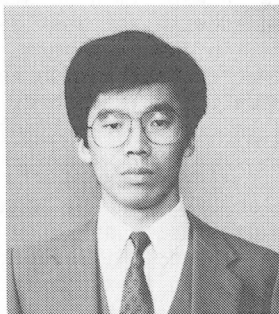
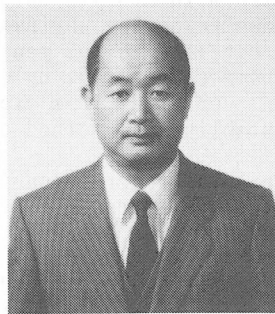


INFLUENCE OF NON-UNIFORM SHRINKAGE STRESS
ON FLEXURAL STRENGTH OF CEMENTITIOUS MATERIAL

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SYNOPSIS

In order to generate non-uniform self stress that is different in area of working zone, partial surface provided by controlling coating area of mortar beams was exposed to drying condition at first. Distribution of self stress thus generated was experimentally determined by measuring variation in strain when a part of a specimen was cut or notched. Next, beams were flexurally loaded and failure condition was examined. It was proved that the maximum tensile stress obtained by superposing stress due to external load upon the self stress did not serve as a failure criterion.

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1. INTRODUCTION

Self stress is generated in materials which are not subjected to external load. With regard to concrete, self stress is caused by drying shrinkage, temperature variation, chemical prestress and so on. For example, when drying shrinkage occurs in concrete specimen, tensile self stress is generated near the surface and compressive self stress in the inner part due to internal restraint. Tensile or flexural strength and cracking tendency are considerably influenced by self stress, and many experimental and analytic studies have been conducted, especially on drying shrinkage[1]~[6]. But measurement of this kind of self stress is very difficult, because it does not correspond to observed strain and it is not uniform in cross section. There is an experimental study in which self stress due to drying shrinkage was measured with stress gauge recently developed[7], but it is not generally used. In many papers, quantitative analyses have been carried out by method of numerical simulation but the results have not been verified from experimental results. Therefore, it is very important to establish the method of measuring this kind of self stress.

Failure criterion of concrete subjected to self stress has been obtained from the maximum principal stress theory : failure occurs when the maximum tensile stress obtained by superposing stress due to external load upon self stress reaches the tensile strength. On the other hand, Hillerborg et. al.[8] explained with fictitious crack model that decrease in flexural strength due to drying shrinkage increased with increase in specimen size. And Kuwahara[9] showed that the decrease in split tensile strength due to thermal stress was only 30% of the strength obtained for control specimen, even if maximum thermal stress in the specimen reached the tensile strength. These test results suggest that the maximum principal stress theory does not serve as a failure criterion for materials subjected to non-uniform self stress[10].

The purposes of this study are to establish the method of measuring non-uniform self stress in mortar and concrete and to make clear the influence of self stress on flexural strength on the basis of experimental results. In order to get various magnitude and distribution of self stress, partial surface provided by controlling coated area was exposed to drying condition. It was thought that the area of working zone of self stress was dependent upon the area of exposed surface, and that magnitude and distribution of self stress were varied with distribution of moisture in cross section.

In this study, self stress was measured by mechanical methods [11],[12] which have been used for measuring self stress in metals due to processing or welding. Self stress due to drying shrinkage of mortar and concrete was experimentally determined by measuring elastic strain which was generated when self stress was partially released by cutting or notching the specimen. Furthermore, flexural strength of mortar was measured after the specified period of drying, and influence of self stress on flexural strength was experimentally investigated.

2. EXPERIMENTAL PROCEDURES

2.1 Materials and Mix Proportions

Mortar and concrete were prepared from the materials, properties of which are shown in Table 1. Maximum size of fine aggregate was 5 mm. Mix proportion of mortars and concrete are shown in Table 2 and 3 respectively. In the case water-cement ratio (W/C) was 0.50 or 0.70, lignin based air-entraining and water-reducing agent was used. For W/C=0.30, naphthalene based superplasticizer and condensed silica fume were used. Compressive strength (measured with

ø7.5x15 cm cylinder), modulus of elasticity for compression (1/3 secant modulus, measured with ø10x15cm cylinder) and split tensile strength (measured with ø10x15 cm cylinder) are shown in Table 4. These tests were carried out immediately after 7 days of water curing.

2.2 Mixing

With W/C=0.50 or 0.70, mortars and concretes were mixed with a forced mixing type mixer (capacity: 0.05 m³, rotation: 56rpm), and with W/C=0.30, mortars were mixed with Hobert type mixer (capacity: 0.01 m³, rotation: 280 rpm, revolution: 80 rpm). All the materials were mixed for 2 minutes after they were put into the mixer.

2.3 Curing and Drying Conditions

Mortar and concrete specimens were demolded after two days from casting and were cured in water (20°C) for five days. Then specimens were stored in the atmospheric condition of 20°C and 50 % R.H.. In order to get self stress which had different magnitude and distribution in cross section, width of exposed surface "a" was varied as 0, 1, 2, 4, 10 and 40 cm. For mortar specimens, specified area was sealed with aluminum tape (0.05 mm in thickness). For concrete specimens, the specified area was covered with gummed cloth tape at first, next coated with epoxy resin, and then wrapped with vinylidene chloride sheet.

Table 1 Materials

Cement	High-early-strength portland cement	
Fine aggregate	Weathered granite pit sand (Maximum size:5 mm)	
Chemical admixture	Lignin-based air entraining and water reducing agent	
	Naphtalene-based superplasticizer	
Admixture	Condensed silica fume (Blaine: 2x10 ⁵ cm ² /g)	Only for W/C=30%

Table 2 Mix proportion of mortar

W/C (%)	S/C	Unit content (kg/m ³)				
		W	C	S	Si	ad. (cc)
30	1.00	288	883	981	98	9800(sp)
50	2.50	282	563	1408	0	1408(wr)
70	3.12	317	452	1408	0	704(wr)

W:Water S:Sand Si:Silica fume

SP:Naphtalene-based superplasticizer

WR:Lignin based air entraining and water reducing agent

Table 3 Mix proportion of concrete

G _{max} (mm)	Slump (cm)	Air (%)	W/C (%)	s/a (%)	Unit content kg/m ³				ad. ml/m ³
					W	C	S	G	
10	3±1	4±1	50	46	180	360	780	959	1260

Table 4 Mechanical properties of mortar

W/C (%)	Compressive strength (kgf/cm ²)	Tensile strength (kgf/cm ²)	Modulus of elasticity (kgf/cm ²)
30	673	45.9	2.63x10 ⁵
50	428	38.4	2.40x10 ⁵
70	250	22.5	1.75x10 ⁵

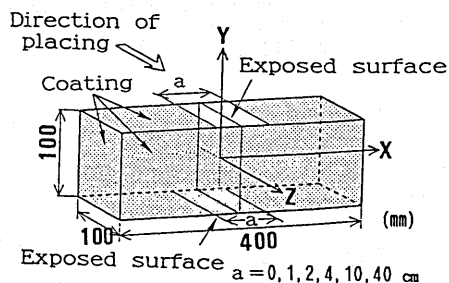


Fig. 1 Specimen for flexural strength test

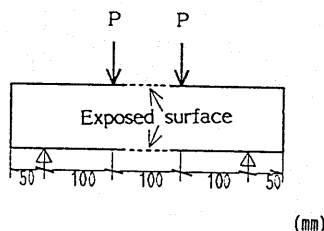


Fig. 2 Method of flexural strength test

2.4 Flexural Strength Test

Mortar specimen for flexural strength test is shown in Fig. 1. After the specified period of water curing and drying, flexural strength of mortar specimens was determined in accordance with JIS A 1106, where load was so applied that two exposed surfaces corresponded to the upper and bottom fibers of the beam, as shown in Fig. 2.

2.5 Test for Weibull's Coefficient of Uniformity

In order to study the influence of loading system on observed flexural strength of mortar, Weibull's coefficients of uniformity were determined. They were obtained from flexural strength tests carried out for thirty 4x4x16 cm beams, which were prepared from a single batch, immediately after water curing.

2.6 Measurement of Self Stress

After the specified period of drying, self stress distributions in mortar and concrete specimens were determined by cutting method for specimens dried with "a"=40 cm, or by notching method for specimens dried with "a"=2~10 cm. In these methods, elastic strain due to self stress was partially released, and the self stress was determined from measured strain by calculation. The detail of the cutting method will be mentioned in chapter 3 and the notching method in chapter 4.

Dimension of the specimen was different for different types of tests: 10x10x40 cm for flexural strength test, 4x10x40 cm for self stress measurement by the cutting method and 2x10x40 cm for self stress measurement by the notching

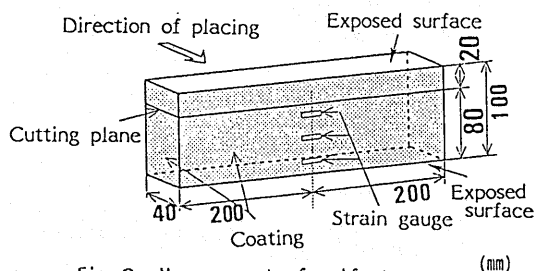


Fig. 3 Measurement of self stress by cutting method (Mortar)

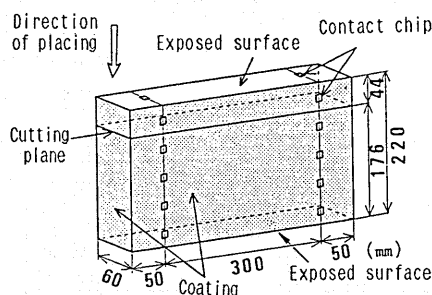


Fig. 4 Measurement of self stress by cutting method (Concrete)

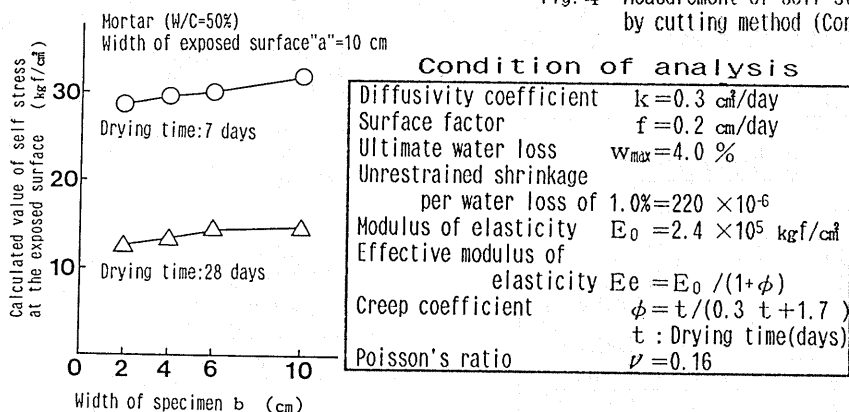


Fig. 5 Influence of specimen width on self stress

method. Influence of width of the specimen on the self stress was estimated by three dimensional elastic analysis, where finite element method was used. Coefficients in the calculation were so fixed that difference between self stress obtained from the analysis and that from the measurement (chapters 3 and 4) might be as small as possible. Influence of width of specimen on the longitudinal self stress is shown in Fig. 5. It is shown in this figure that self stress is not much influenced by the width of the specimen, so far as it is from 2 cm to 10 cm.

3. MEASUREMENT OF SELF STRESS BY CUTTING THE SPECIMEN

Self stress working in a cross section of a specimen should be in equilibrium. When a part of the specimen is cut off, the rest part will elastically deform so that internal stress may be in different equilibrium. By measuring this elastic deformation, self stress of mortar and concrete can be experimentally determined. This method is applicable to the case that the width of drying surface "a" is equal to 40 cm, that is, magnitude of self stress is dependent only upon distance from exposed surface.

3.1 Mortar Specimen

Distribution of longitudinal self stress in the cross section was determined from the observed strain variation due to cutting of a 4x10x40 cm specimen, as shown in Fig. 3. In this measurement, specimens were split by line load which was applied with a set of steel bars of triangular-shaped cross section, and variation in strain was measured by electric strain gauges of 10 mm in length.

It is thought that strain variation in section "A" due to cutting of the specimen is equivalent to the elastic strain which would be generated if self stress in section "B" before cutting was applied to section "A" after cutting as external stress. If this stress is divided into the components of axial force and bending moment, the next equation is obtained.

$$E_c \times \epsilon_x = \frac{P}{A} + \frac{M}{I} \left(y + \frac{d}{2} \right) \dots \dots \dots (1)$$

$$P = \int_{h/2-d}^{h/2} b \cdot \sigma_x dy$$

$$M = \int_{h/2-d}^{h/2} b \cdot \sigma_x (y + d/2) dy$$

- ϵ_x : Elastic strain due to cutting the specimen, σ_x : Self stress,
A : Area of cross section "A", I : Moment of inertia of area,
b : Width of the specimen, h : Height of the specimen,
y : Distance from the center of specimen, E_c : Modulus of elasticity,
d : Distance from cutting plane to the surface of specimen

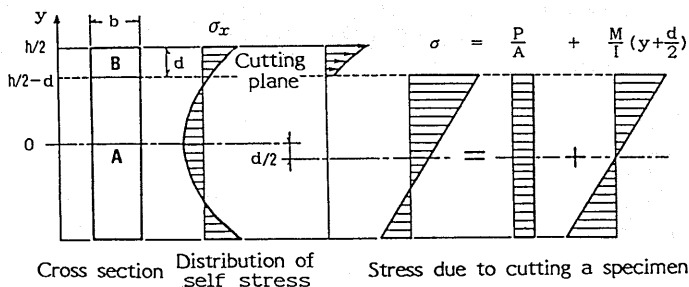


Fig. 6 Outline of calculation of self stress

When distribution of self stress is approximated with a quadratic curve, the following equation can be derived from symmetrical drying condition and the equilibrium of normal stress and moment in a cross section. This equation is effective so far as self stress near the exposed surface does not exceed the tensile strength of the mortar.

$$\sigma_x=6 \sigma_{\max }\left[\left(\frac{y}{h}\right)^2-\frac{1}{12}\right] \cdots \cdots \cdots(2)$$

where, σ_{\max} : Self stress at the exposed surface

Observed strains due to cutting are shown in Fig. 7. Straight lines in these figures are obtained by the least square method and it can be seen that a plane section before cutting remains plane after cutting. Distribution of ϵ_x , which is determined from the regression line, is substituted into eq.(1). Then σ_{\max} can be determined by using eq.(1) and eq.(2). Results are shown in Table 5. It can be seen from this table that measured values are independent of the location of cutting, therefore the approximation with the quadratic curve is judged to be proper under the adopted conditions of the experiments (period of drying, relative humidity and dimension of specimens).

Table 5 Self stress determined by cutting method (Mortar)

W/C (%)	Drying time(day)	No.	Location of cutting:d(cm)	σ_{\max} (Kgf/cm ²)	
					Mean value
3 0	3	1	2.0	32.5	30.3
		2		28.0	
5 0	7	1	2.0	29.7	29.0
		2		28.3	
		3	3.5	24.4	
		4	5.0	28.0	
7 0	7	1	2.0	20.8	18.3
		2		15.7	

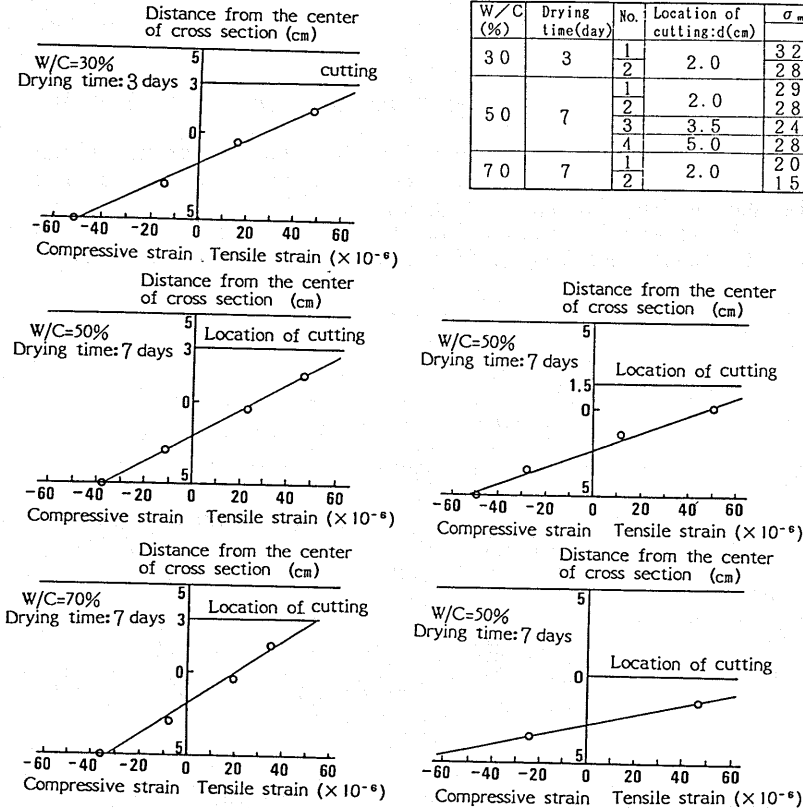


Fig. 7 Strain due to cutting a specimen (Mortar)

Self stress is increased with decrease in water-cement ratio (W/C) under the same drying condition, as shown in this Table. For W/C=0.30, transverse microcracks were observed on the exposed surface in the period of drying, so self stress was thought to be partially released.

Modulus of elasticity of mortar (E_c) is dependent upon various factors such as the sign of stress (compressive or tensile), stress level and moisture content. But it is very difficult to estimate these effects precisely. Therefore, modulus of elasticity in eq.(1) is assumed to be equal to the value for compression measured with wet specimen (Table 4), and to be uniform in the cross section for the simple and practical method of measurement.

3.2 Concrete Specimen

Self stress generated in concrete specimen which had been subjected to drying from two opposite surfaces was determined by the cutting method. Dimension of the specimen was 6x22x40 cm and strain due to cutting was measured with contact strain gauge (test length was 30 cm). Strain variation due to cutting of specimens are shown in Fig. 8. In case 1, the specimen was cut by the plane 4.4 cm away from the exposed surface. In case 2 and 3, the specimen was cut into two equal sections. Modulus of elasticity of concrete (E_c) was assumed to be 3.0×10^5 kgf/cm². The test results are shown in Table 6. It can be seen from Tables 5 and 6 that self stress of mortar and concrete specimen can be experimentally determined by the cutting method with a little experimental error.

4. MEASUREMENT OF SELF STRESS BY NOTCHING THE SPECIMEN

In cases of "a"=2~10 cm, self stress can not be measured by the cutting method, because it is dependent not only upon Y coordinate but also upon X coordinate. Therefore, self stress near the exposed surfaces was measured by the notching method in these cases. Weight loss of mortar specimens with different exposed area is shown in Fig. 9, which suggests that specimens were effectively sealed.

When a specimen is notched from the surface, as shown in Fig. 10, the longitudinal self stress is partially released. The resultant surface strain in the direction of X-axis was

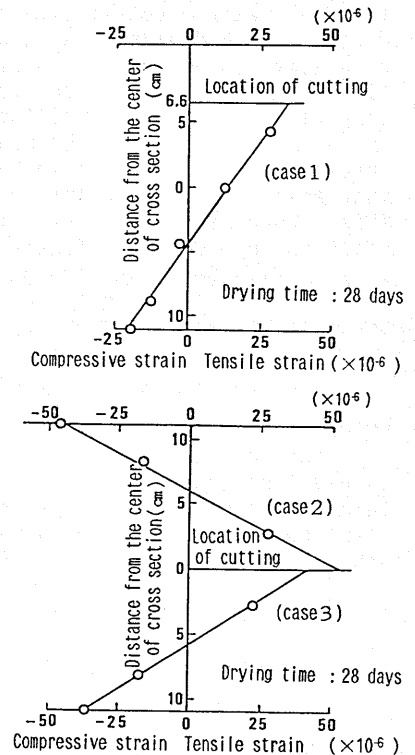


Fig. 8 Strain due to cutting a specimen (Concrete)

Table 6 Self stress determined by cutting method (Concrete)

W/C (%)	Drying time	case No.	σ_{max} (kgf/cm ²)	
				Average
50	28日	1	16.7	17.4
		2	19.6	
		3	15.8	

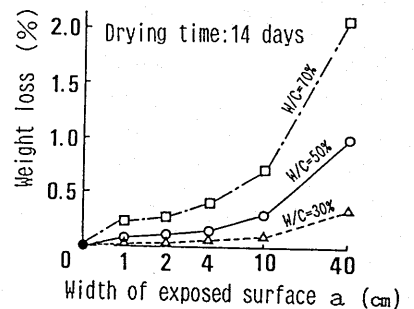


Fig. 9 Relation between exposed area and weight loss of mortar specimen

measured with electric strain gauge (gauge length was 5 mm). As specimen was notched with diamond saw, heat was generated. After notching, specimens were naturally cooled at 20°C until the influence of heat could be ignored. Cooling time was decided to be from 10 to 15 minutes judging from measurement of temperature and strain of the specimen. Examples of recorded strain are shown in Fig. 11. When a specimen was notched from the exposed surface, shrinkage strain was generated due to the release of tensile self stress. This strain was increased with notch depth (y) and it reached a maximum value when y=1~2 cm. This phenomenon agrees with theoretical analysis by means of finite element method (Fig. 12). Relation between notch depth and surface strain can be expressed by the next equation in case of $y \leq 2$ cm.

$$\epsilon(y)=y \cdot \exp (p-q y) \cdots \cdots \cdots(3)$$

Where p and q are constants. Equation (3) is selected as a function with which observed values can be well approximated. Although this equation has no physical meaning, the data and its regression curve resemble each other very much, as shown in Fig. 13.

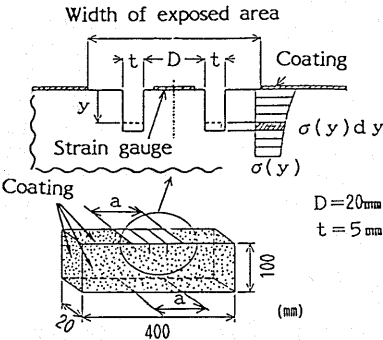


Fig.10 Measurement of self stress by cutting method (Mortar)

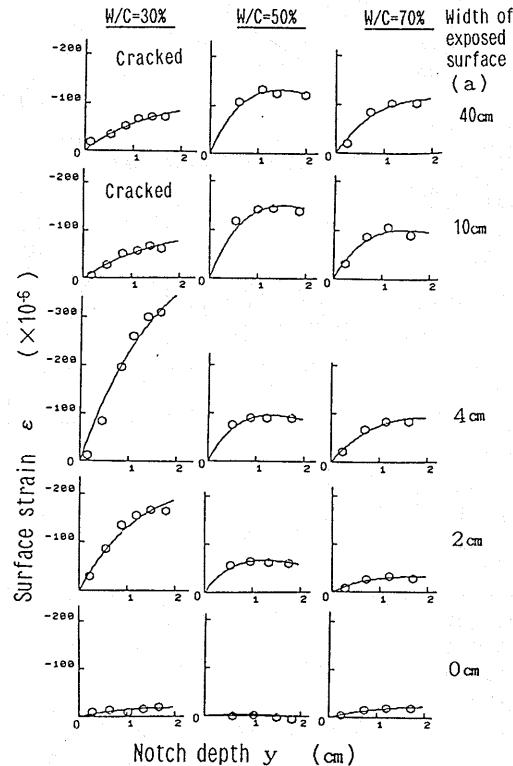


Fig.13 Strain due to nothcing specimens and the regression curves

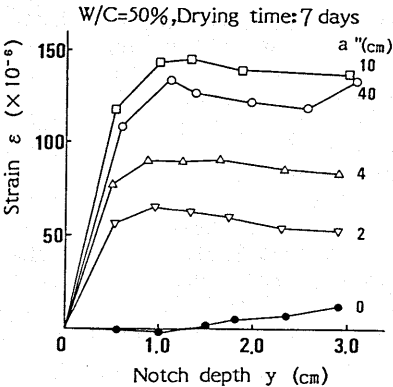


Fig.11 Strain due to notching a mortar specimen

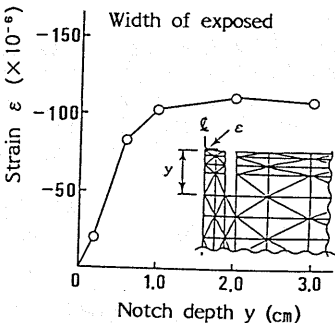


Fig.12 Strain due to notching a mortar specimen (FEM analysis)

When the notch depth is y , small increment of dy makes the equivalent force $(-\sigma(y)dy)$ work at the bottom of the stress released portion. The incremental change in surface strain can be expressed by the next equation, where E denotes the modulus of elasticity.

$$d\epsilon(y) = K_y \cdot (1/E) \cdot \sigma(y) dy \dots\dots\dots (4)$$

Where K_y is a function of D , t , y and the type and arrangement of strain gauge (D , t and y are defined in Fig. 10). Since K_y is independent of self stress distribution, $\sigma_0(y) - \epsilon_0(y)$ relation for another specimen can be expressed by equation (5) using the same K_y as in equation (4).

$$d\epsilon_0(y) = K_y \cdot (1/E_0) \cdot \sigma_0(y) dy \dots\dots\dots (5)$$

In eq.(5), stress distribution determined by the cutting method for the specimen with $W/C=0.50$, " a "=40 cm and "drying time"= 7 days is taken to be $\sigma_0(y)$. From eq.(4) and (5), the next equation can be obtained.

$$\sigma(y) = \frac{E d\epsilon/dy}{E_0 d\epsilon_0/dy} \sigma_0(y) \dots\dots\dots (6)$$

Where, $\sigma_0(y) = 6 \times 29.0 \{ (y/h)^2 - 1/12 \}$

If both $\epsilon(y)$ and $\epsilon_0(y)$ are expressed as equation (3), self stress near the exposed surface (" a "=2~10 cm) can be obtained from equation (6). The results are shown in Fig. 14, and self stress at the exposed surface σ_{max} are shown in Fig.15. The value of σ_{max} decreases with decrease in " a " in case of " a "=2~10 cm. In case of " a "=10 cm, σ_{max} is almost the same as in case of " a "=40 cm. σ_{max} increases with decrease in water-cement ratio under the same drying condition. In case of $W/C=0.70$ and " a "=40 cm, σ_{max} measured by the notching method is 76% of that measured by the cutting method (Table 3).

Surface strain is maximized (ϵ_{max}) when notch depth is between 1 cm and 2 cm. If self stress at the exposed surface is completely released when the surface strain is maximized, σ_{max} is also obtained from the next equation.

$$\sigma_{max} = E \times \epsilon_{max} \dots\dots\dots (7)$$

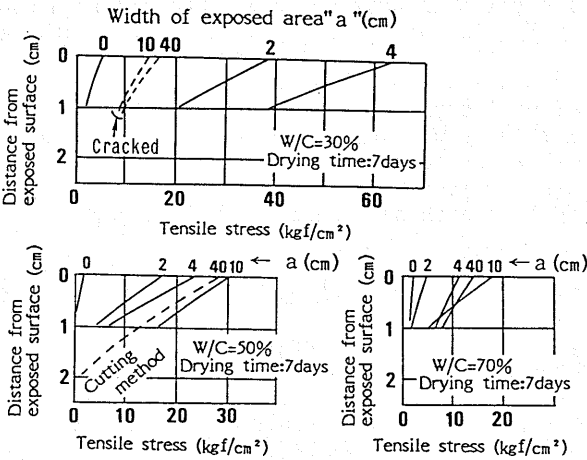


Fig.14 Self stress near the exposed surface measured by the notching method

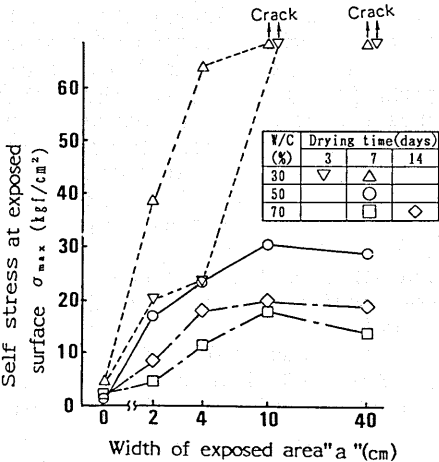


Fig.15 Self stress at the exposed surface measured by the notching method

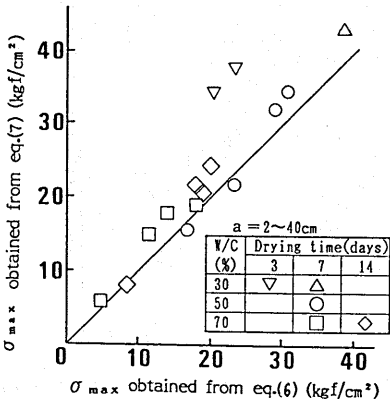


Fig.16 Relation between self stress obtained from eq.(6) and that from eq.(7)

The relation between σ_{\max} obtained from equation (6) and equation (7) is shown in Fig.16 for all the cases, and the both values are almost the same.

In case of $W/C=0.30$ ("a"=10, 40 cm), transverse surface cracks were observed after 3 days of drying, therefore self stress was underestimated. The notching method is not applicable when self stress exceeds tensile strength or shrinkage crack is observed. It is proved from these experimental results that self stress locally distributed in mortar specimen can be experimentally determined by the notching method.

5. INFLUENCE OF SELF STRESS ON FLEXURAL STRENGTH OF MORTAR

It is well known that flexural strength is one of the typical properties which are significantly influenced by self stress due to drying shrinkage. In this study, relation between self stress and decrease in flexural strength is investigated on the basis of measured values of self stress by the cutting method and the notching method.

Change in flexural strength with the time elapsed, which was observed in mortar specimens with different area of exposed surface are shown in Figs. 17-19. In all the cases except the case of $W/C=0.70$ and "a"=1 cm, flexural strength decreases with progress of drying until it reaches a minimum value when drying time is 7 days, and then it gradually recovers. The reason why flexural strength decreases is that tensile self stress is superposed upon tensile bending stress at the bottom fiber of the beam. Self stress has the components not only in the longitudinal direction but also in the transverse direction. Influence of transverse self stress on flexural strength is ignored, because it has been reported that flexural strength of concrete under bi-axial tension is almost the same as that under uni-axial tension[13],[14].

When the mortar specimen with $W/C=0.30$ was sealed on all the surfaces after water curing, flexural strength considerably decreased in 3 or 14 days although no weight loss was observed. It was proved from the observation of the cross section immediately after water curing that the color of surface layer with thickness of 7 mm was darker than that of the inner part. It is because the rate of permeation of curing water into mortar with very low water-cement ratio is much later than the rate of intrinsic voids formation due to hydration of cement (self desiccation)[15]. Therefore, swelling near the surface of specimen is restrained by the inner

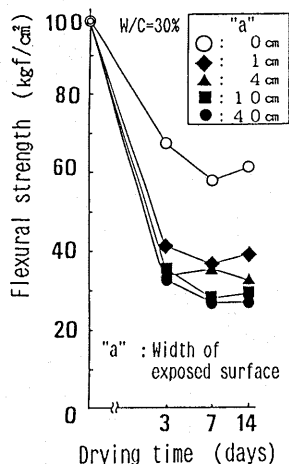


Fig. 17 Change of flexural strength with elapsed time ($W/C=30\%$)

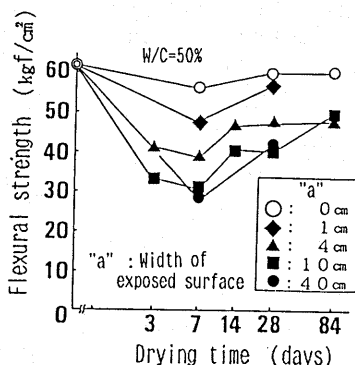


Fig. 18 Change of flexural strength with elapsed time ($W/C=50\%$)

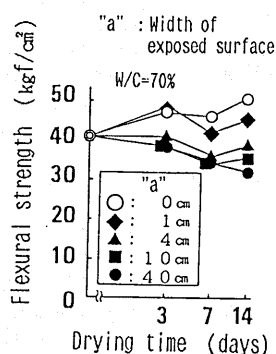


Fig. 19 Change of flexural strength with elapsed time ($W/C=70\%$)

part of the specimen, and compressive self stress is generated at the surface of the specimen. It is thought that the compressive self stress decreases due to redistribution of moisture after sealing and flexural strength decreases without moisture loss. Similar phenomenon has been reported[16]: flexural strength of dried concrete specimen temporarily increased when it was immersed in water.

It is seen from Fig.19 that flexural strength of mortar with W/C=0.70 and "a"=0 cm slightly increased with the time elapsed.

The ratio of flexural strength of sealed specimen to that of dried ones is defined as "relative flexural strength". Relation between relative flexural strength and width of exposed surface "a" of mortar specimen is shown in Fig.20. For any mix proportion, relative flexural strength decreases with increase in "a".

Recorded flexural strength of brittle material is influenced by loading system. From Weibull's theory, flexural strength obtained from the test shown in Fig.21 can be expressed by the next equation[17].

$$f_b = f_o \Gamma(1+1/m) \cdot g(m, \lambda) / V^{1/m} \dots \dots \dots (8)$$

Where, $g(m, \lambda) = [2 \lambda(m+1)^2 / 2 + (m+1)(\lambda-2)]^{1/m}$

- m : Weibull modulus,
- f_o : Standard of flexural strength,
- λ : Factor of loading system (Fig.21),
- V : Volume of specimen

Flexural strength tests were carried out for thirty 4x4x16 cm beams at the end of water curing, and the results are plotted on Weibull's probability graph, as shown in Fig.22. Weibull's coefficients of uniformity (m) can be obtained as the inclination of the regression lines in Fig.22, and are given in Table 7 [18].

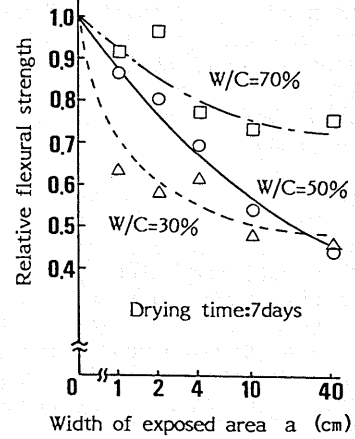


Fig.20 Exposed area--Flexural strength

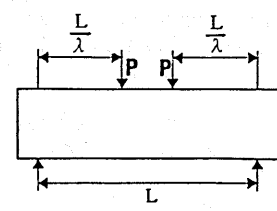


Fig.21 Definition of λ in eq.(8)

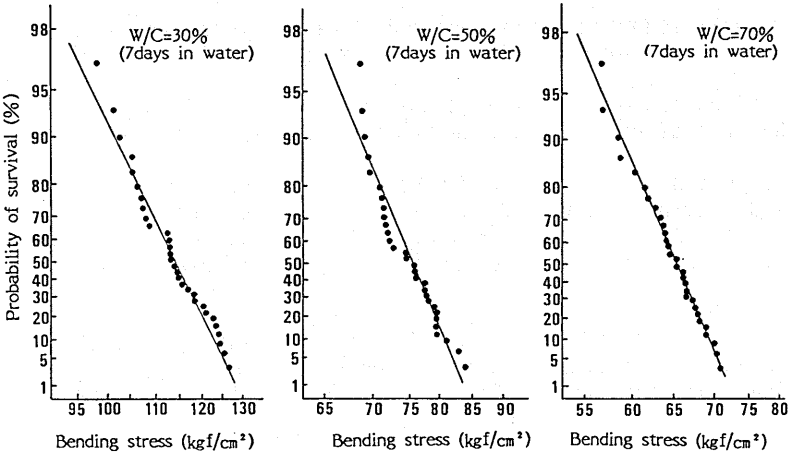


Fig.22 Weibull plot for flexural strength of mortar

Bending span is 10 cm in this experiment, while the maximum self stress in tension occurs at the exposed surface. Therefore the results in Fig. 20 are influenced by loading system. Effect of bending span on flexural strength is estimated by equation (8), and is shown in Table 7. When bending span is increased from 1 cm to 10 cm, corresponding strength reduction is no more than 8 %, therefore influence of loading system is very small compared to the observed value. This fact strongly suggested that there should be another factor to dominate the phenomenon.

Table 7 Influence of bending span on recorded flexural strength

W/C (%)	m	Bending span length (cm)			
		1	2	4	10
30	16.38	1.000	0.982	0.957	0.916
50	18.86	1.000	0.983	0.960	0.934
70	18.09	1.000	0.982	0.959	0.922

Relation between (σ_{\max}/f_t) and relative flexural strength is shown in Fig.22. The values of σ_{\max} were measured by the cutting method or the notching method. In the case when shrinkage cracks were observed, the values of σ_{\max} were assumed to be same as f_t . Strength development after water curing (7 days) is thought to be negligible, because complete hydration was prevented near the exposed surface [19] and high-early strength portland cement was used in this experiment.

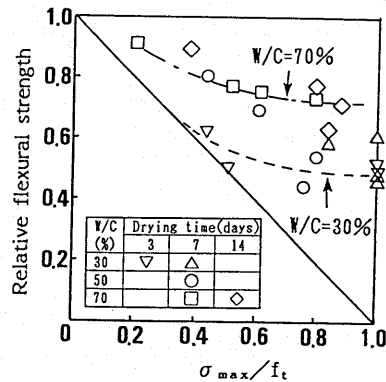


Fig.23 Relation between (σ_{\max}/f_t) and relative flexural strength

Flexural strength decreases with increase in σ_{\max} for any mix proportion. Strength reduction under the same σ_{\max}/f_t is larger for specimen with denser structure. Even if self stress at the exposed surface reaches tensile strength, that is $\sigma_{\max}/f_t=1$, strength reduction due to self stress is only 50% for $W/C=0.30$ and 25% for $W/C=0.70$. From these experimental results, it is proved that the maximum tensile stress obtained by superposing stress due to external load upon self stress does not serve as a failure criterion. Therefore a new criterion reflecting the effect of self stress distribution has to be established.

6. CONCLUSIONS

- (1) Non-uniform self stress in mortar and concrete specimen can be experimentally determined by measuring variation in strain when the specimen is cut or notched.
- (2) Tensile self stress at the exposed surface caused by drying increases with increase in the area of exposed surface and with decrease in water-cement ratio.
- (3) Flexural strength of mortar decreases with increase in tensile self stress at the exposed surface. Even if self stress reaches the tensile strength, strength reduction is less than 50% of the strength obtained for control specimen. This strength reduction increases with decrease in water-cement ratio.
- (4) For mortar specimen subjected to non-uniform self stress due to drying, the maximum tensile stress obtained by superposing stress due to external load upon self stress does not serve as a failure criterion. Therefore, a new criterion reflecting the effect of working zone and distribution of self stress has to be established.

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