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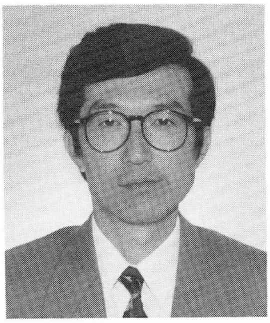
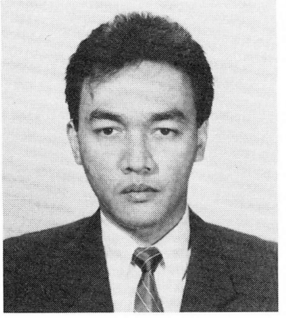
AN ANALYTICAL STUDY OF SHEAR STRENGTH OF PRESTRESSED CONCRETE BEAMS WITHOUT SHEAR REINFORCEMENT

(Translation from Journal of Prestressed Concrete, Japan, Vol.33, No.2, March 1991)





Tamon UEDA   Heru Darjudi Eko PUTRO

SYNOPSIS

Strengths of shear compression failure in prestressed concrete beams are predicted by the finite elements method analysis, in which a main shear crack is modelled as a discrete crack. The predicted effect of prestressing force on the shear strength as well as the shear strength itself agrees well with experimental results. The tip of the main shear crack is located in compression zone. Compression failure of concrete in the maximum moment region causes shear compression failure of the beams. A narrower compression zone is considered to make the shear strength less than the flexural strength. Force transferred along the main shear crack does not affect the shear strength of the beam very much. The shear strength can be predicted by the conventional calculation method for flexural strength using the neutral axis depth found in the finite element analysis.

Keywords: *prestressed concrete, beam without shear reinforcement, shear strength, finite element analysis*

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1. INTRODUCTION

There have been a number of studies on shear strength of concrete beams without shear reinforcement, which clarify many things[1][2]. Because of their experimental approach, however, shear strength predictions are not accurate enough for the cases which the experiments did not cover. It can be said, for example, that in the case of beams subjected to axial force, such as prestressed concrete beams, no satisfactory prediction method has yet been established due to lack of experimental data. Less accuracy is found especially in the cases of axial tension and large axial compression. In this study, therefore, shear strength of prestressed concrete beams without shear reinforcement is analyzed by a nonlinear finite element method (FEM), in which constitutive laws greatly developed in recent years are implemented, and a more rational and general prediction method for the shear strength is sought instead of experimental ones.

2. OUTLINE OF FINITE ELEMENT ANALYSIS

The program used for the finite element analysis is "COMM2"[3] in which Maekawa model (Elasto-plastic fracture model)[4] is applied for the concrete element. The bond link elements which are available in COMM2 are used for force transfer along a shear crack and for bond force between concrete and steel. The model for the bond force is Shima et al's model[5]. The models used for the forces transferred along a shear crack are Li & Maekawa's model (a model for aggregate interlocking)[6] applied for the case that slipping occurs (δ>0.001mm where δ is slip), and Reinhardt et al's model (a model for tension softening)[7] for the case with no slipping (δ<0.001mm and δ/w<0.01 where w is crack width). The details of the analyzed beams, which are specimens in a previous experimental study[8], are shown in Fig.1 and Table 1. In the experiment[8] all the specimens failed in shear and typical shear crack pattern was observed as shown in Fig.2(a). The main shear crack is modeled as a discrete crack in the finite element analysis. The location and configuration of the discrete crack are determined according to the observed shear crack in the experiment as seen in Fig.2(b). An example of meshing pattern for the finite element analysis is given in Fig.3. Only half of each specimen is analyzed. The bond link element installed at the discrete crack is rigid and does not deform before cracking but deforms according to the tension softening model or the aggregate interlocking model after cracking. All

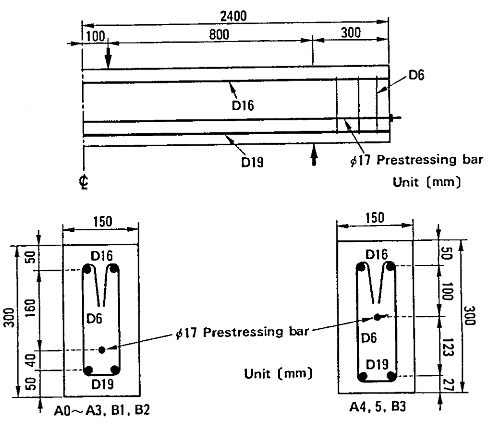
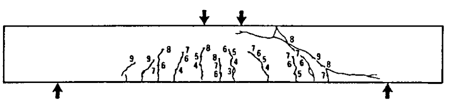
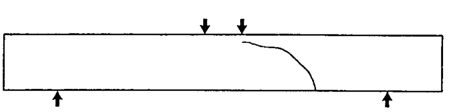


Fig.1 Specimen



(a) Actual cracking pattern of specimen A-5



(b) Idealized cracking pattern of specimen A-5

Fig.2 Cracking Pattern

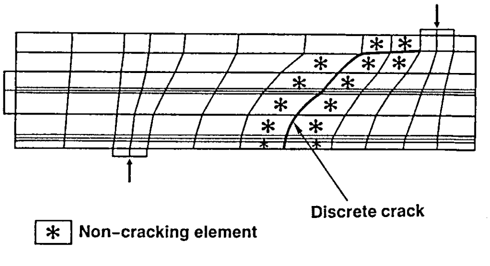


Fig.3 Element Mesh (Specimen A-5)

the concrete elements adjacent to the discrete crack are set not to crack, so that cracking is assured to occur at the discrete crack. In these elements tensile stress equal to the tensile strength of concrete is transferred after the stress reaches the tensile strength. In the other concrete elements crack is modeled as smeared crack which has no tension stiffness in the direction normal to crack and has shear stiffness decreasing with increasing shear strain[3]. The prestressing force is applied by imposing a force at a node of the steel element for the anchor plate in the actual beam[5]. The magnitude of the imposed force is equal to the observed effective prestressing force[5].

Table 1 Specimen and Shear Strength

Specimen	f_c' MPa	P_{eff} kN	x_1 mm	x_2 mm	x_3 mm	δ_{FEM} (mm)	$V_{su,exp}$ kN	$V_{su,FEM}$ kN	$V_{su,cal}$ kN	$\frac{(8)}{(9)}$	$\frac{(8)}{(10)}$	$V_{fu,cal}$ kN	$V_{su,cal2}$ kN	$V_{su,cal3}$ kN
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)			(11)	(12)	(13)
A–0	26.6	0	28	55	119	0.39	44.1	40.5	40.4	1.08	1.09	75.8	42.7	28.1
A–1	24.6	48	43	85	136	0.40	51.5	53.9	54.7	0.96	0.94	76.5	59.1	42.5
A–2	26.6	112	57	90	145	0.25	59.3	54.1	61.9	1.10	0.96	84.8	62.8	52.3
A–3	24.6	152	66	115	162	0.27	72.6	62.6	69.6	1.14	1.04	83.7	76.8	65.1
A–4	36.5	99	62	65	102	0.45	63.3	63.2	64.4	1.00	0.98	83.3	75.6	57.8
A–5	36.5	0	34	55	86	0.49	46.6	63.6	55.5	0.73	0.84	77.7	64.3	38.5
B–1	57.5	148	38	55	84	0.43	89.2	101.4	87.5	0.88	1.02	118.6	110.1	77.9
B–2	57.5	41	–	40	83	0.34	67.7	70.9	65.4	0.96	1.03	117.7	73.9	42.2
B–3	57.5	152	37	60	83	0.31	78.5	79.3	94.6	0.99	0.83	102.5	96.3	70.1

Note (2) Cylinder strength of concrete at test
(3) Effective prestressing force
(4) Observed depth to shear crack at loading point
(5) Depth to neutral axis by FEM in maximum moment region at shear failure
(6) Depth to neutral axis, calculated by conventional bending theory, in maximum moment region at flexure failure
(7) Deflection of beam at loading point obtained by FEM
(8) Measured ultimate shear strength
(9) Ultimate shear strength obtained by FEM
(10) Calculated ultimate shear strength using x_2
(11) Calculated ultimate flexural strength by conventional bending theory
(12) Shear strength calculated by the method in a previous study[9]
(13) Shear strength calculated by the method in a previous study[10]

3. ANALYTICAL RESULTS

3.1 Prediction of Shear Strength

Comparison of the strengths, $V_{su,FEM}$ predicted by the finite element analysis with the experimental strengths, $V_{su,exp}$, is shown in Table 1 and Fig.4. It can be said that the shear strengths of all the specimens among which prestressing force is varied are predicted with reasonable accuracy by the FEM except specimen A–5 (see columns (8) and (9) of Table 1). It is found in the finite element analysis that the ultimate shear strengths are controlled by compression failure (strain softening) of concrete element in compression zone of the maximum moment region. This failure mode agrees with that observed in the experiment. Although this fact of the concrete compression failure is the same as that in flexural failure of the beams, the measured ultimate strengths are much less than the calculated flexural strengths as shown in columns (8) and (11) of Table 1. The reason for this fact can be found in concrete strain distribution at a cross–section in the maximum moment region. As shown in the columns (5) and (6) of Table 1 and Fig.5, depths to the neutral axis, x_2, obtained from the concrete strain distribution calculated by the FEM, are much less than depths, x_3, calculated by the conventional bending theory for ultimate strength using an equivalent stress block. Although it is considered generally that shear cracks propagate in tension zone below the neutral axis depth, it is seen in columns (4) and (5) of Table 1 and Fig.4 that the depth to the

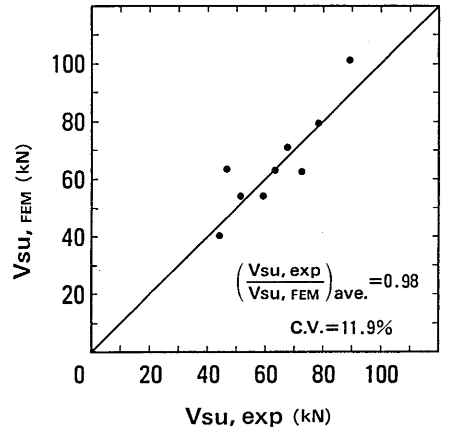


Fig.4 Comparison of Shear Strengths Predicted by FEM with Experimental Values

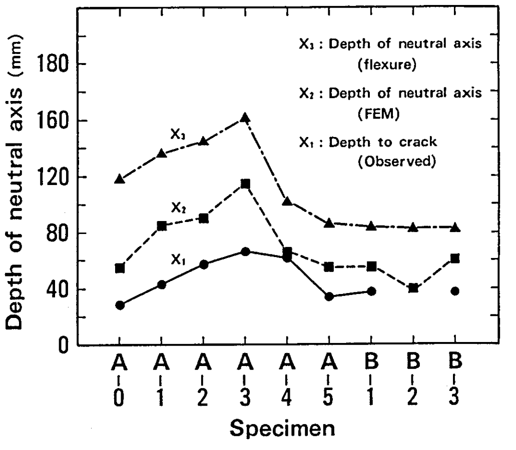


Fig.5 Depth of Compression Zone

neutral axis at shear compression failure is significantly greater than the depth to shear crack tip, x_1. This means that the shear cracks penetrates in the compression zone.

The finite element analysis indicates that the depth of compression zone becomes minimum in the maximum moment region and is much less than that just outside the maximum moment region. Therefore, despite combined flexural moment and shear force working outside the maximum moment region, the principal compressive stress is much less than that in the maximum moment region. This is considered to be the reason why the failure of concrete does not occur outside the maximum moment region. The concrete compression failure in compression zone of the maximum moment region is considered a failure criterion for the case of shear compression failure of a beam. This agrees with the fact that compression failure occurred in and not outside the maximum moment region in the experiment[8].

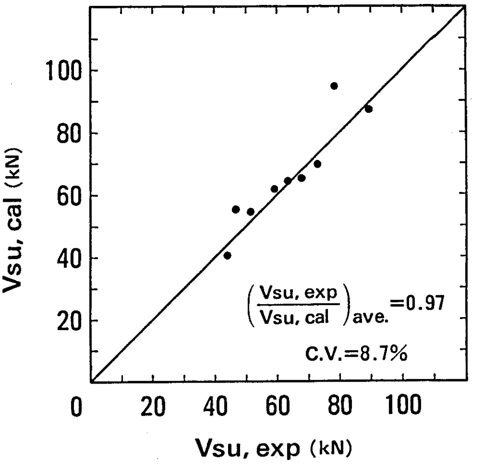


Fig.6 Comparison between Strengths Predicted with Assumed Neutral Axis Depth and Experimental Values

The shear strengths, $V_{su,cal}$, in column (10) of Table 1 and Fig.6 are calculated by the conventional method for ultimate flexural strength with the equivalent stress block, assuming that the neutral axis depth is the one in the maximum moment region, x_2, given by the finite element analysis (see column (5) of Table 1). Similar ways for prediction of the shear strength in which the shear strength is calculated as flexural strength using the neutral axis depth predicted by experimental formulae were proposed in previous studies[9][10]. Despite the simplified assumptions, the calculated shear strengths using the neutral axis depth given by the FEM in this study give better agreement with the experimental ones, $V_{su,exp}$, than those calculated by the formulae in the previous studies as shown in columns (8), (10), (12) and (13) of Table 1. It can be said, therefore, that the ultimate strength for shear compression failure could be estimated easily and accurately if the neutral axis depth could be predicted accurately by some means.

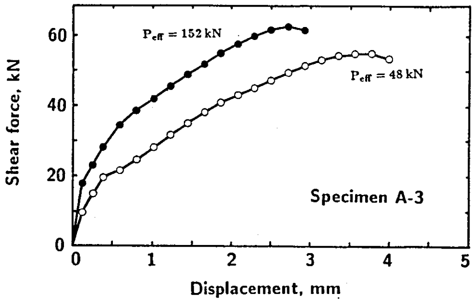
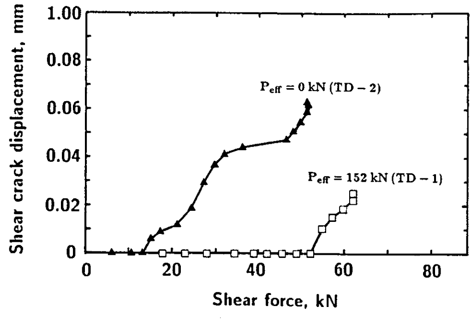
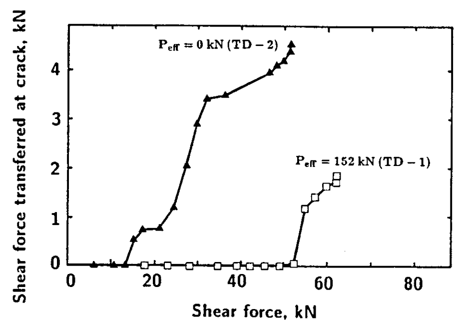


Fig.7 Effect of Prestressing Force on Shear Strength



(a) Displacement at shear crack



(b) Force transferred at shear crack

Fig.8 Effect of Prestressing Force on Displacement and Transferred Force at Shear Crack

3.2 Effect of Prestressing Force on Shear Strength

Figure 7 indicates that the finite element analysis predicts adequately increase of the shear strength with increase of prestressing force as observed in the experiment[8]. Results of the finite element analysis in which the prestressing force, P_{eff}, for specimen A–3 is reduced to 48kN from 155kN are given in Fig.7. It is clearly shown that the increase of prestressing force increases the ultimate strength and decreases the displacement. As shown in Fig.8 it is also indicated by the finite element analysis that the increase of prestressing force delays occurrence of shear cracking and decreases displacements at the shear crack (crack width and slip) and the force transferred by aggregate interlocking. In the cases of TD–1 and TD–2 only the prestressing force is changed while the configuration of shear crack remains the same (see Table 2). Table 1 and Fig.5 indicate that among specimens A–0 to A–3 the calculated depth to the neutral axis, x_2, increases with increase of the prestressing force. This increase of neutral axis depth is considered to cause increase of the shear strength.

The observed depth to shear crack tip, x_1, increases with increase of the prestressing force as shown in Fig.5. In order to see the effect of depth to

Table 2 Effect of Prestressing Force and Depth to Shear Crack Tip on Shear Strength

Case (1)	f_c' (MPa) (2)	P_{eff} (kN) (3)	X_1 (mm) (4)	δ_{FEM} (mm) (5)	$V_{su,FEM}$ (kN) (6)
TD–1	24.6	152	70	0.27	62.1
TD–2	24.6	0	70	0.44	51.5
TD–3	24.6	152	20	0.29	62.5
TD–4	24.6	0	20	0.42	49.4

Note:
(2) Cylinder strength of concrete
(3) Effective prestressing force
(4) Depth to shear crack at loading point
(5) Displacement of beam at loading point obtained by FEM
(6) Ultimate shear strength obtained by FEM

shear crack tip on the shear strength, the shear strengths are calculated by the FEM for two cases of the depths of 20mm and 70mm (see the cases of TD−1 and TD−3 and of TD−2 and TD−4 in Table 2). The calculated strengths, however, are rather close. It seems that the difference in the depth to shear crack tip hardly causes the difference in the shear strength.

3.3 Effect of Force Transferred along Shear Crack on Shear Strength

The finite element analysis is conducted with different force transfer models along a discrete shear crack in order to see its effect on the shear strength of beams. The standard model is a combined model of Li and Maekawa's model and Reinhardt et al's model (see Chap.2). In the cases where the model's stiffness is 10%, 1% and 0% of the standard model the shear strengths are found to be 91%, 89% and 95% of that with the standard model (see TD−12, TD−13 and TD−14 as well as TD−11 in Table 3). The displacements at the ultimate strength are also approximately the same. The stiffness of the force transfer model at shear crack does not affect the overall behavior of beams.

From the finite element analysis it is observed that slip (δ>0.001mm) takes place at most of the part along the shear crack including the vicinity of the crack tip. In the analysis, therefore, Li & Maekawa's model which is an aggregate interlocking model is used mostly (see Chap.2). Another analysis in which Reinhardt et al's model (a tension softening model at crack) is applied under any amount of slip is conducted (see TD−13 in Table 3), so that tensile force would be transferred along the shear crack instead of compressive force which is transferred as in Li and Maekawa's model. As shown in Fig.9 the predicted shear strength in this case is 97% of that in the original case, and the displacements of beams are very close. This indicates that overall behavior of beams would not be influenced by whether compressive force or tensile force is transferred in the direction normal to a shear crack.

From the above mentioned discussion it can be said that effect of the force transferred along a main shear crack on the shear strength is negligibly small for the case of prestressed concrete beams without shear reinforcement whose shear span to effective depth ratio is 3.34. The reason for this may be the fact that the transferred stresses along the shear crack are considerably smaller than the concrete compressive stresses in the vicinity of the shear crack. Main stress flow in concrete does not go through the shear crack.

Table 3 Effect of Type of Force Transfer Model at Shear Crack and Shear Crack Model

Case (1)	f_c' (MPa) (2)	P_{eff} (kN) (3)	Force transfer model (4)	Crack model (5)	δ_{FEM} (mm) (6)	$V_{su\,FEM}$ (kN) (7)
TD−11	36.5	0	Li−Maekawa & Reinhardt	discrete	0.45	63.6 (100%)
TD−12	36.5	0	10% stiffness of TD−11	discrete	0.45	58.2 (91%)
TD−13	36.5	0	1% stiffness of TD−11	discrete	0.42	56.9 (89%)
TD−14	36.5	0	0% stiffness of TD−11	discrete	0.47	60.1 (95%)
TD−15	36.5	0	Reinhardt	discrete	0.47	61.7 (97%)
TD−16	36.5	0	none	smeared	0.40	49.8 (78%)

Note:
(2) Cylinder strength of concrete
(3) Effective prestressing force
(4) Model for force transfer at discrete shear crack
(5) Model for shear crack
(6) Displacement of beam at loading point obtained by FEM
(7) Ultimate shear strength obtained by FEM

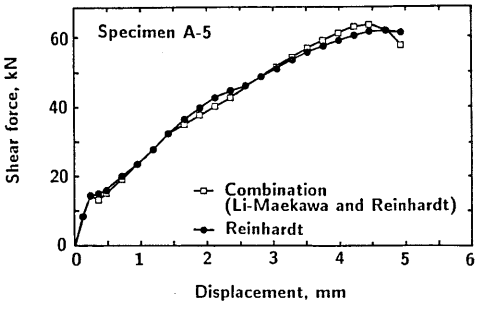
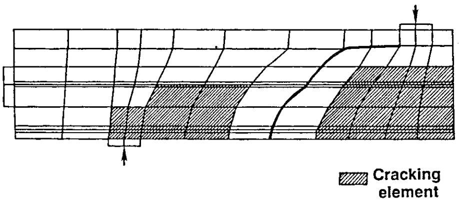
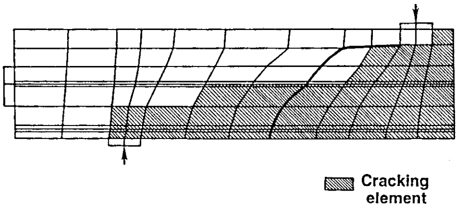


Fig.9 Effect of Type of Force Transfer Model at Shear Crack on Shear Strength



(a) Discrete crack (case TD−11)



(b) Smeared crack (case TD−16)

Fig.10 Difference in Cracking Zone Due to Difference in Type of Shear Crack Model

3.4 Effect of Crack Model Type on Shear Strength

For finite element analysis there are two types of crack model, smeared crack and discrete crack. In this study a main shear crack is modeled as a discrete crack while concrete elements next to the main shear crack are non−cracking element (where tensile stress is limited to the concrete tensile strength) and smeared crack is adopted in the other concrete elements. Another analysis in which smeared crack is applied for all the elements and no discrete crack is implemented is conducted to compare the results (see TD−11 and TD−16). The predicted strength in this case is 78% of the standard case (TD−11), and the predicted displacement at the strength is 89%. Some reduction in the strength can be seen. It seems that the reduced size of compression zone in the maximum moment region due to widening of cracking concrete zone causes early failure (see Fig.10).

3.5 Effect of Shear Cracking on Shear Strength

It is known that ultimate strength for shear tension failure is influenced greatly by the time at which shear cracking occurs. For the prestressed concrete beams examined in this study shear compression failure is observed. Whether the shear cracking strength influences the ultimate strength or not in this case is examined as follows. A beam in which a main shear crack exists before loading is analyzed. In this analysis the routine to check cracking in the link element inserted at the discrete crack is skipped, and the model for force transfer is applied from the beginning. The calculated strength and displacement are the same as those of the identical beam in which, however, shear crack does not exist from the beginning. It can be said that the influence of shear cracking strength on the ultimate strength is not found in the beams in this study.

3.6 Effect of Bond Model on Shear Strength

For the finite element analysis in this study bond between concrete and steel is modeled by the bond link element. The influence of the bond model is examined by changing the stiffness of the model. The calculated shear strength of the beam in which the stiffness of the bond model is reduced to 10% of the stiffness of the standard model (Shima et al's model) is found to be 101% of the strength of the beam with the standard bond model. The displacement is also very close. The influence of the bond model is also negligible.

4. CONCLUSIONS

The finite element method is applied to analyze prestressed concrete beams without shear reinforcement. In the analyses the concrete elements equipped with the elasto-plastic fracture model[4], the bond link elements and a discrete shear crack with crack link elements inserted for expressing force transfer through the crack are applied. The following conclusions are derived from the analyses.

(1) The results of the finite element analyses can predict the strengths for shear compression failure with reasonable accuracy.
(2) The finite element analyses indicate that shear compression failure is caused by compression failure of concrete in compression zone in the maximum moment region. This agrees with the observed results in the previous experiment[8].
(3) The depth of the compression zone in the maximum moment region increases with increase in the prestressing force. This increasing depth increases the shear strength. The depth of compression zone, however, is less than that in flexural failure, which makes the shear strength less than the flexural strength.
(4) Using the depth of compression zone obtained by the finite element analyses, the strength for shear compression failure can be calculated by the conventional prediction method of flexural strength.
(5) For shear compression failure of prestressed concrete beams (including reinforced concrete beams) without shear reinforcement, shear cracking strength and force transferred at a main shear crack do not influence the ultimate shear strength.

Acknowledgements

This study was conducted while the authors were with Asian Institute of Technology and supported by the Japan International Cooperation Agency. The experimental study[8] which actually initiated this study was supported by the Saito Memorial Research Foundation for Prestressed Concrete Technology. The authors would like to express their gratitude to these sponsors.

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