CONCRETE LIBRARY OF JSCE NO. 21, JUNE 1993

AN ANALYTICAL EVALUATION OF THE DUCTILITY OF REINFORCED CONCRETE MEMBERS

(Reprinted from Transaction of JSCE, Vol.420/V-16, 1992)



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SYNOPSIS

Effects of geometrical nonlinearities and material nonlinearities of reinforced concret(RC) columns are analyzed to determine its ductility. Axial stress, web reinforcement ratio, longitudinal reinforcement ratio and shear span and beam depth ratio were the factors treated in the analysis. The simplified equation to calculate ductility is proposed based on the results of the analysis. The applicability is shown by the comparison with as many experimental results as can be collected and it was shown that the proposed equation can estimate the ductility for RC members accurately far better than any proposed models in the past.

Keywords: reinforced concrete member, ductility ratio, finite element analysis

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1. Introduction

The seismic design of RC structures requires the estimation of its ductility as accurate as possible. Many experimental studies for RC members subjected to cyclic loads were carried out for this objective and a variety of evaluation methods of the ductility have been proposed on the basis of the results of experiments of RC members. For example, Arakawa et al.[1] proposed an equation to evaluate the ductility for RC member in buildings. On the other hand, Ishibashi et al.[2], Ohta[3], Higai et al.[4], and Machida et al.[5] tried to evaluate the ductility of RC bridge piers. The proposed methods mentioned above, however, are obtained from a small number of experimental data and they do not have wide applicability for RC members.

Moreover, the dimensions of specimens and load conditions are different between civil structures and buildings. For instance, RC members considered in building column are subjected to large axial force and its effect on ductility is dominant. On the contrary, for RC members such as bridge pier considered in civil structures in which its axial force is comparatively small, axial force is not an influential factor on ductility. Since the factors of structural dimensions besides the load conditions are different, the experimental results are not fully utilized up to the present. For this reason, the methods to evaluate ductility for all RC members correctly have not yet been established.

It is obvious that we need a method to calculate ductility of RC members comprehensively in both fields. To obtain the applicable results for all RC members, it is natural to consider that ductility should be evaluated analytically. Analytical study may make possible to combine the experimental results obtained from RC members having the different dimensions and may give us valuable information in establishing a generalized method to evaluate ductility.

The purpose of this paper is to propose an equation to evaluate the ductility correctly for all kinds of RC members. Firstly, the finite element analysis based on the finite displacement theory is performed considering the factors related to the plastic deformation. It has been shown that the finite displacement analysis is able to define the behavior of RC structures up to the ultimate state[6]. Then, the effects of variable factors on ductility are investigated analytically and a simple equation to evaluate ductility was formulated based on analytical results. Finally, reliability of the proposed equation is assured by the vast comparison with the experimental data.

2. Calculation Method

(1) Stiffness Equation

Only a brief outline of the calculation method is described here, since it is presented in detail in reference[6]. The method adopts the finite element analysis based on the finite displacement theory for RC members in which layered beam element are used considering the material nonlinearities accurately.

The element used is a RC beam-column element subjected to axial force, shear force and bending moment at the node. The incremental stiffness equation used in the analysis is as follows.

$$([K] + [K_g])^{(n)} \{ \Delta d \}^{(n+1)} = \{ \Delta F \}^{(n+1)} + \{ F_r \}^{(n)}$$
(1)

where [K] denotes the stiffness matrix, $[K_g]$ denotes the geometric matrix, $\{\Delta d\}$ denotes nodal displacement increment and $\{\Delta F\}$ denotes nodal force increment and $\{F_r\}$ denotes the unbalanced force vector which is introduced when equilibrium in the previous load step is not satisfied strictly.





Fig.-1(b) Stress-Strain Relation for Reinforcement

(2) Material Modeling

Stress-strain relation for concrete used is shown in Fig.-1(a). In zone of compression, the relation is represented by a second degree parabola up to strain of ε_{co} and a linear falling branch up to the strain of ε_u after this point. The slope for the falling branch is determined by Kent and Park model[7]. In zone of tension, the stress increases linearly with a constant proportionality of $2f'_c/\varepsilon_{co}$ up to the tensile strength. And after that, it decreases linearly to the strain of 0.002. The properties of concrete material used in the analysis are as follows. $f'_c($ compressive strength)=30MPa, $f_t($ tensile strength)=3MPa, $\varepsilon_{co}($ strain corresponding to $f'_c)=2000\mu$.

Stress-strain relation for reinforcement is shown in Fig.-1(b). It is assumed that the stress is proportional to the strain with initial stiffness up to the yielding point, and increases linearly with a slope one hundredth of the initial stiffness after that in both tension and compression zone. The material properties of reinforcement used are assumed that the yielding stress (f_y) is 400MPa and the yielding strain (ε_y) is 2500μ .

(3) Calculation Method

A computer program is developed according to the aforementioned discussions. Since the stress and Young's modulus vary with the depth of the element, an element is subdivided into a number of layers. That is, a layered beam element shown in Fig.-2 is used. Integration to obtain the matrices and the unbalanced force can be represented by the sum of the amount in each subdivided element in which the material nonlinearity is assumed. Young's modulus used in the stress-strain relation for each layer is the tangential modulus. Nonlinear behavior of RC member is thus considered accurately.

In the analysis, the stiffness equations are solved iteratively using Newton-Raphson method to compute displacement increment. The analysis was performed by updating stiffness matrix at every load step within each step until the norm of the unbalanced force relative to applied force becomes smaller than 10^{-4} .

3. Effect on Ductility Ratio of Variable Factors

We have showed in the paper[6] that the effect of variable factors for the ultimate behavior of RC members can be represented rationally in the analysis with comparison between the experimental and the numerical results. Moreover, we have reported that the lateral displacement, where the restoring force decrease rapidly in the cyclic loading tests of RC members, correspond

to the lateral displacement of the maximum moment point in the analysis under monotonous loading. Hence, we define the analytical ductility ratio of RC members subjected to cyclic loads by Eq(2), i.e. the ductility in the analysis is so defined as the ratio of the lateral displacement corresponding to the maximum moment point (δ_{Mmax}) and to the displacement at the yielding of the reinforcement in a member(δ_n)[6].

$$\mu = \delta_{Mmax} / \delta_y \tag{2}$$

Once the ductility ratio is defined analytically, the effects on the ductility of variable factors can be investigated numerically.

Figure-3 shows an example comparison of the experimental and the numerical results, in which the web reinforcement ratio were changed. In the figure, solid lines show the skeleton curve of the experimental values with web reinforcement ratio(ρ_w) of 0.077(%), 0.12(%), 0.23(%). The maximum moment points obtained by the analysis are shown with the mark " \bullet " and the characteristic points in experiment, which the restoring force decrease rapidly, are shown with the mark " \blacktriangle ". It can be seen that both points are in good agreement for every web reinforcement ratio.

A model used in the analysis to evaluate ductility ratio is a RC member having a cross section of $20 \times 15(cm)$ and the beam depth of 16(cm), which is shown in Fig.-4. The reason why only one model is used is that the equation to evaluate ductility will be represented with non-dimensional form and will be compared with many experimental results in which the factors were varied widely. The factors selected in the analysis are axial stress(σ_0), web reinforcement ratio(ρ_w), longitudinal reinforcement ratio(P_t) and shear span ratio(A/D). It is generally reported that these factors are closely related to the ductility ratio of RC members.



Fig.-2 Layered Beam Element





Ig.-4 Dimension of Analytical Model(unit;*mm*)

Fig.-3 Comparison between Experimental and Calculated values in which Web Reinforcement Ratio Is Changed



Fig.-5 Relation of Web Reinforcement Ratio and Ductility Ratio

In the analysis, the axial stress range from -2.0MPa to 8.0MPa, the web reinforcement ratio of 0.0% to 1.0%, the longitudinal reinforcement ratio of 0.3% to 1.5% and shear span ratio of 1 to 6 are used. The range of these factors are decided considering the test specimens of experiments carried in the past.

Furthermore, we verify the ductility equations proposed in the past, and obtain the result that axial stress is most important factor to evaluate the ductility of RC members comprehensively. That is, the proposed equations for RC members such as bridge pier underestimate the effect of axial force, since the most experiments were carried out under the small axial force. On the contrary, the experiments for the building columns are carried out under the large axial force and in the proposed equations, axial force is main factor and the effects of factors other than the axial force can not be evaluated accurately.

To investigate the effect of axial force more accurately, the effects of the web reinforcement ratio(ρ_w), the longitudinal reinforcement ratio(P_t) and the shear span ratio(A/D) are investigated in terms of the axial stress(σ_0). That is, the effects on the ductility ratio(μ) of each factor are investigated always in combination with the axial stress, " ρ_w - σ_0 - μ relation" for example. The following is the effects on ductility ratio of each factor and the formulation of the results obtained from the analysis.

The relationship(μ_w) between the web reinforcement ratio and the axial stress ratio(σ_0/f'_c) is formulated first, and the effects of P_t and A/D are given as the coefficient ($\beta_{P_t} = \mu/\mu_{P_t=1.0}, \beta_{AD} = \mu/\mu_{AD=4}$) of μ_w . Therefore, following discussions are based on these forms.

(1) Effect of Web Reinforcement $\text{Ratio}(\rho_w)$ and $\text{Axial Stress}(\sigma_0)$

The analysis which varied the web reinforcement $ratio(\rho_w)$ and the axial $stress(\sigma_0)$ are performed under the condition that the longitudinal reinforcement ratio is 1.0% and the shear span ratio is 4. The effect of the web reinforcement ratio is taken into consideration by varying the slope of a falling branch for concrete using Kent and Park model.

The results of the analysis are shown in Fig.-5. The ductility ratio increase in proportion to the increase of the web reinforcement ratio, which tendency is identified with experimental results. Moreover, it can be understood from the figure that the ductility ratio becomes smaller relatively and the increase rate of the ductility ratio for the web reinforcement ratio also become smaller when the axial stress increase.

The following equation is obtained assuming that the relation between the web reinforcement

ratio and the ductility ratio is linear for each axial stress level. In the equation, the effect of axial stress is considered by the ratio of axial stress(σ_0) and compressive strength of concrete(f'_c) in order to make the equation in non-dimensional form.

$$u_{w} = a + b\rho_{w}$$
(3)

$$a = 2.9e^{-10/3(\sigma_{0}/f_{c}')}$$

$$b = 7.0e^{-7.0(\sigma_{0}/f_{c}')}$$

The example of Eq(3) for σ_0 of 0.0MPa and 8.0MPa are shown in Fig-5 by a solid and a broken line, respectively. It is noted that since the equation is formulated from the results of analysis in which ρ_w is varied from 0.0% to 1.0%, the application range may be below 1.0%.

(2) Effect of Longitudinal Reinforcement $\text{Ratio}(P_t)$ and $\text{Axial Stress}(\sigma_0)$

The analysis which varied the longitudinal reinforcement ratio and the axial stress are performed keeping ρ_w of 0.2% and A/D of 4. The relationships between the longitudinal reinforcement ratio and the ductility ratio for any axial stress level are illustrated in Fig.-6.

Machida et al.[5] reported based on experimental results that the ductility ratio increase with the decrease of the longitudinal reinforcement ratio, and a remarkable effects of longitudinal reinforcement ratio can be seen when P_t is less than 1.0%. The numerical results also show the same tendency, as far as the axial stress is small. The remarkable effect on the ductility ratio of P_t , however, disappear with the increase of the axial stress even if P_t is less than 1.0%.

Figure-7 shows the relationship between β_{P_t} and P_t . Here, β_{P_t} is defined by the ratio of the ductility ratio for P_t of 1.0% and for any longitudinal reinforcement ratio. The effect of the longitudinal reinforcement ratio and the axial stress is formulated by means of β_{P_t} as follows.

$$\begin{aligned}
\beta_{P_t} &= a(P_t)^b \\
a &= 1.03 \\
b &= -0.85e^{-9.0(\sigma_0/f_t')}
\end{aligned}$$
(4)

Here, β_{P_t} is a coefficient to explain the influence of P_t for the ductility ratio. Although Eq(4) is obtained from method of least squares, the equation overestimates the results of analysis in a smaller P_t and underestimates in a larger P_t due to the character of the function. Therefore, the range of β_{P_t} is set as



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$$\beta_{P_t} \leq 2.8$$

The range should differ according to the axial stress states fundamentally, but it is formed independently of σ_0 considering the simplification of the equation. Equation(5) is determined by considering that β_{P_t} hardly change the range of $P_t \ge 1.0$, and Eq(6) is determined by the maximum value obtained from the analysis.

(3) Effect of Shear Span Ratio(A/D) and Axial Stress(σ_0)

The effects of the shear span ratio and the axial stress on ductility are investigated under the condition that the longitudinal reinforcement ratio is 1.0% and the web reinforcement ratio is 0.2%. The shear span ratio, then, is varied to six cases from 1.0 to 6.0 and the axial stress is varied to five cases from 0.0MPa to 8.0MPa.

The results of analysis are shown in Fig.-8. The analytical values decrease in proportion as the shear span ratio decreases for any axial stress level. Especially, it decreases rapidly when A/D is less than 2. The effect, however, does not appear clearly for $A/D \ge 4.0$. Figure-9 shows the effect of the shear span ratio on the ductility ratio using β_{AD} , which is defined by $\mu_{AD}/\mu_{AD=4}$, at the different axial stress state. It is shown that the ductility ratio decreases are appeared by the shear span ratio decreases are appeared by the shear span ratio appeared by the s rapidly in proportion as the shear span ratio decreases. And the increase of the axial stress reduces the effect of the shear span ratio on the ductility, which is the same as in the case of P_t . Finally, Eq(7), which represents the coefficient of μ_w , is obtained as the effects of the shear span ratio on the ductility ratio.

$$\beta_{AD} = a(A/D)^{b}$$

$$a = 5/7e^{8/7(\sigma_{0}/f_{c}^{\prime})}$$

$$b = 7/30e^{-4.2(\sigma_{0}/f_{c}^{\prime})}$$
(7)

However, we add following condition in order to reduce the error of method of least squares considering that A/D over 4 does not influence the ductility ratio.

$$\beta_{AD} \le 1.0 \tag{8}$$

(4) Effect of Strength of Concrete (f'_c)

Since strength of concrete(f'_c) is given a constant value(30MPa) in the analysis, the effect is not investigated directly. The effects of variable factors, however, is evaluated with the ratio of axial stress and strength of concrete(σ_0/f'_c) in previous equations. Therefore, the effect of strength of concrete may be considered indirectly in these equations. The ductility ratio are estimated small values relatively for smaller values of f'_c , because the effect can be identified with a larger axial stress using the axial stress ratio (σ_0/f'_c) . And the ductility ratio is estimated a large value for a larger f'_c . These facts are doubtlessly similar to the experimental results.



(5) Effect of Axial Stress(σ_0)

The effect of the axial stress is already mentioned in the effects of ρ_w , P_t , A/D. That is, the effect on ductility is remarkable and the effects of other factors disappear accordingly as the axial stress increase. It is reported that the axial stress is the most influential factor and the other factors hardly influence in the experiment for RC members such as columns in buildings subjected to large axial force. The analytical results confirmed this fact. On the contrary, for RC members such as bridge piers in which the axial force level is small, the effects of the factors other than the axial stress must be evaluated correctly.

The results when the axial stress are changed from -2.0*MPa* to 12.0*MPa* with constant values of A/D = 4, $P_t = 1.0\%$, $\rho_w = 0.1\%$, are shown in Fig.-10. The ductility ratio become smaller with increasing axial stress. It is noted that the longitudinal reinforcement in compression yield earlier than in tension for the axial stress over 10MPa ($\sigma_0/f'_c \ge 1/3$) and the ductility ratio increases greatly under the tensile axial force in the analysis. Although such cases are ignored in the proposed equations, some detailed investigation may be needed.

4. Proposed Equation to Evaluate Ductility

(1) Simple Equation to Evaluate Ductility

The effect on the ductility of variable factors for RC members are evaluated quantitatively based on the results of the analysis described in the previous paragraph. Basing on these results, we propose the equation to evaluate the ductility ratio, which is formed to combine a series of equations to estimate ductility as a term of ductility factors.

$$\mu = \mu_{w} \cdot \beta_{P_{t}} \cdot \beta_{AD}$$

$$\mu_{w} = a + b\rho_{w}$$

$$a = 2.9e^{-10/3(\sigma_{0}/f_{c}^{t})}$$

$$b = 7.0e^{-7.0(\sigma_{0}/f_{c}^{t})}$$

$$\beta_{P_{t}} = a(P_{t})^{b}$$

$$a = 1.03$$
(9)

$$b = -0.85e^{-9.0(\sigma_0/f_c')}$$

$$\beta_{P_t} \ge 1.0$$

$$\beta_{P_t} \le 2.8$$

$$\beta_{AD} = a(A/D)^b$$

$$a = 5/7e^{8/7(\sigma_0/f_c')}$$

$$b = 7/30e^{-4.2(\sigma_0/f_c')}$$

$$\beta_{AD} \le 1.0$$

Every terms of the Eq(9) are represented as the function of the axial stress ratio (σ_0/f'_c) , since it is considered that the axial stress is the most important factor for ductility ratio in this study. The proposed equation is formulated such that the ductility ratio of RC member having A/D = 4.0, $P_t = 1.0\%$ is calculated by μ_w which is the function of ρ_w and σ_0/f'_c , the effects of longitudinal reinforcement ratio (P_t) and the shear span ratio (A/D) being given by β_{P_t} and β_{AD} as the coefficient of μ_w . The equation can explain easily the effect on ductility of each factor and can calculate the ductility ratio by hand. This is a feature of the equation.

	Machida	Higai	Ishibashi	Ozaka	Ohta	Akimoto	Arakawa
Pt (%)	0.59~1.06	0.40~1.90	0.12~1.66	0.32~1.26	0.82	0.82~0.86	0.34~0.81
pu(%)	0.00~0.23	0.00~0.99	0.00~0.58	0.18~0.36	0.04~0.16	0.16~0.32	0.15~1.50
a/d	3.0~6.0	3.29~6.25	1.5~4.0	4.0	4.0	4.0	1.7~2.8
σ₀(MPa)	0.0~2.0	0.0	0.0~4.0	0.0~4.0	1.0	0.0~1.0	2.5~7.0
f'c(MPa)	20.3~41.3	15.6~55.2	24.0~42.0	20.2~33.8	29.2	30.5~31.5	20.4~34.5
B x H (cmxcm)	15x20	50x40, 20x30 50x28, 20x20 50x23, 15x30 50x33, 13x30	40x40 60x40 40x50	40x40	80x40	40x40	25x25
number of specimens	15	37	38	12	5	13	50
$\frac{\mu_{cal}}{\mu_{car}} = \frac{\mu_{cal}}{\mu_{car}} + \frac{\mu_{cal}}{\mu_{car}} = \frac{1}{\mu_{car}} = \frac{1}{\mu_{ca$	0.3 0.6 Web reinfor	$\frac{1}{2} \frac{1}{2} \frac{1}$	2 1.5 ; ρ _w (%)		8	$\frac{1}{2} - \frac{1}{2} - \frac{1}{2}$	
0.4 0.8 1.2 1.6 2.0 2.0 4.0 6.0 8.0 Longitudinal reinforcement ratio; $P_w(\%)$ Shear span ratio; A/D							

Table-1 Outline of Specimens

Fig.-11 Ratio of Estimated and Experimental Values(Proposed Equation)



Fig.-12 Ratio of Estimated and Experimental Values(Machida's Equation)



Fig.-13 Ratio of Estimated and Experimental Values(Arakawa's Equation)

(2) Reliability of Proposed Equation

The proposed equation verified its reliability by the vast comparison with the experimental data. The experimental data used are the data obtained by Machida et al.[8], Ozaka et al.[9], Ishibashi et al.[2], Ohta[3], Akimoto[10], Higai et al.[4, 11, 12], and Arakawa et al.[1, 13 – 16]. It is noted that the data is used by the original definition of each researcher. Outline of specimens is shown in Table-1. The total number of experimental data is 170. Here, the data of Machida et al., Higai et al., Ozaka et al., Ishibashi et al., Ohta and Akimoto are obtained from the RC members such as bridge pier. On the other hand, Arakawa's data are obtained from the RC members such as columns in buildings, which are characterized with the larger values for σ_0 and ρ_w and smaller values for A/D in comparison with the data for RC bridge piers. All experimental results were obtained from specimens which were subjected to more than three

cycles at every loading cycles. Note that the data used are in great numbers and the range of each factor is also wide.

The ratio of estimated and experimental values of ductility ratio for the various factors are shown in Fig.-11. Figure-12 and Figure-13 show the results obtained from the equations proposed by Machida and Arakawa, respectively. In these figures, the experimental results for the bridge piers are marked with " \triangle " and marked with " \bigcirc " for the building columns.

The equation proposed by Machida et al. can estimate the ductility ratio when the longitudinal reinforcement ratio and the shear span ratio are large, and the web reinforcement ratio is small. The equation, however, does not estimate the ductility ratio except for the above range. On the contrary, Arakawa's equation can estimate the ductility ratio correctly when the longitudinal reinforcement ratio and the shear span ratio are small, and the axial stress and the web reinforcement ratio are large. This is due to the fact that the equations proposed in the past are based on a small number of experimental data and the applicability is limited within the range where the experiments were carried out. Since Machida's equation was formulated from the experimental results aimed at bridge pier, it can not apply to the RC members such as building columns. The Arakawa's equation was based on the results of experiment for the columns in buildings and it does not estimate correctly for RC members such as bridge pier. On the other hand, it can be seen that the proposed equation can estimate the ductility ratio more accurately than the other equations for every factors and for wide range of values of factors.

Figure-14 shows the comparison between the estimated values using the proposed equation and experimental values. The mean of the ratio of estimated and experimental value is 0.99, the coefficient of variation is 26.7(%) and the multiple correlation coefficient is 0.80. We can doubtlessly understand the effectiveness of proposed equation. Furthermore, the Committee of "Ductility of Concrete Structures and Its evaluation" performed a multiple regression analysis in which A/D, ρ_w/P_t , P_t , ρ_w , D and σ_0 are selected as a variable, using the same experimental data as we use. Then, the multiple correlation coefficient was 0.66[17]. We can also understand the effectiveness of the analytical method and the proposed equation. The proposed equation is formulated by the analytical results only which is obtained independently from experimental values.



Fig.-14 Comparison between Estimated and Experimental Ductility Ratio

Finally, the ductility ratio of real-scale RC structure which is impossible to obtain experimentally is analytically obtained and compared with the prediction of the proposed formula to demonstrate the real usefulness of the proposal. The model is a RC bridge pier having a cross section of $1.5 \times 1.5m$ and the height of 5.2m. The longitudinal reinforcements of D32 are arranged around and web reinforcement ratio of D13 are arranged with spacing of 12.5cm. The dimensions of the model are almost same with real RC bridge pier. It is assumed that the maximum compressive strength of concrete is 30MPa and axial stress is 1.0MPa. The ductility ratio obtained from the analysis is 4.5 and the ductility ratio obtained from the proposed equation is $4.9(P_t = 0.55\%)$, $A/D = 3.7, \ \rho_w = 0.14\%, \ \sigma_0 = 1.0 MPa, \ f_c' = 30 MPa).$ The proposed equation can also estimate the ductility ratio of real-scale structures.



6. Conclusions

Fig.-15 Analytical Model(unit;cm)

(1) The finite element analysis for RC members considering the geometrical nonlinearities and the material nonlinearities are carried out. Then, the ductility of RC members was evaluated analytically using the ductility ratio which is defined as the ratio of the lateral displacement corresponding to the maximum moment point and to the one for the yielding of the reinforcement in a member.

(2) The effects on the ductility of variable factors were investigated from the results of the parametric analysis in which the web reinforcement ratio, the longitudinal reinforcement ra-tio, the shear span ratio and the axial stress are varied. The analytical results showed that the ductility ratio increase in proportion to the increase of the web reinforcement ratio and increase with the decrease of the longitudinal reinforcement ratio. Furthermore, the ductility ratio decrease in proportion as the shear span ratio decrease. However, if the axial stress is large, the effect of the axial stress is most influential and the effect of the web reinforcement ratio, the longitudinal reinforcement ratio and the shear span ratio do not appear clearly.

(3)A simple equation to evaluate the ductility ratio is proposed on the bases of the analytical results. The equation is represented as the function of the axial stress, the web reinforcement ratio, the longitudinal reinforcement ratio, the shear span ratio and the compressive strength of concrete in which every terms of the equation include the effect of the axial stress ratio (σ_0/f'_c) .

(4) The accuracy of the proposed equation is verified by comparison with as many experi-mental results as can be collected. The total number of experimental data is 170. The mean of the ratio of estimated and experimental value is 0.99 and the multiple correlation coefficient is 0.80. The proposed equation can estimate ductility ratio correctly for all dimensions of RC members. It can estimate the ductility ratio of RC members accurately far better than any proposed models in the past and has wide applicability.

References

Arakawa, T., Arai, Y., Egashira, K., Ohkubo, S., "Cyclic behavior and evaluation of inelastic capacity of reinforced concrete columns", Proc. of JCI, No.5, pp.309-312, 1983.
 Ishibashi, T., Yoshino, S., "Study on deformation capacity of reinforced concrete bridge piers

under earthquake", Proc. of JSCE, No.390, pp.57-66, 1988.

[3] Ohta, M., "An experimental study on the behavior of reinforced concrete bridge piers under

cyclic loadings", Proc. of JSCE, No.292, pp.65-74, 1979. [4] Higai,T., Niwa,J.,Okamura,Y.,"Considerations on the post-yield shear failure mechanism of

[4] Higat, I., Miwa, J., Okamura, I., Considerations on the post-yield shear failure mechanism of RC members", Proc. of JCI, No.9, pp.329-334, 1987.
[5] Machida, A., Mutsuyoshi, H., Toyoda, K., "Evaluation of ductility of reinforced concrete members", Proc. of JSCE. No.378, pp.203-212, 1987.
[6] Nakamura, H., Niwa, J., Tanabe, T., "Analytical study on the ultimate deformation of RC columns", Proc. of JSCE. No.420, pp.63-76, 1990.
[7] Kent, D.C., Park, R., "Flexural members with confined concrete", Proc. of ASCE, Vol.97, No.577, pp.169-1900.

[7] Kent, D.O., 1 arkjur, 112aura Linear L forced concrete columns under static alternating cyclic loads", Proc. of JSCE, No.372, pp.45-54, 1986.

[10] Akimoto, Y., Yuki, M., Proc. of the 33 the annual conference of the JSCE, pp.331-332, 1978. [11] Higai, T., S, Rizkalla, H.Ben-Omran, F.Saadat, "Shear failure of reinforced concrete members subjected to large deflection reversals", Proc. of JCI, No.6, pp.506-508, 1984.

[12] Higai.T,"Ductility of reinforced concrete members failing in shear due to large deflection

reversals", Proc. of JCI, No.8, pp.769-772, 1986. [13] Arakawa,T., Arai,Y., Fujita,Y., "Effectiveness of shear reinforcement on the deformation behavior of reinforced concrete columns under cyclic loadings", Proc. of JCI, No.2, pp.457-460, 1980.

[14] Arakawa, T., Arai, Y., Fujita, Y., Mizoguchi, M., "Evaluation for deformation behavior of re-inforced concrete columns under cyclic loadings", Proc. of JCI, No.3, pp.449-452, 1981.
[15] Arakawa, T., Arai, Y., Egashira, K., Fujita, Y., "Effects of the rate of cyclic loading on the load carrying capacity and inelastic behavior of reinforced concrete columns", Proc. of JCI,

No.4, pp.325-328, 1982. [16] Arakawa,T.,Tsunoda,N.,Fujita,Y.,Egashira,L.,"Hysteretic behavior of reinforced concrete columns subjected to dynamic lateral loads", Proc. of Annual meeting of architectural institute of Japan, 1979.

[17] Committee report on "Ductility of concrete structures and its evaluation", JCI, p.126,1988.