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INVESTIGATION OF THE COMPRESSIVE FATIGUE CHARACTERISTICS OF CONCRETE AND THE CHARACTERISTIC VALUE OF FATIGUE STRENGTH

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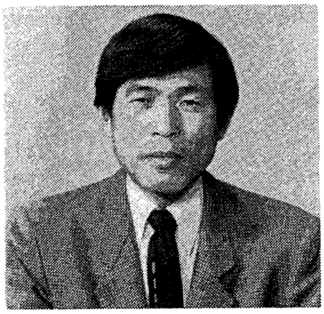

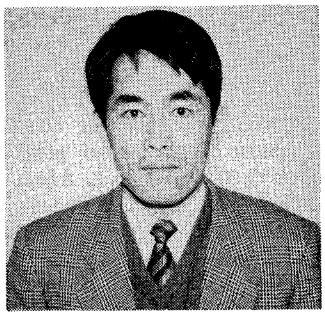

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SYNOPSIS

Compressive fatigue tests were carried out on about 240 concrete cylinders and the following characteristics were investigated: i) the form of the fatigue-life probability distribution; and ii) the effects of specimen size, concrete properties (including static strength), and frequency of loading on the fatigue strength and the scatter in fatigue life.
Test results indicate that fatigue-life distributions are logarithmic normal distributions or 3-parameter Weibull distributions, and that if fatigue tests on specimens with a diameter-to-maximum aggregate-size ratio of five or more are conducted, at loading frequencies from 0.1 to 15 Hz, factors such as concrete strength, specimen size, and loading frequency hardly affect the fatigue strength, though the scatter in static strength significantly affects the scatter in fatigue life.

Keywords: Fatigue, S-N curve, Fatigue life, Fatigue strength, P-S-N curve

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1. INTRODUCTION

Many researchers have noted that large variations are measured in the fatigue life of concrete even when fatigue tests are performed under identical conditions. This large scatter has been considered an intrinsic property of the fatigue behavior of concrete. Consequently, in order to stipulate rational fatigue limits in the design of concrete structures and to improve the accuracy of fatigue safety checks, it is important to clarify the probabilistic behavior of a concrete's fatigue life.

The fatigue limit was added to serviceability and ultimate limit states as a stipulation in the Standard Specifications for Design and Construction of Concrete Structures published by the JSCE in 1981 (called JSCE's Specifications in this paper) [1]. Also the stipulated provisions for fatigue limit state is that (i) an examination of fatigue safety must be performed when the ratio of variable load to total load, or the number of applied cycles, is large, and (ii) a characteristic value of fatigue strength derived from tests which consider the type of concrete and its exposure conditions shall be used in examinations of fatigue safety.

However, these provisions entrust the actual examination of fatigue safety to the engineer. Today, new materials and new types of structure are being developed all the time, and the number of concrete structures put up in untested environments (such as at sea) is increasing every year. Consequently, durability design is now being put in practice, and these circumstances demand better engineering judgment, which should include planning and execution of experiments and analysis of the results.

In this paper, the form of the probability distribution for concrete fatigue life — necessary to determine the characteristic value of lcompressive fatigue strength — is investigated. That is, we investigate which of several proposed probability distribution functions best matches the measured data. In addition, the effects of specimen size, concrete properties (including static strength and its scatter), and frequency of loading on the fatigue strength and the scatter in fatigue-life is also investigated.

2. EXPERIMENTS

2.1 Mix proportions and specimens

The concrete mix proportions, the size of cylindrical specimens made from each mix, and the compressive strength measured during fatigue testing are summarized in Table 1. Ordinary portland cement was used in this study. The coarse aggregate was crushed stone and the fine aggregate was a mixture of river sand and crushed sand. The mix design conditions used to select the mix were as

Table 1 Mix proportion

Mix	M.S. (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit weight (kg/m³) W	C	S	G	Size of specimen	f_c (kgf/cm²)	V[f_c] (%)	n
Mix I	20	5±1	5±1	66	43	165	250	786	1074	φ10 × 20 cm	246 (24.1)	5.0	15
Mix II	15	5±1	5±1	61	46	170	280	823	1004	φ7.5× 15 cm	417 (40.9)	3.5	15
Mix III	15	5±1	5±1	65	43	162	250	848	1034	φ7.5× 15 cm	247 (24.2)	6.4	12
Mix IV	20	5±1	5±1	58	43	163	280	780	1070	φ10 × 20 cm	424 (41.6)	6.7	8

( ): Compressive strength in Mpa   V[f_c]: Coefficient of variance   n : Number of specimen for static test follows: slump=5 + 1, air content=5%, target compressive strength at the age of 28 days, fc_{28}=210 kgf/cm^2 (for Mix I and Mix III shown in Table 1) and fc_{28} = 320 kgf/cm^2 (for Mixes II and IV). The concrete was mixed in a 100 l open-pan mixer. The concrete was poured into moulds in two layers, and each layer was compacted with a rod and then by vibration.

The specimens were cured for 27 days in water maintained at 20+1 °C after removal from the moulds at the age of 1 day. Thereafter, the specimens were stored in the laboratory under normal conditions until the time of the fatigue tests (at more than 100 days).

2.2 Test procedure

Before beginning the fatigue tests, the mean static strength ($\overline{fc}$) was determined in static compression tests using 8~15 cylinder specimens selected randomly for each mix proportion (refer to Table 1).

In the fatigue tests, the maximum stress ratio (S) which was determined as a percentage of the mean of static strength ($\overline{fc}$ in Table 1) for each mix proportion, was varied from about 70 ~ 90% of $\overline{fc}$, while the minimum stress ratio was fixed at 10% of $\overline{fc}$ in all tests. The fatigue tests were carried out using a pulsator fatigue testing machine with a capacity of 20 kN. A load varying sinusoidally with time was used.

a) Investigation of the form of fatigue-life probability distribution
Specimens 10 cm in diameter and 20 cm in height made from Mix I were used in this experiment. The fatigue test was carried out with the fatigue testing machine set to a constant frequency of 5 Hz. Four levels of S (85, 80, 75, and 70%) were selected and fatigue tests using about 30 specimens for each maximum stress ratio were conducted.

b) The effects of specimen size and concrete strength on fatigue life
Specimens used in this test were 10 cm in diameter and 20 cm in height (for Mixes I and IV in Table 1) and 7.5 cm in diameter and 15 cm in height (for Mixes II and III). Fatigue tests were carried out using a pulsator testing machine as well as a servo-hydraulic-control fatigue testing machine with a capacity of 25 kN at Kyoto University. In these tests, six levels of maximum stress ratio were selected and the loading frequency was fixed at 5 Hz.

c) Estimation of the effects of loading frequency on fatigue life
Specimens used were of Mix II (size of specimen: ϕ7.5X15 cm). These fatigue tests were conducted using a 25 kN electric servo-hydraulic-control fatigue testing machine at maximum constant stress ratio of S=90%. This test was performed at loading frequencies y of R=0.01, 0.1, 0.5, 1, and 5 Hz.

3. RESULTS AND DISCUSSION

The results of the fatigue tests are summarized in Table 2. In the table, R is the loading frequency, N is fatigue life, r is the order statistic; that is ordinal numbers of fatigue life arranged in order from young to old.

According to the mean rank method [2], the probability of survival (P(N)) corresponding to the rth specimen arranged in increasing order of age of total number (L) of specimens tested under the same test conditions is calculated from Eq.(1).

$$P(N) = 1 - r/(L+1) \qquad (1)$$

Table 2 Results of fatigue test

(a) R = 5 Hz

r	Mix I : φ10 x 20 cm				Mix II : φ7.5 x 15 cm		
	S=85%	S=80%	S=75%	S=70%	S=82%	S=77%	S=72%
	N(x10)	N(x10)	N(x10)	N(x10)	N(x10)	N(x10)	N(x10)
1	7	223	3215	8785	87	932	16834
2	12	260	3267	9640	98	1296	20987
3	14	290	4270	12332	139	1521	28974
4	16	318	4780	20400	187	1900	40230
5	22	390	5190	24669	222	2318	43581
6	27	391	7096	25950	294	2818	51825
7	29	420	7226	26000	325	3579	75943
8	33	485	7928	28600	386	4317	91325
9	34	500	10050	33628	463	4600	104765
10	41	517	10480	34350	582	4653	156882
11	43	600	10780	37320	805	6065	200000
12	43	614	12140	38280	1084	8995	
13	47	652	13000	39644	1294	10684	
14	49	796	13930	41740	1441	12885	
15	53	852	15640	43257	1854	23164	
16	56	900	16700	45150			
17	58	1074	17275	45870			
18	59	1120	18300	48290			
19	64	1275	19050	57384			
20	76	1420	22640	76105			
21	76	1668	28320	85860			
22	84	1680	29835	92821			
23	105	1728	33780	96070			
24	105	1980	38000	96070			
25	125	2230	41985	152779			
26	128	2400	43820	166689			
27	180	2554	49820	185280			
28	153	2781	58950	202630			
29	178	3180	66155	205921			
30	217	3882	100000	225000			
31	243						
32	271						
33	275						
34	540						

(b) Mix II, φ7.5 x 15 cm, S = 90 %

R (Hz)	r	N
5	1	50
	2	90
	3	140
	4	190
	5	320
	6	460
	7	960
	8	1880
	9	4230
1	1	28
	2	50
	3	80
	4	180
	5	340
1	6	670
	7	1240
	8	2330
0.5	1	24
	2	45
	3	80
	4	90
	5	130
	6	150
	7	230
	8	330
	9	440
	10	630
	11	940
0.5	12	1360
	13	1780
	14	2510
	15	3730
0.1	1	18
	2	38
	3	50
	4	85
	5	120
	6	170
	7	190
	8	310
	9	360
	10	400
0.1	11	510
	12	600
	13	760
	14	940
0.01	1	10 (8)
	2	16 (16)
	3	26 (27)
	4	32 (35)
	5	38 (35)
	6	43 (40)
	7	51 (54)
	8	67 (72)
	9	98 (88)

( ) : Fatigue life obtained by Mix IV

(c) φ7.5 x 15 cm, R = 5 Hz, S = 77 %

Mix III ·fc=250		Mix II fc=400	
r	N	r	N
1	3890	1	9320
2	18880	2	12960
3	24155	3	15210
4	33420	4	19100
5	43451	5	23180
6	49659	6	28184
7	62087	7	35790
8	93325	8	43170
9	115611	9	46000
10	206538	10	46530
		11	60650
		12	89950
		13	106840
		14	128850
		15	231640

3.1 Investigation of the form of the fatigue life probability distribution

As is evident from Table 2, measurements of the fatigue life of concrete exhibit great variability even if the test conditions are identical. A probabilitic treatment is thus the best way to study fatigue life.

Various probability distribution models which directly express the measured fatigue-life distribution have been proposed in the past. When applying these probability distribution models to fatigue life, the relationship between fatigue life (N) at each stress ratio and the probability of survival (P(N)) calculated by Eq.(1) can be linearized as shown in Eqs.(2) ~ (6).

Exponential distribution [3] : $\ln P(N) = -A \cdot N + B$ ------------- (2)

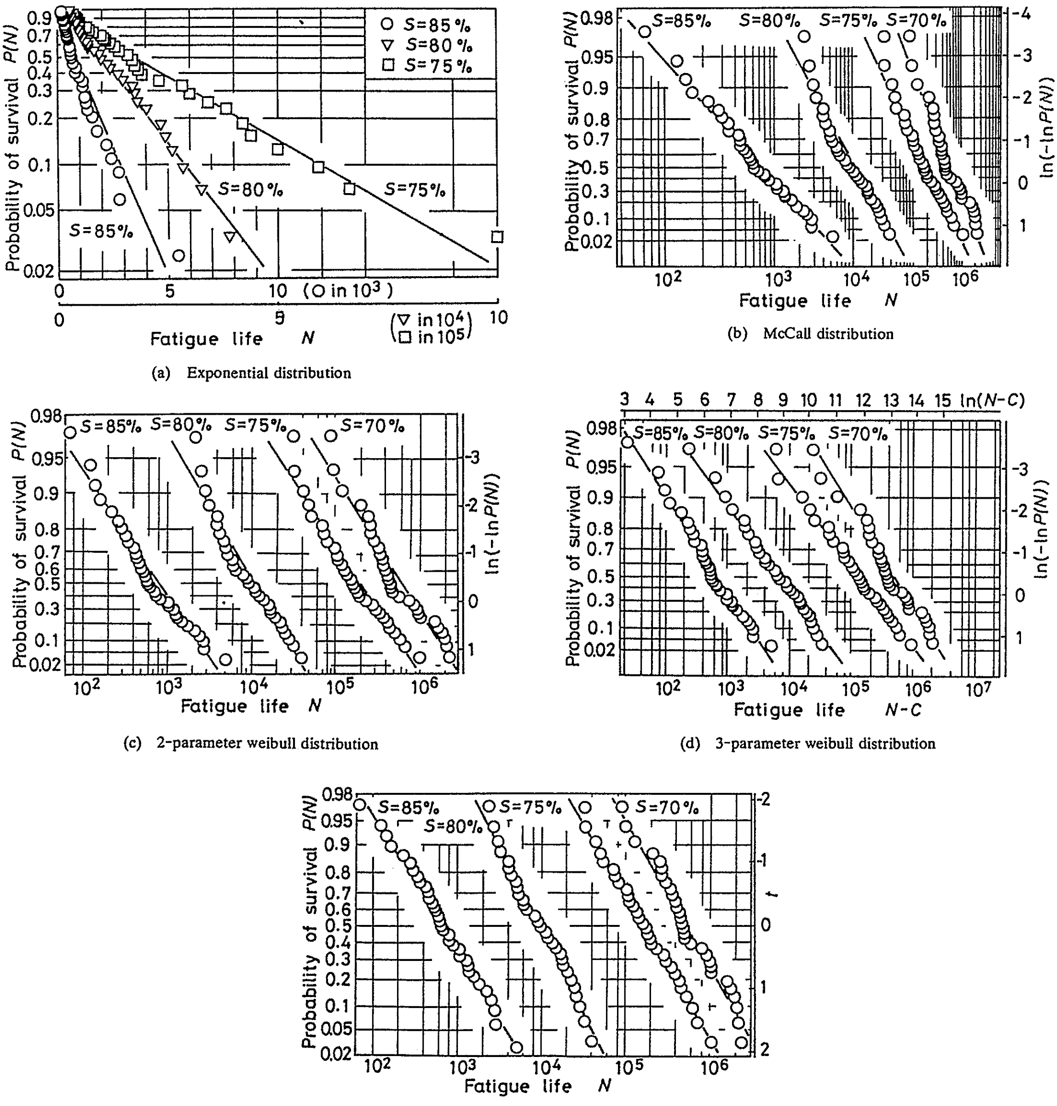
Distribution proposed by McCall (hereinafter called McCall distribution) [4] :
$\ln(-\ln P(N)) = A \cdot \ln(\ln N) + B$ ------------- (3)

2-parameter Weibull distribution : $\ln(-\ln P(N)) = A \cdot \ln N + B$ ------- (4)

3-parameter Weibull distribution : $\ln(-\ln P(N)) = A \cdot \ln(N-C) + B$ ---- (5)

Logarithmic normal distribution [5],[6],[7] :
$t = A \cdot \log N + B, \quad t = \Phi^{-1}(1-P(N))$ --------------- (6)

Here, t is the standard normal variate; that is, the distance from the axis of symmetry of the standard normal probability density function. It can be obtained from the tables of cumulative distribution function, $\Phi(t)$, by substituting a certain value of probability of survival (P(N)=P) for P(N) in Eq.(6).



(a) Exponential distribution

(b) McCall distribution

(c) 2-parameter weibull distribution

(d) 3-parameter weibull distribution

(e) Logarithmic normal distribution

Fig.1 $P(N)-N$ curves

Figures 1(a)~(e), in which the various forms of fatigue-life probability distribution are investigated, show the relationships corresponding to Eqs.(2)~(6) between fatigue life (N) for individual maximum stress ratios (S) and the probability of survival (P(N)) calculated by Eq.(1). They are plotted on probability paper.

Firstly, consider the exponential distribution, as shown in Fig.1(a), and the fatigue failure process of concrete. The value of N corresponding to P(N)=1 represents the potential (critical) cycle (Nc) in which no fatigue failure occurs for applied cyclic loads of less than Nc. The slope of points plotted represents the failure rate (λ(N)) which is defined as the ratio of the number

Table 3 Regression analysis and results of Kolmogrov-Smirnov test

S (%)	Distribution	Regression equation	N: P(N)=0.5	N: P(N)=0.977	D_{max}	D_{ℓ}^{α}	$D_{max}/D_{\ell}^{\alpha}$	ℓ
85	Exponential	$\ln P(N)=-7.7\times10^{-4}\ N-0.174$	674	*	0.176	0.182	0.97	34
	McCall	$\ln(-\ln P(N))=7.228\ \ln(\ln N)-13.942$	693	60	0.090		0.49	
	2-Weibull	$\ln(-\ln P(N))=1.141\ \ln N-7.914$	746	64	0.118		0.65	
	3-Weibull	$\ln(-\ln P(N))=1.005\ \ln(N-47.04)-6.893$	708	70	0.096		0.53	
	Log. normal	$t=2.154\ \log N-6.046$	641	76	0.060		0.53	
80	Exponential	$\ln P(N)=-8.7\times10^{-5}\ N+0.131$	9470	1770	0.081	0.190	0.43	30
	McCall	$\ln(-\ln P(N))=12.265\ \ln(\ln N)-27.596$	9940	1080	0.112		0.59	
	2-Weibull	$\ln(-\ln P(N))=1.341\ \ln N-12.764$	10350	824	0.122		0.64	
	3-Weibull	$\ln(-\ln P(N))=0.902\ \ln(N-2007)-8.362$	9080	2170	0.063		0.33	
	Log. normal	$t=2.560\ \log N-10.137$	9120	1510	0.082		0.43	
75	Exponential	$\ln P(N)=-3.8\times10^{-6}\ N+0.047$	170000	*	0.122	0.190	0.64	30
	McCall	$\ln(-\ln P(N))=14.418\ \ln(\ln N)-36.307$	178900	21550	0.108		0.57	
	2-Weibull	$\ln(-\ln P(N))=1.200\ \ln N-14.921$	185100	10940	0.122		0.64	
	3-Weibull	$\ln(-\ln P(N))=0.883\ \ln(N-26168)-10.822$	165000	29140	0.081		0.43	
	Log. normal	$t=2.288\ \log N-11.911$	160600	21460	0.067		0.35	
70	Exponential	$\ln P(N)=-1.3\times10^{-6}\ N+0.002$	531700	16360	0.137	0.190	0.72	30
	McCall	$\ln(-\ln P(N))=16.184\ \ln(\ln N)-42.188$	568800	46380	0.167		0.88	
	2-Weibull	$\ln(-\ln P(N))=1.230\ \ln N-16.702$	585900	37100	0.179		0.94	
	3-Weibull	$\ln(-\ln P(N))=1.007\ \ln(N-63597)-13.536$	541900	76710	0.145		0.76	
	Log. normal	$t=2.332\ \log N-13.308$	509000	70640	0.122		0.61	

2- and 3- : 2 and 3 parameter Weibull, Log. : Logarithmic * : show negative value α : significant level

of specimens which fail during a single loading cycle — from cycle n to cycle (n+1)— to the number of specimens surviving until the nth cycle. This figure indicates that (i) regardless of the value of fatigue life, the failure rate for a particular maximum stress ratio is almost constant ($\lambda(N)=A$: constant), and (ii) the value of Nc is close to zero. This behavior of $\lambda(N)$ is similar to the creep failure rate for brittle materials such as glass ($\lambda(T)$), in which the parameter is creep failure time, T [3]. This indicates that the physical quantity which controls concrete fatigue failure under repeated constant-amplitude compressive loading may be considered to be the rate of failure.

In the case of concrete, there are unfortunately few studies which attempt to clarify the behavior of failure rate ($\lambda(N)$), though it is thought to be directly connected with the occurrence of fracture cracks and their propagation rate. Consequently, for practical purposes, a number of distribution models have been proposed to fit the measured distribution; there have been few studies of theoretical distributions obtained from an analysis of $\lambda(N)$. In order to clarify the form of the measured fatigue-life distribution of concrete against this background, tests of the other distribution models for goodness of fit are carried out, including one on the exponential distribution.

The relationships plotted in Figs. 1(a)～(e) are almost linear and the distribution of fatigue life at each maximum stress ratio is reasonable in all the distribution models proposed. However, which model exhibits the best goodness of fit as regards the form of the measured fatigue-life distribution must be judged by a test for goodness of fit.

Regression equations, shown in Table 3, are calculated by the least squares method from Eqs.(2)～(6), and the corresponding curves are shown in Fig.1. In Table 3, the Kolmogrov-Smirnov test [5] is used to determine how well the various theoretical distributions fit the measured fatigue-life distributions. In this study, theoretical distributions are expressed by the regression equations in Table 3. It is clear from Table 3 that all measured fatigue-life distributions for individual maximum stress ratios can be considered as obeying the theoretical distributions at a significance level of α=20%, because the maximum difference between the theoretical distribution and measured distribution (Dmax) is less than the critical value (D_ℓ^α) obtained from the Kolmogrov-Smirnov Table for a significant level of α=20%. However, looking at the values of Dmax/D_ℓ^α , those for the 3-parameter Weibull and logarithmic normal distributions are smaller than for the exponential, McCall, and 2-parameter Weibull distribution; as a result, the measured fatigue-life distribution can be expressed to better accuracy by using the 3-parameter Weibull or logarithmic normal distribution. The probable fatigue lives, obtained by substituting P(N)=0.5 (t=0) and P(N)=0.977 (t=-2) into P(N) of the regression equations, are also given in the table. It is clear that the probable fatigue life in the region of P(N) below P(N)=0.977 is not significantly different from the values calculated from the logarithmic normal distribution and the 3-parameter Weibull distribution.

The experimental constants A and B in Eq.(6) of the logarithmic normal distribution can be calculated directly by regression analysis using the least squares method only. On the other hand, constants A, B, and C in the 3-parameter Weibull distribution cannot be found directly, and must be calculated by the following procedure: (i) calculations of a set of three constants A, B, and C in Eq.(5); and (ii) finding of a set of constants value of A, B and C which the absolute value of correlation coefficients has a maximum. The value of A and B for a given value of C , when the value of C changes, can be calculated by using least squares method. This calculation is considerable complicated. Therefore, Considering simplicity, convenience, and utility, it is concluded that the most appropriate distribution for the measured fatigue-life is the logarithmic normal distribution.

3.2 Variation in fatigue life and its mean

When the distribution of fatigue life for each maximum stress ratio obeys the logarithmic normal distribution as given by Eq.(6), the mean fatigue life (the value of N corresponding to P(N)=0.5 (t=0)), represented by $\tilde{N}$, and the mean and standard deviation of logN, m[logN], and V[logN], are calculated as follows:

$$\tilde{N} = 10^{-B/A}, \quad m[\log N] = -B/A, \quad V[\log N] = 1/A \qquad \text{----------- (7)}$$

Figure 2 gives the relationship between fatigue life for each maximum stress ratio and the survival probability calculated by Eq.(1). It is plotted from the results of a fatigue test conducted at the minimum stress ratio of Smin=10% and in the region of maximum stress ratio, S=72~85%. Specimens made with Mix I and

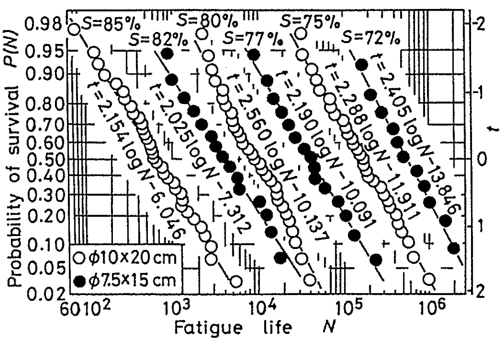


Fig.2 $P(N)-N$ curves

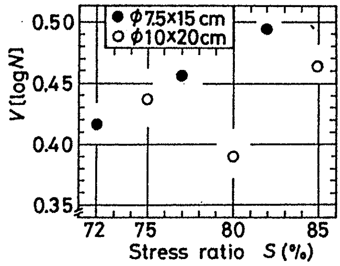


Fig.3 $V[\log N]$ for each fatigue test

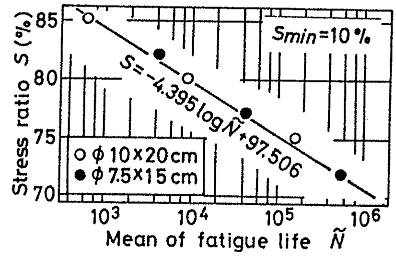


Fig.4 $S-\tilde{N}$ curve

Mix II are used. The regression equations are calculated by the least squares method from Eq.(6) and the corresponding curves are shown in Fig. 2.

Figures 3 and 4 show, respectively, the relationship between maximum stress ratio and standard deviation of logN, V[logN], and mean of fatigue life, $\tilde{N}$; these are obtained by substituting the known constants A and B as in Fig. 2 into Eq.(7). Figure 3 demonstrates that the values of V[logN] for a specimen of diameter 7.5 cm (φ7.5 cm) are somewhat larger than those for a specimen of φ10 cm, though the coefficient of variance in static compressive strength for the φ7.5 cm specimen is smaller than that for the φ10 cm specimen (refer to Table 1). This indicates that the variation in fatigue life becomes larger as the specimen size decreases as in the case of static compressive strength. With regard to the effects of the magnitude of stress ratio on value of V[logN] when the coefficient of variance in static strength is constant, V[logN] increases as the stress ratio rises.

On the other hand, with regard to the mean of fatigue life ($\tilde{N}$), we conclude from the S-$\tilde{N}$ equation in Fig. 4 that $\tilde{N}$ is not affected by differences in mix proportion and specimen size. Accordingly, the relationship between S and $\tilde{N}$, which is common to both specimens used in this study, is the following:

$$S = -4.395 \log \tilde{N} + 97.506 \qquad (8)$$

The fatigue strength at $2X10^6$ cycles, which is obtained by substituting $\tilde{N}=2X10^6$ into the S-$\tilde{N}$ equation (8), is S=69.8%. This value lies in the region between the maximum value, S=70%, and the minimum value, S=60% of fatigue strength obtained in tests within the region of minimum stress ratio (Smin=(2 ~ 22%)) by Sakata et al [5], Matusita et al [6], Raju [9], Ople et al [10], Antrin et al [11], and Bennet et al [12]. Similarly, although the slope of the S-$\tilde{N}$ equation (-4.395) in this study lies between minimum value (-6.369) and maximum value (-4.292) suggested by Raju [9] and Bennet et al [12], it approaches the upper limit of this range. The values of these slopes indicate that the fatigue life of concrete under repeated compressive loading is increased by approximately one order as the maximum stress ratio falls by 5%. The fatigue life is also sham to be very sensitive to small changes in maximum stress ratio.

3.3 Effect of factors such as concrete strength, specimen size, and loading frequency of loading on fatigue life

According to the JSCE's specifications, it is desirable to determine fatigue characteristics through tests using specimens which are similar to the actual structure in all respects. This includes strength and size. However, most fatigue testing machines are limited to a capacity of about 20 kN at most, while the compressive strength of concrete in recent civil structures is 30 MPa or more. Thus, it is almost impossible to carry out fatigue tests on the

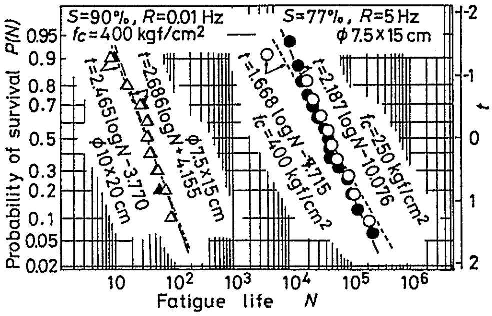


Fig.5 $P(N)-N$ curves

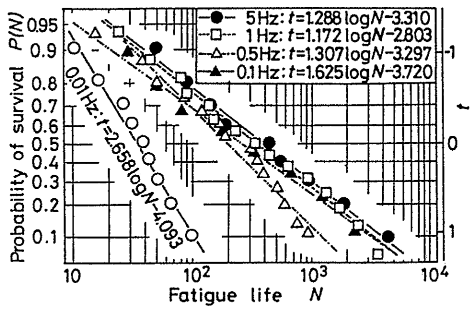


Fig.6 $P(N)-N$ curves

standard types of cylindrical specimens (φ15X30 cm and φ10X20 cm). In order to address this problem and thereby reduce testing time and experimental costs, we here examine the effects of factors such as concrete strength, specimen size, and frequency of loading on the fatigue life.

These tests were carried out on specimens with an air content of 5% and with a specimen diameter to maximum size of coarse aggregate ratio of five. This excludes the effects of both air content and maximum aggregate size on fatigue life, as pointed out by Bennet [13].

Figure 5 shows the relationship between fatigue life and probability of survival in two cases: (a) upper stress ratio of S=77%, loading frequency of R=5 Hz, specimen size of φ7.5X15 cm, concrete strength of about fc=250 kgf/cm^2; and (b) maximum stress ratio of S=90%, loading frequency of R=0.01 Hz, concrete strength of fc=400 kgf/cm^2, specimen sizes of φ7.5X15cm and φ10X20cm. Table 4 gives the results of a variance analysis conducted to examine whether or not there is a difference in the value of logN for the two cases at a selected significant level,α. This analysis of fatigue life variance indicates that no significant difference is exhibited in the fatigue life of each case at the 95% significance level.

Besides, noting the S-$\tilde{N}$ diagram shown in Fig. 4, almost linear relationships are obtained for both φ15cm and φ7.5cm specimens and, if fatigue tests are conducted at the same stress ratio, the mean fatigue life for the two kinds of specimens are almost equal. From these results, we judge that if the applied maximum stress ratio is equal, differences in concrete strength and specimen size barely affect the mean fatigue life.

Figure 6, in which the effects of loading frequency on the fatigue life are examined using data corresponding to the stress ratio of S=90% presented in Table 2, shows the relationship between fatigue life (N) and survival probability on logarithmic normal probability paper. The regression equations calculated by the least squares method from Eq.(6), and the corresponding curves, are shown. It is clear from this figure that the fatigue life corresponding to a particular survival probability (P(N)=P) is extended with an increase in the loading frequency (R) in the region R=0.01～0.1 Hz but is holds almost the same values in the region R=0.1～5 Hz.

In order to quantitatively clarify this characteristic a variance analysis was conducted to determine whether or not the difference in the value of logN in the two cases presented in Table 5 exists at a selected 95% significance level. The two cases were (a): loading frequencies of R=0.1 and 0.01 Hz; and case (b): four loading frequencies of R≥0.1 Hz. The variance analysis in Table 5 shows no

Table 4 Analysis of variance ((a) : ϕ7.5 X 15 cm. R=5 Hz, S=77% (b) : f_c=400 kgf/cm², R=0.01 Hz, S=90%)

	Variance	Sum of squares T	Degrees of freedom f	Mean square S=T/f	Ratio $F_o=S_A/S_E$	$F^{\nu-1}_{k-\nu}$ (α=5%)	Num. of specimen k
(a) f_c=250, 400 kgf/cm²	Between Within	0.017 3.644	$\nu-1=1$ $k-\nu=23$	$S_A=0.017$ $S_E=0.158$	0.11	4.28	25
(b) ϕ10 x 20 cm ϕ7.5 x 15 cm	Between Within	0.001 1.579	$\nu-1=1$ $k-\nu=16$	$S_A=0.001$ $S_E=0.099$	0.01	4.49	18

Table 5 Analysis of variance

Frequency of loading R (Hz)	Variance	Sum of squares T	Degrees of freedom f	Mean square S=T/f	Ratio $F_o=S_A/S_E$	$F^{\nu-1}_{k-\nu}$ (α=5%)	Number of specimen k
0.01, 0.1	Between Within	3.068 4.285	$\nu-1=1$ $k-\nu=21$	$S_A=3.068$ $S_E=0.204$	15.03	4.32	23
0.1, 0.5 1, 5	Between Within	0.593 16.167	$\nu-1=3$ $k-\nu=42$	$S_A=0.198$ $S_E=0.385$	0.51	2.83	46

significant difference among fatigue lives at the loading frequency of R=0.1, 0.5, 1, and 5 Hz but there is a significant difference between fatigue lives at 0.1 Hz and 0.01 Hz.

Considering these variance results and the report by Kesler, et al [14] which says the loading frequency hardly affects fatigue life in the region R=1.2～15 Hz, it is inferred that loading frequencies within the region R=0.1～15 Hz have little effect on fatigue life.

3.4 Characteristic S-N curves

In order to produce a characteristic S-N curve, information on variations in fatigue life is required. Generally speaking, since the variation in static strength of concrete is greater than that of metals, the following two sources are generally thought to contribute to variations in the fatigue life of concrete: (A) variations due to natural phenomena included in fatigue failure process of concrete and metal; and (B) variations induced by variations in the static strength of concrete. For this reason, compressive fatigue for concrete and tensile fatigue for metal are commonly investigated. Particular differences in the S-N equations for concrete and metal are (i) the magnitude of fatigue strength (in MPa) of concrete in the same cycle is lower by one order than that of metal, (ii) S-N curves for concrete give a straight line on semi-log graph paper, where S is plotted on the normal scale, but with metal the line is straight when plotted on log-log scales. This indicates that the fatigue life of concrete is more sensitive to small changes in stress than that of metal, and variations in concrete fatigue are mainly a results of (B) above.

With respect to compressive fatigue failure using cylindrical specimens, cracks propagate continuously in the circumferential direction and specimens eventually separate into two conical shapes in failure. Thus, it can be assumed that this fracture process is not a local failure due to stress concentration which induces metal fatigue under repeated tensile stress, but is rather complete failure after the stress is fully redistributed. Consequently, from the viewpoint of (A), complete failure makes the fatigue life of concrete uniform; that is, it gives rise to less variation in fatigue life. From the viewpoint of

(B), complete failure gives rise to an equivalent effect; making the apparent variation in static strength small.

The variation in fatigue life of concrete is thought to be closely related to the complex behavior described above. However, it is generally agreed that the variation is a results of (A) when the variation in static strength of the concrete is very small or nonexistent and a result of only (B) when the variation in static strength lies within a certain range.

The validity of this argument, that is, whether the variation in fatigue life can be predicted by variations in the static strength only or not, is examined using concrete with a coefficient of variance in static strength of V[fc]=3.5 ~13.5%. The effect of variations in static strength on the variation in fatigue life is then examined by assuming that the relationship between mean stress ratio, $\overline{S}$, defined by the ratio of applied stress, σ_{max}, to the mean static strength, $\overline{fc}$, and the most probable value of fatigue life, $\overline{N}=N(\overline{S})$, is already known and that this $\overline{S}$-$\overline{N}$ relationship is described by Eq.(8) or Eq.(9) as a Goodman type of $\overline{S}$-$\overline{N}$ equation. A design S-N equation based on the results of the variations in static strength and fatigue life will be considered later.

$$\log \overline{N} = \log N(\overline{S}) = \frac{A1\cdot\overline{S} + B1}{K\dfrac{1-\overline{S}}{1-\overline{S}min}} \quad \text{or} \qquad \text{------------------ (9)}$$

where $\overline{S}min$ is the mean of minimum stress ratio and A1, B1, and K are the already-known experimental constants.

In performing the calculation, the assumptions made for the concrete are that ① the distribution of static strength obeys a normal distribution, and ② the distribution of fatigue life obeys a logarithmic normal distribution.

According to assumption ①, when a sample corresponding to a survival probability of P[fc]=P is taken from the concrete population whose mean and coefficient of variance of static compressive strength are known as $\overline{fc}$ and V[fc]=V, the corresponding strength of the sample specimen, fc(t), is given by Eq.(10).

$$fc(t) = \overline{fc}\,(1 + t\cdot V/100)\,, \qquad t = \Phi^{-1}(1-P), \qquad \overline{fc} = fc(0) \quad \text{-----} \quad (10)$$

When the applied maximum stress, σ_{max}, is constant, the true stress ratio for sample specimen, S(t), is calculated using Eq.(11).

$$S(t) = \frac{\sigma max}{fc(t)} = \frac{S(0)}{(1+ t\cdot V/100)}\,, \qquad \overline{S} = S(0) = \sigma max/fc(0) \quad \text{-----} \quad (11)$$

The most probable value of fatigue life, N(S(t)), corresponding to the true stress ratio, S(t), can be obtained by substituting S(t) for $\overline{S}$ in Eq.(9), that is, from Eq.(12).

$$N(S(t)) = \frac{10^{\,A1\cdot S(t)+B1}}{10^{\,K(1-S(t))/(1-Smin(t))}} \quad \text{or} \qquad \text{------------} \quad (12)$$

where Smin(t) is the value of S(t) calculated by replacing σ_{max} in Eq.(11) with minimum stress, σ_{min}.

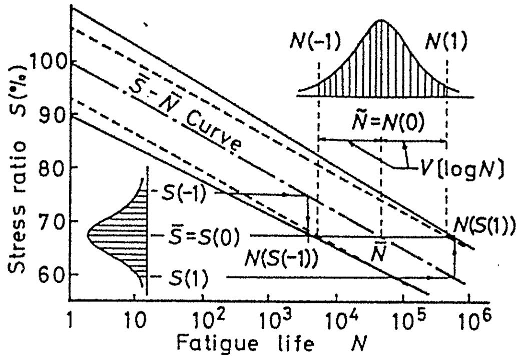


Fig.7 **Relationships between scatter in fatigue life and scatter in static strength**

Consequently, the effects of variations in static strength on the variation in fatigue life can be evaluated from the S(0)-N(S(t)) curve, in which N(S(t)) is calculated using Eq.(11) and Eq.(12) for a certain value of S(0) and t, as shown in the rough sketch in Fig. 7. In this figure, the dash-dot line represents the S(0)-N(0) equation corresponding to P=0.5 (t=0) and the solid lines represent S(0)-N(S(t)) curve at t= $\pm$ 1 (P=0.84, 0.16).

On the other hand, according to assumption ②, when the parameters of logarithmic normal distribution for the mean stress ratio S=S(0), m[logN]= -B/A, and V[logN]=1/A, have been determined from the experimental data as shown in Fig. 2 or Table 3, the probable fatigue life corresponding to a certain probability of survival of P(N)=P (or t= $\Phi^{-1}(1-P)$), N(t), is calculated using Eq.(13).

$$N(t)=10^{m[\log N]+V[\log N]\cdot t} \qquad (13)$$

The broken lines in Fig. 7 represent the S(0)-N(t) curve, which is obtained by substituting t= $\pm$ 1 into Eq.(13).

Holmen [15] has proposed that the S(0)-N(S(t)) curve corresponding to an arbitrary value of t almost agrees with the S(0)-N(t) curve; that is, if the equation S(0)-N(S(0)) obtained, the S(0)-N(t) curve corresponding to an arbitrary survival probability P (t= $\Phi^{-1}(1-P)$), can be predicted from the S(0)-N(S(t)) curve.

In Fig. 8 (A), the validity of Holmen's proposal is examined for the results of this study. The plotted points in the figure are probable fatigue lives, N(t), obtained by substituting the known values of A and B (as shown in Fig. 2) and t=-1 into Eq.(13), the dash-dot line represents equation S(0)-N(S(0)) (coinciding with the S-Ñ equation in Fig. 4), and the solid lines represent S(0)-N(S(t)) obtained by substituting t=-1 and V[fc]=3.5% (for a ϕ7.5X15cm specimen, as shown in Table 1) or 7% (for a ϕ10X20cm specimen) as the coefficient of variance in Eq.(11) and Eq.(12). Figures 8 (B) and (C) show the results of treating Sakata, et al [5] and Matusita, et al [6] using the same procedure.

In all results of this study using ϕ7.5cm specimens and most results of Matusita's study using the same size specimens, the plotted points on the S(0)-N(t=-1) curve almost fit with the solid line corresponding to the S(0)-N(S(t=-1)) curve, so indicating that Holmen's proposal is valid.

On the other hand, in Sakata's results and the results of this study using

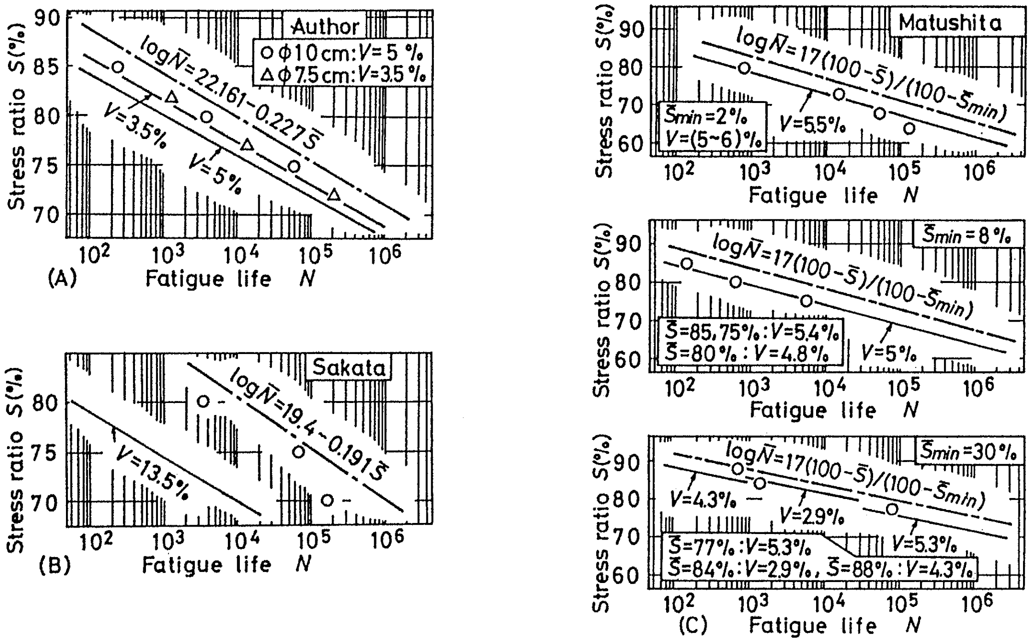


Fig.8 Scatter in fatigue life compared with scatter in static strength

φ10X20 cm specimens, the S(0)-N(S(t=-1)) curve estimated from the variations in static strength is larger than the S(0)-N(t=-1) curve estimated from the variation in fatigue life. Thus, when fatigue tests using specimens bigger than φ7.5X15 cm are conducted, the P-S-N curve for an arbitrary survival probability may be predicted with considerable accuracy and on the safe side by substituting the S(0)-N(S(t)) curve calculated from Eqs. (11) and (12) for the P-S-N curve.

In cast-in-situ concrete, since its coefficient of variance in static strength (V[fc]=V2) is generally considered to be larger than that of laboratory test concrete (V[fc]=V1), the P-S-N curve for the cast-in-situ concrete can be estimated by substituting V=V2 into V in Eq.(11).

Finally, let's consider a design S-N equation. One specification of the JSCE's Standard Specifications is that when no fatigue test to confirm fatigue performance is carried out, the design fatigue life, N_d, should in general be computed using Eq.(14).

$$\log N_d(S(t)) = K \frac{1 - \gamma_c \cdot S(t)/k1}{1 - \gamma_c \cdot Smin(t)} \qquad (14)$$

where γ_c=1.3 is a material factor and k1=0.85, and K=17 for compression.

Putting k1=1, γ_c=1 in Eq.(14), it agrees with Eq.(12). In this case, it is clear, as stated before that the P-S-N curve for an arbitrary value of survival probability, P, can be estimated with considerable accuracy from Eq.(12). Examining the S-N equation obtained from tests conducted in a region of lower stress ratio, Smin=2~22%, the mean fatigue life becomes longer (or shorter) by approximately one order with each 5% decrease (increase) in maximum stress ratio. Still requiring examination is the most appropriate value of γ_c for the fatigue limit state, but the design fatigue life, $N_d(S(t))$, may be assumed to be about 10^6 smaller than N(S(t)) in the case of γ_c=1.3 and k1=1.

4. CONCLUSIONS

This study was carried out to clarify the characteristics of probability distribution in compressive fatigue life, and to determine quantitatively the effect of specimen size, concrete static strength and its scatter, loading frequency, and other factors in fatigue testing on the mean fatigue life and the scatter in fatigue life.

The following is a summary of the results obtained:

(1) The distribution of fatigue life under compressive repetitive loading was confirmed to an exponential distribution, the distribution proposed by McCall, the 2- or 3-parameter Weibull distribution, and the logarithmic normal distribution. Distributions with the best fit are the 3-parameter Weibull distribution and the logarithmic normal distribution. Considering simplicity, convenience, and utility, however, the most appropriate is the logarithmic normal distribution.

(2) When concrete specimens of different sizes have the same coefficient of variance and standard deviation of logN (V[logN]) is used to indicate the amount of variation in fatigue life, the value of V[logN] becomes somewhat larger with decreasing of specimen size.

(3) When fatigue tests are carried out on specimens with a ratio of height to diameter of 2 and with a ratio of specimen diameter to maximum aggregate size of 5, differences in specimen size and concrete static strength have hardly any effect on the mean fatigue life, $\tilde{N}$.

(4) Data obtained from fatigue tests at a high stress ratio of S=90%, loading frequency from 0.01 Hz to 0.1 Hz have some effect on fatigue life, but from 0.1 Hz to 15 Hz, when considering this experimental results and the Kesler's report [14], hardy affect fatigue life.

(5) Variations in fatigue life are closely related to variations in static strength. If the $\bar{S}$-$\bar{N}$ relationship between stress ratio based on the mean static strength and the mean fatigue life corresponding to a stress ratio is known, together with the mean and standard deviation in static strength, a P-S-N curve taking account of survival probability, P, can be estimated with considerable accuracy from Eq.(11) and Eq.(12).

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