrups uniformly distributed throughout the portion of the model between the support and the joint of the frame members. Such reinforcement should be designed such that, at yield, it would be capable of sustaining a total tensile force $T_{sv} = A_{sv}f_{yv}$, applied half way between the support and the joint, the contribution of which to the moment capacity of the critical section (s) is $M_{sv} = T_{sv}a/2 = M_a - M_c$ (see **Fig.A3**). Hence, the total amount of stirrups required to provide the critical section with a moment of resistance larger than, or equal to, the applied moment will be

$$A_{sv} \geqq T_{sv}/f_{yv} = \frac{2(M_a - M_c)}{a \cdot f_{yv}} \cdots\cdots (A\text{-}4)$$

(A3) Design calculations for slender beam in Fig.12 in accordance with proposed

model

a) Longitudinal (tension) reinforcement

Assessment is based on information provided in **Fig.12**. All safety factors are taken as 1.0. The applied bending moment is

$M = wl^2/8 = 85 \times 5.0^2/8 = 266$ KN · m

Use 2-D32. Then, $A_s = 1588\text{mm}^2$ and $T_u = 1588 \times 0.39 = 619$ KN. From the condition $C = T$, it is found that $z = 440$mm and, thus, flexural capacity $M_u = T_u \times z = 619 \times 0.44 = 272\text{KN}\cdot\text{m} > M = 266$ KN ·m O.K.

b) Transverse reinforcement

From **Fig.12**, $L/d = 5000/470 = 10.6 > 8$. Hence, beam exhibits type II behaviour (slender beam) as defined in **Fig.9**. For such a beam, the joint between the inclined and the horizontal member of the model lies at a distance $a = 2d = 2 \times 470 = 940$ mm from the support (see **Fig.8**).

The maximum transverse tensile force V_c that can be sustained by concrete alone in the region of the joint is given by eqn. (A-2), i.e.

$V_c = M_c/s$

where M_c is given by eqn. (A-1).

For $d = 470$ mm, $s = 2d = 940$ mm, $b_1 = 200$ mm, $M_u = 272$ KN·m, $z = 440$ mm, $\rho_w = 0.034$, $f_y = 390$ N/mm^2, eqn. (A-1) gives $M_c = 131$ KN·m. Hence,

$V_c = 131/0.94 = 139$ KN

The applied transverse force in the region of the joint is essentially the shear force V_a at a distance $s = 940$ mm from the support, i.e.,

$V_a = 213 - 85 \times 0.94 = 133$ KN $< V_c = 139$ KN

As a result, the nominal links following current design practice are provided over a whole length of the beam. Hence,

$A_{sv} = 0.0015 \times b_0 \times a = 0.0015 \times 100 \times 350 = 53$ mm^2

a : Spacing of link

Provide nominal links ϕ 9@350 ($A_{sv} = 128$ mm^2) following current design practice as indicated in **Fig.12**. (Note that, for comparison purposes, **Fig.12** also includes the link arrangement of the beam as established by following current design practice.)

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コンクリート構造の終局限界状態設計の統一化に関する検討

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小論は，構造用コンクリートの終局限界状態の照査を単純かつ統一的に取扱える手法の開発を目指すものである．その手法は，圧縮力経路に基づくものであり，単純梁の場合，従来の設計法に比べ，より経済的かつ安全性の高い結果をもたらす．
本設計手法は，単純梁に限らず，種々の骨組み構造物にも適用可能であり，この場合，各構成部材は隣接したモーメントの反曲点間の要素としてモデル化し，反曲点では，せん断補強鉄筋を用いることにより「内部ヒンジ支承」を形成する．