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A NON-LINEAR CREEP PREDICTION EQUATION FOR CONCRETE

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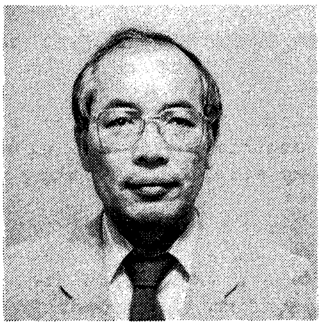

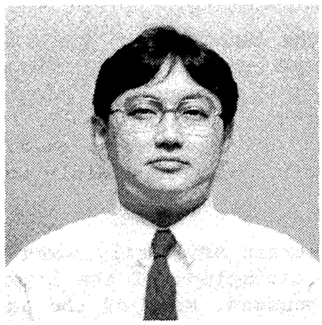

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SYNOPSIS

Concrete is considered to be an aging linear visco-elastic material. Hence, the creep behavior of concrete under constant sustained stress is represented by either the creep coefficient or specific creep, both of which are based on the assumption of a linear relationship between creep strain and externally applied stress. What is in doubt, however, is the upper limit of this assumption. In terms of the stress/strength ratio, an upper limit between about 0.23 and 0.75 has been observed. The purpose of this study is to clarify the non-linearity of creep strain of concrete under constant sustained stress. We verify that there is a significant difference among the concrete creep coefficients under various levels of constant stress. Furthermore, we propose a non-linear creep prediction equation which can accurately represent the results of creep experiments.

Keywords : creep strain, stress/strength ratio, non-linear, creep compliance, modified Bailey equation

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1. INTRODUCTION

The relationship between creep strain and stress of concrete is assumed to be linear. Therefore, the creep prediction equations now in use are based on the linear creep compliance.

Many investigations have been performed to clarify the validity of this assumption. But the applicable upper limit of this assumption is in dispute. In terms of the stress/strength ratio, upper limits between 0.23 and 0.75 have been observed [1]. Thus, this upper limit varies according to the researcher. It seems that there is no definite upper limit for this linearity.

However, the linearity of creep to stress/strength ratio up to 0.4 is defined in the standard specification for design and construction of concrete structure published by JSCE concrete committee [2] and in the CEB/FIP model code 1978 [3]. When the application service loads exceed the valid upper limit for linearity, creep prediction equations based on the linear assumption cannot give correct results. It is also impossible to predict the creep strain over a long period of time.

As creep strain ordinarily seems to occur under very low stress, a lower limit, where the assumption of the linearity of creep to stress is accurate, has been rarely discussed. Most of the investigations to confirm the proportionality of creep to stress were carried out by using concrete which had reached hygral equilibrium with surrounding medium prior to the application of load [4] ~ [6]. In those cases, the regression line which represents the relationship between creep strain and stress almost crosses the origin. But as pointed in the investigation by L'Hermite [7], if concurrent shrinkage occurs during the period under loading, the value obtained from the curve to represent the relationship between time-deformation (creep-plus-shrinkage) and stress, is smaller than the shrinkage of an unloaded companion specimen at zero stress. This suggests that the approximate line which does not intersect the vertical axis is inadequate for representing the relationship between creep strain and stress.

The creep prediction equation modeled by CEB/FIP in 1990 [8] has taken the non-linearity of creep strain into account when the applied stress exceeds 40% strength. However, when the applied stress is below 40%, the linear assumption has remained in this prediction equation as before. If the upper or the lower limit occurs at a stress value smaller than 40% strength, the non-linearity of creep strain must be considered even in cases where the stress is below 40% strength.

As mentioned earlier, the linear assumption between creep strain and stress applies approximately to concrete without reliable evidence. The purpose of this study is to clarify experimentally the non-linearity of creep strain of concrete under constant stress and to establish a non-linear creep prediction equation.

2. THE APPLICABILITY OF LINEAR CREEP PREDICTION

In order to express the proportionality of creep strain to stress, the creep strain of concrete under constant stress is represented by specific creep or creep coefficient. The specific creep is defined as the ratio of creep strain to sustained stress, as follows;

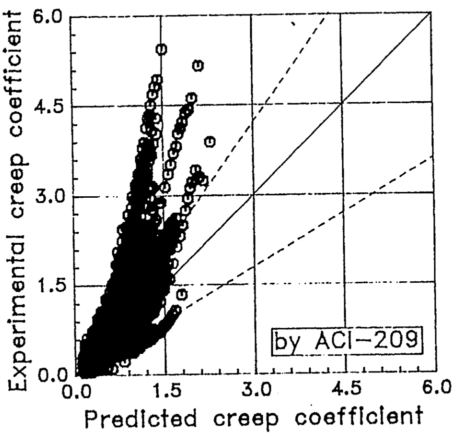

Fig.1 Creep prediction by ACI Model.

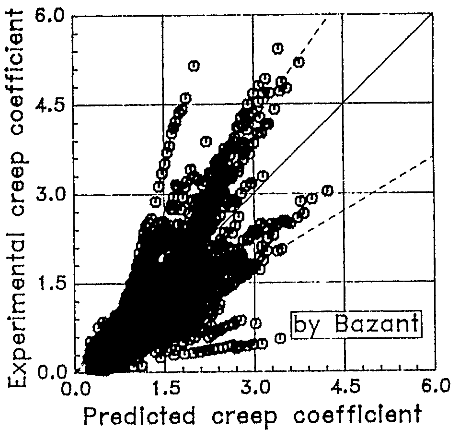

Fig.2 Creep prediction by Bazant Model.

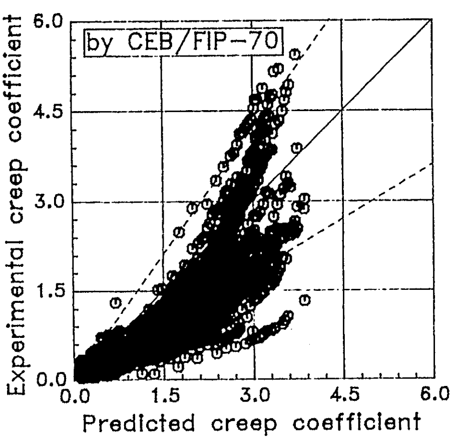

Fig.3 Creep prediction by CEB/FIP-70 Model.

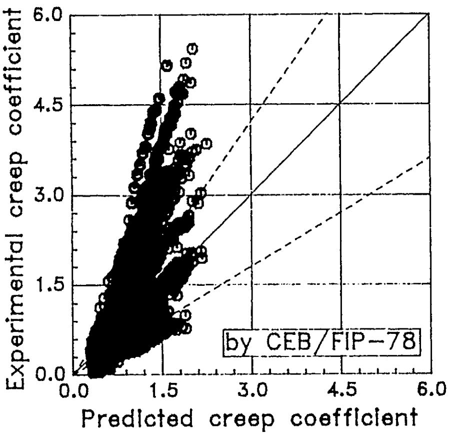

Fig.4 Creep prediction by CEB/FIP-78 Model.

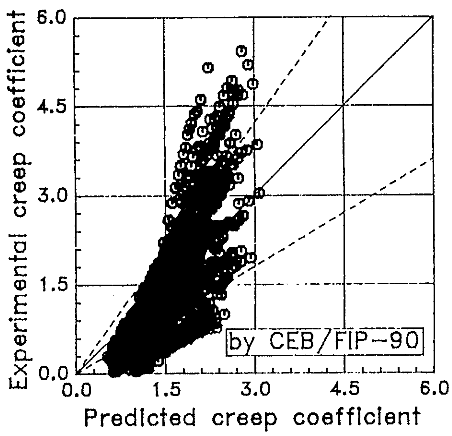

Fig.5 Creep prediction by CEB/FIP-90 Model.

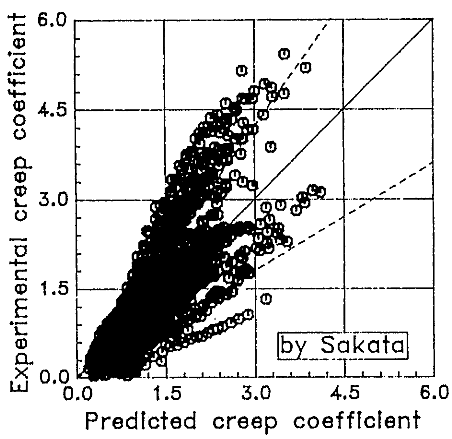

Fig.6 Creep prediction by Sakata Model.

$$C(t,t')=\varepsilon_{cr}(t,t')/\sigma_0 \tag{1}$$

where, $C(t,t')$: specific creep
$\varepsilon_{cr}(t,t')$: creep strain
σ_0 : sustained stress
t : age of the concrete
t' : age at the first application of load

The creep coefficient is defined as the ratio of the creep strain to elastic strain, as follows;

$$\phi(t,t')=\varepsilon_{cr}(t,t')/\varepsilon_0=C(t,t')\times E(t') \tag{2}$$

where, $\phi(t,t')$: creep coefficient
$E(t')$: modulus of elasticity
ε_0 : elastic strain

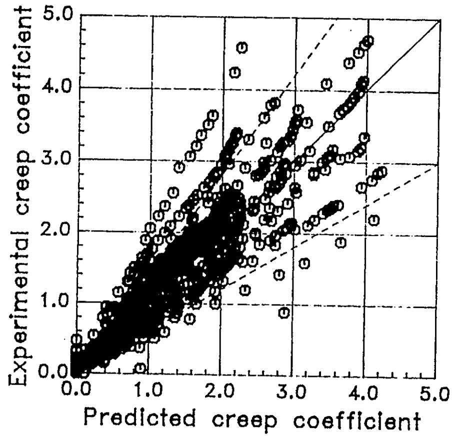

Fig.7 Creep prediction by CEB/FIP-90 Model.

In general prediction equations for creep, the creep coefficient or specific creep is given in terms of the basic properties of the concrete. In this section, we investigate the applicability of usual creep prediction equations in order to confirm the suitability of linear creep prediction.

Figures 1 ~ 6 show the comparison between experimental data [13] and predicted data by the ACI-209 model [9], Bazant model [10], CEB/FIP 1970 model [11], CEB/FIP 1978 model [3], CEB/FIP 1990 model [8] and the authors' model [12], respectively. The total number of specimens used in this experiment was 104. The horizontal axis in these figures shows the creep coefficient predicted by each model. The vertical axis shows the experimental creep coefficient. The broken line shows 40% variation of the predicted data from the experimental data. As is evident from these figures, we can predict the creep accurately enough by any of these model. However, the scatter between the predicted and experimental data becomes larger with the lapse of time.

Figure 7 shows the comparison between the prediction by CEB/FIP 1990 model and the experimental data [14] used for the establishment of this prediction equation. The broken line has the same meaning as above. The predictions are scattered within ±40% around the experimental data.

It is generally acknowledged that linear creep prediction has the error of 40% due to the linear assumption of the relationship between creep strain and applied stress.

3. THE NON-LINEARITY OF CREEP STRAIN OF CONCRETE

In this section, we examine the non-linearity of creep in the difference among the creep coefficients yielded by stresses of various magnitude.

3.1. Experiment outline

The type of cement used was normal portland cement (specific gravity : 3.15). The fine aggregate was river sand (specific gravity : 2.60, water absorption : 2.08, F.M. : 3.10), and the coarse aggregate was crushed stone (specific gravity : 2.74, water absorption : 1.14, F.M. : 6.55). The strength of the concrete after curing was 25.1 MPa. The mix proportion of the concrete is shown in Table 1.

Table 1 Mix proportion of concrete.

Max size (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit weight(kg/m^3)			
					W	C	S	G
20	4 ~ 5	0.7	66.1	44.0	185	280	808	1083

The size of the prism specimen for measuring creep strain was 10cm×10cm×38cm. The size of the prism specimen for measuring shrinkage strain was 10cm×10cm×40cm. At about 24 hours after casting, the specimens were removed from the mold and cured in water for two days. After that, the specimens were cured for 25 days in a constant temperature and constant relative humidity room at 20±1 ℃, 68±5%. The total curing period was 28 days. Two pairs of point gauges were put on each surface, except for the treated surface and the side opposite the treated surface. Measurements were made of strain by a Whittemore strain meter with minimum divisions of 1/1000mm. The experiment was performed in a constant temperature and constant relative humidity room at 20±1℃, 68±5%. Stresses of 10%, 20%, 30%, 40% and 50% strength were applied to each specimen. The specimen used for measuring concrete strength had the same shape and size as the specimen used for measuring shrinkage strain, and was cured in the same method as the specimen used for measuring creep strain. The mean value obtained from 3 specimens was regarded as the strength of the concrete. The strength of concrete was 25.1 MPa. The total number of the specimens subjected to each stress was 3, 16, 3, 15 and 18, respectively. This was due to experimental circumstances. In measuring the strength, we obtained the stress-strain curve, too. From this stress-strain curve, we determined the strain, which was yielded when the required stress was applied. To make this elastic strain yielded in the specimen used for measuring creep strain, we judged that the required stress was applied. We applied stresses of 0.2 ~ 0.3 MPa per second to specimens for measuring concrete strength and creep strain. Because of the loss of prestress due to shrinkage, creep and relaxation of steel which occurs with time, each specimen was prestressed again on the 3rd, 10th and 30th day from the first application of load. The permissible error of applied stress was 2%.

In accordance with Eq. (3), the loss of prestress by the stress of 50% strength is calculated as follows; loss of 1.11 Mpa occurs in the period between the first application and 3rd day, 0.86 Mpa between 3rd day and 10th day, 1.15 Mpa between 10th day and 30th day, 0.60 Mpa between 30th day and 49th day. The loss of prestress by the stress of 50% strength is the largest among the losses of prestress by the other stress level. However, the largest loss of prestress is 4.6% in terms of stress/strength ratio and is less than half of the stress level 10%. Therefore, the experiment can be considered to have been performed under constant stress.

$$\Delta \sigma_c = \frac{1}{A_c} \times \left(A_p \cdot E_p \cdot \frac{l_c}{l_p} \cdot \Delta \varepsilon_c \right) \tag{3}$$

where, A_c : area of the concrete
A_p : area of the prestressing steel
E_p : modulus of elasticity of the prestressing steel
l_c : length of the concrete
l_p : length of the prestressing steel
$\Delta \varepsilon_c$: both creep strain and shrinkage strain which occurs in the period between repeated prestressings

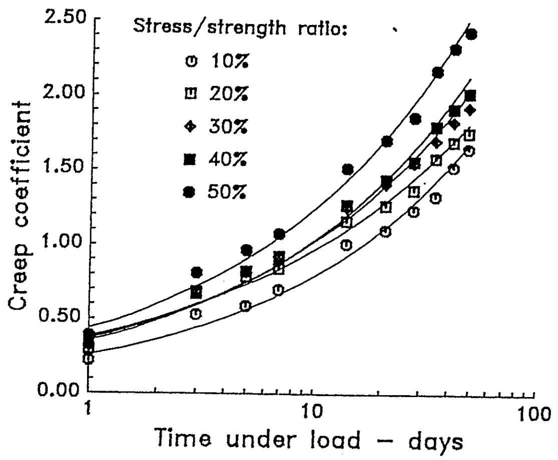

Fig.8 Results of creep test.

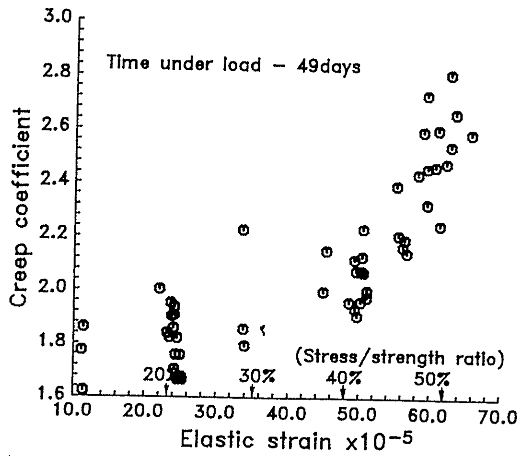

Fig.9 Scatter of creep coefficients.

3.2. Results

(a) Investigation of non-linearity of creep strain

Figure 8 shows the change over time of creep coefficient for each stress. The symbols "○", "□", "◇", "■" and "●" are the mean values of the creep coefficients subjected to stresses of 10%, 20%, 30%, 40% and 50% in terms of stress/strength ratio, respectively. If the relationship between creep strain and stress of concrete under constant stress were a linear phenomenon, the change over time of creep coefficients should be represented by only a single curve, irrespective of the magnitude of applied stress. But, as is evident from Fig. 8, the change over time of creep coefficient for each stress do not coincide. In particular, the creep coefficient for the stress of 50% strength is much larger than others. Also, the difference between creep coefficients produced by stresses of 10% and 40% strength is constant during the applied period.

Figure 9 shows the relationship between the creep coefficient and elastic strain at the 49th day after the first application of load. If the relationship between creep strain and stress of concrete under constant stress were a linear phenomenon, the creep coefficient must be constant for any elastic strain. But, as is obvious from Fig. 9, the larger the elastic strain, the larger the creep coefficient. Furthermore, it is also clear that not only the mean but also the scatter of the creep coefficients by the stresses larger than 40% of strength is larger than the others.

Figure 10 shows the comparison of creep coefficients obtained by two approaches. The creep coefficients represented by the horizontal axis in this figure are the optimal slopes obtained by regression of the line expressed by Eq. (4) at every measuring period. The creep coefficients represented by the vertical axis are the creep strain divided by elastic strain.

$$\varepsilon_{cr} = \phi \cdot \varepsilon_0 \tag{4}$$

where, ε_{cr}: creep strain
ε_0 : elastic strain
ϕ : creep coefficient

The dot-dash lines show 40% variation of the optimal slope from the creep strain divided by elastic strain. And, the broken lines show 20% variation of the optimal slope from the creep strain divided by elastic strain. Figure 10 suggests that the confidence limit of linear creep compliance is 40%, and shows that the creep coefficients by the optimal slopes of regression line result in an overestimation compared to the experimental data when applied stress is small and an underestimation when applied stress is large.

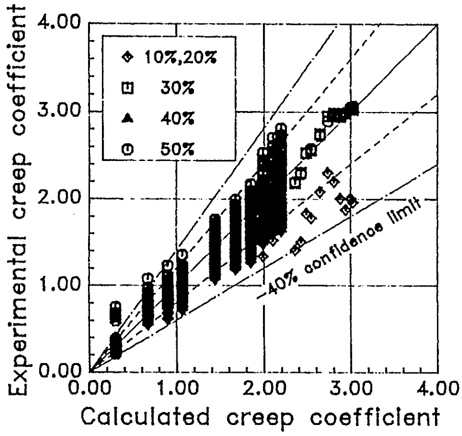

Fig.10 The comparison between calculated data by linear creep compliance and test data.

(b) Non-linear creep compliance

Figure 11 shows the relationship between creep strain and elastic strain at the 28th day from the first application of load. The solid curve in this figure is obtained by regression of the curve expressed by Eq. (5), which is called Bailey equation. For steel creep, the Bailey equation is often used to represent the relationship between creep strain and elastic strain.

$$\varepsilon_{cr} = a\,\varepsilon_0^{\,b} \qquad (5)$$

where, ε_{cr} : creep strain

ε_0 : elastic strain

a and b : indeterminate coefficients obtained by a non-linear least squares method. In this study, we use the hybrid method [15] derived from the combination of the Gauss-Newton method and the steepest descent method.

From Fig. 11, it appears that the regressed values by the Bailey equation are in good agreement with experimental data when the stresses are 20%, 30% and 50% in terms of stress/strength ratio. However, the regressed values by the Bailey equation are less than the experimental data when the stresses are 10% in terms of stress/strength ratio and larger than the experimental data when the stresses are 40%. This is confirmed at another measuring period. Namely, the curve regressed by the Bailey equation can represent the nonlinearity of creep strain accurately enough.

The creep coefficients represented by the horizontal axis in Fig. 12 are the calculated creep strain by the Bailey equation divided by elastic strain. The optimal value of the indeterminate coefficients in the Bailey equation are obtained by regression based on experimental data at every measuring period. The creep coefficients represented by the vertical axis are experimental creep strain divided by elastic strain subjected to each stress at every measuring period. The variation of calculated creep coefficients from experimental ones in Fig. 12 is smaller than that in Fig. 10. Therefore, it is clear that the Bailey equation can represent the relationship between creep strain and elastic strain more precisely by taking into account the nonlinearity of creep strain. However, when applied stress is 10% and 20% in terms of stress/strength ratio, the creep coefficients calculated by the Bailey equation exceed the confidence limit of 40%.

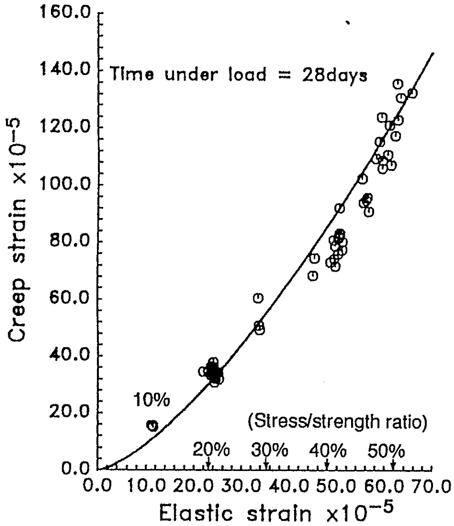

Fig.11 The relationship between elastic strain and creep strain by the Bailey equation.

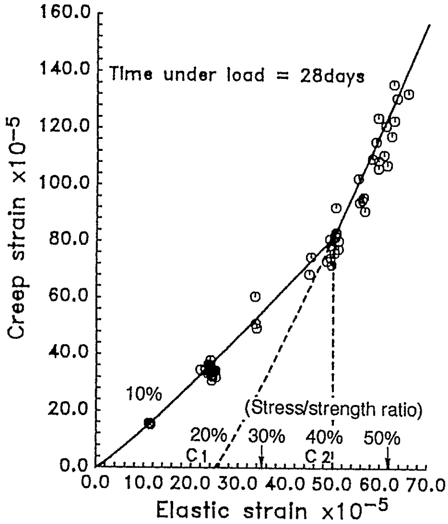

Fig.13 The relationship between elastic strain and creep strain by the Modified Bailey equation.

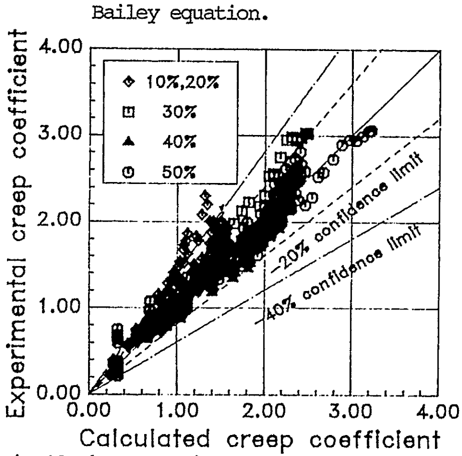

Fig.12 The comparison between calculated data by the Bailey equation and test data.

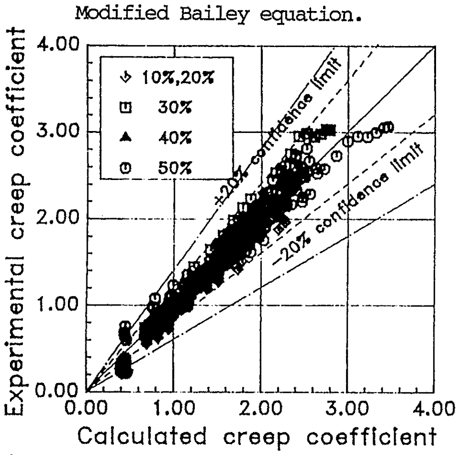

Fig.14 The comparison between calculated data by the Modified Bailey equation and test data.

In order to express the turning point of the relationship between creep strain and elastic strain occuring at the stress of 40%, a new non-linear creep compliance which introduces indeterminate coefficients-c_1 and c_2 into the Bailey equation is proposed. This new non-linear creep compliance is shown in Eqs. (6) and (7) and referred to as the Modified Bailey equation in this paper.

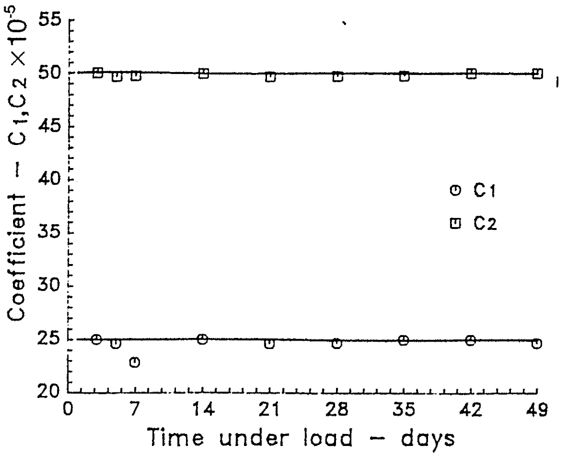

Fig.15 The optimal value of coefficient-c1,c2.

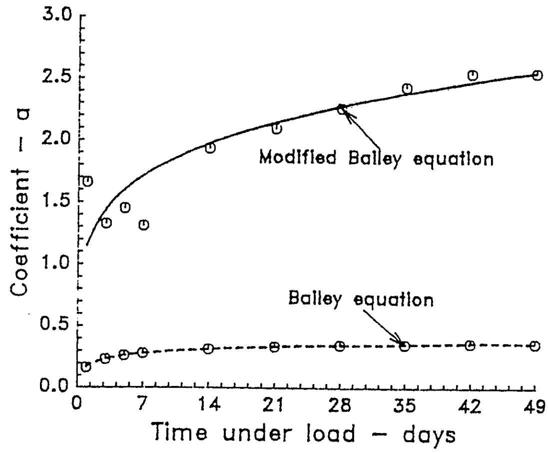

Fig.16 The optimal value of coefficient-a.

In the case that $\varepsilon_0 < c_2$;

$$\varepsilon_{cr} = a\left(\frac{c_2-c_1}{c_2}\varepsilon_0\right)^b \qquad (6)$$

In the case that $\varepsilon_0 \geqq c_2$;

$$\varepsilon_{cr} = a\left(\varepsilon_0-c_1\right)^b \qquad (7)$$

The solid curve in Fig. 13 is drawn according to the Modified Bailey equation whose indeterminate coefficients are calculated by the regression. The relationship between creep strain and elastic strain shown in Fig. 13 is obtained at 28th day from the first application of load. As is evident from Fig. 13, the Modified Bailey equation proposed in this paper can accurately model the actual relationship between creep strain and elastic strain.

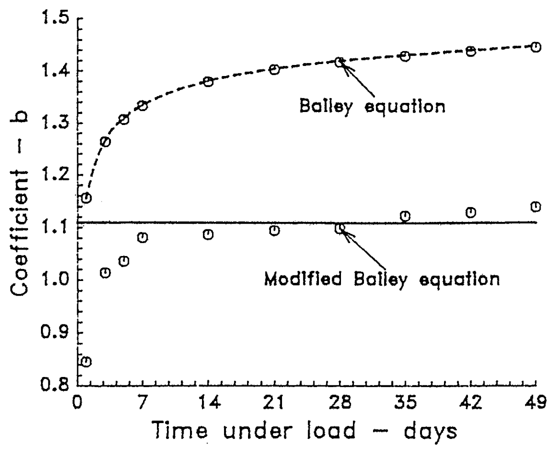

Fig.17 The optimal value of coefficient-b.

Figure 14 shows the comparison between the creep coefficients given by the Modified Bailey equation and the experimental creep coefficients. The optimal value of indeterminate coefficients, such as a, b, c_1 and c_2, in the Modified Bailey equation are obtained by regression based on experimental data at every measuring period. In Fig. 14, the variation of the data calculated by the Modified Bailey equation from the experimental data is within 20% of the confidence limit. We can therefore say that the nonlinear characteristics of creep strain are accounted for in the Modified Bailey equation by the new coefficients c_1 and c_2.

Figure 15 shows the optimal values of indeterminate coefficients c_1 and c_2 in the Modified Bailey equation. The horizontal axis in this figure shows the time under load. These optimal values are nearly constant irrespective of time. The optimal value of coefficient c_2 corresponds to the elastic strain produced by the stress of 40% strength. In other words, it is verified that the coefficient c_2 which appears in the Modified Bailey equation can represent the turning point

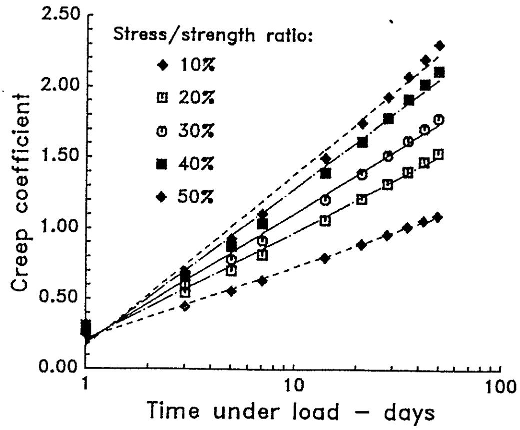

Fig.18 The creep coefficients calculated by Bailey equation.

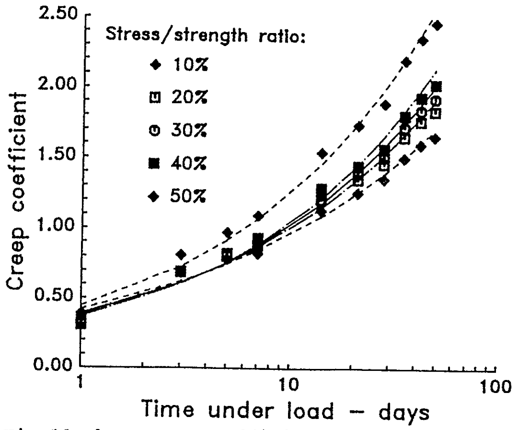

Fig.19 The creep coefficients calculated by Modified Bailey equation.

of the relationship between creep strain and elastic strain lying at the stress of 40% in terms of stress/strength ratio.

Figure 16 shows the optimal values of the indeterminate coefficient a which appears in the Bailey equation and Modified Bailey equation. Figure 17 shows the optimal values of the coefficient b. The horizontal axis in these figures shows the time under load. The optimal coefficient a which appears in the Bailey equation is smaller than that appers in the Modified Bailey equation. The optimal coefficient b which appears in the Bailey equation varies with time, whereas that appears in the Modified Bailey equation is constant independent of time. And, the optimal coefficient b which appears in the Bailey equation is larger than that appers in the Modified Bailey equation.

Figure 18 shows the creep coefficients calculated by the Bailey equation whose indeterminate coefficients are the optimal values determined by regression of the experimental data. As is obvious from this figure, the relationships between the creep coefficients calculated by the Bailey equation and time under load are logarithmic and considerably different from that obtained by experiment as shown in Fig. 8. This means that the error between the calculated data and experimental data becomes larger with time. On the other hand, Fig. 19 shows the creep coefficients calculated by the Modified Bailey equation whose indeterminate coefficients are the optimal values determined by regression of experimental data. The relationships between the creep coefficients calculated by the Modified Bailey equation and time under load are approximated by power expression and is very similar to that obtained by experiment as shown in Fig. 8. The creep coefficients for the stress of 50% strength are considerably larger than the others. However, in the calculated results, the difference between the creep coefficients for the stress of 10% strength and the creep coefficients for the stress of 40% strength is larger with time, which disagrees with the experimental observations. Except for this single inconsistency, however, creep phenomena based on the Modified Bailey equation agree sufficiently with the results of the experiment. Therefore, it is clear that Modified Bailey equation for nonlinear creep prediction is accurate enough to describe the nonlinear behaviour of creep strain of concrete under constant stress.

4. PROPOSITION OF A NON-LINEAR CREEP PREDICTION EQUATION

By using the Modified Bailey equation, it is possible to represent the relationship between the creep strain and elastic strain of concrete. But the Modified Bailey equation will not be useful unless it is a function of time, especially when it comes to calculating the creep strain of concrete under variable stress. Furthermore, it may be meaningless to represent the relationship between creep strain and elastic strain by the Modified Bailey equation if the coefficient c_2 appearing in this equation always corresponds to the elastic strain produced by the stress of 40% strength as the applied service load is usually below 40% in terms of stress/strength ratio.

In this section, we establish a more general creep prediction equation to incorporate the effects of the age at the first application of load, drying time and the period of the application of load into the Modified Bailey equation. Furthermore, we investigate the effects of both the water curing period and the drying period on the nonlinearity of creep of the concrete.

Table 2 Mix proportion of concrete.

Max size (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit weight(kg/m³)			
					W	C	S	G
20	9~12	1.2	60.0	47.7	200	333	842	963

Table 3 Water curing period and drying time.

Water curing period (days)	Age at the loading (days)				
3	3	10	24	52	94
7	7				
14	14	21	35	63	105
28	28				
56	56	63	77	105	

4.1 Experiment outline

The type of cement used was normal portland cement (specific gravity : 3.15). The fine aggregate was river sand (specific gravity : 2.62, water absorption : 1.78, F.M. : 2.81), and the coarse aggregate was crushed stone (specific gravity : 2.73, water absorption : 0.76, F.M. : 6.68). The mix proportion of the concrete is shown in Table 2.

The experiment was performed in a constant temperature and constant relative humidity room at 20±1℃., 68±7%. The size of the prism specimen used for measuring creep strain was 10cm×10cm×38cm. The size of the prism specimen used for measuring shrinkage strain was 10cm×10cm×40cm. Two pairs of point gauges were put on each surface, except for the treated surface and the side opposite the treated surface. Measurements of strain were made by using a Whittemore strain meter with minimum divisions of 1/1000mm. The measuring period was 200 days. Because of the loss of prestress due to shrinkage, creep and relaxation of the steel which occurs with time, each specimen was prestressed again on 3rd, 10th, 30th and 70th day from the first application of load. The permissible error of applied stress was 2%. Table 3 shows the water curing period and the age at the first application of load. The total number of specimens subjected to each stress of 10%, 20%, 30%, 40% and 50% were 1, 2, 2, 2 and 2, respectively. But when the strength of concrete exceeded 30 Mpa, a specimen subjected to the stress of 30% strength was added. And when the strength of concrete was below 30 Mpa, a specimen subjected to the stress of 50% strength was added.

4.2 Results

Figures 20 and 21 show the relationship between creep strain and elastic strain of concrete, with the stress being applied immediately after water curing for 3 days and 56 days, respectively. Figure 22 shows the relationship between creep strain and elastic strain of concrete whose water curing period is 3 days and whose age at the first application of load is 94 days. Figure 23 shows the

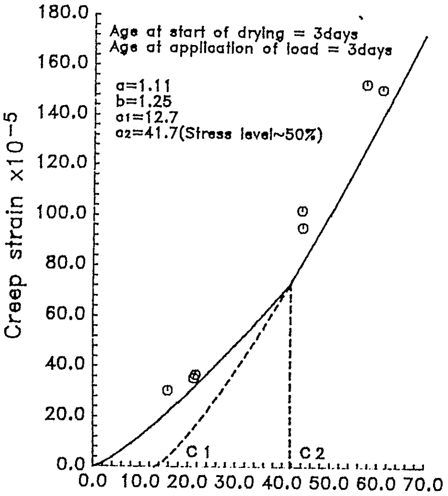

Fig.20 The relationship between creep strain and elastic strain.

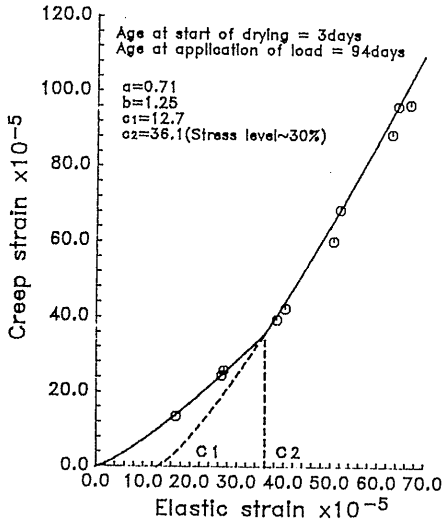

Fig.22 The relationship between creep strain and elastic strain.

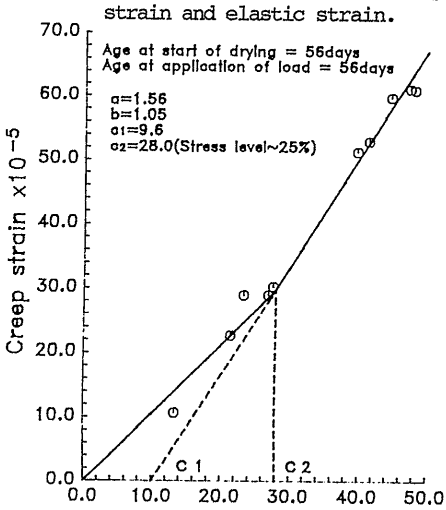

Fig.21 The relationship between creep strain and elastic strain.

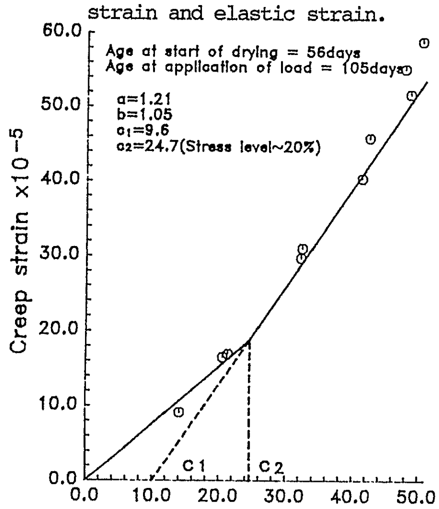

Fig.23 The relationship between creep strain and elastic strain.

relationship between creep strain and elastic strain of concrete whose water curing period is 56 days and whose age at the first application of load are 105 days. The data in these figures are obtained at 30th day after the first application of load. As is evident from these figures, the Modified Bailey equation represents the experimental results very well.

From the optimal value of indeterminate coefficient b shown in Figs. 20 ~ 23, it is clear that the longer the water curing, the smaller the nonlinearity of creep strain. And, it is confirmed that the nonlinearity of creep strain is same, even if the age at the first application of load is different, if the duration of water curing is the same.

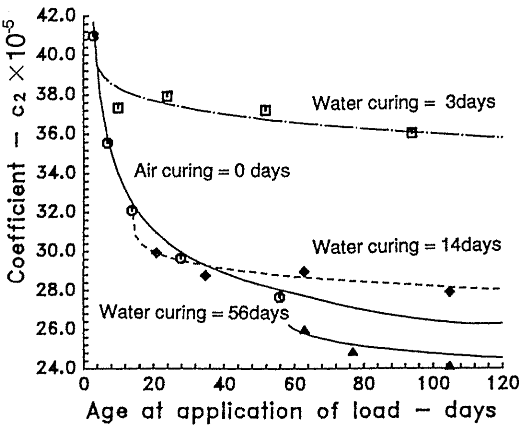

Fig.24 The change of c2 with curing period.

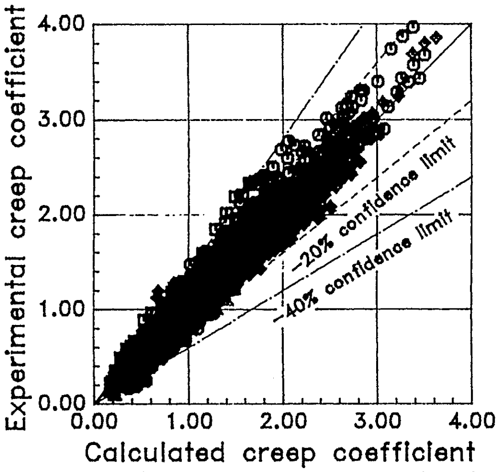

Fig.25 The comparison between calculated data by the present non-linear creep prediction equation and test data.

The Modified Bailey equation which incorporates the effects of the age at the first application of load, drying time and time under load is given by Eqs. (8) and (9). The coefficients involved in Eqs. (10) ~ (13) are calculated by the hybrid method from the experimental data of concrete cured in accordance with Table 3.

in the case that $\varepsilon_0 < c_2(t',t_0)$;

$$\varepsilon_{cr}(\varepsilon_0,t,t',t_0) = a(t,t',t_0)\left(\frac{c_2(t',t_0)-c_1(t_0)}{c_2(t',t_0)}\varepsilon_0\right)^{b(t_0)} \tag{8}$$

in the case that $\varepsilon_0 \geqq c_2(t',t_0)$;

$$\varepsilon_{cr}(\varepsilon_0,t,t',t_0) = a(t,t',t_0)\times(\varepsilon_0-c_1(t_0))^{b(t_0)} \tag{9}$$

in which,

$$a(t,t',t_0) = 2.64\,t_0^{0.114}\{0.002(t'-t_0)+1\}^{-2.9}\left(\frac{t-t'}{262+(t-t')}\right)^{0.434} \tag{10}$$

$$b(t_0)=0.285\exp(-0.047t_0)+1 \tag{11}$$

$$c_1(t_0)=\left(\frac{9.62}{t_0}+9.81\right)\times10^{-5} \tag{12}$$

$$c_2(t',t_0)=47.1\times10^{-5}\{\log_e(t_0+1)\}^{-0.372}\exp\{-0.055(t'-t_0)^{0.214}\} \tag{13}$$

where, $\varepsilon_{cr}(t,t',t_0)$: the virgin creep strain ($\times10^{-5}$)
t : the age of the concrete (days, $t \geqq t'$)
t': the age at the first application of load (days, $t' \geqq t_0$)
t_0 : the age at the start of drying (days)
ε_0 : the elastic strain produced by stress ($\times10^{-5}$)

The optimal values of coefficients b, c_1 and c_2 calculated by the hybrid method are regarded as constants independent of time. The drying time affects all optimal indeterminate coefficients in the Modified Bailey equation, and the age at the first application of load influences both coefficients a and c_2.

Figure 24 shows the relationship between the optimal value of coefficient c_2 and the curing period. From this figure, the longer curing period, the smaller the optimal value of coefficient c_2. The optimal values of coefficient c_2 shown in Fig. 24 never correspond to the elastic strain produced by the stress of 40% strength, and are smaller than those shown in Fig. 15. This means that the optimal values of coefficient c_2 are affected by curing period, curing method, mix proportion of the concrete and so on, and that the turning point of the relationship between creep strain and elastic strain of concrete under constant stress exists below the stress level of 40% in terms of stress/strength ratio.

Figure 25 shows the comparison between the experimental creep coefficients and the creep coefficients calculated by the Modified Bailey equation given by Eqs. (8) and (9). The total number of experimental data points is 3,200. It is evident that Eqs. (8) and (9) represent the experimental result as precisely as the case shown in Fig. 14, in spite of the fact that Eqs. (8) and (9) involve the age "t", age at the first application of load "t'" and age at the start of drying "t_0".

5. CONCLUSION

It has been confirmed that an explicit upper limit and lower limit of proportionality of concrete creep strain to stress do not exist. It is also clear that the creep strain of concrete does not increase uniformly with stress, but rather that there is turning point in the relationship between creep strain and stress. In representing such a nonlinear phenomenon of creep strain of concrete by the Modified Bailey equation presented by the authors, the variation of the calculated data from the experimental data is within 20%, which is half the range covered by the linear creep compliance.

The Modified Bailey equation may be imperfect, since it does not involve properties of concrete such as strength, mix proportion, curing method, environmental conditions and so on. However, by investigating the effect of basic properties on the coefficients in the Modified Bailey equation, we will be able to develop a creep prediction equation which is more precise than the linear creep prediction equation.

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References

[1] Freudenthal, A. M. and Roll, F., Creep and Creep Recovery of Concrete under High Compressive Stress, ACI Journal, 54, pp. 1111 ~ 1142, 1958

[2] Concrete Committee of JSCE, "Standard Specification for Design and Construction of Concrete Structures", Concrete Library Special Publication, No. 1, pp. 32 ~ 35, 1986

[3] CEB-FIP, Model Code for Concrete Structures, Vol. II, International System of Unified Standard Codes of Practice for Structures, [Comite Euro-International du Beton-Federation International de la Precontrainte], 1978

[4] L'Hermite, R. G., Volume Changes of Concrete, Proc. Fourth Int. Symp. on the Chemistry of Cement, Vol. 2, Washington DC, pp. 659 ~ 694, 1960
[5] Sheikin, A. A. and Baskakov, N. J., The Influence of Mineralogical Composition of Portland Cement on the Creep of Concrete in Compression, Stroitel'naya Promyshlennost, No. 9, pp. 39 ~ 40, 1955, Translation No. 236, Department of Scientific and Industrial Research, 1956
[6] Klieger, P., Early High-strength Concrete for Prestressing, Proc. World Conf. on Prestressed Concrete, San Francisco, pp. A5-1 ~ A5-14, 1957
[7] L'Hermite, R. G. and Mamillan, M., Further Results of Shrinkage and Creep Tests, Proc. Int. Conf. on the Structure of Concrete, Cement and Concrete Association: London, pp. 423 ~ 433, 1968
[8] CEB-FIP, Model Code 1990, [Comite Euro-Internatinal du Beton-Federation International de la Precontrainte], (Draft), 1990
[9] ACI Committee 209, Prediction of Creep, Shrinkage and Temperature Effects on Concrete Structures, ACI-SP-76, 1982
[10] Bazant, Z. P. and Panula, L., Simplified Prediction of Concrete Creep and Shrinkage from Strength and Mix, Structural Engineering Report No. 78-10/6405, Department of Civil Engineering, Northwestern University, Evanston, Illinois, pp. 24,1978, Oct.
[11] CEB-FIP, Internatinal Recommendations for the Design and Construction of Concrete Structures, [Comite Europeen du Beton-Federation Internatinale de la Precontrainte], 1970
[12] Sakata, K., Ayano T. and Hiromura O., Prediction of Creep of Concrete, Proceedings of the Japan Concrete Institute, Vol. 10-2, pp. 271 ~ 276, 1988 (in Japanese)
[13] Sakata, K. and Kohno, I., Prediction of Shrinkage and Creep of Concrete, Memoirs of the School of Engineering, Okayama University, Vol. 21-1, 1986, pp. 57 ~ 80
[14] Dilger, W. H, Private communication
[15] Nakagawa, T. and Koyanagi, Y., "The Analysis of Experimental Data by Least Squares Method," Tokyo Univ. Press, 1989, pp. 110 ~ 119. (in Japanese)