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STUDY ON A DESIGN METHOD FOR RC MEMBERS SUBJECTED TO FLEXURE-SHEAR AND AXIAL LOAD

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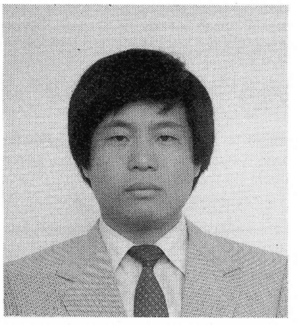


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SYNOPSIS

The influences of the factors affecting the shear capacity of reinforced concrete members, which are taken into consideration in various design codes and so on, are discussed in order to point out the features and problems of current evaluation equations. Also, both the evaluation of the shear capacity of reinforced concrete members subjected to flexure-shear and axial load and the interaction among the three capacities of the members which were derived theoretically on the basis of the energy principle, were taken into consideration in order to develop a new shear design method. In this new design method, the effect of the shear force as well as of the bending moment and axial load on the longitudinal reinforcement is taken into consideration, and therefore it is noticeable that the increase of stress on longitudinal reinforcement is considered in this new design method.

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1. INTRODUCTION

As for RC members, web reinforcement should be arranged so as to prevent shear failure due to the propagation of diagonal cracks. The effect of web reinforcement for the prevention of shear failure has been investigated by means of the truss analogy by various researchers.

It is clear that tension stress on web reinforcement is less than that calculated by the truss analogy [1] and that shear capacity may be provided by components other than web reinforcement. Therefore, in the current shear design method shear capacity is expressed as a sum of capacity provided by web reinforcement and that provided by the member itself. The current shear design method is simple and presents no problems if the design factors remain in the application range. But such a design equation is not necessarily thought to have a theoretical basis.

In recent years, various studies have attempted to evaluate shear capacity analytically [2],[3], and to reestimate the effect of web reinforcement so as to realize a decrease in the amount of web reinforcement [4]. Moreover, a theory [5] that web reinforcement is only one factor in the transmittance of load to support, neglecting the truss action of the cracked member, has also been proposed.

On the other hand, the design method of the RC member subjected to combined bending moment and axial force has been established on the basis of the interaction relationship between these two action effects. But as for the RC member subjected to combined bending moment and shear force, bending and shear behavior may be thought to be independent of each other, and their interaction curve has not been taken into consideration. In the current design method, the effect of shear force on the longitudinal reinforcement seems to be evaluated indirectly by means of the increase of the anchoring length of longitudinal reinforcement.

However,because the effect of the shear force on tension reinforcement is experimentally [6] and analytically [7] verified, it is desirable to base practical design on the interaction relationship between bending moment and shear force. Therefore, it is necessary to derive their interaction relationship analytically and to establish a design method based on this relationship.

In light of the above, the authors have proposed a method of evaluating the shear capacity and interaction relationship of the RC member subjected to flexure-shear and axial load on the basis of the ultimate equilibrium method of the upper bound theorem of the plastic theory [8],[9]. In this paper, in order to examine the features and problems of the current shear design method and the proposed shear design method, the effect of the main factor influencing shear capacity is evaluated qualitatively. Moreover, on the basis of the proposed equation of shear capacity and the interaction relationship among those capacities, a new shear design method for the RC member subjected to flexure-shear and axial load is proposed.

2. FEATURES OF THE CURRENT SHEAR DESIGN METHOD

The shear capacity of the RC member subjected to shear force has been studied for the past forty years and many experimental results have been reported. However, no theory nor model for the shear failure mechanism of the RC member has been established despite intensive study because of the complexity of the failure mechanism.

At present, there are two basic approaches to the design of the RC member subjected to shear force.

As mentioned above, the first method employs an experimental equation based on the existing shear test data and Ritter-Mörsch's truss analogy. This design method is presently the most widely used. In it, the shear capacity (V_u) is given as the sum of the shear strength provided

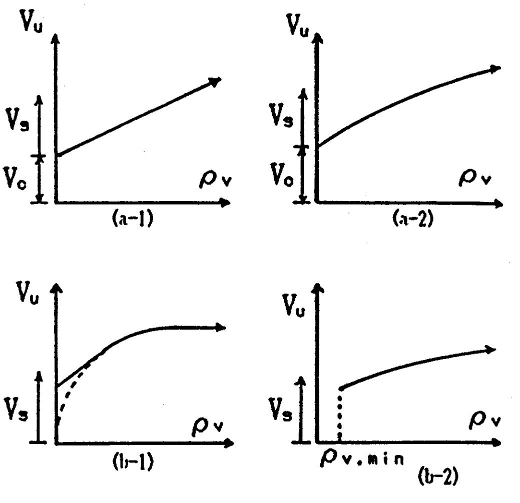

Fig. 1 Methods of estimating shear capacity

by the member itself (V_c) of the RC member without web reinforcement, and shear strength is provided by the web reinforcement (V_s), as follows in **Eq.(1)** .

$$V_u = V_c + V_s \quad \cdots\cdots (1)$$

Because **Eq.(1)** is based on the truss analogy, namely, that the inclination of a diagonal crack, that is to say, the angle of concrete strut, is 45°, most studies seem to focus on estimation of shear capacity of the member itself.

The second design method is based on shear capacity derived by the analytical approach, independent of existing experimental data. This method is based on the plastic theory, focusing only on the ultimate state rather than the total deformation process, and estimating force and deformation at the failure zone. Also, in this method the shear strength provided by concrete is almost completely ignored, as shown in **Eq.(2)** below, and the shear strength provided by web reinforcement is not increased in proportion with the increase of web reinforcement, but changes according to the amount and arrangement of the reinforcement.

$$V_u = V_s \quad \cdots\cdots (2)$$

In recent years, a method used to estimate the inclination of diagonal crack on the basis of the deformation mechanism at which the applied force becomes maximum [10], and a method used to determine the shape of the truss mechanism by considering the stress-strain relationship of concrete [2][7], and so on, have been proposed. These approaches have led to good results in the estimation of shear capacity and have increased basic knowledge of the mechanism of shear failure. But research results have not been applied to design formulae, with the exception of one part of the codes [11][12].

The two methods of estimating shear capacity mentioned above are schematically presented in **Fig.1** .

Fig.1(a-1) shows the estimation method of the current shear design method, and **Fig.1(a-2)** shows that used to estimate the effect of web reinforcement as the square root or the 5/8 root, such as the Ono–Arakawa equation [13] or Muguruma–Watanabe equation [14]. **Fig.1(b-1)** shows the estimation method employing Nielsen's theory based on the plastic theory and truss analogy, and **Fig.1(b-2)** shows that based on the theory for the RC member with minimum web reinforcement, such as the strut and tie model [15]. The method of estimating web reinforcement described in **Fig.1(a-1)** is different from the other three methods, but shear capacities estimated by all these method in the case of low web reinforcement ratio are nearly equal. However, the differences among methods become large in proportion to the increase of web reinforcement ratio.

As mentioned above, there are differences among the four methods of estimating shear capacity. The basic difference between **(a)** and **(b)** is whether or not the effect of shear force on the longitudinal reinforcement is considered. In the first approach the design is carried out on the assumption that the effects of bending moment and shear force are mutually independent, that is to say, tension reinforcement is calculated for only the bending moment and web reinforcement for the shear force, respectively, without evaluating the interaction relationship between bending moment and shear force. On the other hand, in the second approach, the interaction relationship between bending moment and shear force, and the stress increase of tension reinforcement due to the action of the shear and axial forces are taken into consideration.

If we examine the two approaches with regards to practical design for beam or column, the difference is not so large, but in case of a deep beam it is known to be difficult to evaluate the effect of shear force by means of the concept of anchoring length of longitudinal reinforcement [7] and that the stress increase of tension reinforcement due to the action of the axial force can not be estimated.

The authors' design method [8],[9] used to estimate the shear capacity analytically on the basis of the interaction relationship between flexure-shear and axial force corresponds to the second approach mentioned above, and can be used to evaluate the effect of not only shear force but also axial force on tension reinforcement.

3. PROPOSED SHEAR CAPACITY EQUATION

The authors proposed a new method of estimating shear capacity [8] and the interaction relationship among capacities analytically [9], for the RC member with more than minimum web reinforcement. The approach of this method corresponds with the second approach mentioned above, on the basis of the ultimate equilibrium method of the upper bound theorem of the plastic theory.

A summary of the two papers mentioned above is as follows, and details can be found in Ref.8 and 9.

3.1 Derivation of the shear capacity

The shear capacity equation based on the energy principle is derived on the assumption that the diagonal crack propagates in the direction which minimizes the internal resistance, as shown in **Eqs.(3)** and **(4)** .

$$\cot\alpha = \frac{-a}{h-d_p} + \sqrt{\left(\frac{-a}{h-d_p}\right)^2 + \frac{2s}{(h-d_p)^2 A_v f_{vy}} *}$$

$$\overline{* \left\{ A_l f_{ly}(d - \frac{d_p}{2}) + A_l' f_s'(\frac{d_p}{2} - d') \right\} - \frac{2Ns(d - d'' - d_p/2)}{(h-d_p)^2 A_v f_{vy}}} \quad \cdots\cdots (3)$$

$$V_u = \frac{A_v f_{vy}}{s}\left[\sqrt{a^2 + \frac{2s}{A_v f_{vy}}\left\{A_l f_{ly}(d - \frac{d_p}{2}) + A_l' f_s'(\frac{d_p}{2} - d')\right\} - \frac{2Ns(d - d'' - d_p/2)}{A_v f_{vy}}} - a\right] \cdots (4)$$

where α is the inclination of the diagonal crack, V_u is the shear capacity, A_l is the amount of tension reinforcement, f_{ly} is the yield strength of tension reinforcement, A_l' is the amount of compression reinforcement, f_{ly}' is the yield strength of compression reinforcement, A_v is the amount of web reinforcement, f_{vy} is the yield strength of web reinforcement, s is the spacing of web reinforcement, d_p is the height of concrete of compressive zone, d' is the cover of compression reinforcement, d'' is the distance from the plastic centroid to the tension reinforcement, a is the length of shear span, h is the member height, d is the effective height of member, f_s' is the stress of compression reinforcement, f_s is the stress of tension reinforcement, and N is the axial force.

Substituting **Eq.(3)** into **Eq.(4)** , **Eq.(5)** is derived. It is similar to the equation based on the truss analogy.

$$V_u = \frac{A_v f_{vy}}{s}(h - d_p)\cot\alpha \quad \cdots\cdots (5)$$

The shear capacity derived by the ultimate equilibrium method is similar to that by the truss analogy proposed by Nielsen [10]; this is the reason why in the proposed theory, as in Nielsen's theory, shear capacity provided by concrete is neglected and the inclination of the diagonal crack is dependent on the amount of tension and web reinforcement and the shear capacity of the member is estimated by the amount of web reinforcement crossed by the diagonal crack. All main factors affecting the shear capacity of RC members, for example, tension reinforcement ratio, web reinforcement ratio, shear span to depth ratio, and compressive strength of concrete and so on, are taken into consideration in this proposed equation. Also, the effect of compression reinforcement which is generally ignored in the past evaluation equations is included.

3.2 Interaction relationship among capacities

The authors proposed an equation of shear capacity of RC member subjected to flexure–shear and axial force and an equation of interaction relationship among capacities as follows in **Eqs.(6)** and **(7)** . These equations are obtained by transforming the length of shear span a into the ratio of bending capacity and shear capacity, that is to say, M_u/V_u .

$$M_u + \frac{V_u^2 s}{2A_v f_{vy}} + N_u(d - d' - \frac{d_p}{2}) = A_l f_{ly}(d - \frac{d_p}{2}) + A_l' f_s'(\frac{d_p}{2} - d') \quad \cdots\cdots (6)$$

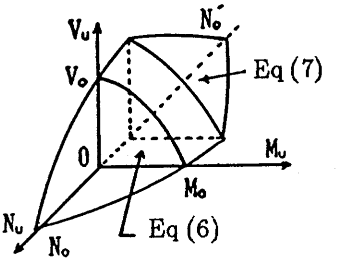

Fig. 2 Interaction relationship between flexure-shear and axial force

$$M_u + \frac{V_u^2 s}{2A_v f_{vy}} + N_u(d - d' - \frac{d_p}{2}) = A_l f_s(d - \frac{d_p}{2}) + A_l' f_{ly}'(\frac{d_p}{2} - d') \quad \cdots\cdots (7)$$

The failure mode which becomes the border between **Eq.(6)** and **Eq.(7)** is the case in which the yielding of tension reinforcement and failure of compressive zone of concrete occur simultaneously. The interaction relationship between flexure–shear and axial force is shown in **Fig.2**.

In the RC member subjected to flexure-shear and axial force, the interaction relationship among capacities may be plotted in the space in which the axes are composed of three capacities. The curve of the interaction relationship between bending moment and shear force is obtained by projecting the interaction relationship among the three capacities on the plane of the interaction relationship between bending moment and shear force.

3.3 Relationship between shear capacity and main factors

One part of the relationship between shear capacity and main factors is described in Ref.[8] and [9]; in this paper the effects of various factors influencing shear capacity, including those mentioned above, are examined. The following dimensionless factors are used.

$$\omega = \tau_u / f_{ck}, \quad \phi = \rho_l f_{ly} / f_{ck}, \quad \psi = \rho_v f_{vy} / f_{ck}, \quad \eta = a/d \quad \cdots\cdots (8)$$

where τ_u is the shear strength, ρ_l is the tension reinforcement ratio, ρ_v is the web reinforcement ratio, and f_{ck} is compressive strength of concrete.

First of all, on the assumption that the shear span to depth ratio and the compressive strength of concrete are constant, the interaction relationship among the dimensionless shear capacity, the tension reinforcement ratio and the web reinforcement ratio is plotted in **Fig.3**.

On the assumption that the tension reinforcement ratio and the web reinforcement ratio are constant, the interaction relationship among the shear capacity, the shear span to depth ratio, and the compressive strength of concrete is plotted in **Fig.4**. As for **Fig.4**, non-dimensional shear capacity is not used, because if it is divided by concrete compressive strength, its effect is included in the term of shear capacity.

Next, the interaction relationship among shear capacity, axial force, tension reinforcement ratio, and web reinforcement ratio is examined. Since there are four variables in this case, the ratio

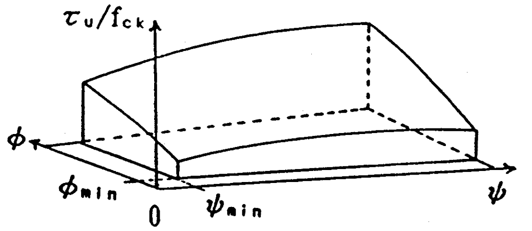

Fig. 3 Interaction relationship among shear capacity, tension reinforcement ratio and web reinforcement ratio

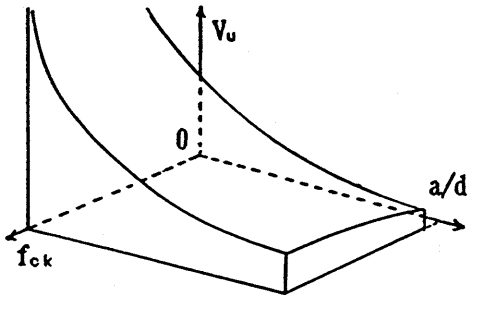

Fig. 4 Interaction relationship among shear capacity, shear span to depth ratio, and compressive strength of concrete

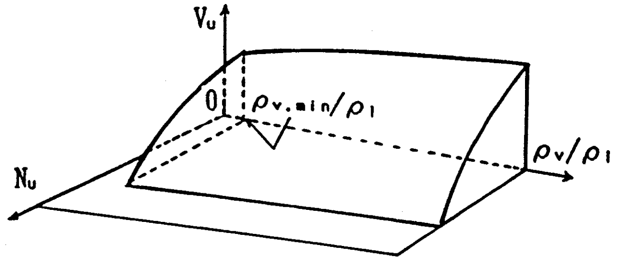

Fig. 5 Interaction relationship among shear capacity, axial force, tension reinforcement ratio, and web reinforcement ratio

of tension reinforcement ratio to web reinforcement ratio is used. Their interaction relationship is plotted in **Fig.5**.

3.4 Considerations

According to the results, on the basis of the same shear span to depth ratio and the same compressive strength of concrete, the larger the web reinforcement ratio, the larger the shear capacity. However, the tendency for the shear capacity to increase may not be linear. Although it is clear that the shear capacity has an upper limit for the amount of web reinforcement, the upper limit cannot be quantitatively estimated because of the effects of other factors.

As for the RC member subjected to tension force, the larger the tension force, the smaller the shear capacity, and the degree of decrease of the shear capacity becomes large in case of the small shear span to depth ratio. However, as for the case subjected to compressive force, the larger the compression force, the larger the shear capacity. But if compression force is greater than the certain limit of compressive force, conversely the shear capacity decreases.

4. ESTIMATION OF FACTORS AFFECTING THE CURRENT SHEAR CAPACITY EQUATIONS AND THE PROPOSED EQUATION

As most shear design codes of various countries employ the shear capacities provided by web reinforcement based on the truss analogy, there is little difference between their calculated values. However, the shear capacity provided by the member itself is quite different, as shown in **Table 1**, because that capacity is based on experimental data.

It is well known that numerous factors influence shear capacity. In this section, the tension reinforcement ratio (ρ_l), the shear span to depth ratio (a/d), the compressive strength of concrete (f_{ck}), and the web reinforcement ratio (ρ_v), which are thought to be the main factors, are examined to point out the features and problems of the proposed equation and other equations.

4.1 Effect of tension reinforcement ratio(ρ_l)

The tension reinforcement ratio is the main factor influencing bending behavior, but its effect is almost completely ignored in current shear design. Because it is clear that the influence of tension reinforcement on aggregate interaction, dowel action, and stress redistribution is great, the effect of tension reinforcement on shear behavior should be estimated in design.

Generally, the effect of the tension reinforcement ratio is included in the term of shear capacity provided by the member itself (V_c) in the first approach mentioned above. In the case of the second approach, because the shear capacity of the RC member is influenced by the ratio of the tension reinforcement ratio to the web reinforcement ratio,the effect of the tension reinforcement ratio is included in the total shear capacity. Also, the former is expressed by the term of $\rho_l, \rho_l^{1/2}, \rho_l^{1/3}$, and so on, as shown in **Table 1**. There are a number of cases which do not take this effect into consideration.

As for each shear capacity equation shown in **Table 1**, the relationship between tension reinforcement ratio and shear capacity is examined under the conditions that $a/d, f_{ck}$, and ρ_v are constant and ρ_l is 1%, as described in **Fig.6**.

According to the results, the expression of the equation differs owing to differences of evaluation methods. But their evaluation values seem to be almost equal.

However, if the effect of tension reinforcement of the proposed equation is compared with that of other equations, a clear distinction among them can be seen. This is the reason why the

Table 1 Factors and features of various shear capacity equations

influence factor	ρ_l	a/d	f_{ck}	ρ_v
Code				
JSCE(1974)	–	–	△	○
JSCE(1986)(1)	$\rho_l^{1/3}$	–	$f_{ck}^{1/3}$	○
AIJ(1980)(2)	–	$1/(M/Vd+1)$		○
AIJ(1986)(3)	–	$\sqrt{(a/h)^2+1}-a/h$	νf_{ck}	○
ACI(4)	ρ_l	Vd/M	$f_{ck}^{1/2}$	○
BSI-CP110	△	–	△	○
USSR(5)	–	d/a	f_t'	○
CEB(1978)(6)	–	–	$f_{ck}^{2/3}$	○
CEB(1990)(7)	$\rho_l^{1/3}$	–		○
Proposed equation				
Okamura· Higai (8)	$\rho_l^{1/2}$	d/a	$f_{ck}^{1/3}$	
Ono·Arakawa (9)	$\rho_l^{0.23}$	$0.1/(M/V\cdot d+0.1)$	f_{ck}	$(\rho_v f_{vy})^{1/2}$
Muguruma·Watanabe(10)	–	–	f_t'	$(\rho_v f_{vy})^{5/8}$
Nielsen(11)	–	–	νf_{ck}	$\sqrt{\rho_v(1-\rho_v)}$
Zsutty(12)	$\rho_l^{1/3}$	$(d/a)^{1/3}$	$f_{ck}^{1/3}$	
Laupa(13)	–	d/a	f_{ck}^2	
Regan (14)	$\rho_l^{1/3}$	–	$f_{ck}^{1/3}$	

– : not considered, △ : see table, ○ : truss analogy,
ν : concrete effective reduction coefficient, ft' : concrete tensile strength

(note)

(1) $V_u = 0.9\sqrt[4]{\frac{100}{d}}\sqrt[3]{100\rho_l}\sqrt[3]{f_{ck}} + \frac{A_v f_{vy}}{s}\cdot z,$ (2) $V_u = \left\{\alpha\cdot f_s + \frac{1}{2}f_t(\rho_v - 0.002)\right\} bjd,$

(3) $V_u = \frac{1}{2}\left\{\sqrt{(\frac{a}{h})^2+1} - \frac{a}{h}\right\}\left\{1 - \frac{(1+\cot^2\phi)}{\nu f_{ck}}\rho_v\right\} bh\nu f_{ck} + \rho_v f_{vy}\cot\phi bj,$

(4) $V_u = \left(0.504\sqrt{f_{ck}} + 176\frac{\rho_l V\cdot d}{M}\right) b_w d + \frac{A_v f_{vy}}{s}\cdot d,$

(5) $V_u = 2\sqrt{2bd^2\frac{A_v f_{vy}}{s}f_t'},$ (6) $V_u = 0.29 f_{ck}^{2/3} bd + \frac{A_v f_{vy}}{s}\cdot 0.9d,$

(7) $V_u = 0.125 b_w\cdot d\left(1+\sqrt{\frac{20}{d}}\right)\sqrt[3]{\frac{100A_l}{b_w\cdot d}f_{ck}}\left(\frac{1}{1-(P+N_{sd})h_p/M_{max}}\right) + \frac{A_v f_{vy}}{s}\cdot 0.9d,$

(8) $V_u = 0.94\sqrt[3]{f_{ck}}\left(\sqrt{\rho_l} + \sqrt[4]{\frac{1}{d}} - 1\right)\left(0.75 + \frac{1.4}{a/d}\right) + \frac{A_v f_{vy}}{s}\cdot z,$

(9) $V_u = \left\{\frac{0.115\cdot k_u\cdot 0.82\rho_l^{0.23}}{M/V\cdot d + 0.12}(f_{ck}+180) + 2.7\sqrt{\rho_v f_{vy}}\right\} b\cdot z,$

(10) $V_u = \left(\frac{7}{48}\right) f_t' bd(C_w/s) + \{1 - 0.785(d - 0.5h)nA_l d/I_{eq}\}V_0 + 3(\rho_v f_{vy})^{5/8} b\cdot j\cdot d,$

(11) $V_u = bh\nu f_{ck}\sqrt{\frac{\rho_v f_{vy}}{\nu f_{ck}}\left(1 - \frac{\rho_v f_{vy}}{\nu f_{ck}}\right)},$ (12) $V_u = 10.04\sqrt[3]{f_{ck}\rho_l\frac{d}{a}}bd,$

(13) $V_u = \frac{k}{a}bd^2 f_{ck}\left(0.57 - \frac{6.39 f_{ck}}{10000}\right)\left(1 + \frac{2.84\rho_v f_{vy}}{100}\right),$ (14) $V_u = 1.18\sqrt[3]{f_{ck}100\rho_l}bd + \rho_v f_{vy} b\cdot c$

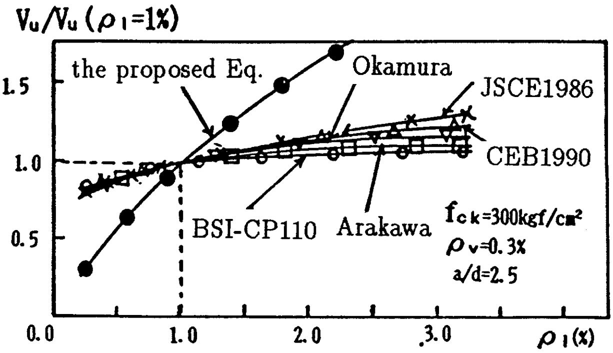

Fig. 6 Relationship between the shear capacity and the tension reinforcement ratio

effect of tension reinforcement is taken into consideration for the evaluation of shear capacity of RC member without web reinforcement in other equations, and conversely why the proposed equation is derived for the RC member with more than minimum web reinforcement. Because the effect of tension reinforcement in the proposed equation is influenced by other factors, quantitative comparison with other equations is impossible.

4.2 Effect of shear span to depth ratio(a/d)

The shear span to depth ratio is determined by the loading position. If the loading position approaches the support, vertical compressive stress which is ignored in flexural theory should not be ignored, so as to avoid the occurrence and propagation of diagonal crack. Therefore, in design of the member with a/d of less than 3, this effect must necessarily be taken into consideration. However, the dominant appearance of the effect of a/d is limited to the case in which compressive force is produced by the support reaction in the direction perpendicular to the axis of the member. Therefore, it is clear that the member subjected to force indirectly through another member is not influenced by the effect of a/d.

The relationship between a/d and shear capacity of various published equations in **Table 1** is shown in **Fig.7**. Estimation is carried out under the conditions that ρ_l, ρ_v, and f_{ck} are constant and a/d is 3. Because the application range of a/d is not clearly specified in the code, the estimation is carried out under the same conditions.

Generally, an equation derived on the basis of experimental data includes the term of shear span to depth ratio. However, practical members are subjected to combined forces, so the effect of the position at which loading force is applied, that is to say, the effect of shear span to depth ratio, cannot be included in the design equation. Therefore, in the current shear design method, the effect of shear span to depth ratio is not evaluated in terms of a/d, conservative values are used or the effect of a/d is transformed into the form of M/Vd (see **Table 1**), using the ratio of design bending moment M to design shear force V.

According to the results, in most of the shear capacity equations, the larger the a/d, the smaller the shear capacity. But the effect of this is almost completely neglected in case a/d is over 3. In the proposed equation, however,the decrease of shear capacity in case that a/d is over 3 is also shown.

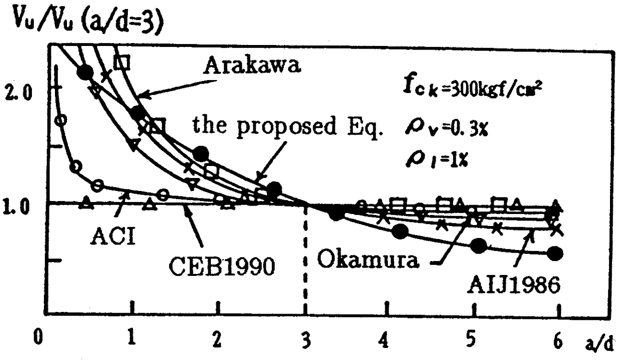

Fig. 7 Relationship between shear capacity and shear span to depth ratio

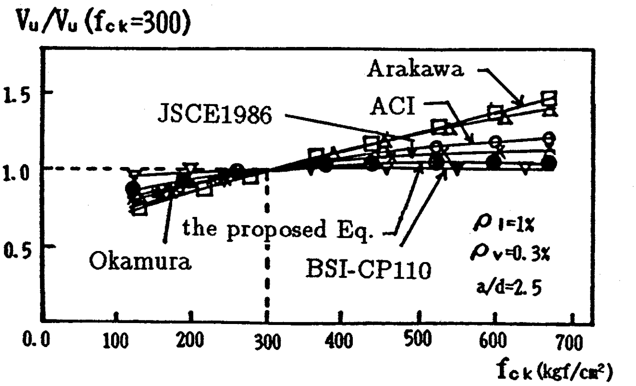

Fig. 8 Relationship between shear capacity and compressive strength of concrete

4.3 Effect of compressive strength of concrete (f_{ck})

It is quite natural to consider the appearance of diagonal cracks, which is the cause of shear failure of the RC member, to be related to tensile strength of concrete, so past shear design equations were expressed as a function of tensile strength of concrete. However, it is not always suitable to use the concrete tensile strengths directly in design, so compressive strength of concrete is usually used instead.

The effect of compressive strength of concrete is represented in the form of $f_{ck}^{1/3}$, $f_{ck}^{1/2}$, $f_{ck}^{2/3}$, and so on. But in the evaluation based on the plastic theory, the yielding of reinforcement is the main estimation factor, and therefore the term of compressive strength of concrete is not directly used in the shear capacity equation. Rather, the effectiveness factor of concrete is used indirectly to describe the strength of web concrete.

The relationships between compressive strength of concrete and shear capacity are shown in **Fig.8**. The examination was carried out under the conditions that ρ_l, ρ_v, and a/d are constant and f_{ck} is 300 kgf/cm^2.

As shown by our results, there is little difference among the various equations; however, the Arakawa equation [13] overestimates the effect of compressive strength of concrete in comparison with other equations, and the BSI-CP110 equation [17] almost completely ignores its

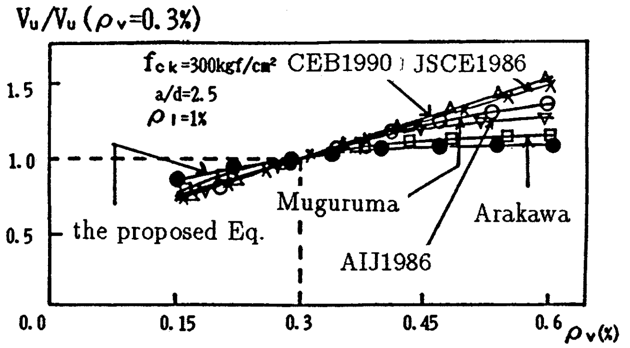

Fig. 9 Relationship between shear capacity and web reinforcement ratio

effect. Though the proposed equation has the same tendency as other equations with regard to the effect of compressive strength of concrete, the influence is less than that with the other equations. Besides, compressive strength of concrete as a main influencing factor is limited to estimation of maximum shear capacity of the member itself, so most model codes represent it as the form of the function of compressive strength of concrete in order to prevent shear failure.

4.4 Effect of web reinforcement ratio(ρ_v)

It is specified in the model code that RC member with more than minimum web reinforcement must be designed so as to prevent abrupt shear failure. Web reinforcement plays several important roles; tension strut in truss analogy, restraint of expansion of width of diagonal cracks, promotion of aggregate interlocking on the crack surface and shear transfer of upper concrete by restrain of propagation of cracks to the upper concrete. In addition, it also acts to promote dowel action by restraining crack propagation along tension reinforcement.

The relationship between the web reinforcement ratio and shear capacity is shown in **Fig.9**. The examination was carried out under the conditions that $\rho_l, a/d$, and f_{ck} are constant and ρ_v is 0.3%.

Most shear design equations are derived on the basis of truss analogy, so the shear capacity has a tendency to increase in proportion to the web reinforcement ratio. However, Arakawa's equation [13], and Muguruma and Watanabe equation [14] evaluate the effect of web reinforcement ratio in the form of $\rho_v^{1/2}$ or $\rho_v^{5/8}$ by regression analysis of the experimental data. Comparing the calculated values by these equations with those derived by the truss analogy, it can be seen that the difference between them is not so great, but it increases in proportion to the increase of the web reinforcement ratio. As Arakawa's equation and Muguruma-Watanabe's equation are derived experimentally, evaluation of the shear capacity provided by web reinforcement in proportion to ρ_v may result in overestimation. Therefore, it is necessary to further study the effect of web reinforcement.

Other equations evaluate the effect of web reinforcement ratio and that of other factors independently. Because the web reinforcement ratio, the tension reinforcement ratio and the shear span to depth ratio influence one another in the proposed equation, the relationship between shear capacity and web reinforcement ratio is not quantitatively. However, it is said that the increase of the shear capacity calculated by the proposed equation decreases gradually in proportion to the increase of web reinforcement ratio as in Arakawa's equation.

4.5 Considerations of various shear specifications

The shear capacities provided by concrete in shear design codes of various countries are quite similar despite differing methods of estimation due to different backgrounds and data on shear behavior,as shown in **Table 1**. Among 15 equations used for examination, there are 7 equations which evaluate the effect of tension reinforcement and 13 which evaluate the effect of compressive strength of concrete. Regarding application of experimental equations to design, it is known that most of the various equations employ a conservative method or use the ratio of bending moment to shear force at the critical section instead of the shear span to depth ratio. Furthermore, the size effect of the member (the effect of effective height, d) is taken into consideration in 6 equations, so as to estimate shear capacity more accurately.

On the other hand, as for the evaluation of the effect of web reinforcement, except in a few cases of equations based on experimental data, most model codes have adopted equations derived by the 45° truss analogy. According to analytical considerations and test results, it is clear that the evaluation of the effect of web reinforcement by the 45° truss analogy likely leads to overestimation. However, since estimation based on the 45° truss analogy can be easily applied in practical design, that method is widely used despite the problems involved. In order to alleviate the above-mentioned problem in some model codes (for example, the refined method of CEB-FIP MODEL CODE of 1978, general method of CANADA STANDARD CODE), both the design method based on the 45° truss analogy and the one derived using the analytical approach are used in combination.

5. PROPOSED NEW SHEAR DESIGN METHOD

5.1 Basic concept of design method

Shear failure is the failure mode which takes place due to the propagation of diagonal cracks before the member reaches the ultimate state of bending. In the current shear design method, web reinforcement is designed to prevent shear failure, using experimental equations and the truss analogy. As for the estimation of shear capacity, however, the yielding of tension reinforcement is ignored in design methods based on the truss analogy. Judging from the report that concrete in the compressive zone of the RC member with web reinforcement fails due to uniaxial stress, it is not necessarily ensured in the current design method that the RC member with web reinforcement will fail ductilely. Therefore, it seems necessary to include the condition of yielding of tension reinforcement, which is a prerequisite for ductile failure, in any shear design method.

From this viewpoint, the authors propose a new shear design method in which the amount of web reinforcement is calculated in order to allow tension reinforcement to reach yielding strength. In this design method, the shear capacity of the member is determined by the amount of tension and web reinforcements, so it is necessary to consider the interaction relationship between bending moment and shear force.

5.2 Stress increase of tension reinforcement due to the effect of shear force and axial force

Currently, in the shear design method, the increase of stress of the tension reinforcement due to the effect of shear force and axial force is not taken into consideration, but the Variable Angle Truss Analogy [11],[18], in which the angle of the concrete strut is affected by the stress state of the member, and Diagonal Compression Field Theory [2],[12] partially evaluate its effect.

In this section this effect is taken into consideration by the proposed equation derived on the basis of the ultimate equilibrium method.

For the failure section, the external moment (M_{ext}) due to external force must be equilibrated with the internal resistance moment (M_{int}) (details are described in Ref.[9]).

$$M_u + V_u(h - d_p)\cot\alpha + N_u(d - d'' - d_p)$$

$$= A_l f_{ly}(d - d_p) + A_l' f_s'(d_p - d') + A_v f_{vy}(h - d_p)^2 \cot^2\alpha/2 \cdot s + k_1 f_{ck} b d_p^2/2 \quad \cdots\cdots (9)$$

In order to simplify the expansion of the equation, we examine only the case in which the effect of compression reinforcement is neglected.

Eliminating the term of compressive strength of concrete in **Eq.(9)**, the following equation is derived.

$$M_u + V_u(h - d_p)\cot\alpha + N_u(d - d'' - d_p)$$

$$= A_l f_{ly}(d - d_p/2) + A_l' f_s'(d_p/2 - d') + A_v f_{vy}(h - d_p)^2 \cot^2\alpha/2 \cdot s - N_u d_p/2 \quad \cdots\cdots (10)$$

On the other hand, substituting **Eq.(5)** into **Eq.(10)**, the following equation is derived.

$$M_u + V_u^2 s/2A_v f_{vy} + N_u(d - d'' - d_p/2) = A_l f_{ly}(d - d_p/2) \quad \cdots\cdots (11)$$

Assuming that the distance $(h - d_p)$ from the end of tension zone to the neutral axis is the same as the arm length $(d - d_p/2)$ of the couple moment, **Eq.(11)** is equal to the equation derived by the truss analogy.

Substituting the equation of inclination (α) of diagonal crack at which the shear resistance becomes minimum in **Eq.(11)**, the following equation is derived.

$$\frac{M_u}{(d - d_p/2)} + \frac{(h - d_p)}{(d - d_p/2)} \cdot \frac{V_u}{2}\cot\alpha + \frac{(d - d'' - d_p/2)}{(d - d_p/2)} N_u = A_l f_{ly} \quad \cdots\cdots (12)$$

In the above equation, the second term concerns on the stress increase of tension reinforcement due to the effect of shear force, and the third term concerns such increase due to the effect of the axial force.

5.3 Proposed shear design method

As described in **Eq.(4)**, the shear capacity does not have a one-to-one relationship with the web reinforcement ratio. Rather, it is also related to the tension reinforcement ratio, web reinforcement ratio, and shear span to depth ratio. Moreover, one parameter has an influence on the others, so it is difficult to adopt the proposed equation as is for design purposes. This is the reason why the ultimate equilibrium method cannot be used to estimate the size of the member or the amount of the reinforcement, but only to check the safety of a member whose design factors have already been decided.

However, two methods are thought to be possible for purposes of adopting the capacity equation for use as a design equation. One is a method in which the value of the inclination of the diagonal crack $\cot\alpha$, which is the function of various factors, is obtained on the basis of experimental data for the use of design, resulting in a one-to-one correspondence of shear capacity and web reinforcement ratio as in **Eq.(5)**. The other is a method that assumes the size of the member and the amount of reinforcement, calculates the ultimate capacity, and determines a more reasonable member size and amount of reinforcement by comparison with design force. That is to say, the former is a method that employs the existing shear design method and takes into account the increase in the amount of tension reinforcement, and the latter is a method which is different from the existing shear design method and which takes into consideration the interaction relationship between bending moment and shear force. With the former, the shear design can be simply carried out by calculation of the value of $\cot\alpha$. The latter has a demerit of requiring considerable time for repeating calculations. Therefore, the authors propose a new shear design method corresponding with the first method mentioned above.

As mentioned above, on the assumption of an interaction relationship between bending moment and shear force, the new shear design method estimates the stress increase of tension reinforcement due to shear and axial force. That is, the amount of tension reinforcement should be calculated on the basis of not only the bending moment but also shear and axial force.

First of all, the amount (A_{lf}) of tension reinforcement necessary to resist bending moment may be written as follows, utilizing the existing flexural theory.

$$A_{lf} = \frac{M_u}{f_{ly}(d - d_p/2)} \quad \cdots\cdots \quad (13)$$

Next, the amount (A_v) of web reinforcement needed to resist shear force may be written as follows, using in **Eq.(5)**

$$A_v = \frac{V_u \cdot s}{f_{vy}(h - d_p)} \tan\alpha \quad \cdots\cdots \quad (14)$$

The amount (A_{lv}) of tension reinforcement necessary to resist both shear and axial force may be written as follows.

$$A_{lv} = \frac{(h - d_p)V_u \cot\alpha}{(d - d_p/2)2f_{ly}} + \frac{(d - d'' - d_p/2)N_u}{(d - d_p/2)f_{ly}} \quad \cdots\cdots \quad (15)$$

Therefore, the amount (A_l) of tension reinforcement required for the RC member subjected to flexure-shear and axial force may be written as follows, using **Eqs.(13)** and **(15)**. The section position for calculation of A_l is the same as that for calculating the tension reinforcement due to bending moment.

$$A_l = \frac{M_u}{f_{ly}(d-d_p/2)} + \frac{(h-d_p)V_u \cot\alpha}{(d-d_p/2)2f_{ly}} + \frac{(d-d''-d_p/2)N_u}{(d-d_p/2)f_{ly}} \quad \cdots\cdots (16)$$

5.4 Relationship between the minimum amount of reinforcement and the inclination of the diagonal crack

As shown in **Eqs.(14)** and **(16)**, the total amount of tension and web reinforcement to be arranged along the member is influenced by the inclination (α) of the diagonal crack. For example, the smaller the value of α, the less the amount of web reinforcement needed to resist shear force; however, the amount of tension reinforcement needed to resist bending moment should be allowed to increase. Conversely, the amount of tension reinforcement can be reduced by increasing the value of α, but that of web reinforcement should be allowed to increase.

Considering the economy which can be realized when the required total amounts of reinforcement are minimum under conditions of the same section and the same shear capacity, the inclination of the diagonal crack at which the amount of reinforcement required by the whole member becomes minimum is examined.

As for the RC member subjected to flexure-shear and axial force, the volume (V_t) of the required reinforcement per unit length is written as follows based **Eqs.(14)** and **(16)**.

$$V_t = \frac{M_u}{f_{ly}(d-d_p/2)} + \frac{V_u(b_v+2d_v)}{f_{vy}(h-d_p)}\tan\alpha + \frac{V_u(h-d_p)\cot\alpha}{2f_{ly}(d-d_p/2)} + \frac{(d-d''-d_p/2)N_u}{(d-d_p/2)f_{ly}} \quad \cdots\cdots (17)$$

where, b_v denotes the width of web reinforcement and d_v denotes the height of web reinforcement.

The inclination of the diagonal crack (α_0) at which the required amount of reinforcement of the whole member becomes minimum is obtained by differentiating **Eq.(17)** with respect to the inclination of the diagonal crack (α).

$$dV_t/d\alpha = 0 \quad \cdots\cdots (18)$$

$$\tan\alpha = \sqrt{\frac{(h-d_p)^2 f_{vy}}{2(d-d_p/2)(b_v+2d_v)f_{ly}}} \quad \cdots\cdots (19)$$

Substituting **Eq.(19)** into **Eqs.(14)** and **(16)**, the minimum required amount (A_l) of tension reinforcement and that (A_v) of web reinforcement can be written as follows.

$$A_v = V_u\sqrt{\frac{s^2}{2f_{ly}f_{vy}(d-d_p/2)(b_v+2d_v)}} \quad \cdots\cdots (20)$$

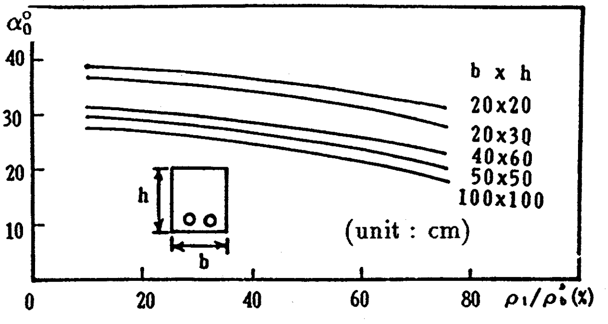

Fig. 10 Relationship between α_0 and ρ_l/ρ_b

$$A_l = \frac{M_u}{f_{ly}(d-d_p/2)} + V_u\sqrt{\frac{s^2}{2f_{ly}f_{vy}(d-d_p/2)(b_v+2d_v)}} + \frac{(d-d''-d_p/2)N_u}{(d-d_p/2)f_{ly}} \quad \cdots\cdots\cdots\cdots\cdots (21)$$

That is to say, if the amount of web reinforcement required to resist shear force is equal to that of the tension reinforcement added to withstand the influence of shear force, the required reinforcement of the whole member could be minimum.

In order to determine the inclination (α_0) of the diagonal crack at which the required amount of reinforcement of the whole member becomes minimum, RC member with the rectangular section should be examined. Such examination was carried out by calculating the balanced reinforcement ratio ρ_b for each member size and the inclination of the diagonal crack by using **Eq.(19)** for each ρ_l/ρ_b. The results are shown in **Fig.10**.

The inclination (α_0) of the diagonal crack is not influenced by f_{ck} and A_v, but by ρ_l/ρ_b and member size (b, h). Namely, as for the member with the same section, the larger the ρ_l/ρ_b, the smaller the α_0. Also, as for the member with the same ρ_l/ρ_b, the larger the member size, the smaller the α_0. Kupfer [19] reported the range of inclination of the diagonal crack to be $35° \sim 40°$, applying minimum strain energy theory on the assumption of the truss analogy.

5.5 Design example

The proposed shear design method is compared with the current shear design method through numerical examples. The design example is RC simple beam of T type. The yield strength (f_{sy}) of reinforcement and width (b) of the section are constant, but the section height (h) and the span length (L) are chosen as variables. The design load is decided on the basis of the Highway Bridge Code [20] and the member is designed as a first class bridge.

Fig.11 shows the changes of the ratios of the amounts of tension reinforcement and web reinforcement calculated by the proposed design method in comparison with those of the current design method. ks denotes the ratio of the amount (ρ_l - ρ_l') of increase of tension reinforcement by the new design method to that (ρ_l') by the current design method, and k_v denotes the ratio of the amount ($\rho_v - \rho_v'$) of decrease of web reinforcement by the new design method to that (ρ_v') by the current design method. The symbol (+) in **Fig.11** indicates the increase.

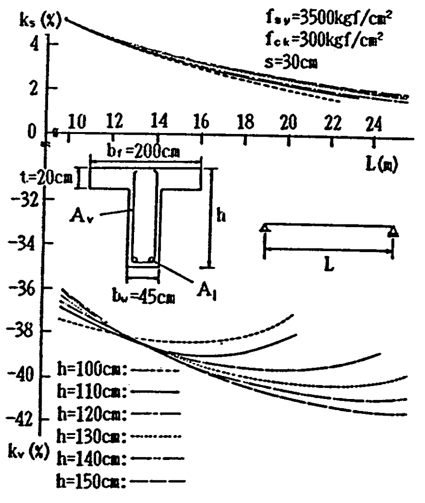

Fig. 11 Relationships between L and k_s, k_v

According to **Fig.11**, the longer the span length, the smaller the k_s. In other words, there is little difference between the amount of tension reinforcement in the new design method and that in the current design method in the case of a larger span. But the longer the span, the bigger the decrease of the k_v.

6. CONCLUSIONS

The object of this study was to examine the features and problems of various equations proposed by others and to propose a new shear design method based on the basis of the equation of shear capacity and interaction relationship among capacities derived for the RC member subjected to flexure-shear and axial force. Obtained results are as follows.

(1) As for the shear capacity provided by concrete in shear design codes of various countries despite the differences in estimation methods due to differing background and shear behavior data, there is little difference in values.

(2) The effect of the shear force influencing tension reinforcement was verified experimentally and analytically, so a design method based on their interaction relationship can be developed.

(3) A new analytically based shear design method which takes the interaction relationship between bending and shear into account is proposed.

(4) As for the RC member subjected to flexure-shear and axial force, the amount of tension reinforcement should be calculated taking into consideration the effect of shear and axial force, as well as that of bending moment.

(5) Under conditions of the same section and same shear capacity, if the amount of web reinforcement required to resist shear force is equal to that of the tension reinforcement added to

withstand the influence of shear force, the total amount of reinforcement required by the whole member becomes minimum. The inclination (α_0) of the diagonal crack is largely influenced by the ratio (ρ_l/ρ_b) of the tension reinforcement ratio to the balanced tension reinforcement ratio and member size (b,h).

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