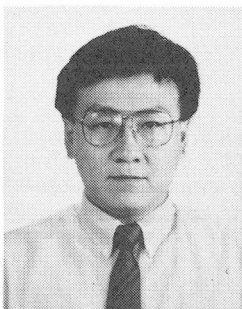


**NONLINEAR COUPLING ANALYSIS OF HEAT CONDUCTION AND  
TEMPERATURE-DEPENDENT HYDRATION OF CEMENT**

(Translation from Proceedings of JSCE, No.426, February 1991)



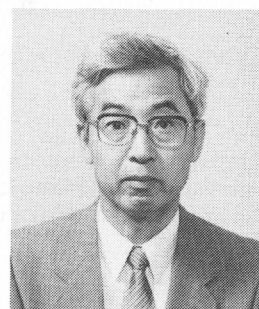
Shusuke HARADA



Koichi MAEKAWA



Yukikazu TSUJI



Hajime OKAMURA

**SYNOPSIS**

This study was made to present a method for nonlinear coupling analysis of heat conduction and temperature-dependent hydration of cement and to indicate the difference in results between the conventional linear method, using an adiabatic temperature rise curve, and the proposed nonlinear method, using the temperature dependent heat hydration model. For the cement hydration model, the heat generation rate which depends on the temperature and past hydration process was adopted. This model also includes a function of temperature and accumulated heat. The nonlinear analysis results are considerably different from that of the conventional linear analysis in the case of thin-wall type structures. The proposed nonlinear analysis is able to express the actual behavior occurring in the concrete structure with high fidelity.

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S.Harada is a manager and senior researcher at the Cement/Concrete Research Laboratory of Sumitomo Cement Co.,Ltd., Chiba, Japan. His research interests include basic characteristics of early aged concrete and durability.

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K.Maekawa is an Associate Prof. in the Dept. of Civil Eng. at the Univ. of Tokyo, Tokyo, Japan. He received his Doctor of Eng. Degree from the Univ. of Tokyo in 1985. His research interests include constitutive models for concrete, structure analysis of RC and the development of high performance concrete.

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Y.Tsuji is a Prof. in the Dept. of Civil Eng. at Gunma Univ., Gunma, Japan. He received his Doctor of Eng. Degree from the Univ. of Tokyo in 1974. His research interests include behavior of reinforced concrete structures, chemically pre-stressed concrete, and properties of fresh concrete.

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H.Okamura is a Prof. in the Dept. of Civil Eng. at the Univ. of Tokyo, Tokyo, Japan. His research interests include fatigue and shear of RC members, durability design of RC structures and application of FEM to RC.

## 1. INTRODUCTION

On the basis of the results of research on thermal stress in mass concrete structures conducted by the Research Committee on Mass Concrete Thermal Stress of Japan Concrete Institute [1], a system has been established to evaluate the probability of thermal cracking induced by heat of hydration. This system, in turn, has been used in various fields as a basis for measuring the effects of mixing factors (cement quality, cement content per unit volume of concrete, etc.) on thermal stress or thermal cracking. Quantification of the possibility of thermal cracking occurrence may be said to enable, for the first time, the limit state design method concerning occurrence of thermal cracking. An approach through the above-described quantitative evaluation will contribute greatly to the enhancement of the durability and reliability of concrete structure in future.

The accuracy and reliability of techniques for determining the probability of thermal cracking depend, to a large extent, upon the kind, shape and dimensions of structures, and the characteristics of concrete used. These, in turn, depend upon the kinds of assumptions made about the total process, such as: (1) the cement heat generation process and thermal properties of concrete, (2) the constitutive equation and mechanical properties for concrete at an early age, which take into account aging and temperature, and (3) criteria for evaluating the occurrence of thermal cracking. The accuracy of analysis of the thermal stress, depends, first of all, upon the accuracy of the basic assumptions and their range of application.

Though important research achievements have been reported by many researchers, their assumptions contain unknown factors and the accuracy of analysis and the range of application of these techniques have not yet reached a satisfactory level. Consequently, calculation of the thermal cracking probability results in either underestimation or overestimation, depending upon the situation, with accuracy of analysis varying to a considerable degree. Needless to say, prediction of the temperature distribution induced by heat generation of cement in a structure can exert considerable influence on the ability to predict the possibility of thermal cracking.

At the present level of technology, engineers know from experience that the results from theoretical temperature analysis differ frequently from the practical, in-the-field results. They are also familiar with the fact that results from analysis can be made to agree with practical results by modifying the analytical model in one way or another.

In current practice, during thermal analysis of concrete, the heat generation curve obtained through an adiabatic temperature rise test is expressed as a function of time, which is then used as the heating curve at all positions within the concrete structure. In other words, the heat generation term of heat conduction analysis has been given, not as a function of location, but as a function of time only. The adiabatic temperature rise, however, is due to a cement hydration process which occurs in a special environment having no heat transfer. The position near the center of a real structure with an extremely large cross-section can be deemed to be a condition which is similar to that in the adiabatic temperature rise test. However, concrete existing near the structure surface and the concrete located at the forcibly cooled sections undergo a hydration process and heat generation different from those in the adiabatic condition.

In performing a more accurate heat conduction analysis, it is necessary to calculate the heat generation rate at specific points in the structure because

the cement hydration itself depends upon the temperature hysteresis of the position concerned. In other words, we are required to identify the hydration process for each different point in a structure and the resultant heat generation rate. Suzuki et al have already presented an approach to determine quantitatively the heat generation process of cement per unit volume in concrete, while at the same time, considering the temperature hysteresis [2].

In other words, the cement hydration reaction itself is dependent on the temperature history of concrete at a particular location. Therefore, the result of heat conduction analysis is indispensable to the prediction of the heat generation rate and it is necessary to trace the mutually dependent behavior between heat transfer and heat generation during thermal analysis.

This study proposes a nonlinear thermal analysis method, taking into account the effects of temperature hysteresis on the cement heat generation due to hydration, for the purpose of enhancing the accuracy of thermal cracking probability determination. The coupled behavior (coupled problem) of the heat transfer and the heat generation are analyzed as a nonlinear problem in terms of temperature that is discrete for space and time. Concerning the thermal analysis of relatively thin, wall-type structures, we suggest that an essential condition in improving the accuracy of the calculation is to consider a coupled problem of the heat transfer and the cement hydration process during heat generation.

The methods for setting up to study heat generation by hydration in the nonlinear thermal analysis method proposed in this study are basically the same as in that of the conventional linear thermal analysis method. The setup procedure differs depending only on the degree of accuracy required in the final result. In other words, in general cases, the heat generation model of concrete may be set up, on the basis of the deduction method discussed in the previous paper [2] and using the adiabatic temperature rise rate already known. When a more detailed study is required, it is possible to set up a more accurate hydration heat generation model, by obtaining an adiabatic temperature rise curve of higher accuracy from an experiment in which the initial temperature is set at several different levels. Consequently, the nonlinear thermal analysis proposed here requires no greater effort than that for the conventional thermal analysis.

## 2. COUPLED NONLINEAR EQUATION

The governing equation of heat conduction is expressed by the law of energy conservation and the heat flux constitutive equation. Assuming thermal flux to be isotropic and linear in relation to the temperature gradient, the governing equation and the boundary condition are already established as follows:

Law of energy conservation

$$c\rho \cdot \frac{dT}{dt} = -\text{div } q + H \quad \dots\dots\dots (1)$$

Heat flux equation

$$q = -k \cdot \text{grad } T \quad \dots\dots\dots (2)$$

Boundary condition

$$q \cdot n = m(T - T_{\text{ext}}) \quad \dots\dots\dots (3)$$

where t: time, k: heat conductivity,  $\rho$ : unit volume weight, c: specific heat, n: outward unit normal vector defined at the surface, m: heat transfer coefficient; for concrete, these are normally given as constants. Furthermore, q, T,

$T_{ext}$ ,  $H$  are respectively the heat flux vector, temperature, external environment temperature, and heat generation rate per unit time and unit volume. These are functions of space and time.

The conventional method of analysis based upon the adiabatic temperature rise test treats the heat generation rate  $H$  as though it was a function of time only. This is equivalent to the assumption that  $H$  is equal in any portion of the domain to be analyzed. In this case, Eq.(1) and Eq.(2) consists of an operator allowing linear combination for the temperature  $T$ . Hence, the equations, even if discretized, are converted into linear algebraic equations.

Uchida et al. proposed a heat generation model by hydration taking temperature hysteresis into account, which enables a determination of the heat generation rate by hydration for any temperature hysteresis, including an adiabatic state, from the heat generation rate by hydration of cement paste under isothermal conditions [3]. The present study employs Suzuki's method and also incorporates Uchida's model as a heat generation by hydration model to trace the heat generation process at different points of a concrete structure. This model of heat generation due to hydration is shown below;

$$H = \underline{H}(Q, T) = \underline{H}_s(Q, T_s) \exp \left[ -\frac{E(Q)}{R} \left( \frac{1}{T} - \frac{1}{T_s} \right) \right] \dots\dots\dots (4)$$

$$Q = \int \underline{H} dt, \quad H = \underline{H} \cdot C \dots\dots\dots (5)$$

where  $T_s$  : a reference temperature,  $Q$  : an accumulative heat generation,  $R$  : the gas constant (1.986 cal/deg.mol),  $C$ : the unit cement content, and  $\underline{H}$  : a heat generation rate per unit weight of cement.

In the above equations,  $E(Q)$  (the activation energy) and  $\underline{H}_s(Q, T_s)$  (reference heat generation rate at  $T_s$ ) should be given as material functions respectively corresponding to cement in concrete. These material functions are influenced greatly by the conditions for dispersion of cement particles and by their contact with water. Consequently, the effects of the mixing efficiency of cement particles on the hydration reaction,  $E(Q)$  and  $\underline{H}_s(Q, T_s)$  which are obtained through a cement paste test cannot be assumed to be the same as the characteristics in concrete. Suzuki et al have developed a method of determining the activation energy  $E(Q)$  and reference heat generation rate  $\underline{H}_s(Q, T_s)$  by means of the adiabatic temperature rise test with concrete of the same composition and the same mixing procedures, but with differing initial temperatures [2]. The analysis by this model makes it possible to deal with the differences in the heat generation characteristics which vary with concrete composition. The research was started with a nonlinear thermal analysis, using the material functions which represent the actual heat generation process of concrete quantified by Suzuki's method.

The heat generation rate  $\underline{H}$  represented by Eq.(4) is a function of the temperature  $T$  and accumulative heat generation  $Q$ . Since the temperature  $T$  is a function of time and space, the heat generation rate  $\underline{H}$  and accumulative heat generation  $Q$  vary also with time and space. In this case, the accumulative heat generation  $Q$  represents the degree of hydration varying in each position of a structure, while the time integral in Eq.(5) is a parameter representing the path-dependency which reflects the differences in terms of the temperature and hydration hysteresis.

By continuously following up the hydration hysteresis at all positions, the coupled problem of the heat conduction and the heat generation process can be

worked out from Eq.(5) after solving a system of Eq.(1) to Eq.(4) in terms of space and time. In this process, Eq.(4) being composed of a nonlinear operator in terms of T, the group of governing equations is not a linear system in terms of the temperature T, but a system of nonlinear equations.

### 3. DISCRETIZATION AND NONLINEARITY OF THE TEMPERATURE

Substituting Eq.(2) and Eq.(4) into Eq.(1), we have the following nonlinear differential equation to be discretized in space and time;

$$c\rho \cdot \frac{dT}{dt} = k \cdot \nabla^2 T + H(Q, T) \quad \dots\dots\dots (6)$$

Using the weight residual method, which can be applied independently of the nonlinearity of governing equations, leads us to a system of nonlinear equations as shown below [4];

$$[K]\{T\} - [C]\left\{\frac{dT}{dt}\right\} + \{H(Q, T)\} = 0 \quad \dots\dots\dots (7)$$

where [K] is the heat conduction matrix and [C] the heat capacity matrix. Here, these matrices are added by the element matrix, using the shape function  $N_i$  composed of m nodes as shown below;

$$\left. \begin{aligned} [K] &= \sum_{\text{element}} \int k [B]^T [B] dV \\ [C] &= \sum_{\text{element}} \int c [L]^T [L] dV \end{aligned} \right\} \quad \dots\dots\dots (8)$$

$$[B]^T = \begin{vmatrix} dN_1/dx & dN_1/dy & dN_1/dz \\ dN_2/dx & dN_2/dy & dN_2/dz \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ dN_m/dx & dN_m/dy & dN_m/dz \end{vmatrix} \quad [L]^T = \begin{vmatrix} N_1 \\ N_2 \\ \cdot \\ \cdot \\ N_m \end{vmatrix}$$

$\{H(Q, T)\}$  is the heat generation vector as defined below;

$$\{H(Q, T)\} = \sum_{\text{element}} \int [L]^T H(Q, T) dV \quad \dots\dots\dots (9)$$

Introducing the time difference scheme of the Crank-Nicolson method, the following nonlinear equation is obtained in terms of  $\{T\}$  which is the nodal temperature vector at the time  $(t_0 + \theta \Delta t)$ ;

$$R(\{T\}) = 0 \quad \dots\dots\dots (10)$$

$$R = \left[ [K] - \frac{1}{\theta \Delta t} [C] \right] \{T\} + \frac{1}{\theta \Delta t} [C] \{T_{00}\} + \{H(Q, T)\}$$

Here, the nodal temperature vector  $\{T_{00}\}$  at  $t_0$  is a known value corresponding to the initial temperature at the first step.

Since the hydration heat generation vector  $\{H(Q, T)\}$  is a nonlinear function of the unknown  $\{T\}$ , and the solution  $\{T\}$  which satisfies Eq.(10) can not be obtained explicitly, it should be determined by an iterative procedure. This research employed the Newton method for an iterative calculation. When we partially differentiate Eq.(10) with respect to  $\{T\}$ , we have Eq.(11) from Eq.(9);

$$dR=[D]d\{T\} \dots\dots\dots(11)$$

$$[D]=\left[ [K]-\frac{1}{\theta\Delta t}[C]+\sum_{\text{element}}\int\frac{dH}{dT}[L]^T[L]dV \right]$$

Consequently, if the convergence conditions ( $R \ll 0$ ) with a nodal vector of  $\{T^{(i)}\}$  are not satisfied, calculation should be repeated in the following procedure until the result is converged.

$$\{T^{(i+1)}\}=\{T^{(i)}\}+[D]dR \dots\dots\dots(12)$$

From the nonlinear solution  $\{T\}$  obtained at the time  $(t_0+\theta\Delta t)$ , the nodal temperature vector  $\{T_d\}$  at the time  $(t_0+\Delta t)$  is given as follows through assumed linear interpolation;

$$\{T_d\}=\{T_{i0}\}+[\{T\}-\{T_{i0}\}]/\theta \dots\dots\dots(13)$$

It is not necessary to keep in memory the hysteresis of heat generation and temperature at the numerical integration point in each element. The past process of the temperature and the heat generation is explicitly expressed by the accumulative heat generation  $Q$  given by Eq.(5). Hence, we may sequentially calculate  $Q$  newly by numerically integrating Eq.(5) for each time step:

$$Q_i=Q_{i0}+\Delta t \cdot H(Q_{i0}, T) \dots\dots\dots(14)$$

The frequency of iteration on the temperature varies, depending on the interval of the time steps, but the result can be converged in two or three times when the time interval is used conventionally.

The water in concrete moves according to the difference of the hydration path as well as the shape of the concrete structure. It can be thought that the heat generation process of concrete changes along with the concrete water content. Strictly speaking, this analysis should be considered as a system of coupled equations, including a governing equation concerning the water movement in concrete, evaluating a parameter of the water content for the heat generation model. Solution of this kind of multi-phase, mutually-coupled problem is in itself an established technique of numerical analysis regardless of linear or nonlinear type. However, the water movement in young concrete or in concrete of a hydration heat generation model and the water content-dependency of the heat generation are left unknown at the moment. In this study, therefore, the effect of water movement during the hydration process on nonlinear thermal analysis is deemed sufficiently small and can be neglected. But, in the future, the three factors of water movement, thermal energy movement, and hydration process should be solved as a system of coupled equations for thermal analysis of concrete structures of relatively thin cross section using ultra rapid hardening cement.

#### 4. NONLINEAR THERMAL ANALYSIS OF CONCRETE WALL

Through a process of nonlinear thermal analyses, structures of five different thickness were analyzed using the proposed model which couples heat generation and heat conduction behavior in concrete. These results were compared to those of the conventional analysis model. Conventional methods have worked well with structures of large cross section, but not so well with thin wall-type structures. The new proposed model attempts to analyze structures of all sizes in a single model.

In conventional analysis, it is assumed that the adiabatic temperature rise will

occur of all positions in a structure. Empirical evidence shows that this analysis does not accurately describe conditions as structures become thinner in cross section [5]. Wall type structures of 1 m or less in thickness, using rich concrete, are expected to increase in the future. The conventional analysis will not provide the degree of accuracy needed for this type structure.

#### 4.1 Objects to be analyzed

In addition to the proposed nonlinear thermal analysis which links heat generation with heat conduction, the conventional analysis model is applied to check the effect of path dependency of heat generation along with heat transfer. The analysis is conducted on two kinds of concrete, 250 kg/m<sup>3</sup> and 350 kg/m<sup>3</sup> of unit cement content, using the heat generation model of Eq.(4) which considers the temperature dependency. Table 1 shows the mix proportion of concrete. For each mix proportion, the adiabatic temperature rise test is made by differing the concreting temperature to 238 °K, 293 °K, and 303 °K. The result is shown in Fig.1. The accuracy of the adiabatic temperature rise testing apparatus used in this study was thoroughly proved using large concrete blocks [6].

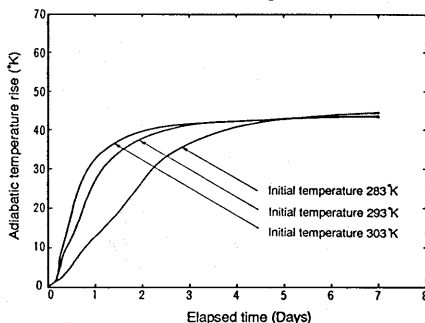
Fig.2 shows the results of calculations of the activation energy  $E(Q)$  and reference heat generation rate  $H_s(Q, T_s)$  of

Table 1 Mix proportions used in the experiment

Concrete temperature (°K)	Max. size of aggregate (mm)	Range of slump (cm)	Range of air content (%)	Water cement ratio (%)	Sand aggregate ratio (%)	Unit weight (kg/m <sup>3</sup> )				
						Water	Cement	Fine aggregate	Coarse aggregate	Chemical* admixture
283	40	8 ± 1	4 ± 1	60.8	42.4	152	250	810	1117	0.625 —
	40	8 ± 1	4 ± 1	43.4	39.0	152	350	713	1129	0.875 0.7
293	40	8 ± 1	4 ± 1	61.2	42.3	153	250	807	1117	0.625 0.25
	40	8 ± 1	4 ± 1	43.7	38.9	153	350	710	1129	0.875 0.35
303	40	8 ± 1	4 ± 1	60.0	42.5	150	250	815	1117	0.625 0.5
	40	8 ± 1	4 ± 1	42.9	39.2	150	350	718	1129	0.875 0.7

\* AE & water-reducing agent

(1-a) Unit cement content = 250 kg/m<sup>3</sup>



(1-b) Unit cement Content = 350 kg/m<sup>3</sup>

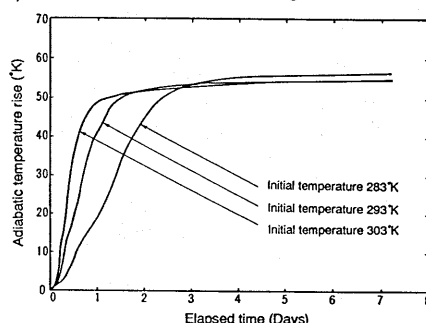
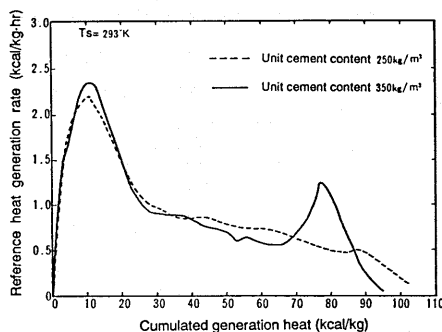


Fig.1 Adiabatic Temperature Rise Data

(2-a) Reference heat generation rate



(2-b) Activation energy

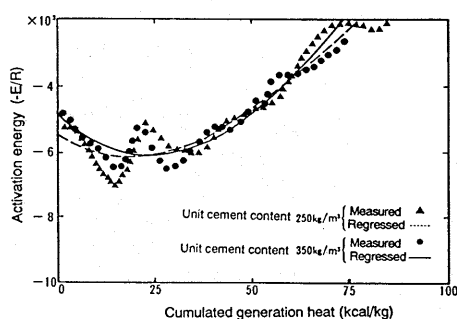


Fig.2 Activation Energy and Reference Heat Generation Rate (C = 250,350 kg/m<sup>3</sup>)

cement in concrete used for Eq.(4) according to the method of Suzuki. The data used in this analysis is determined through spline interpolation from the result of the adiabatic temperature rise test of Fig.1. Details of the method to derive  $E(Q)$  and  $H_s(Q,T_s)$  are described in the previous report [2].

Fig.3 shows the comparison between the analytical value obtained through path integration of Eq.(4) under adiabatic conditions and the measured values. Both show satisfactory agreement with each other.

In order to consider the contribution of nonlinearity of the temperature in the heat generation process, the simplest one dimensional heat conduction state was assumed as condition for analysis. Namely, as shown in Fig.4, our attention is paid to the heat transfer in the direction of wall thickness for a wall with sufficient plane width. Here, five wall thicknesses from 0.1 m to 2.0 m are studied to set the boundary conditions for radiation from both sides of the wall.

In the thermal analysis of a wall structure, the heat conductivity and the outer environmental conditions should be taken as important factors. But there exist many uncertain factors for quantification of the effects of wind, temperature dependency of heat conductivity, and the effect of radiation. In view of the

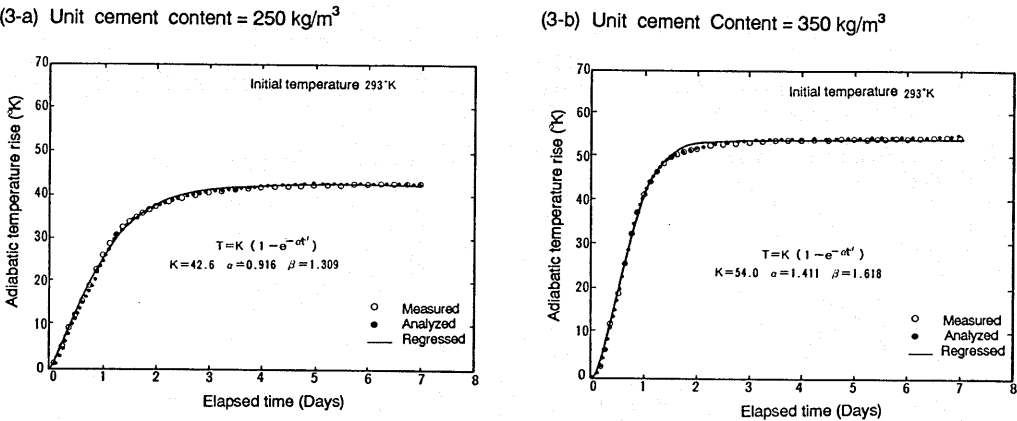


Fig.3 Comparison of Theoretical and Measured Adiabatic Temperature

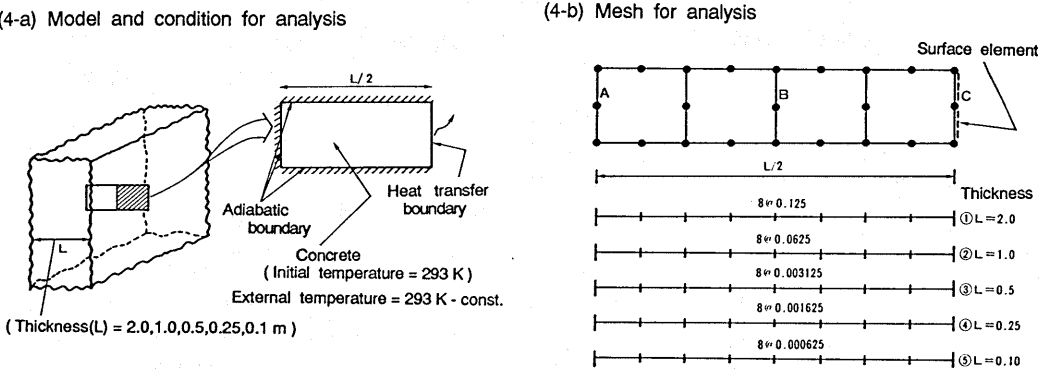


Fig.4 One-Dimensional Heat Transfer Model Using the Analysis



object of this analysis, that is, the investigation of the coupled effect in the heat conduction analysis of the heat generation process of cement depending on the temperature, the simple boundary conditions are set up with the outside air temperature kept at 293 °K constant (equal to the concreting temperature) and the heat conductivity kept constant, also.

Table 2 shows the thermal constant used in the thermal analysis. The specific heat is calculated from the concrete mix proportion. The same thermal constant is applied for each of the nonlinear thermal analyses considering the temperature dependency of the heat generation process and the conventional linear thermal analysis. Also, the same time step is used for calculation. In the conventional linear thermal analysis, the adiabatic temperature rise curve is expressed by the exponential function of three coefficients which have been proven to be compatible with the actual measurement of adiabatic temperature rise and was proposed by the authors in the previous report [7]. The average error of the adiabatic temperature rise obtained through the exponential function from the experimental value is 0.2 °K or less while the maximum error remains about 1 °K. Linear and nonlinear analyses on the temperature, which are dealt with for comparison in this research, agree well with each other in the adiabatic state, as shown in Fig.3.

Table 2 Analysis Cases and Thermal Constants

No.	Unit cement content (kg / m <sup>3</sup> )	Thickness (m)	Thermal conductivity (kcal / m·hr·°C)	Specific Heat (kcal / kg·°C)	Unit weight (kg / m <sup>3</sup> )	Heat transfer coefficient (kcal / m <sup>2</sup> ·hr·°C)	Time step (day)
1	250	2.00	2.55	0.2526	2327	10	0.005
2		1.00					0.005
3		0.50					0.005
4		0.25					0.005
5		0.10					0.0025
6	350	2.00	2.55	0.2523	2342	10	0.005
7		1.00					0.005
8		0.50					0.005
9		0.25					0.005
10		0.10					0.0025

#### 4.2 Coupling effect on the temperature rise

Fig. 5 shows the result of analysis on temperature change with time. In this figure, temperature change at a central position of the wall (Point A), the surface (Point C), and an intermediate point (Point B) is shown for different wall thicknesses and cement contents.

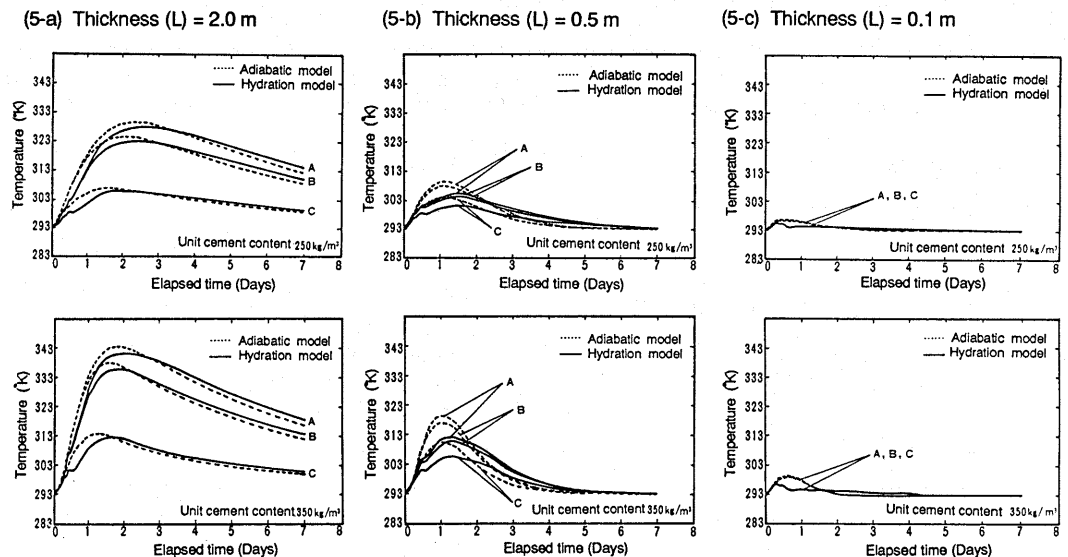


Fig.5 Temperature Change with Time under One-Dimensional Heat Transfer Conditions

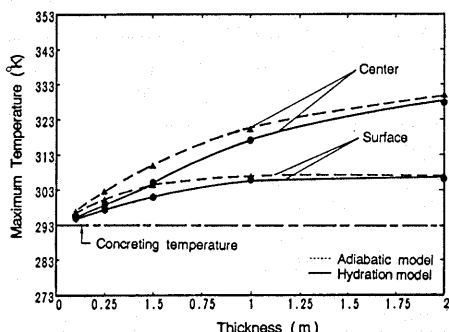
In the case of a wall of 2.0 m in thickness, when we compare the conventional analysis results which give the heat generation rate from the adiabatic temperature rise with those considering the temperature dependency of the heat generation, almost no difference is found according to the differences in mix proportion of concrete and position in the wall. However, if the wall thickness becomes smaller, they differ from each other. Here, it should be noted that the maximum temperature rise of the conventional linear thermal analysis exceeds that of the analysis considering the temperature dependency of the heat generation process in all positions.

Fig.6 shows the maximum temperature rise in the center of the wall and at the wall surface using both analysis methods. Fig.7 shows the difference in the maximum temperature rise between the two analyses considering the temperature dependency of the heat generation process and conventional analysis on the center and surface of the wall, respectively for each wall thickness. The most significant difference is observed to appear between both analytical results with respect to the thickness range of 0.25 - 0.5 m which is widely used in concrete structures. Such differences decrease when the wall thickness approaches 0.1 m.

The concrete near the wall surface in contact with the outside air is exposed to cooling effects and shows a hydration heat generation process different from the adiabatic state. Since radiation to the air is relatively small compared to the hydration heat generated from the entire concrete structure with a thick cross section, the temperature hysteresis and heat generation state are nearly equivalent to the adiabatic state in almost all ranges. On the other hand, the region exposed to the cooling effect by outside air increases with respect to the whole structure as thickness decreases, and the heat generation corresponding to the adiabatic state cannot be applied to analyze entire section. This may result in a substantial difference in the result of analysis. If the sectional thickness decreases further, the hydration heat moves to the surface quite rapidly, resulting in a reduced difference in the result. In an extreme case with zero thickness, the temperature in the structure becomes equal to the outside air temperature theoretically, with the temperature rise due to heat generation by hydration becoming zero in both analytical models.

As shown in Fig.7, for the concrete of  $350 \text{ kg/m}^3$  in unit cement content, the maximum temperature rise increases above that of concrete with small cement content. Moreover, it may be found that the difference in both analytical

(6-a) Unit cement content =  $250 \text{ kg/m}^3$



(6-b) Unit cement content =  $350 \text{ kg/m}^3$

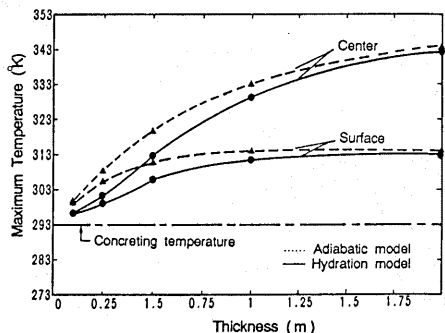
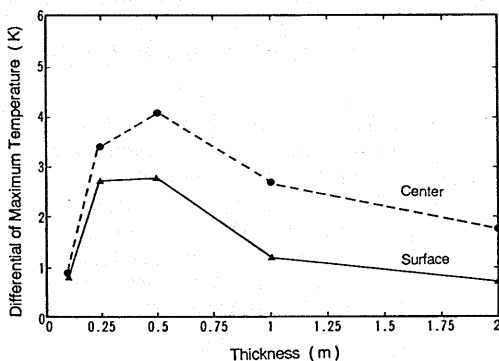


Fig.6 Relationship between the Maximum Temperature Rise and the Wall Thickness

results has been further amplified due to the effect of thickness. For example, the analytical value of the maximum temperature for the thickness of 0.25 m is lower by 7 °K than that of the conventional analysis at the center point, showing the difference in maximum temperature rise by an amount as large as 30%. This indicates that the temperature dependency of the hydration heat generation process may possibly affect substantially the thermal distribution of the structure in a case of concrete with a high heat generation rate. The maximum temperature rise discussed above is an amount directly related to the thermal stress induced by the external restraining action of the wall (e.g., already hardening concrete) during the cooling process.

Fig.8 shows the analytical results of the temperature distribution of the wall for each analysis method and mix proportion. Here, this figure shows the result of each analysis method for the time point when the temperature gradient is largest. Regarding the average temperature gradient in the center of the wall

(7-a) Unit cement content = 250 kg/m<sup>3</sup>



(7-b) Unit cement content = 350 kg/m<sup>3</sup>

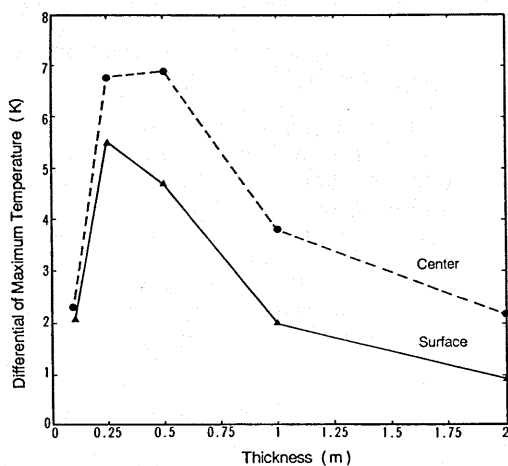
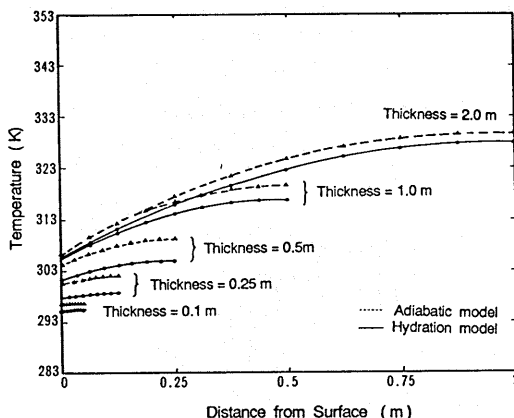


Fig.7 Differences in Maximum Temperature between the Analysis Methods

(8-a) Unit cement content = 250 kg/m<sup>3</sup>



(8-b) Unit cement content = 350 kg/m<sup>3</sup>

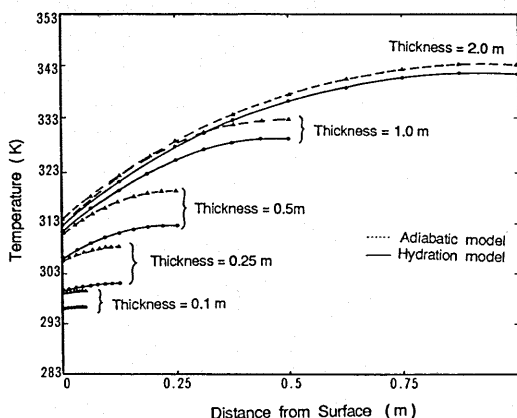


Fig.8 Temperature Distribution at the Age of Maximum Temperature Rise

and the wall surface, no substantial variance due to differences in the analysis methods appears. This means the internal restraining action at the thickest cross section may not lead to substantial variation in the results of the proposed analysis as compared to the conventional analysis.

## 5. VERIFICATION OF NONLINEAR THERMAL ANALYSIS METHOD

Nonlinear thermal analysis, which considers the temperature dependency of the heat generation process of cement, and conventional thermal analysis without such consideration are made for wall type structures of differing thicknesses. It is known from analytical results with wall type structures that the method coupling the cement heat generation process and heat conduction (a method considered to simulate the practical state faithfully) differs greatly from the conventional approaches. But, the effectiveness of the thermal analysis method cannot be discussed solely by comparing analytical results. The effectiveness can be verified by matching analytical results and the actual measurements taken from a practical structure. Here, in order to verify the effectiveness of a nonlinear thermal analysis, comparison is made between the actual measurement of temperature hysteresis of a test specimen in a laboratory and the corresponding analytical results.

The test specimen used in temperature measurement is a cube of about 0.6 m on each side as shown in Fig.9. Two kinds of test specimen were prepared to vary the surface thermal radiation conditions : one specimen was covered with heat insulating material over the entire surface and the other was without such coverage. The concrete used for the test specimen is normal portland cement, with the unit cement content being  $500 \text{ kg/m}^3$  for the covered specimen and  $350 \text{ kg/m}^3$  for the other specimen. Foamed styrol of 8 cm in thickness was used as insulating material. The test specimens were left in the laboratory with less air flow to measure the change in the interior temperature of the specimens and the room temperature.

Among thermal characteristic values used for analysis, the heat transfer coefficient is determined analytically from the temperature changes of test specimens and the fluctuation of the outside air temperature during the two week period after concreting. Temperature change of test specimens in the stage when the concrete heat generation rate had decreased sufficiently was assumed to be governed by temperature fluctuation of the outside air. The heat transfer coefficient is determined indirectly so that the analytical value agrees with the actual measurement of temperature two weeks after concreting. The heat transfer coefficient was calculated to be  $0.425 \text{ kcal/m}^2 \text{ hr}^\circ\text{C}$  when the test speci-

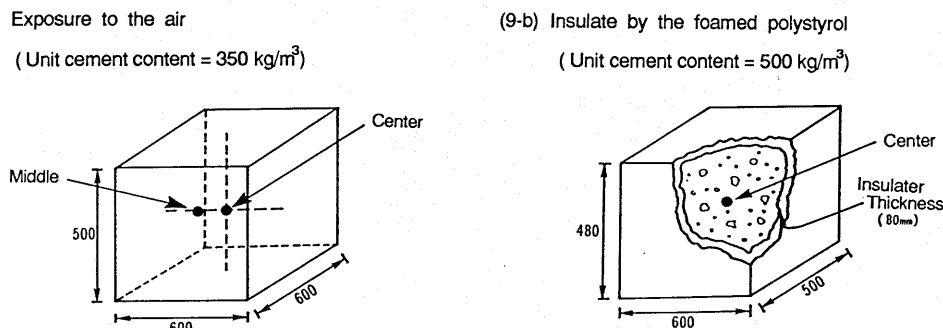


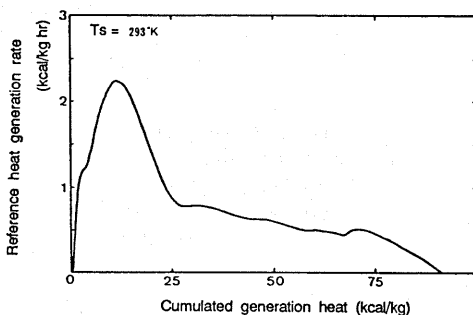
Fig.9 Diagram of Test Specimens used to measure the Temperature Rise

men was covered with insulating material and  $10 \text{ kcal/m}^2 \text{ hr } ^\circ\text{C}$  when not covered. The specific heat was determined through a calculation which included the specific heat factors for each component material in the respective mix proportion [2]. The value generally applied to conventional thermal analysis, that is,  $2.0 \text{ kcal/mhr}^\circ\text{C}$  was used as the heat conductivity. For the heat generation characteristics of concrete, the cement activation energy  $E(Q)$  and reference hydration heat generation rate  $H_s(Q, T_s)$ , which were calculated according to the method of Suzuki et al [2], were used. Fig.10 shows the concrete heat generation characteristics used in analysis when the unit cement content was  $500 \text{ kg/m}^3$ .

Fig.11 shows mutual comparison of the actual temperature measured and the analytical value from nonlinear thermal analysis considering temperature dependency of the heat generation process of cement. In this case, the unit cement content is  $350 \text{ kg/m}^3$  and the test specimen surface was exposed to air. The figure shows a comparison of temperature variation at two points (center and intermediate point between the center and the surface) of the test specimen. In each position, the measured value and the analytical value agree quite well. In this way, the proposed thermal analysis can properly estimate that the hydration reaction in cement was suppressed and the heat generation rate slowed down when radiation from the test specimen surface was large and temperature rise was low.

Fig.12 shows a comparison between the actual measured temperature and the analytical value at the center of the test specimen when the unit cement content was  $500 \text{ kg/m}^3$  and the test specimen was covered with insulating material over

(10-a) Reference heat generation rate



(10-b) Activation energy

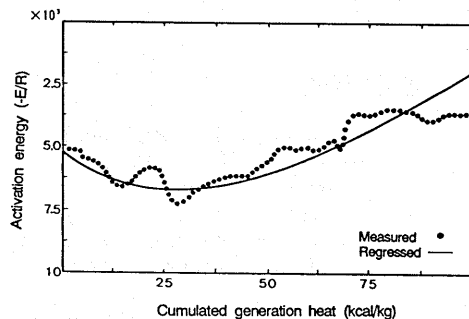


Fig.10 Reference Heat Generation Rate and Activation Energy ( $C = 500 \text{ kg/m}^3$ )

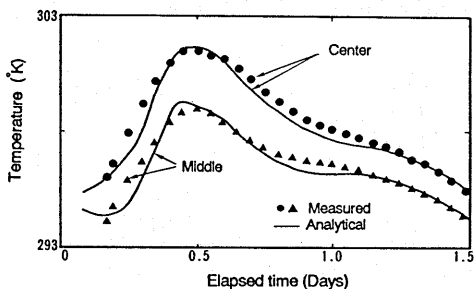


Fig.11 Temperature Change with Time in the Exposed Surface Condition

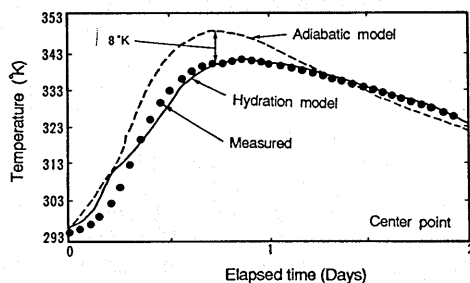


Fig.12 Temperature Changes with Time in the Insulated Surface Condition

it's surface and was exposed to the air. In this case, the analytical value obtained from the conventional thermal analysis was also shown for a comparison with the nonlinear thermal analysis which considered the temperature dependency of the heat generation process.

This figure indicates relative agreement between the measured temperature and the result of nonlinear thermal analysis. It is known, however, concerning the conventional linear thermal analysis based on the adiabatic temperature rise that the results exceed the measured value from early age and is greater by about 8°K in maximum temperature. The concrete used in this experiment included a superplasticizer for the water reducing agent, but material functions  $E(Q)$  and  $H_s(Q, T_s)$  used in the analysis are given for cement in concrete using a AE water reducing agent. Accordingly, the analysis accuracy in 0.3 days after concreting was more or less lower.

The surface insulated test specimen (Fig.9b) simulated conditions, under surface heat radiation was low, causing heat accumulation effects. This condition was further enhanced by the high unit cement content causing a high temperature rise rate. Under these conditions making the assumption that an adiabatic state heat generation process existed led to considerable inaccuracy using the conventional model of heat analysis. The proposed nonlinear analysis led to more accurate prediction.

As is evident from above analytical and experimental investigation, introduction of different heat generation processes into analysis of heat conduction for all locations of a concrete structure concerned, that is, modeling oriented more to the practice, may become indispensable for enhanced accuracy of thermal analysis in future.

## **6. CONCLUSION**

In this research, the nonlinear thermal analysis method is presented, which takes coupling of the heat conduction equation and temperature-dependent heat generation process into account. Where the effect of radiation is relatively large as in the case of wall type structures, the conventional linear thermal analysis method, which assumes the heat generation process under adiabatic state for all locations, is considered not feasible enough.

The necessity of a practice-oriented, nonlinear thermal analysis method tracing strictly individual heat generation processes in each location of a structure is obvious, due to the wide range of structural dimensions and the need for various concrete mix proportions found in modeling construction. This fact indicates that the conventional analysis method is not enough for structures under forced cooling or under conditions with large heat movements. Conclusions obtained in this research are summarized as follows:

(1) A coupled problem of the temperature hysteresis dependent hydration heat generation process and heat conduction equation was expressed as a nonlinear equation for temperature. In this case, nonlinear iterative calculation according to the Newton method proves effective.

(2) When the structural thicknesses exceed 1 m, nearly equal temperature rise values and temperature distribution values can be obtained from both the nonlinear thermal analyses proposed in this study and the conventional linear thermal analysis assuming similar adiabatic heat generation for all locations.

(3) For structural thicknesses of 25 - 50 cm which are commonly used, there is

a difference of about 30% in the prediction of maximum temperature rise between the conventional linear method and proposed nonlinear method. Consequently, the conventional linear thermal analysis proves to be insufficient for structures exposed to forced cooling and where heat movement is large.

(4) The difference in analysis methods as pointed out in (3) above becomes more evident with increasing unit cement content.

(5) As regards temperature gradient occurring between the center and the surface of the wall, the difference between analysis methods is relatively small with the respect to the maximum temperature rise.

(6) The concept of setting a concrete heat generation model in the proposed nonlinear thermal analysis is basically the same as the conventional method and the labor necessary for the proposed method is nearly the same as that of the conventional method.

In the analysis of relatively large wall structures or block concrete structures, using concrete with high cement content, the accuracy of thermal analysis can be increased by considering coupling of the temperature dependent heat generation process and heat conduction.

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#### REFERENCES

- [1] Subcommittee of JCI, "Report of the Subcommittee for Research on Thermal Stress in Mass Concrete", Journal of JCI Vol.23, No.11, November 1985 (in Japanese)
- [2] Suzuki.Y., Tsuji.Y., Maekawa.K., Okamura.H., "Quantification of Hydration-Heat Generation Process of Cement in Concrete", Concrete Library of JSCE, No.16, pp.111-124, December 1990
- [3] Uchida.K., Sakakibara.H., "Formulation of the Heat Liberation Rate of Cement and Prediction Method of Temperature Rise based on Cumulative Heat Liberation", Concrete Library of JSCE, No.9, pp.85-95, 1987
- [4] O.C.Zienkiewicz, "The Finite Element Method In Engineering Science", McGraw-hill, 1977
- [5] Suzuki.Y., Harada.S., Maekawa.K., Tsuji.Y., "Applicability of Adiabatic Temperature Rise for Estimating Temperature Rise in Concrete Structures", Transactions of the Japan Concrete Institute, Vol.7, pp.49-56, 1985
- [6] Suzuki.Y., Harada.S., Maekawa.K., Tsuji.Y., "Method for Evaluating Performance of Testing Apparatus for Adiabatic Temperature Rise of Concrete", Concrete Library of JSCE, No.14, pp.151-162, March 1990
- [7] Suzuki.Y., Harada.S., Maekawa.K., Tsuji.Y., "Evaluation of Adiabatic Temperature Rise of Concrete measured with the New Testing Apparatus", Concrete Library of JSCE, No.13, PP.71-84, June, 1989