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STUDY OF THE CREEP STRAIN OF CONCRETE UNDER THE VARIOUS STRESS HISTORIES

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SYNOPSIS

Most of the widely-used creep analysis methods for concrete are based on the principle of superposition, and the creep coefficient used in the analysis is constant irrespective of applied stress. Hence, non-linear creep phenomena are frequently neglected in the actual design. In this paper, we clarify that the creep strain of concrete under the various stress histories is a non-linear phenomena strongly affected by the maximum previously applied stress. When we treat creep as a non-linear phenomena, we need a creep hardening rule to calculate the creep strain under varying stress. Many creep hardening rules have been already proposed; for example, Strain-hardening rule, Time-hardening rule, Work-hardening rule and so on. However all of these are for creep phenomena of steel materials rather than concrete. We propose a creep hardening rule which is applicable to concrete under any stress conditions.

Keywords : creep strain, creep analysis, stress history, non-linear, creep hardening rule

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1. INTRODUCTION

At service load, creep strain of concrete causes increases in deflection and curvature, loss of prestress, redistribution of stresses, and internal actions. In order to design concrete structures with high accuracy in size, it is very important to accurately predict the creep strain.

To accurately account for the effects of creep strain in the design of concrete structure, two basis prerequisites must be satisfied.
(1) Reliable prediction equations or the experimental data for the creep strain under constant stress.
(2) An analytical procedure to calculate the creep strain under various stress histories.
Usually, the stresses acting on concrete are not constant due to the redistribution of stress between concrete and steel, even under constant load conditions. So, if either of the two prerequisites is inadequate, it is impossible to efficiently predict the creep strain of concrete under ordinary stress conditions. As for the creep prediction equations shown in (1), many have been proposed and adopted in various practical design codes, for instance, the ACI-209 model [1], CEB/FIP model [2] and Bazant-Panula model [3]. The analytical procedure given in (2) is for calculating the creep strain under varying stress with the creep prediction equations shown in (1). In present-day designs, the 'Effective Modulus method' (EM method) [4], 'Rate of Creep method' (RC method) [5], 'Trost-Bazant method' (TB method) [6], 'Step-by-step method' [7] and so on are used.

In most widely-used design methods, the relationship between the creep strain and the stress is assumed to be linear if the stress/strength ratio is below 40% [8]. Nevertheless, we have already observed cases in which the non-linearity with respect to creep inducing stress is present. Jones and Richart [9] have clarified that the relationship between the creep strain and the stress/strength ratio is approximated by two straight lines whose slopes are different from each other. Furthermore, we have clarified that these lines intersect below stress/strength ratio of 40% [10].

The purpose of this study is to clarify the non-linearity of concrete creep strain under varying compressive stress, and to establish a rule which can express the creep behavior of concrete under various compressive stress histories on the basis of the creep hardening rule. The creep hardening rule gives nonlinear creep compliance for concrete under variable stress.

2. CONCRETE CREEP ANALYSIS METHOD UNDER VARIABLE STRESS

First of all, we introduce a method of concrete creep analysis under variable stress conditions and outline the purpose of this study. Figure 1 shows the calculation process of the creep strain of concrete under varying stress with the creep strain of concrete subjected to constant stress.

Conventionally, the creep behavior of concrete under constant stress is represented by either the creep coefficient or the specific creep. The creep strain of concrete under varying stress is calculated by the method of the superposition. Both of these are based on the assumption of a linear relationship between creep strain and externally-induced stress. The difference between the EM method, RC method and TB method shown in Fig. 1 is the assumption of the age in which the application of load effects the creep strain under the constant stress. Therefore, the creep strain under varying stress can be represented by

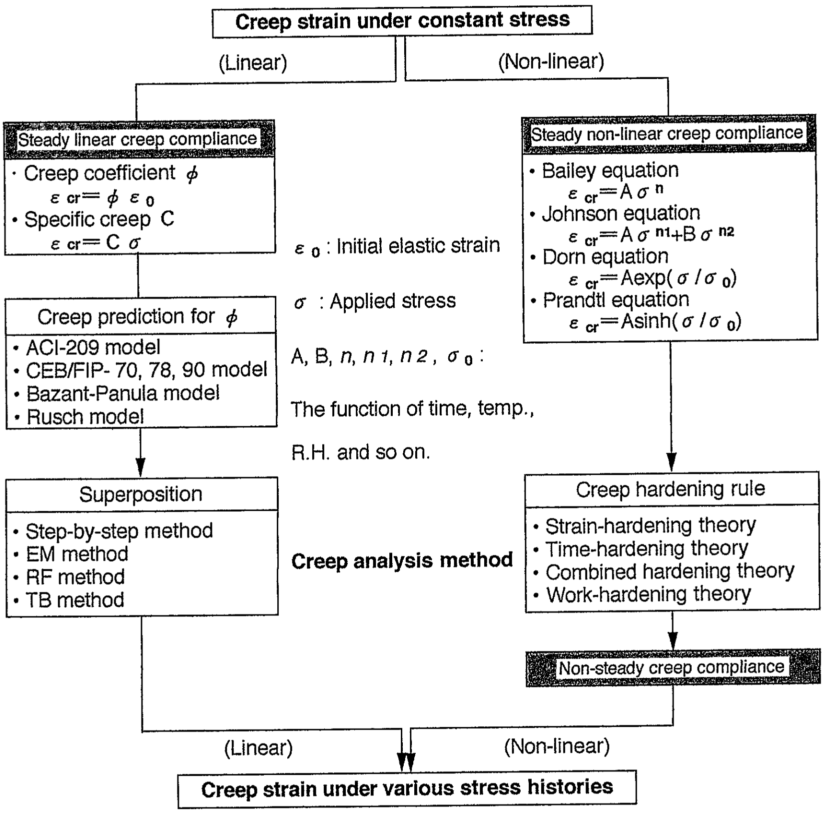

Fig.1 Creep analysis chart.

the steady linear creep compliance independent of time, as well as the creep strain under constant stress.

As for steel creep strain, when the relationship between creep strain and applied stress is regarded as nonlinear, creep compliance used under varying stress is different from that used under constant stress [11]. Under constant stress conditions, the relationship between the creep strain and the stress must be represented by steady nonlinear creep compliance, such as the Bailey equation, Johnson equation, Prandtl equation and so on. Under variable stress conditions, the relationship between the creep strain and the stress must be represented by non-steady creep compliance which varies with the change of applied stress. In order to deduce the non-steady creep compliance from the steady nonlinear one, a creep hardening rule is applied. These rules include the time-hardening rule, strain-hardening rule, combined hardening rule, work-hardening rule, and so on. These creep hardening rules are proposed for steel materials whose special qualities never change under the application of load. So, when we treat concrete creep strain as a nonlinear phenomenon, we need a steady nonlinear creep compliance and creep hardening rule suitable for concrete.

3. CREEP PROPERTIES OF CONCRETE SUBJECTED TO VARIOUS STRESS HISTORIES.

3.1. Experiment outline

The type of cement used was normal portland cement (specific gravity : 3.15). The fine aggregate was river sand (specific gravity : 2.60, water absorption : 2.08, F.M. : 3.10), and the coarse aggregate was crushed stone (specific gravity : 2.74, water absorption : 1.14, F.M. : 6.55). The strength of the concrete after curing is 25.1 MPa. The mix proportion of the concrete is shown in Table 1.

Table 1 Mix proportion of concrete.

Max size (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit weight(kg/m^3) W	C	S	G
20	4 ~ 5	0.7	66.1	44.0	185	280	808	1083

The size of the prism specimen used for measuring creep strain was 10cm×10cm×38cm. The size of the prism specimen used for measuring shrinkage strain was 10cm×10cm×40cm. At about 24 hours after casting, the specimens were removed from the mold and cured in water for two days. After that, the specimens were cured for 25 days in a constant temperature and constant relative humidity room at 20±1℃, 68±5%. The total curing period was 28 days. Two pairs of point gauges were put on each surface, except for the treated surface and the side opposite the treated surface. Measurements were made using a Whittemore strain meter with minimum divisions of 1/1000mm. The experiment was performed in a constant temperature and constant relative humidity room at 20±1℃, 68±5%. Because of the loss of prestress due to shrinkage, creep and relaxation of steel which occurs with time, each specimen was prestressed again at 3rd, 10th and 30th day from the beginning of each cycle. The permissible error for applied stress was 2%.

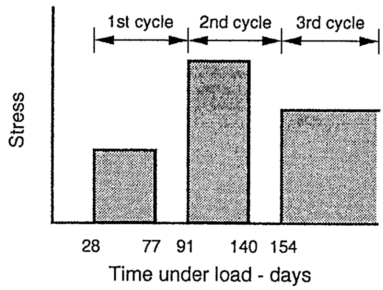

Fig.2 Stress history.

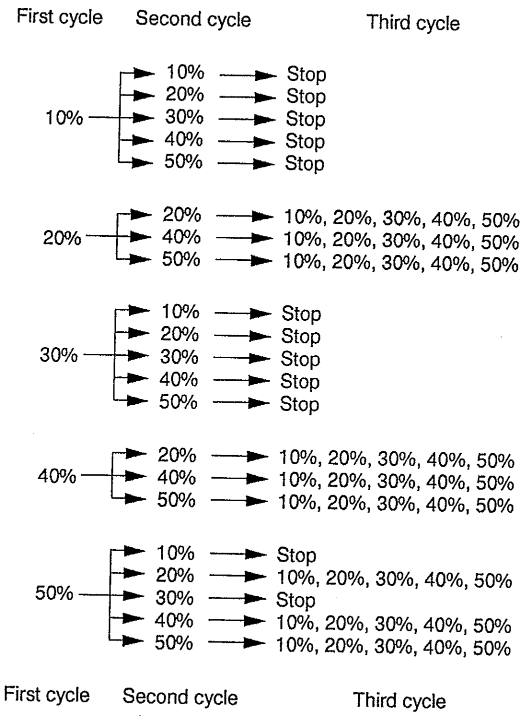

Fig.3 Stress history.

Each specimen was subjected to a stress history of three cycles as shown in Figs. 2 and 3. The period of each cycle was 63 days. For the first 49 days, each specimen was under sustained loading. After that, each specimen was unloaded. The unloaded period was used to avoid the effect of recoverable creep. The age at which the first load was applied was 28 days, enough to avoid the effect of aging. The ages at the second and third application were 91 days and 154 days, respectively. The stresses applied at each cycle are shown in Fig. 3 in terms of stress/strength ratio.

The specimen used for measuring concrete strength had the same shape and size as the specimen used for measuring shrinkage strain, and was cured in the same method as the specimen used for measuring creep strain. The mean value obtained from 3 specimens was regarded as the strength of concrete. When we measure the strength, we obtained the stress-strain curve. From this stress-strain curve, we determined the strain, which was yielded when the required stress was applied. To make this elastic strain yielded in the specimen used for measuring creep strain, we judged that the required stress was applied. We applied a stress of 0.2 ~ 0.3 MPa per second to the specimens to measure concrete strength and creep strain.

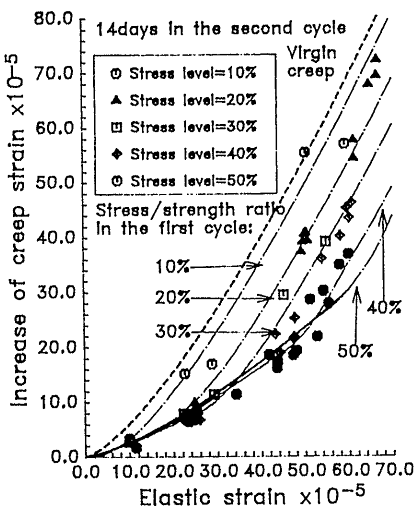

Fig.4 The relationship between creep strain and elastic strain in the second cycle.

3.2. Results

Figure 4 shows the relationship between creep strain and elastic strain after 14 days in the second cycle. The horizontal axis in Fig. 4 shows the elastic strain produced by applied stress in the second cycle. The vertical axis shows the increment of creep strain in the second cycle. The virgin creep curve (broken line) shows the relationship between creep strain and elastic strain of concrete for which the age at the first application of load is 91 days; i.e. the beginning of the second cycle in Fig.2. The symbols "○", "△", "□", "◇" and "●", represent the data for stresses of 10%, 20%, 30%, 40% and 50% strength in the first cycle, respectively. Each approximate curve presented by the dot-dash line shows that the turning point of the relationship between the increase of creep strain and elastic strain lies in the elastic strain yielded by the same stress subjected to in the first cycle. Namely, the slopes of the creep curves, which present the relationship between the creep strain and elastic strain of concrete subjected to stress larger than that in the first cycle, are the same as that of the virgin creep curve. Each creep curve which represents the relationship between the creep strain and elastic strain of concrete subjected to stress smaller than that in the first cycle has the same slope.

Figure 5 represents conceptually the relationship between creep strain and elastic strain of concrete subjected to different stress in the second cycle. This figure is based on the results shown in Fig. 4. The curve O-A'C' in Fig. 5 represents the relationship between creep strain and elastic strain of concrete of which the elastic strain in the first cycle is ε_D'. The relationship between O-ABC and O-A'C' is represented by using ε_D' , ε_D and Eq. (1), provided $\overline{DA}=\overline{DA'}$. Equation (1) is the creep compliance of the virgin concrete.

$$\varepsilon_{cr} = f\ (\varepsilon_0, t, t') \tag{1}$$

Where, ε_{cr} : the virgin creep strain
ε_0 : the elastic strain produced by stress in the second cycle
t : the age of the concrete
t' : the age at the first application of load

When the applied stress in the second cycle is larger than that in the first cycle, the relationship between creep strain and elastic strain of concrete under the varying stress history is the same as the relation for virgin concrete with a shift in the horizontal direction. The curve A'-C' in Fig. 5 presents the curve A-BC shifted in the horizontal direction. The curve A'-C' in Fig. 5 can be represented by Eq. (2),

$$\varepsilon_{cr} = f\ (\varepsilon_D+(\varepsilon_0-\varepsilon_D'),t,t_2) \qquad (2)$$

Where, σ_1 : applied stress in the first cycle

σ_2 : applied stress in the second cycle

t_2 : beginning time of the second cycle

ε_D' : elastic strain produced by applied stress in the first cycle

ε_D : The stress σ_D which produces the same magnitude of creep strain in virgin concrete as the stress σ_D' produces in concrete subjected to the stress σ_D' in the first cycle. The elastic strain ε_D' is given by stress σ_D', and the elastic strain ε_D is given by stress σ_D.

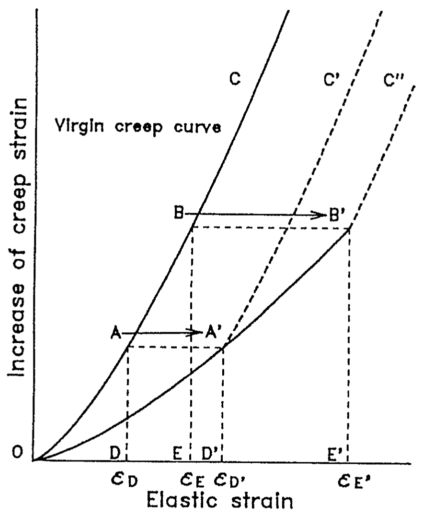

Fig.5 The relation between non-virgin creep strain and elastic strain.

The relationship between creep strain and elastic strain can be expressed by only one curve when the applied stress in the second cycle is smaller than that in the first cycle. If the horizontal axis of curve O-A is expanded by $\varepsilon_D/\varepsilon_D'$, it will coincide with the curve O-A', because O-AA'$\backsim$O-BB'. This is why the curve O-A' is represented by Eq. (3).

$$\varepsilon_{cr} = f\ ((\varepsilon_D/\varepsilon_D')\times\varepsilon_0,t,t_2) \qquad (3)$$

The dot-dash lines in Fig. 4 are calculated by Eqs. (2), (3) and (4). Equation (4) is the creep prediction for virgin concrete under constant stress. We can see that each dot-dash line agrees with the experimental data.

$$f\ (\varepsilon_0,t,t') = a\varepsilon_0^{\,b} \qquad (4)$$

in which, $a=0.44(t-t')^{0.12}t'^{-0.21}$, $b=1.25\{\log_e(t-t'+1)\}^{0.11}$. Where, ε_0 is elastic strain produced in the second cycle($\times10^{-5}$), t is age of the concrete (days), t' is age at the first application of load (days).

To obtain Eq. (4), we use the creep strain data of virgin concrete which has the same mix proportion and under the same conditions as concrete under varying stress. Each indeterminate coefficient in Eq. (4) is calculated by the hybrid method [12], which is a non-linear least squares method derived from the combination of the Gauss-Newton method and the method of steepest descent.

Figures 6 and 7 show the relationship between creep strain and elastic strain after 28 days in the third cycle. The horizontal axis of Figs. 6 and 7 shows the elastic strain produced by the applied stress in the third cycle. The vertical

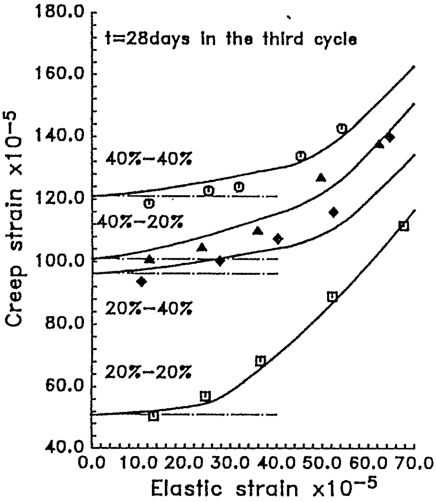

Fig.6 The relationship between creep strain and elastic strain in the third cycle.

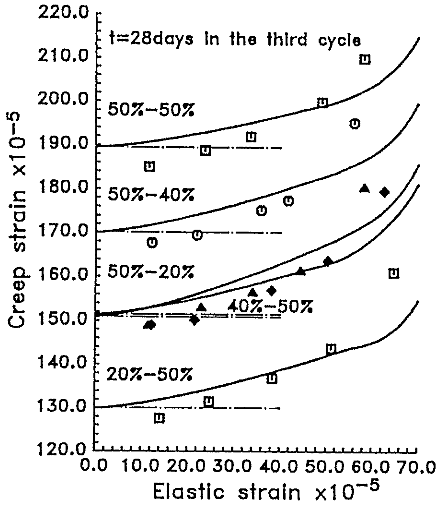

Fig.7 The relationship between creep strain and elastic strain in the third cycle.

axis of Figs. 6 and 7 shows the creep strain increment in the third cycle. The 20%-50% in this figure means that the stress applied to the specimen is 20% strength in the first cycle and 50% strength in the second cycle. The solid lines in these figures are the calculated results of Eqs. (5) and (6). The virgin creep strain is calculated by Eq. (4). Equations (5) and (6) reflect the relations correctly as is evident in Figs. 6 and 7. Therefore, it was confirmed that concrete creep strain under varying stress histories is a non-linear phenomenon and is strongly affected by the maximum applied stress to which the specimen is subjected to in the previous cycles. In addition, since Eqs. (5) and (6) are the generalized equations, we can calculate the creep strain in the preceding cycle by using them.

In the case that $\sigma_3 \geqq \sigma_{max}'$

$$\varepsilon_{cr} = f\,(\varepsilon_{max}+(\varepsilon_0-\varepsilon_{max}'),t,t_3) \tag{5}$$

and in the case that $\sigma_3 < \sigma_{max}'$

$$\varepsilon_{cr} = f\,((\varepsilon_{max}/\varepsilon_{max}')\times\varepsilon_0,t,t_3) \tag{6}$$

where, ε_0 : elastic strain produced by stress in the third cycle ($\times 10^{-5}$)
t : age of the concrete (days)
t_3 : age at the beginning of the third cycle (days)
σ_3 : applied stress in the third cycle
σ_{max}' : $\sigma_1 \geqq \sigma_2$ then $\sigma_{max}'=\sigma_1$, $\sigma_1<\sigma_2$ then $\sigma_{max}'=\sigma_2$
ε_{max}' : elastic strain produced by applied stress σ_{max}'
ε_{max} : The stress σ_{max} which produces the same magnitude of creep strain in virgin concrete as the stress σ_{max}' produces in concrete subjected to the stress σ_{max}' in the previous cycle. The elastic strain ε_{max}' is given by stress σ_{max}', and the elastic strain ε_{max} is given by stress σ_{max}.

4. THE PREDICTION OF CREEP STRAIN OF CONCRETE UNDER VARIOUS STRESS HISTORIES

We verified that the creep strain of concrete under varying compressive stress histories is a non-linear phenomenon which is affected by the maximum stress in the previous cycle. We also established a creep hardening rule (Eqs. (5) and (6)), by which the non-linearity of creep strain under a varying compressive stress history is described. But in Eqs. (5) and (6) the effects of the recoverable creep strain, which occurs when the applied load is removed, is neglected. So, those equations may not be accurate when the recoverable creep strain is large. Therefore, in this section, we present a creep hardening rule which includes the effects of recoverable creep strain. Furthermore, we compare experimental data under varying stress with the results calculated by superposition, the usual creep hardening rule and the authors' creep hardening rule. We find that the authors' creep hardening rule gives better results than those of the usual creep analysis method. However, we used the time-hardening rule and the strain-hardening rule as the usual creep hardening rule. These are used in steel creep problems and are easy to calculate.

4.1. Experiment outline

The type of cement used was normal portland cement (specific gravity : 3.15). The fine aggregate was river sand (specific gravity : 2.62, water absorption : 1.78, F.M. : 2.81), and the coarse aggregate was crushed stone (specific gravity : 2.73, water absorption : 0.76, F.M. : 6.68). The mix proportion of the concrete is shown in Table 2. The experiment was performed in a constant temperature and constant relative humidity room at 20±1℃, 68±7%. The size of the prism specimen used for measuring creep strain was 10cm×10cm×38cm. The size of the prism specimen used for measuring shrinkage strain was 10cm×10cm×40cm. Two pairs of point gauges were put on each surface, except for the treated surface and the side opposite the treated surface. Measurements were made using a Whittemore strain meter with minimum divisions of 1/1000mm.

Table 2 Mix proportion of concrete.

Max size (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit weight(kg/m³) W	C	S	G
20	9~12	1.2	60.0	47.7	200	333	842	963

Table 3 Water curing period and age at application of load.

Water curing period (days)	Age at the loading (days)				
3	3	10	24	52	94
7	7				
14	14	21	35	63	105
28	28				
56	56	63	77	105	

We applied six types of varying stress to the specimens. The specimens were cured for 14 days in water. After that, the specimens were cured for 21 days in a constant temperature and constant relative humidity room at 20±1℃, 68±5%. The total curing period was 35 days. Monotonically increasing stress and monotonically decreasing stress were immediately applied to the specimens after water curing whose period was 3 or 56 days. The varying stress-time curve was a sinusoidal curve. The minimum varying stress was 10% of strength, and the maximum varying stress was 50%. We applied stress of 0.2 ~ 0.3 MPa per second to the specimens.

4.2. Results

(a) The non-linear steady creep compliance

The non-linear steady creep compliance, which represents the creep strain of concrete under constant stress, is expressed by Eqs. (7) and (8). To obtain these equations, we used the creep strain data of virgin concrete which had the same mix proportion and is under the same conditions as the concrete subjected to varying stress. The water curing period and the age at the first application of load for the virgin concrete are shown in Table 3. These data were obtained from the specimens whose stress-strength ratio ranged from 10% to 50%. We measured the creep strain for 200 days. The total number of specimens was 160. It was confirmed that the deviation of the calculated data using Eqs. (7) and (8) from the experimental data is within 20%.

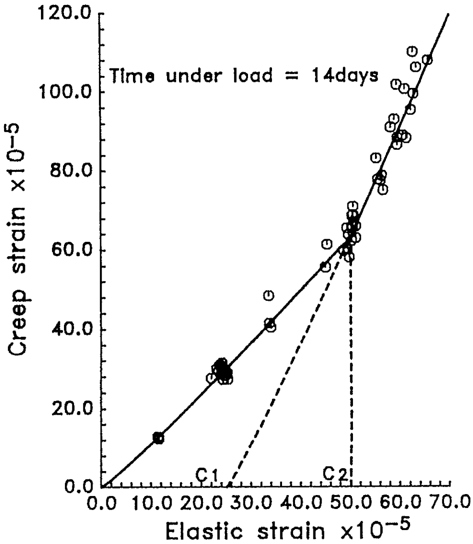

Fig.8 The relationship between virgin creep strain and elastic strain.

In the case that $\varepsilon_0 < c_2(t',t_0)$;

$$\varepsilon_{cr}(\varepsilon_0,t,t',t_0) = a(t,t',t_0)\left(\frac{c_2(t',t_0)-c_1(t_0)}{c_2(t',t_0)}\varepsilon_0\right)^{b(t_0)} \tag{7}$$

and in the case that $\varepsilon_0 \geqq c_2(t',t_0)$;

$$\varepsilon_{cr}(\varepsilon_0,t,t',t_0) = a(t,t',t_0)\times(\varepsilon_0-c_1(t_0))^{b(t_0)} \tag{8}$$

in which,

$$\left.\begin{aligned} a(t,t',t_0) &= 2.64\, t_0^{0.114}\{0.002(t'-t_0)+1\}^{-2.9}\left(\frac{t-t'}{262+(t-t')}\right)^{0.434} \\ b(t_0) &= 0.285\exp(-0.047t_0)+1 \\ c_1(t_0) &= \left(\frac{9.62}{t_0}+9.81\right)\times 10^{-5} \\ c_2(t',t_0) &= 47.1\times10^{-5}\{\log_e(t_0+1)\}^{-0.372}\exp\{-0.055(t'-t_0)^{0.214}\} \end{aligned}\right\} \tag{9}$$

where, $\varepsilon_{cr}(t,t',t_0)$: the virgin creep strain ($\times10^{-5}$)
t : the age of the concrete (days, $t\geqq t'$)
t' : the age at the first application of load (days, $t'\geqq t_0$)
t_0 : the age at the start of drying (days)
ε_0 : the elastic strain produced by stress ($\times10^{-5}$)

Figure 8 shows the creep strain and elastic strain of concrete under constant stress. The application period is 14 days. In this figure, symbol "○"

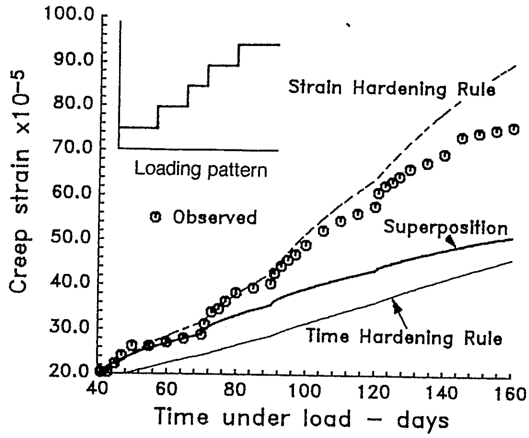

Fig.9 The prediction of creep strain under the monotonically increasing stress (t'= 56 days).

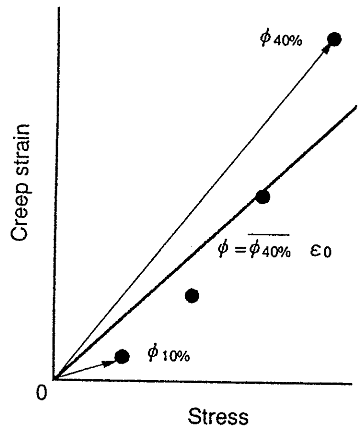

Fig.10 The relationship between creep strain and elastic strain.

represents the experimental data. The solid line represents the calculated results of Eqs. (7) and (8). The coefficient c_2 which appears in Eqs. (7) and (8) reflects the turning point of the relationship between creep strain and elastic strain of concrete under the constant stress.

(b) The applicability of each creep analysis method

Figure 9 shows the creep strain of concrete subjected to monotonically increasing stress history. The age at the first application of load is 56 days. The symbol "○" in Fig. 9 represents the experimental data. The broken line, the solid line and the bold solid line in Fig. 9 represent the results calculated by the strain-hardening rule, the time-hardening rule and superposition, respectively. The results calculated by the superposition and time-hardening rule are considerably smaller than the experimental data.

The reason for this can be explained by using Fig. 10 as follows.

The symbols $\phi_{40\%}$ and $\phi_{10\%}$ in Fig. 10 represent the creep coefficients of concrete subjected to stresses of 40% and 10% of the strength, respectively. $\overline{\phi}$ is the optimal slope of the regression line which does not intersect the vertical axis. If the creep strain of concrete is linear with stress, the calculated results by superposition are all same independently of the creep coefficient used and agree with the experimental data. But in reality, since the concrete creep strain under constant stress is nonlinear as shown in Fig. 10, the relation among $\phi_{40\%}$, $\overline{\phi}$ and $\phi_{10\%}$ is as $\phi_{40\%} > \overline{\phi} > \phi_{10\%}$. Therefore, the results calculated by superposition with $\phi_{10\%}$ are smaller than the experimental data whereas the results calculated by superposition with $\phi_{40\%}$ are larger than the experimental data. The result calculated by superposition in Fig. 9 is obtained by using $\phi_{10\%}$. Thus, we can observe remarkable nonlinearity of creep strain even under small stress when varying stress is applied.

Figure 11 shows the results calculated by the strain-hardening rule and the creep hardening rule presented by the authors. The age at the first application of load is 56 days. As shown in this figure, the strain-hardening rule has the

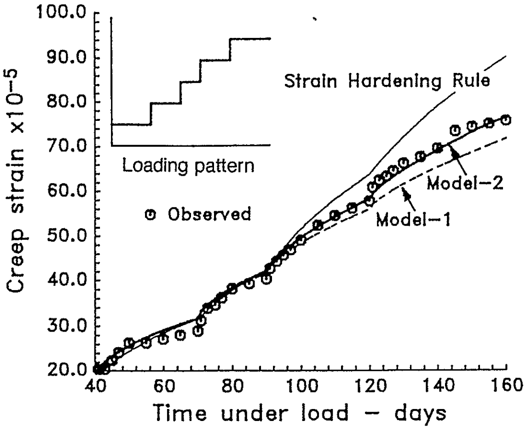

Fig.11 The prediction of creep strain under the monotonically increasing stress (t'= 56 days).

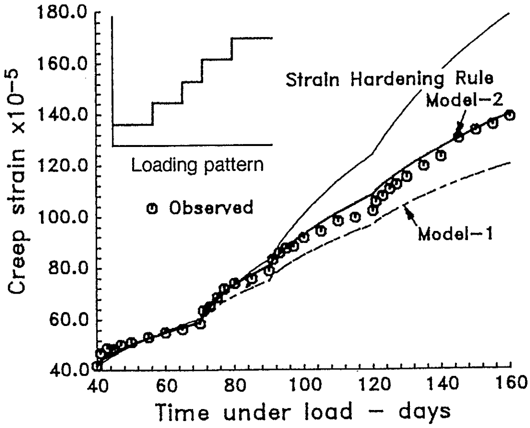

Fig.12 The prediction of creep strain under the monotonically increasing stress (t'= 3 days)

same accuracy as the creep-hardening rule presented by the authors.

Figure 12 shows the creep strain of concrete whose first application of varying stress is at 3rd day. Model-1 and Model-2 shown in this figure are the authors' creep hardening rules. The result calculated by the strain-hardening rule is larger than the experimental data as the applied stress becomes larger, although the result calculated by the authors' creep-hardening rule fits with the experimantal data very well independent of the applied period. It seems that the rule cannot estimate the change in creep properties of concrete under load because the usual creep-hardening rule uses only one creep curve obtained at the first application of load for calculation. So, when the age at the first application of load is young, the result calculated by the strain-hardening rule will be an overestimation. When the age at the first application of load is old, the result calculated by the strain-hardening rule agrees with the actual creep phenomena.

Model-1 in Figs. 11 and 12 is established by the experimental fact that the creep strain of concrete under varying stress is a nonlinear phenomenon which is affected by the maximum stress applied in the past. The result noted by Model-1 is calculated from Eqs. (5) and (6).

Model-2 is a modified version of Model-1. Model-2 is a creep hardening rule which takes into account the effect of recoverable creep strain and creep strain in the case that the elastic strain is beyond c_2 which appears in Eqs. (7) and (8). Equations (10), (11) and (12) are based on Model-2 to obtain the creep strain of concrete under varying stress. However, ε_{max} and ε_{max}' included in these equations are same as that appear in Eqs. (5) and (6).

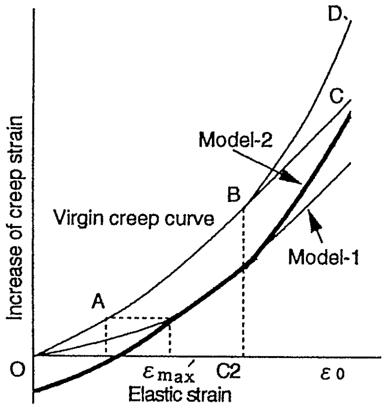

Fig.13 The relation between creep strain and elastic strain by Model-1 and Model-2.

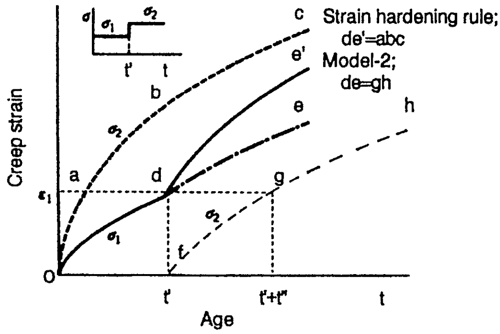

Fig.14 Creep strain calculated by Eq.(12)

In the case that $\varepsilon_0 \geqq \varepsilon_{max}$;

$$\varepsilon_{Hc}(t) = \varepsilon_{crs}\ (\varepsilon_{max} + (\varepsilon_0 - \varepsilon_{max}'), t, t', t_0) \tag{10}$$

In the case that $\varepsilon_0 < \varepsilon_{max}$;

$$\varepsilon_{Hc}(t) = \left(\frac{\varepsilon_{crs}(\varepsilon_{max}, t, t', t_0)}{\varepsilon_{crs}(\varepsilon_{max}, t, t', t_0) - \varepsilon_{crs}(\varepsilon_{max}/\varepsilon_{max}' \times \varepsilon_E, t, t', t_0)} \right) \times \left\{ \varepsilon_{crs}(\varepsilon_{max}/\varepsilon_{max}' \times \varepsilon_0, t, t', t_0) - \varepsilon_{crs}(\varepsilon_{max}/\varepsilon_{max}' \times \varepsilon_E, t, t', t_0) \right\} \tag{11}$$

Furthermore, in the case that $\varepsilon_0 \geqq c_2$;

$$\begin{aligned} \varepsilon_{Hc}(t) = &\{\varepsilon_{Hc}(t) \text{ obtained by Eq. (10) or Eq. (11)}\} \\ &+\{\varepsilon_{crL}(\varepsilon_0, t+t'', t', t_0) - \varepsilon_{crs}(\varepsilon_0, t+t'', t', t_0)\} \\ &-\{\varepsilon_{crL}(\varepsilon_0, t'', t', t_0) - \varepsilon_{crs}(\varepsilon_0, t'', t', t_0\} \end{aligned} \tag{12}$$

Here, ε_{crs} and ε_{crL} are the virgin creep curves shown in Fig. 13 and given by Eqs. (13) and (14).

$$\varepsilon_{crs}(\varepsilon_0, t, t', t_0) = a\ (t, t', t_0) \left(\frac{c_2(t', t_0) - c_1(t_0)}{c_2(t', t_0)}\ \varepsilon_0 \right)^{b(t_0)} \tag{13}$$

$$\varepsilon_{crL}\ (\varepsilon_0, t, t', t_0) = a\ (t, t', t_0) \times (\varepsilon_0 - c_1(t_0))^{b(t_0)} \tag{14}$$

$a(t,t',t_0)$, $b(t_0)$, $c_1(t_0)$ and $c_2(t',t_0)$ are the coefficients obtained from Eq. (9).

Figure 14 shows the calculation method based on Model-2 when the elastic strain of concrete is larger than c_2 which appears in Eqs. (7) and (8). The time length t" shown in Fig. 14 is same as that involved in Eq. (12), which is the application period to make the virgin concrete yield the same creep strain as the concrete under varying stress. The creep-time curve 'de' represented by the second and third terms of Eq. (12) is obtained by shifting the curve 'gh' in the negative horizontal direction through the time length t".

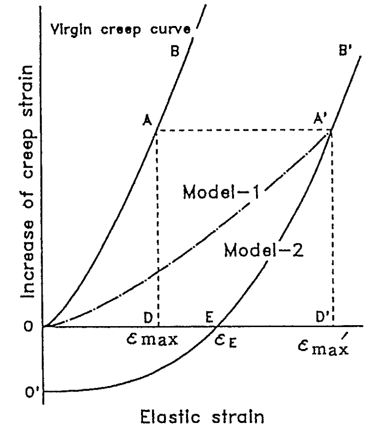

Fig.15 The effect of recoverable creep strain calculated by Eq.(11)

Figure 15 is a conceptual figure which shows the method used to account for the effects of the recoverable creep strain based on Model-2. The curve O'-EA' represents the relationship between the creep strain and the elastic strain of concrete which is subjected to smaller stress than the maximum previously applied stress. This curve is represented by Eq. (11). The relation among ε_{max}, ε_E and ε_{max}' is as follows;

$$\varepsilon_{max} = \varepsilon_{max}' - \varepsilon_E \tag{15}$$

The curve based on Model-1 passes through the point (0,0). But the value calculated by Model-2 shows recoverable creep strain at ε_0=0.

Figures 16 and 17 show the creep strain of concrete subjected to a monotonically decreasing stress history. The age at the first application of load is 3 days. In cases where the stress decreases, the creep strain produced by the initial load is considerably larger than that produced by following stresses. Therefore, there is no difference among the results based on the various creep analysis methods.

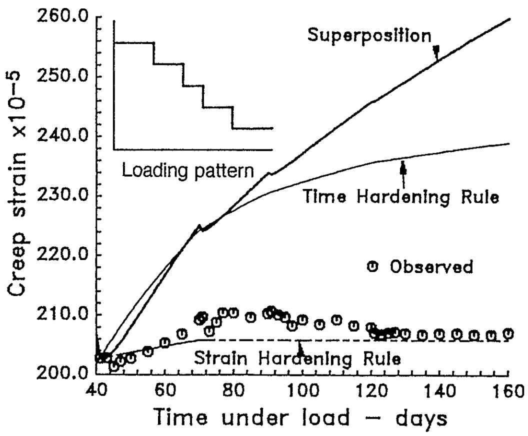

Fig.16 The prediction of creep strain under the monotonically decreasing stress.

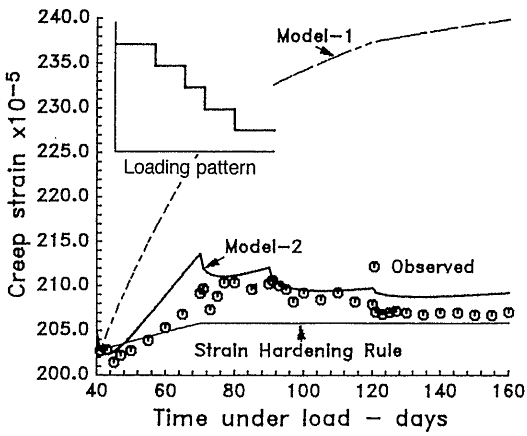

Fig.17 The prediction of creep strain under the monotonically decreasing stress.

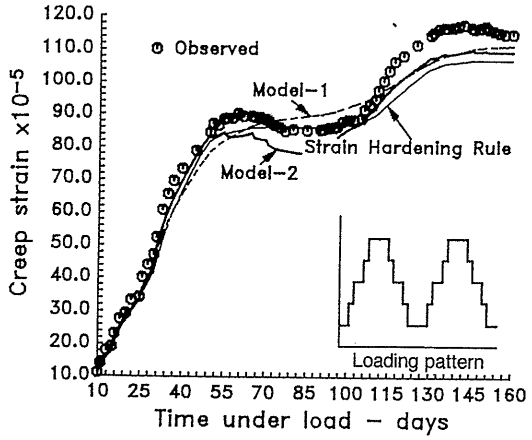

Fig.18 The prediction of creep strain under various stress histories.

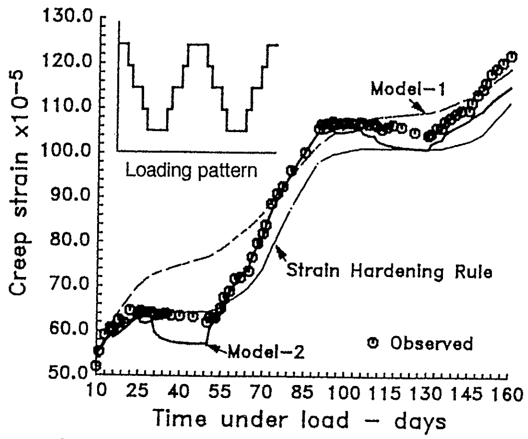

Fig.19 The prediction of creep strain under various stress histories.

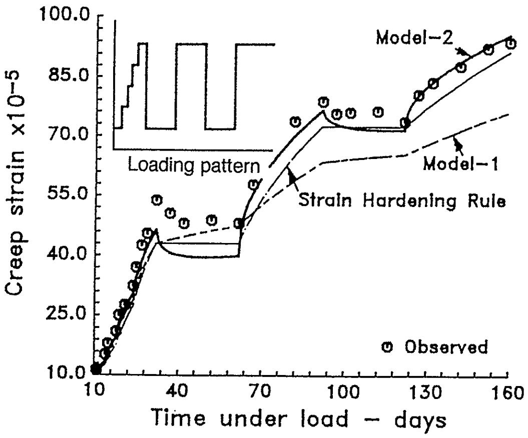

Fig.20 The prediction of creep strain under various stress histories.

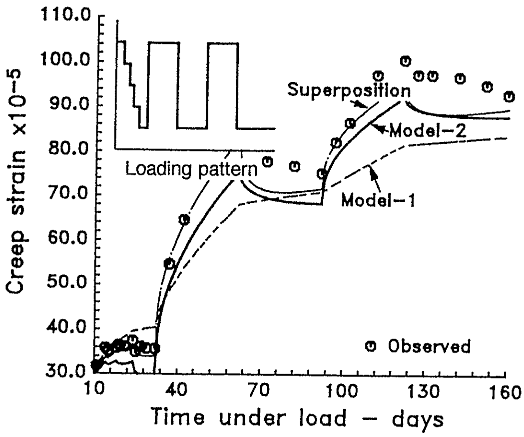

Fig.21 The prediction of creep strain under various stress histories.

Figures 18 to 21 show the accuracy of the authors' creep-hardening rule under various stress histories. The results based on usual creep analysis methods shown in these figures are only those which best fit the experimental data. As shown in these figures, the result calculated by the strain-hardening rule best fit the experimental data, except in Fig. 21. The results calculated by the authors' creep hardening rule is in good agreement with experimental data under any varying stress history.

5. CONCLUSION

It was verified that results calculated by superposition considerably underestimate the creep strain of concrete under monotonically increasing stress history. This means that creep under varying stress behaves in a nonlinear manner. We investigated the relationship between creep strain and elastic strain of concrete under varying compressive stress and confirmed that the creep strain of concrete under varying stress history is a nonlinear phenomenon which is affected by the maximum previously applied stress. Furthermore, we established a creep-hardening rule for concrete with which experimental data under the various stress histories with the results calculated by superposition, strain-hardening rule, time-hardening rule are compared. We confirmed that the authors' creep-hardening rule gives better results than those from the usual creep analysis methods.

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