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LIQUEFACTION OF FRESH CONCRETE DUE TO VIBRATION AND THE SPHERE OF ACTION OF INTERNAL VIBRATOR

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SYNOPSIS

In the consolidation of the concrete by vibrators, concrete moves and settles when it is liquefied by the vibration. This paper involves the liquefying action of vibration to fresh concrete and its liquefaction as a response. These two factors dominating the process of compaction of the concrete were theoretically analyzed. The results, together with that on the propagation of the vibration, made it possible to estimate how far from an internal vibrator the completion of compaction could be achieved. The effects of a plate of form work and of vertical reinforing steel bars on the distribution of intensity of the liquefying action were also calculated and illustrated.

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1. INTRODUCTION

For the effective use of concrete vibrators, numerous investigations have been made with respect to the action of vibration on fresh concrete. As to internal vibrators, our chief concern is the radius of action in relation to the time of vibration and the vibration intensity as well as to the consistency of the concrete. However, the studies so far have gone no further than experimenting on the compaction of a slab without any effects of reinforcements or form work.

In this study, the consolidation of concrete is understood as the consequence of flowing and settling of liquefied concrete by its own weight, and the process of compaction is analyzed. Thus, the present paper gives equational expressions of the liquefying action of vibration, the response of concrete to it, the conditions required for the completion of consolidation, and the relationship between the time of vibration and the distance from the vibrator within which the concrete can be fully compacted.

2. LIQUEFYING ACTION OF VIBRATION TO FRESH CONCRETE

2.1 Equation Expresssing the Vibration from an Internal Vibrator

According to the results of analysis and experiments[4], in the vibration of a poker housing a rotating eccentric shaft, the phase of its displacement is $\pi/2$ behind that of the rotation of the gravity centroid, and the displacement transmitted to surrounding concrete travels through it as a wave with the angular frequency correspondent to the angular velocity ω of the shaft. If the present time is t and the wave velocity is c, the wave front which just passes a point P (Fig.1) is supposed to have been emitted from the poker surface at the time t_1 written as $t_1=t-l/c=t-(r_1-R_v)/c$.

When the time is measured from a certain instant at which the gravity centroid W comes right in the direction of x axis, then the position of W is $\omega t_1+\theta_w$, and the direction of the maximum displacement θ_m will be $\omega t_1+\theta_w-\pi/2$ (Fig.2). Accordingly the displacement, $\psi_{\theta 1}$,in the direction of θ_1 at t is expressed as

$$\begin{aligned}\psi_{\theta 1}&=\psi_0 \cos(\theta_m-\theta_1)\\&=\psi_0 \sin\{\omega t-k(r_1-R_v)+\theta_w-\theta_1\}\end{aligned}$$

in which ψ_0 : maximum displacement
$k=\omega/c$:wave number

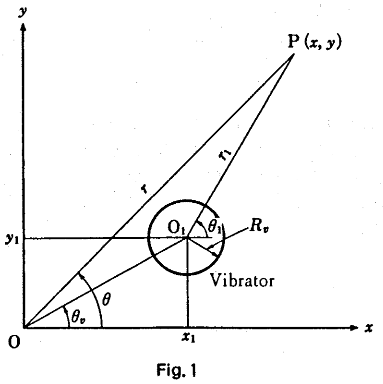

Fig. 1

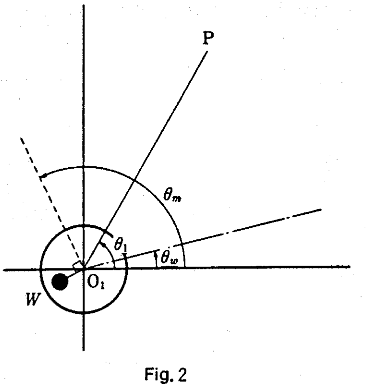

Fig. 2

In consideration of the reduction of energy density due to geometical dilution and viscosity, the wave ψ_P at the point P is finally written as

$$\psi_P=\psi_0\sqrt{R_v/r_1}\ e^{-\beta(r_1-R_v)}\sin\{\omega t-k(r_1-R_v)+\theta_w-\theta_1\}\ \cdots\cdots(1)$$

When the origin of the coordinates coincides the center of the poker O_1 , Eq.(1) becomes Eq.(2)

$$\psi_P=\psi_0\sqrt{R_v/r}\ e^{-\beta(r-R_v)}\sin\{\omega t-k(r-R_v)-\theta\}\ \cdots\cdots(2)$$

These equations express whirly waves.

2.2 Estimation Method for Liquefying Action of Vibration

Considering a square element $PQRS$ at an arbitrary point $P(r,\theta)$ (Fig.3), if the displacement at the point is   at a certain time, displacements at the points Q, R, S are

$$Q: \psi+\frac{\partial\psi}{\partial\theta}\Delta\theta$$

$$R: \psi+\frac{\partial\psi}{\partial\theta}\Delta\theta+\frac{\partial\psi}{\partial r}\Delta r$$

$$S: \psi+\frac{\partial\psi}{\partial r}\Delta r$$

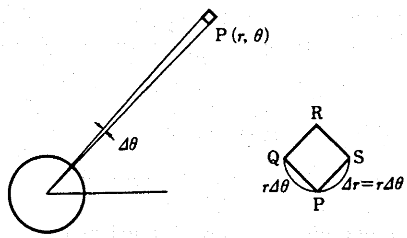

Fig. 3

Consequently, the positions of P, Q, R, S after displacing are the points P', Q', R', S' respectively in Fig.4(a). In Fig.4(b) with P' moved to fall on P ,$P'Q'R'S'$ is $P''Q''R''S''$ As is seen, the square is deformed into a parallelogram through expanding (or contracting) and shearing deformations. The strains in expansion and shear ε , γ are

$$\varepsilon=\frac{SS''}{PS}=\frac{SS'-PP'}{PS}=\frac{\{\psi+(\partial\psi/\partial r)\Delta r\}-\psi}{\Delta r}=\frac{\partial\psi}{\partial r}$$

$$\gamma=\frac{QQ''}{PQ}=\frac{QQ'-PP'}{PQ}=\frac{\{\psi+(\partial\psi/\partial\theta)\Delta\theta\}-\psi}{r\Delta\theta}$$

$$=\frac{1}{r}\frac{\partial\psi}{\partial\theta}$$

and the strain of the diagonal is

$$\frac{\Delta l}{l}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}+\frac{\partial\psi}{\partial r}\right)=\frac{1}{2}(\gamma+\varepsilon)\cdots\cdots(3)$$

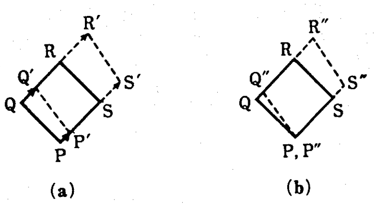

Fig. 4

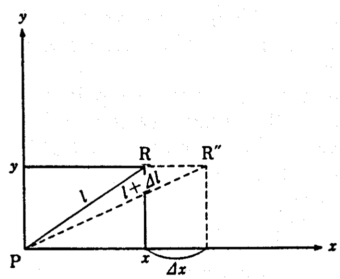

Fig. 5

Evaluating the liquefying action, L , of vibration to fresh concrete by the product of the diagonal strain and frequency f

$$L=f\cdot\frac{\Delta l}{l}=\frac{\omega}{4\pi}\left(\frac{1}{r}\frac{\partial\psi}{\partial\theta}+\frac{\partial\psi}{\partial r}\right)\cdots\cdots\cdots\cdots\cdots\cdots\cdots(4)$$

When a wave function ψ is Eq.(2),

$$\psi=A\sin\phi$$

in which $A=\psi_0\sqrt{R_v/r}\; e^{-\beta(r-R_v)}$

$$\phi=\omega t-k(r-R_v)-\theta,\quad k=\omega/c=2\pi f/c$$

In that case,

$$\frac{1}{r}\frac{\partial\psi}{\partial\theta}+\frac{\partial\psi}{\partial r}=-A\left\{\left(k+\frac{1}{r}\right)\cos\phi+\left(\beta+\frac{1}{2r}\right)\sin\phi\right\}=-AL_r\sin(\phi+\alpha)$$

where $L_r=\sqrt{\left(k+\frac{1}{r}\right)^2+\left(\beta+\frac{1}{2r}\right)^2}\cdots\cdots\cdots\cdots\cdots\cdots\cdots(5)$

$$\alpha=\tan^{-1}\frac{2(kr+1)}{2\beta r+1}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(6)$$

and L is obtained as

$$L=-\frac{\omega A}{4\pi}L_r\sin(\phi+\alpha)\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(7)$$

To represent the intensity of liquefying action, the magnitude of , designated "Liquefaction Value : value ", might as well be used, that is,

$$L_q=\frac{Af}{2}L_r=\frac{\omega A}{4\pi}L_r\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(8)$$

If a is the acceleration amplitude of vibration and a_G , the ratio to the gravity g

$$L_q=\frac{a}{4\pi\omega}L_r=\frac{g a_G}{4\pi\omega}L_r\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(9)$$

$$L_r=\sqrt{\left(k+\frac{1}{r}\right)^2+\left(\beta+\frac{1}{2r}\right)^2}$$

If necessary c can be estimated by Eq.(10)[4].

$$c=\sqrt{\frac{1}{A_l(1-A_l)}\frac{K_A}{\rho_0}}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(10)$$

in which A_l is air content (volume ratio), ρ_0 is density of concrete with air content of 0%, and K_A is bulk modulus of elasticity of air.

In case the accelerations have been measured at different distances from the vibrator, the acceleration of concrete at the contact surface with the poker a_0 and attenuation coefficient β are obtained simultaneously[4]. Therefore the value at an arbitrary position is calculated by the following formula.

$$L_q=\frac{a_0 L_r}{4\pi\omega}\sqrt{\frac{R_v}{r}}\,e^{-\beta(r-R_v)}$$

$$=\frac{a_0}{4\pi\omega}\sqrt{\frac{R_v}{r}\left[\left(k+\frac{1}{r}\right)^2+\left(\beta+\frac{1}{2r}\right)^2\right]}\,e^{-\beta(r-R_v)}\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots(11)$$

Fig. 6 shows examples of L_q values varying with the distance from vibrator. Values of a_0 are 12, 15, and 20 g for the vibrators with diameters of 40, 50, and 60 mm respectively. Other parameters used are listed in Table 1.

Table 1 f, c, β

Case	Concrete		f (Hz)	c (m/s)	β (cm^{-1})
	Air	Consistency			
NST-200	Non-air-entrained	Stiff	200	50	0.034
NPL-150	Non-air-entrained	Plastic	150	50	0.017
NPL-200	Non-air-entrained	Plastic	200	50	0.017
ASP-200	Air-entrained	Stiff plastic	200	30	0.017

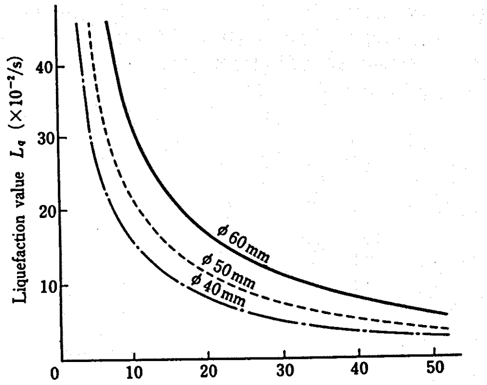

Fig. 6 Examples of L_q values.

2.3 Liquefying Action of a Plane Wave

Where the distance from the vibrator is as great as the following assumption can be taken as reasonable,

$$k \gg \frac{1}{r}, \quad k \gg \beta + \frac{1}{2r}$$

Eq. (9) becomes

$$L_q = \frac{ak}{4\pi\omega} = \frac{a}{4\pi c} \cdots\cdots (12)$$

This shows that the liquefaction value L_q is proportional to the wave velocity.

On the other hand, for a plane wave or a cylindrical wave given by

$$\psi = A \sin \omega\left(t - \frac{r}{c}\right) \cdots\cdots (13)$$

$\Delta l / l$ has only a component of expansion or contraction, then

$$\frac{\Delta l}{l} = \frac{1}{2}\frac{\partial \psi}{\partial r} = -\frac{\omega A}{2c} \cos \omega\left(t - \frac{r}{c}\right)$$

$$\therefore \quad L = f \cdot \frac{\Delta l}{l} = -\frac{\omega^2 A}{4\pi c} \cos \omega\left(t - \frac{r}{c}\right)$$

$$= -\frac{a}{4\pi c} \cos \omega\left(t - \frac{r}{c}\right)$$

Hence, the liquefaction value is identical with Eq. (12), that is,

$$L_q = \frac{a}{4\pi c}$$

Fig. 7 shows the changes in L_r/k to see the effect of the shearing strain. The values of c, β, f used to calculate L_q by Eq. (5) are listed in Table 1. In this figure, L_r/k decreases rapidly with increase in r, for instance, it is about 1.2 at $r = 20$ cm

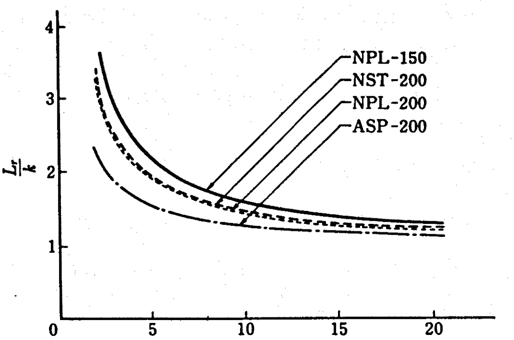

Fig. 7 L_r/k vs r.

Fig. 8 is the changes in L_r with r. Further, Fig. 9 is the relationship between r and L_{q0} that is L_q obtained by substituting $1g$ into a_0 of Eq. (11). Other parameteres are listed in Table 1. Keeping a_0 and r be constant, L_q value is greater for ASP-200 than NST-200, which indicates the effective sphere of an internal vibrator stretches far wider for plastic AE concrete than for stiff plain concrete.

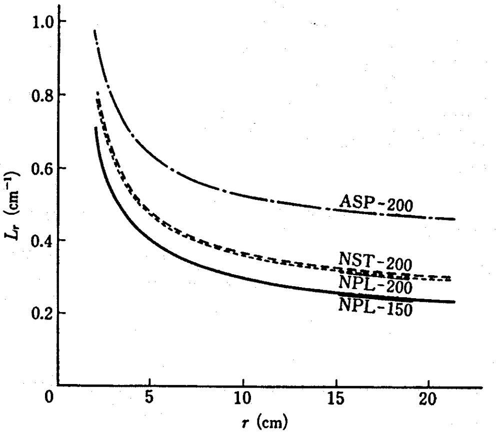

Fig. 8 L_r vs r.

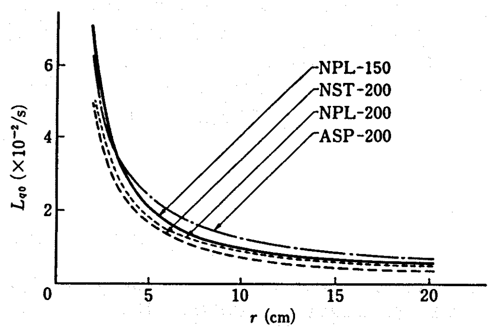

Fig. 9 L_{q0} vs r.

3. LIQUEFACTION OF CEMENT PASTE BY VIBRATION AND THE REQUIREMENTS FOR CONSOLIDATION TO BE COMPLETED

3.1 Conditions for Solid Particles to be Liquefied by Vibration

As known through experiments on the rate of dissolution of powdered materials, the surface of a solid particles in the water attracts the thin layer of water called diffusion layer which would not move off by stirring, and its thickness δ_0 has been estimated to be approximately 10^{-3} cm regardless of the substance of the particle.

Considering this fact, when a couple of particles contacting with each ather go apart by $2\delta_0$ at their contact points, the connections of the particles are assumed to be released and brought into the liquefied state. The liquefying action L introduced in 2.2 is a periodic quantity with a dimension of the rate of strain s^{-1}. Since the average of its magnitude is $2L_q/\pi$, Δd, the change in the distance between the centers of the particles may be accumulated during vibration time t to

$$\Delta d=\frac{2}{\pi}L_q t d_c \cdots\cdots(14)$$

where d_c is the diameter of the particles.

To extend the interparticle distance, the minimum irreducible rate of strain must be required as to be discussed later in 4.1. Taking this factor into consideration, Eq. (14) yields

$$\Delta d=\frac{2}{\pi}(L_q-L_0)t d_c \cdots\cdots(15)$$

L_0 in Eq.(15) is designated "Liquefaction Resistance Value". In the case that there exists a water layer with thickness of δ on the surface of cement particles, the condition under which liquefaction should occur is $\Delta d \geqq 2\delta_0 - \delta$. By substituting Eq.(15) into this relation, it becomes

$$(L_q - L_0)t \geqq \frac{\pi(2\delta_0 - \delta)}{2d_c} \cdots\cdots (16)$$

The value of δ can be estimated roughly as follows from the water cement ratio W/C and the specific surface area of the cement S_0.

$$\delta = \frac{W - Ce/g_c}{CS_0} = \left(\frac{W}{C} - \frac{e}{g_c}\right) / S_0 \cdots\cdots (17)$$

where e : void ratio of cement
g_c : specific gravity of cement
S_0 : specific surface of cement

Assuming e and g_c to be 0.50 and 3.17 respectively, Eq.(17) is

$$\delta = \left(\frac{W}{C} - 0.16\right) / S_0 \cdots\cdots (18)$$

Further, with W/C of 0.50 and S_0 of 3000 cm^2/g, δ is 1.1×10^{-4} cm rather small as compared with $2\delta_0$. If δ is taken as negligible, Eq.(16) will be simplified into Eq.(19).

$$(L_q - L_0)t \geqq \frac{\pi}{d_c} \times 10^{-3} \cdots\cdots (19)$$

As Eq.(16) and Eq.(19) show, the smaller the particles are the less easily liquefied they are. It follows that under the condition of a fixed $(L_q - L_0)t$ liquefied and unliquefied particles are mixed in the cemnet paste.

Now let a density function $f(D)$ express the diameter distribution of a particle cluster and d_m be the maximum diameter, then the weight ratio of the particles over d_c to the whole is

$$F(d_c) = \int_{d_c}^{d_m} f(D)dD \cdots\cdots (20) \qquad \int_0^{d_m} f(D)dD = 1$$

Hence, $F(d_c)$ means the portion of liquefied particles in $F(d_c)$ satisfies

$$d_c \geqq \frac{\pi(2\delta_0 - \delta)}{2(L_q - L_0)t} = \frac{\pi\delta_0}{(L_q - L_0)t}\left(1 - \frac{\delta}{2\delta_0}\right) \cdots\cdots (21)$$

In case the size distribution of the cement is known, the proportion of liquefired particles can be determined for a given $(L_q - L_0)t$. By neglecting $\delta/2\delta_0$, formula (21) is reduced to

$$d_c \geqq \frac{\pi\delta_0}{(L_q - L_0)t} = \frac{\pi}{(L_q - L_0)t} \times 10^{-3} \text{ (cm)} \cdots\cdots (22)$$

3.2 Liquefaction Ratio of Cement Paste

Fig. 10 is a particle size distribution of Portland cement. The curve is obtained from the experimental data shown in the well-known technical books [5], [6] as typical examples.

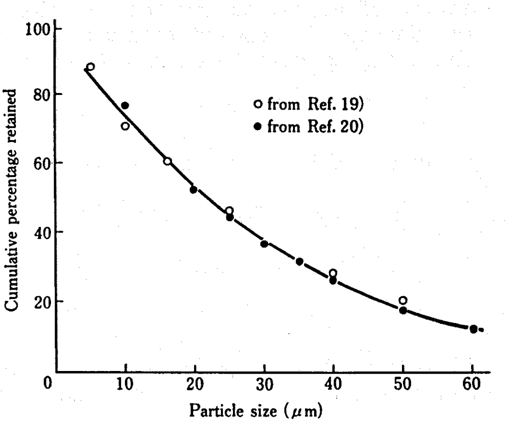

Fig. 10 Particle size distribution of typical Portland cement.

According to this curve, $F(d_c)$, the proportion of liquefied particles is estimated to be about 15% for d_c of 50 μm and 85% for 5 μm. The values of $(L_q-L_0)t$ corresponding to these are calculated by Eq.(19) as 0.63 and 6.28 respectively.

Fig.11 shows a relationship between $(L_q-L_0)t$ and $F(d_c)$. When the proportion of liquefied particles is termed liquefaction ratio L_p denotative of the degree of liquefaction, Fig.12 is obtained as a linear expression of $(L_q-L_0)t \sim \log_e(0.86-L_p)$.

$$L_p=0.86-e^{-0.661(L_q-L_0)t} \quad \cdots\cdots(23)$$

Taking δ into consideration, Eq.(23) becomes

$$L_p=0.86-\exp\left\{-0.661(L_q-L_0)t\Big/\left(1-\frac{\delta}{2\delta_0}\right)\right\} \quad \cdots\cdots(24)$$

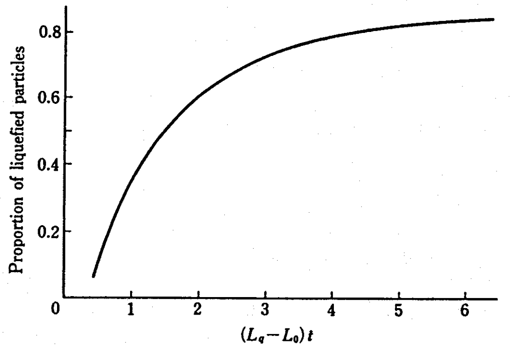

Fig.11 Relationship between $(L_q-L_0)t$ and proportion of liquefied cement particles.

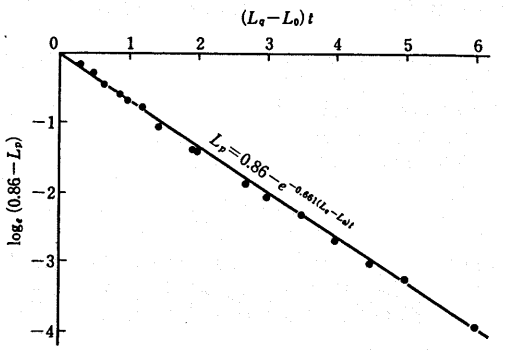

Fig.12 Relationship between $(L_q-L_0)t$ and liquefaction ratio of cement paste.

3.3 Required Condition for Consolidation to be Completed

The progress of the consolidation of concrete is dominated by the liquefying action of vibration and the reaction characteristics of the concrete, so that the degree of consolidation may be evaluated by the settlement z. Supposing the settling rate to be proporational to the liquefaction ratio of cement paste

$$\frac{dz}{dt}=v_0L_p=v_0(b-e^{-at}) \quad \cdots\cdots(25)$$

in which v_0 is settling rate when $L_p=1$, b is 0.86, and a is $0.661(L_q-L_0)$.

$$\therefore \quad z=\int v_0(b-e^{-at})dt=v_0\left(bt+\frac{1}{a}e^{-at}\right)+C_1$$

From initial condition $t=0$, $z=0$

$$z=v_0\left\{bt-\frac{1}{a}(1-e^{-at})\right\} \quad \cdots\cdots(26)$$

If z_0 represents the final setteliment at the completion of consolidation, the required condition for consolidation to be completed is

$$\frac{z_0}{v_0}\leqq bt-\frac{1}{a}(1-e^{-at}) \quad \cdots\cdots(27)$$

This follows that the region expected to be compacted by the vibration of t is predicted through substituting L_q into Eq.(31).

Reversely, to determine the required time of vibration to compact a given area, the corresponding irreduciable L_q calculated by Eq.(9) or (11) is to be substituted into Eq.(30). The minimum necessary vibration time t can be estimated from $T(t, L_q)=0$.

Table 2 shows the required L_q values in relation to vibration time with varying L_0 and T_0 . As described in this chapter, the region where L_q exceeds the required value is supposedto be compacted.

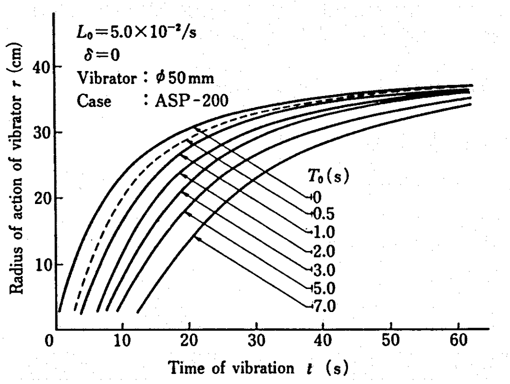

Fig. 13 Relationship between time of vibration and radius of action of vibrator.

The curves in Fig. 13 were obtained for the case ASP-200 by this method.

In practice, the vibration time is about 5 to 20 s for each compaction, and the observed radius of action is usually 13 to 25 cm for stiff plastic concrete[7], which corresponds to the range of T_0=0.5~2.0 s .

Table 2 Required L_q values ($\times10^{-2}$/s).

L_0 (10^{-2}/s)	T_0 (s)	t (s) 5	10	15	20	30	45	60	remarks
3.0	0	12.3	7.6	6.1	5.3	4.5	4.0	3.8	
	0.5	20.3	9.6	6.9	5.8	4.8	4.1	3.8	
	1.0	29.9	11.6	7.8	6.3	5.0	4.2	3.9	
	2.0	*	16.5	9.8	7.3	5.4	4.4	4.0	
	3.0	*	22.5	12.0	8.5	5.9	4.6	4.1	
	5.0	*	41.5	17.6	11.1	6.9	5.0	4.3	
	7.0	*	*	25.8	14.6	8.1	5.5	4.6	$\delta=0$
5.0	0	14.3	9.6	8.1	7.3	6.5	6.0	5.8	Eq.(29)
	0.5	22.3	11.6	8.9	7.8	6.8	6.1	5.8	
	1.0	31.9	13.6	9.8	8.3	7.0	6.2	5.9	
	2.0	*	18.5	11.8	9.3	7.4	6.4	6.0	
	3.0	*	24.5	14.0	10.5	7.9	6.6	6.1	
	5.0	*	43.5	19.6	13.1	8.9	7.0	6.3	
	7.0	*	*	27.8	16.6	10.1	7.5	6.6	
	0	13.8	9.4	7.9	7.2	6.5	6.0	5.8	
	0.5	21.3	11.2	8.7	7.6	6.7	6.1	5.8	
	1.0	30.5	13.2	9.6	8.1	6.9	6.1	5.8	
	2.0	*	17.7	11.4	9.1	7.3	6.3	5.9	$\delta=1.1\times10^{-4}$ cm
	3.0	*	23.4	13.5	10.2	7.7	6.5	6.0	
	5.0	*	41.3	18.8	12.7	8.7	6.9	6.2	
	7.0	*	*	26.6	15.9	9.8	7.4	6.5	

* >50.0×10^{-2}/s

z_0/v_0 means the time necessary to complete the compaction when $L_p=1$, which is principally associated with the consistency and the thickness of the concrete to be compacted, involving the delaying effect of horizontal reinforcing bars. Rewriting Eq.(27) by use of $z_0/v_0=T_0$,

$$T_0-bt+\frac{1}{a}(1-e^{-at})\leqq 0 \quad \cdots\cdots(28)$$

Substituting the values into b, a,

$$T_0-0.86\,t+\frac{1}{0.661(L_q-L_0)}\{1-e^{-0.661(L_q-L_0)t}\}=0 \quad \cdots\cdots(29)$$

This equation gives a relation between the vibration time t and the liquefaction value L_q at the moment the compaction is completed.

4. RELATION BETWEEN CHARACTERISTICS OF CONCRETE AGAINST VIBRATORY COMPACTION AND THE RADIUS OF ACTION OF VIBRATOR

4.1 Parameters Expressing the Characteristics of Concrete Against Vibratory Compaction

L_0 included in Eq.(29) is related to the minimum irreducible rate of strain required to extend the distance between two particles as described in 3.1. While any direct measurements of neat cement paste can not be found, in a study of the rheology of cement paste, it is seen that the paste with W/C of 30% gained viscosity at the strain rate ranging from 0.007 to 0.03 s^{-1} and that the visosity is held constant upto 0.07 s^{-1}, which indicates that the paste behaves as a dilatant material in this range of strain rate. Despite the decrease in the value with increase in W/C, since the paste in concrete may contain less water than the neat cement paste, the minimum irreducible strain rate of the concrete for usual use is supposed to have a magnuitude of this order and L_0 is $\pi/2$ times of it. Therefore L_0 expresses the minimum liquefying action required to initiate the liquefaction of the cement paste.

Another parameter T_0 is the time of vibration necessary to achieve the complete compaction in the case that the paste is perfectly liquefied, that is, $L_p=1$. Hence, it may suitably be designated "Basic Time of Vibration". Thus, the property of concrete for vibratory campaction is expressed by these two parameters. When a fresh concrete is approximated by Bingham body, L_0 is related to yield value. If it is 0, L_0 reduces to 0, while in the case that the viscosity is 0, T_0 to 0.

4.2 Relation Between the Time of Vibration and the Radiur of the Fully Compacted Region

When the parameters L_0 and T_0 are given, the minimum value of L_q of the region expected to be fully compacted with a vibration time of t, i.e. the required liquefaction value to complete the compaction in t can be obtained by the numerical calculation of L_q which satisfies $T(t, L_q)=0$. T is formulated as Eq.(30). Meanwhile L_q at a destance of r from the center of the vibrator is expressed as Eq.(11).

$$T(t,L_q)=T_0-0.86\,t+\frac{1}{0.661(L_q-L_0)}\{1-e^{-0.661(L_q-L_0)t}\} \quad \cdots\cdots(30)$$

$$L_q\leqq\frac{a_0}{4\pi\omega}\sqrt{\frac{R_v}{r}\left\{\left(k+\frac{1}{r}\right)^2+\left(\beta+\frac{1}{2r}\right)^2\right\}}\;e^{-\beta(r-R_v)} \quad \cdots\cdots(31)$$

4.3 Proposed Equation Describing the Condtion of Complete Compaction

Eq.(29) is generalized as Eq.(32), in which and involve the particle size distribution of the cement and the proportion of liquefaction, L_p of Eq.(33).

$$T_0 - bt + \frac{1}{h(L_q - L_0)}\{1 - e^{-h(L_q - L_0)t}\} \leqq 0 \quad \cdots\cdots\cdots\cdots (32)$$

$$L_p = b - e^{-h(L_q - L_0)t} \quad \cdots\cdots\cdots\cdots (33)$$

Taking $b=1$ so that is 1 if $(L_q - L_0)t$ approaches infinity and h is $0.661 \cong 2/3$, Eq.(32) becomes the following proposed equation(34).

$$T(t, L_q) = (t - T_0) - \frac{3}{2(L_q - L_0)}\{1 - e^{-\frac{2}{3}(L_q - L_0)t}\} \geqq 0 \quad \cdots\cdots\cdots\cdots (34)$$

4.4 Method of the Determination of L_0 and T_0

To determine the values of L_0 and T_0 for a given concrete, a compaction test of slab is available. As is seen in Fig.13 , when t is small the curves which express the relation between t and r are apart each other, while they get closer with increase in the time of vibration. Therefore, the following procedure is practical in the case that the characteristics of the vibration, a_0, f, c, β , are known beforehand, otherwise, they can be determined by measuring the change in acceleration with the destance from the vibrator[4].

① to measure R_5, R_{10}, R_{15} and R_{60} , the radii of fully compacted areas at $t=5$, 10 , 15 and 60 s , respectively: ② to calculate $L_5 \sim L_{60}$, the L_q values at $r = R_5 \sim R_{60}$ by Eq.(11): ③ to obtain L_0 by substituting $t=60$, $T_0=0$, $L_q=L_{60}$ into Eq.(34) and resolving it numerically: ④ to obtain T_5, T_{10}, T_{15} by calculating Eq.(35) with $t=5$, 10, 15 s respectively: ⑤ finally to determine T_0 by taking the average of $T_5 \sim T_{15}$ that is $T_0 = (T_5 + T_{10} + T_{15})/3$.

$$T_0 = t - \frac{3}{2(L_q - L_0)}\{1 - e^{-\frac{2}{3}(L_q - L_0)t}\} \quad \cdots\cdots\cdots\cdots (35)$$

Table 3 shows the experimental results and the values of L_0 and T_0 obtained by this means. Fig.14 is the comparison of the calculated and the experimental results of the relation between the time of vibration and the radius of action. Though it is natural that the calculated result agrees with the experiment at particular points because the values of L_0 and T_0 used for the calculation have been obtained from the experiment, the overall agreement of the curves may verify the validity of the arguement above discussed.

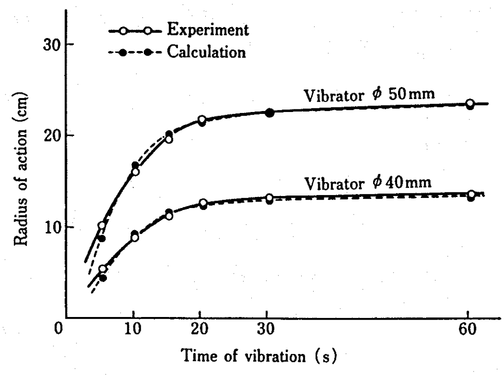

Fig.14 Comparison of calculation with experiment.

Table 3 Values of L_0 and T_0 obtained from experimental results.

Concrete	Vibrator	Characteristics of vibration					Experimental data						Calculated results	
							Time of vibration (s)							
		f (Hz)	a_0 (G)	c (m/s)	β (cm^{-1})		5	10	15	20	30	60	L_0 ($\times 10^{-2}$/s)	T_0 (s)
Non-AE Slump 2.5cm	φ 40mm	200	14.23	50	0.030	radius of action (cm)	5.5	9	11	12.5	13	13.5	8.00	1.27
	φ 50mm		17.03		0.031		10	16	19.5	21.5	22.5	23.5	5.37	0.96

5. EFFECTS OF FORMWORK AND REINFORCEMENT ON THE LIQUEFYING ACTION OF VIBRATION

5.1 Reflected Wave from the Surface of a Vertical Steel Bar

The wave spreading from an internal vibrator is not so simple as plane wave that it is difficult to obtain the general solution for reflected wave. Here, the wave reflected from a point on the surface of a round steel bar is sought by a classical method, and the liquefying actions of the direct and the reflected waves are combined.

Now, the two dimensional coordinates are taken as Fig. 15 with their origin at the center of the vibrator. S shows the position of a steel bar. $P(x, y; r, \theta)$ is the point aimed at, and its position on the basis of $S(x_s, y_s)$ is decided as in Fig. 16

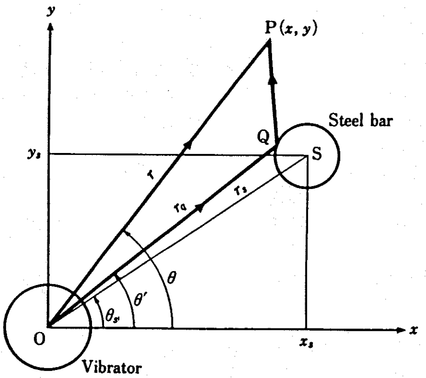

Fig. 15 Positions of vibrator and steel bar.

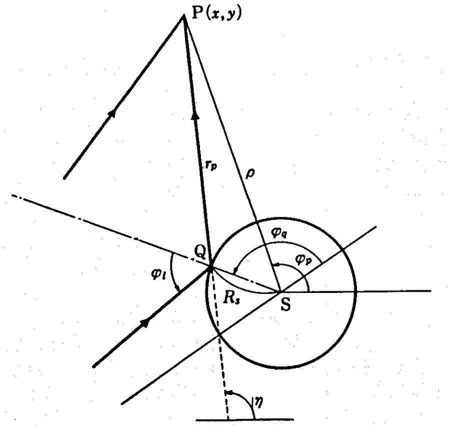

Fig. 16 Position of $P(x, y)$ on the basis of steel bar.

Then the point of reflection Q can be determined as follows.

$$\rho=\sqrt{r^2+r_s^2-2\,r r_s \cos(\theta-\theta_s)} \quad \cdots\cdots (36)$$

$$\varphi_p=\tan^{-1}(l_s/l_c)+\theta_s \quad \cdots\cdots (37)$$

where,

$$l_s=r\sin(\theta-\theta_s)$$
$$l_c=r\cos(\theta-\theta_s)-r_s$$

φ_q is obtained as the solution of Eq(38).

$$f(z)=\frac{1}{\tan z}-\frac{1}{\tan(\varphi_p-\theta_s-z)}+\frac{R_s}{r_s\sin z}+\frac{R_s}{\rho\sin(\varphi_p-\theta_s-z)}=0 \quad \cdots\cdots (38)$$

As a root of the equation exists within a certain range, it is easy to obtain its numerical value by the bisection method. Fig. 17 and Table 4 may serve as the aids to select the initial range. In Table 4 ,

$$r_c=\sqrt{r_s^2-R_s^2} \quad \cdots\cdots (39)$$

$$\theta_c=\tan^{-1}(R_s/r_c) \quad \cdots\cdots (40)$$

$$(x-x_s)^2+(y-y_s)^2>R_s^2 \quad \cdots\cdots (41)$$

Using $m_s = r_s \sin \varphi_q$,
$m_c = r_s \cos \varphi_q + R_s$

$$\varphi_t = -\tan^{-1}(m_s/m_c) \cdots\cdots (42)$$
$$r_q = \sqrt{m_s^2 + m_c^2} \cdots\cdots (43)$$
$$\theta' = \varphi_q + \varphi_t + \theta_s - \pi \cdots\cdots (44)$$
$$r_p = \rho^2 + R_s^2 - 2\rho R_s \cos(\varphi_q - \varphi_p + \theta_s) \cdots\cdots (45)$$
$$\eta = \varphi_q - \varphi_t + \theta_s \cdots\cdots (46)$$
$$r' = r_p + r_q \cdots\cdots (47)$$

Thus, the time when the reflected wave was emitted in the direction of θ' by the vibrator is $t' = t - (r_p + r_q)/c$.

As the decreasing rate of amplitude is

vibrator ~ Q : $\sqrt{\dfrac{R_v}{r_q}}\, e^{-\beta(r_q - R_v)}$

point $Q \sim P$: $\sqrt{\dfrac{R_s}{r_p + R_s}}\, e^{-\beta r_p}$

the wave function for the reflected vibration is obtained as follows.

$$\psi' = A' \sin \phi' \cdots\cdots (48)$$

in which

$$A' = R_f \psi_0 \sqrt{\frac{R_s R_v}{r_q (r_p + R_s)}}\, e^{-\beta(r_p + r_q - R_v)} \cdots\cdots (49)$$
$$\phi' = \omega t - k(r_p + r_q - R_v) - \theta' \cdots\cdots (50)$$

R_f: reflection coefficient

5.2 Liquefying Action of the Reflected Vibration from a Vertical Steel Bar

A square element $PLMN$ is taken as Fig. 18 , in which n is a factor relating to the deviation of reflecting direction due to the difference in the incidence point and varies with the position of Q. Supposing $n=1$, the liquefying action of reflected vibration at point P , L', can be analyzed in the same manner as in 2.2.

$$L' = \frac{\omega}{4\pi}\left(\frac{1}{r'}\frac{\partial \psi'}{\partial \theta'} + \frac{\partial \psi'}{\partial r'}\right) \cdots\cdots (51)$$

By use of the approximation (52), Eq. (49) is reduced to Eq. (53).

$$\sqrt{r_q(r_p + R_s)} \simeq \frac{1}{2}(r_q + r_p + R_s) = \frac{1}{2}(r' + R_s) \cdots\cdots (52)$$

$$A' = 2 R_f \psi_0 \sqrt{R_s R_v}\, \frac{1}{r' + R_s}\, e^{-\beta(r' - R_v)} \cdots\cdots (53)$$

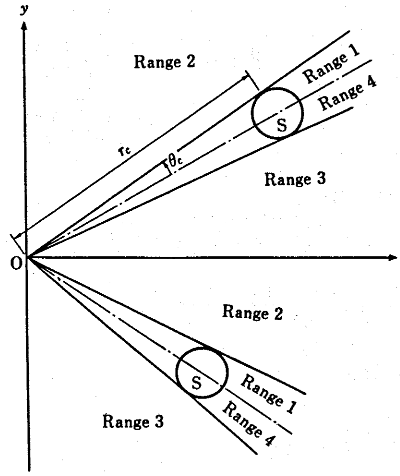

Fig. 17 Divisions of plane.

Table 4 Range of φ_q.

Range	Position of P (r, θ)	Range of φ_q
1	$\theta_s < \theta < \theta_s + \theta_c$ and $r \geqq r_c$	$0 < \varphi_q < \varphi_p - \theta_s$
2	$\theta_s < \theta < \theta_s + \theta_c$ and $r < r_c$	$\varphi_p - \theta_s < \varphi_q < \pi$
	$\theta_s + \theta_c < \theta < \theta_s + \pi$	
	$\theta = \theta_s + \theta_c$	$\varphi_q = \theta_c + \pi/2$
3	$\theta_s - \theta_c + 2\pi < \theta < \theta_s + 2\pi$ and $r < r_c$	$\pi < \varphi_q < \varphi_p - \theta_s$
	$\theta_s + \pi < \theta < \theta_s - \theta_c + 2\pi$	
	$\theta = \theta_s - \theta_c + 2\pi$	$\varphi_q = \theta_c + 3\pi/2$
4	$\theta_s - \theta_c + 2\pi < \theta < \theta_s + 2\pi$ and $r \geqq r_c$	$\varphi_p - \theta_s < \varphi_q < 2\pi$
on the elongation of OS	$\theta = \theta_s$ and $r > r_s$	$\varphi_q = 0$
on the OS	$\theta = \theta_s$ and $r < r_s$	$\varphi_q = \pi$

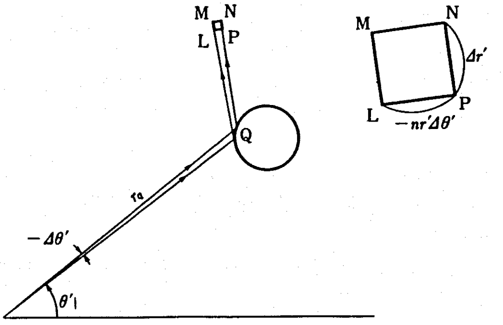

Fig. 18

Then

$$\frac{1}{r'}\frac{\partial\psi'}{\partial\theta'}+\frac{\partial\psi'}{\partial r'}=-\frac{A'}{r'}\cos\phi'-A'\left(\beta+\frac{1}{r'+R_s}\right)\sin\phi'-A'k\cos\phi'$$

$$=-A'\left\{\left(k+\frac{1}{r'}\right)\cos\phi'+\left(\beta+\frac{1}{r'+R_s}\right)\sin\phi'\right\} \cdots\cdots\cdots\cdots(54)$$

Consequently

$$L'=-\frac{\omega A'}{4\pi}\sqrt{\left(k+\frac{1}{r}\right)^2+\left(\beta+\frac{1}{r'+R_s}\right)^2}\sin(\phi'+\alpha') \cdots\cdots\cdots\cdots(55)$$

$$\alpha'=\tan^{-1}\frac{k+1/r'}{\beta+1/(r'+R_s)} \cdots\cdots\cdots\cdots(56)$$

in which

$$A'=R_f\psi_0\sqrt{\frac{R_sR_v}{r_q(r_p+R_s)}}\,e^{-\beta(r'-R_v)} \cdots\cdots\cdots\cdots(57)$$

Taking n into consideration,

$$L'=-\frac{\omega A'}{4\pi}\sqrt{\left(k+\frac{1}{nr'}\right)^2+\left(\beta+\frac{1}{r'+R_s}\right)^2}\sin(\phi'+\alpha') \cdots\cdots\cdots\cdots(58)$$

$$\alpha'=\tan^{-1}\frac{k+1/(nr')}{\beta+1/(r'+R_s)} \cdots\cdots\cdots\cdots(59)$$

As r' is the distance from the vibrator to point P via point Q , $1/r'$ is rather small as compared with k , n may be possibly taken as 1.

5.3 Liquefying Action of Reflected Vibration from Formwork Plate

In Fig. 19 , M is a mirror image of the center of the vibrator with respect to a vertical form work plate. On referring to the figure,

$$\theta'=\tan^{-1}\frac{y}{2x_f-x}$$

$$\eta'=\pi-\theta'$$

$$r'=\sqrt{(2x_f-x)^2+y^2}$$

Then, the reflected wave is

$$\psi'=R_f\psi_0\sqrt{\frac{R_v}{r'}}\,e^{-\beta(r'-R_v)}\sin\{\omega t-k(r'-R_v)-\theta'\} \cdots\cdots\cdots\cdots(60)$$

L' will be obtained by substituting r' for r contained in L_r of Eq.(7).

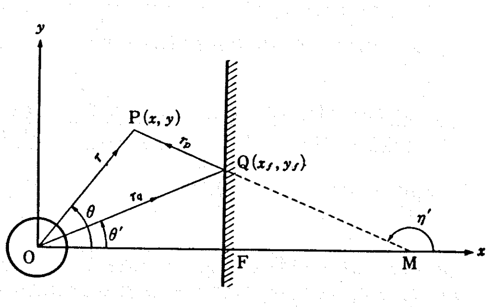

Fig. 19 Reflection of wave at the surface of a solid plate.

6. SPHERE OF ACTION OF AN INTERNAL VIBRATOR UNDER THE INFLUENCES OF REINFORCING BARS AND FORM WORK

6.1 Combined Liquefying Action of Direct and Reflected Vibrations

As shown in Fig. 20 , liquefying actions may be combined as vectors. N is the number of steel bars and L_i indicates the action of reflected vibration from bar No. i .

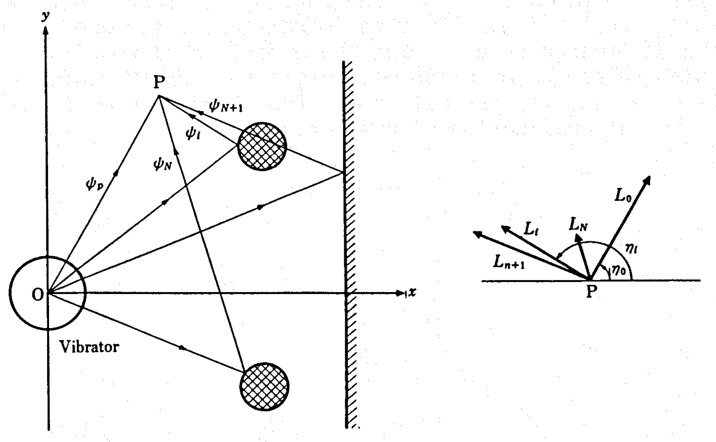

Fig. 20 Waves and their liquefactions as vectors at a point P.

The action of direct vibration from the vibrator is indicated by $i=0$,and that of reflected vibration from a form work plate by $i=N+1$. Then, the combined liquefing action is given by

$$L_q=\left[\sqrt{\left(\sum_{i=0}^{N+1} L_{xi}\right)^2+\left(\sum_{i=0}^{N+1} L_{yi}\right)^2}\,\right]_{\max} \quad \cdots\cdots\cdots\cdots(61)$$

where

$L_{xi}=L_i\cos\eta_i$

$L_{yi}=L_i\sin\eta_i$

L_i : liquefying action of No. i vibration
η_i : angle of the direction of No. i wave

L_q is obtained as the maximum value of $|L|=\sqrt{(\Sigma L_x)^2+(\Sigma L_y)^2}$ in the range of $t=0\sim T$ (T : periodic time).

6.2 Isometric Lines of L_q Value for Several Cases

Fig. 21 ~ Fig. 26 are isometric lines of L_q values for 6 cases listed in Table 5 . Interval of t in the calculation of maximum $|L|$ was $T/60$. Among the results, Fig. 21 ~ Fig. 23 show such an effect of a single reinforcing bar that L_q is remarkably reduced behind it. The left halves of Fig. 21, 22 are the results without the effect of the bar.

Fig. 21 with $c=50$ m/s for Non-AE concrete is influenced by the reinforcement more than Fig. 22 for AE concrete. Fig. 24 illustrates the effect of a form work plate and Fig. 25 , Fig. 26 are the cases with multiple steel bars.

7. CONCLUSIVE REMARK

Although a great deal of relating experimental results have been reported, they cannot be made the best of, because of discrepancy in the situations of investigation such as the characteristics of vibrator, properties of concrete, vibration time, subjects of measurement and so forth. If any common basis to analyze the experiment should be established, this difficulty might be resolved.

With this regard the problem of the liquefaction of fresh concrete by vibration was theoretically dealt with. The three parameters introduced in this paper are able to be determined by analyzing the experimental results of compaction test with a slab. The accumulation and compilation of the informations about the correlation between the parameters and the properties of fresh concrete, will lead to the rationalization of compaction practice.

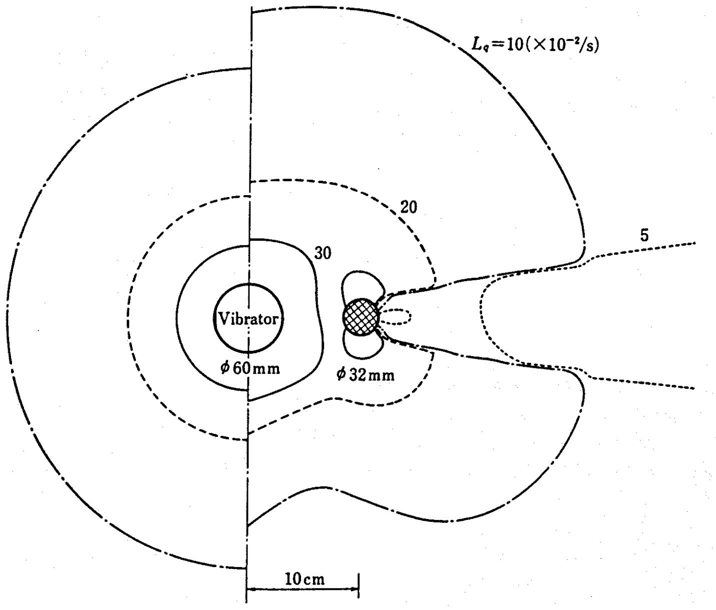

Fig. 21 Isometric lines of L_q value under the influence of a reinforcing steel bar (Non-AE concrete : $c=50$ m/s).

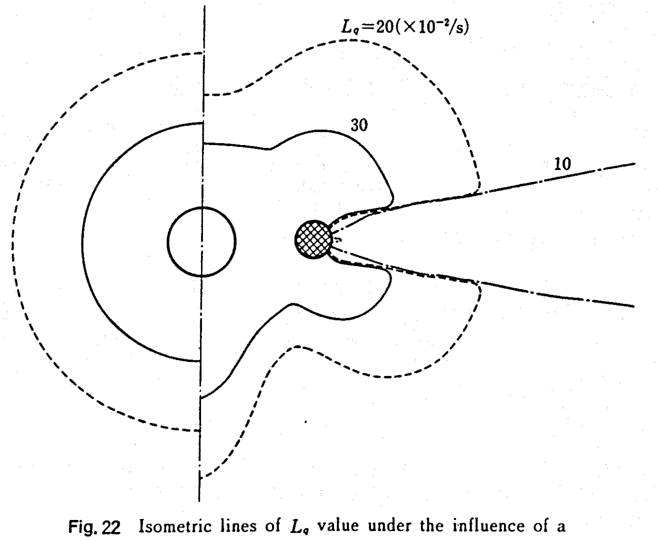

Fig. 22 Isometric lines of L_q value under the influence of a reinforcing steel bar (AE concrete : $c=30$ m/s).

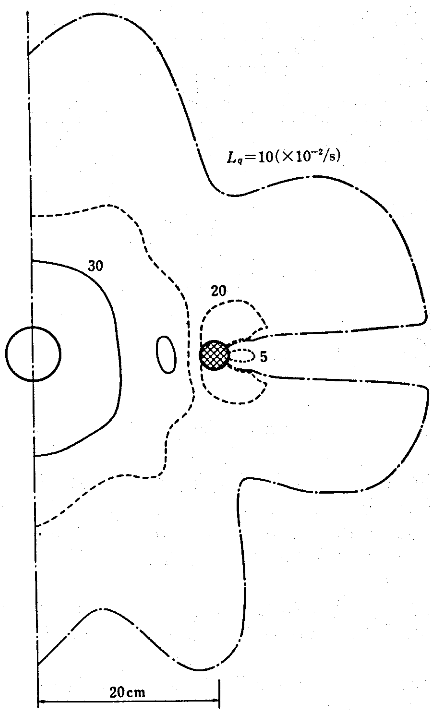

Fig. 23 Isometric lines of L_q value.

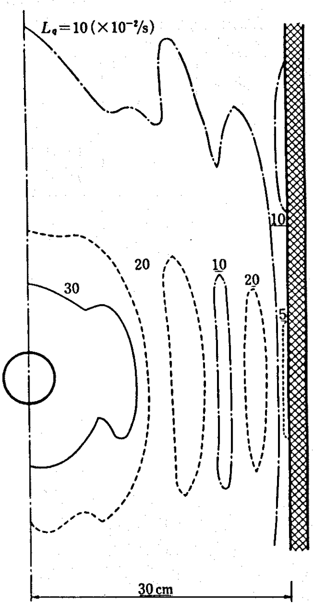

Fig. 24 Isometric lines of L_q value under the influence of a solid form plate (AE concrete).

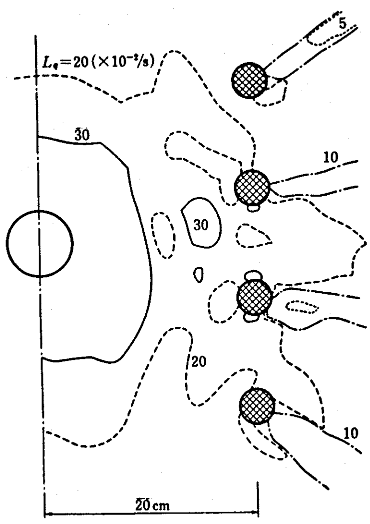

Fig. 25 Isometric lines of L_q value under the influence of reinforcing steel bars arranged in a line.

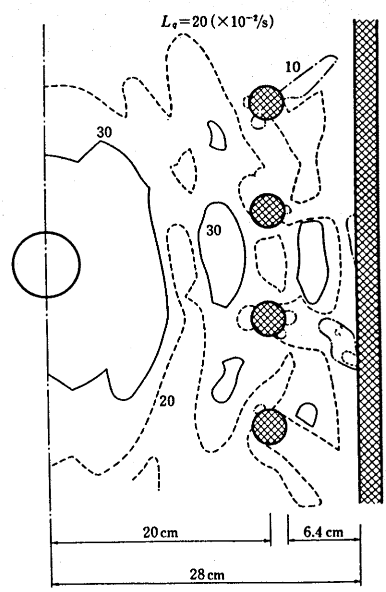

Fig. 26 Isometric lines of L_q value under the influence of reinforcing steel bars and a solid form plate.

Table 5 Assumed situations.

Fig.	Steel bars				Plate	Coeff. of reflection	Vibrator			Propagation	
	Number	Diameter	Position (cm)								
		D_v (mm)	x_s	y_s	x_f	R_f	ϕ (mm)	f (Hz)	a_0 (G)	c (m/s)	β (cm^{-1})
21	1	32	10	0	—	1	60	200	20	50	0.017
22			10		—					30	
23			20		—						
24	—	—	—	—	30						
25	4	32	20	±5, ±15	—						
26					28						

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