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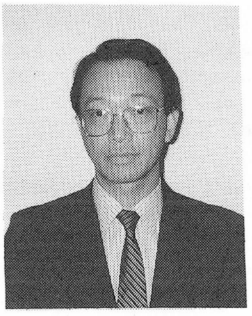
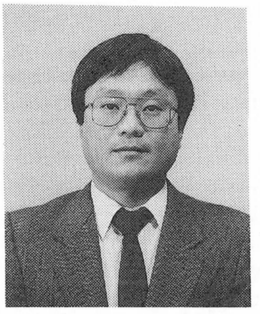
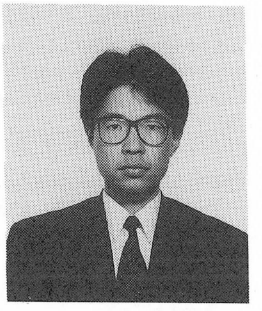
# Study on the Effect of Local Strain of Tensile Reinforcement on the Flexural Behavior of Reinforced Concrete Beams

Shoji IKEDA    Takahiro YAMAGUCHI    Yasuyuki GOTO

## SYNOPSIS

**Experimental studies were carried out to clarify the mechanism of the flexural behavior of reinforced concrete beams with low reinforcement. The experiments showed that the strain of tensile reinforcement at the crack of concrete immediately reached the strain hardening region while the reinforcement at the other portion was under the yield strain. The load-deflection curves after the yielding of the beam were shown to fluctuate due to the local strain hardening and the bond failure near the crack of concrete. An analytical model to evaluate these phenomena was successfully obtained.**

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JSCE Member Shoji Ikeda is a professor of Civil Engineering at Yokohama National University. He is a member of the committee on concrete and a member of the committee on structural engineering in JSCE. He was a director of the Japan Concrete Institute from 1985 to 1987. He is now a director of the Japan Prestressed Concrete Engineering Association. He serves as a president of Japan Gas-Pressure Welding Association. He is a member of IABSE, a fellow of ACI, and vice-president of FIP. He is a graduate of the University of Tokyo in 1960 and obtained doctor of engineering degree in 1974. He specializes in the design and mechanics of reinforced and prestressed concrete and composite structures.

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T. Yamaguchi is a research associate of Civil Engineering at Yokohama National University. His research interests include seismic design of reinforced concrete structures and response behavior of reinforced concrete members under static and dynamic loadings. He is a member of JSCE, JCI and JPCEA.

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Y. GOTO was a graduate student of Civil Engineering at Yokohama National University. He is a member of JSCE and JCI. (After obtaining his Master's degree in 1990, he became a civil engineer at JR Tokai Corporation.)

## 1. INTRODUCTION

Although experimental and analytical studies had been carried out extensively on the flexural behavior of reinforced concrete beams, its mechanism was not sufficientlly clarified. For example, in the case of the reinforced concrete beams with low reinforcement, the experimental yield load is often significantly larger than the calculated values using the nonlinear properties of the materials and these phenomena are scarcely known (1). Hence its mechanism should be clarified in detail. In the conventional analysis, there are various methods which assume the softening of tensile strain on concrete and so on. However, these methods mainly concern with the flexural behavior before the yielding of reinforcement and cannot coincide with the micromechanism of the flexural behavior of reinforced concrete beams including post-yield of reinforcement (2).

In this study, in order to clarify the above mentioned peculiar behavior of reinforced concrete beams with low reinforcement, monotonic flexural loading tests were carried out. From these experimental results, the causes of the peculiar behavior were clarified based on local strain of the reinforcement. An analytical model to evaluate these phenomena was proposed (3).

## 2. FLEXURAL LOADING EXPERIMENT

### 2.1 OUTLINE OF EXPERIMENT

The experimental study consists of two series : Series A aims to obtain the characteristics of the flexural behavior of reinforced concrete beams with low reinforcement (hereafter low reinforced beams). Series B aims to measure the strain of tensile reinforcement at the crack portion in detail. The tensile reinforcement ratio of the low reinforced beam was 0.2 % in order to represent clearly the effect of low reinforcement ratio. The reinforced concrete beams (hereafter RC beams) in which the ratio of tensile reinforcement was 0.8 % were tested to compare with low reinforced beams.

### 2.2 SPECIMEN AND LOADING METHOD

The characteristics of each specimen are summarized in Table 1. Series A has nine specimens named as No.1 to No.9 where the specimens of No.7 to No.9 are additional ones whose age of the concrete is different from that of the other specimens. As shown in Table 1, the major factors of the experiments are the ratio of tensile reinforcement, the compressive strength of concrete, the thickness of concrete cover, the existence of stirrup in the bending span and the number of tensile reinforcement. The characteristics of specimen No.7 is same to the specimen No.3 except the age of concrete.

Table 1 - Characteristics of Specimens

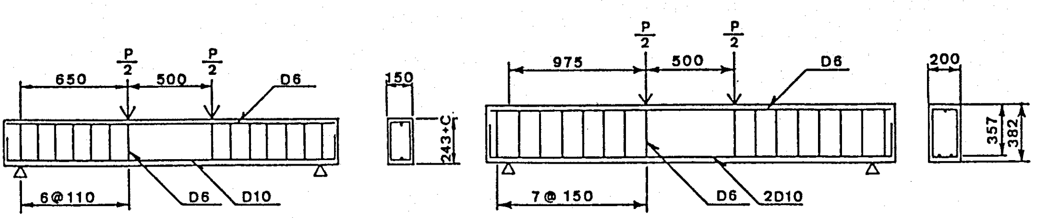
| Specimen | | Tensile Reinforcement | | | Compressive Strength of Concrete | Concrete Cover C | Existence of Stirrups in Bending Span |
|---|---|---|---|---|---|---|---|
| No. | Series | Diameter (mm) *1 | Number of Bars | Ratio (%) | (kgf/cm²) | (mm) | |
| 1 | A | D10 | 4 | 0.8 | 630 | 20 | Not Exist |
| 2 | | | 1 | 0.2 | | 10 | |
| 3 | | | | | | 20 | |
| 4 | | | | | | 50 | |
| 5 | | | | | 200 | 20 | |
| 6 | | | | | | 50 | |
| 7 | | | | | 630 | 20 | |
| 8 | | | | | | | Exist |
| 9 | | | 2 | | | | Not Exist |
| 10 | B | D19 | 1 | 0.2 | 470 | 40 | Not Exist |
| 11 | | | 4 | 0.8 | | | |

*1 : D19 is the threaded bars

Fig.1 shows the details of series A specimens and loading conditions.

D10 (10 mm diameter deformed bar) was used for tensile reinforcement and the number of reinforcement is only one except specimen No.1 and No.9. The shear span-depth ratio (a/d) was 2.73. Sufficient amount of stirrups of D6 was provided at shear span in order to prevent from shear failure, and only specimen No.8 had stirrups in the pure bending span.

Series B has two specimens named as No.10 to No.11 as shown in Table 1. Fig.2 shows the details of series B specimens and loading condition. In order to prevent from the deterioration of the bond strength between concrete and tensile reinforcement due to strain gauges attached on the surface of tensile reinforcement, the threaded bars of D19 were used for tensile reinforcement and the strain gauges which had 0.2 mm length were attached on their concave ribs. The strain gauges were attached closely within the bending span, the spaces of which were 2 cm in the central 20 cm portion and 5 cm in the other portion within the bending span. A stainless plate which was 0.5 mm thick was planted at the span center to determine the crack position as shown in Fig. 2. Sufficient amount of stirrups of D6



(a) Specimen No. 1 ~ No. 8 (b) Specimen No. 9

Fig.1 - Details of Specimen and Loading Condition (Series A)

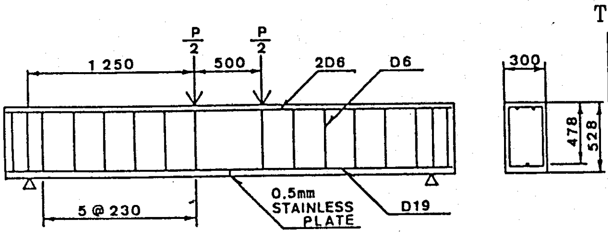


Fig.2 - Details of Specimen and Loading Condition (Series B)

Table 2 - Properties of Materials

| Tensile Reinforcement | | | | |
|---|---|---|---|---|
| Diameter (mm) | Type | Yield Strength | Tensile Strength | Young's Modulus |
| | | (kgf/cm²) | | |
| D10 | SD30 | 3900 | 5660 | $1.95\times10^6$ |
| D19 | | 4060 | 6140 | $2.10\times10^6$ |
| Concrete | | | | |
| Maximum Size of Gravel (mm) | | Compre. Strength | Tensile Strength | Young's Modulus |
| | | (kgf/cm²) | | |
| 13 | | 630 | 38 | $3.17\times10^5$ |
| | | 200 | 18 | $1.79\times10^5$ |
| 20 | | 470 | 34 | $3.13\times10^5$ |

Compre. : Compressive

was provided in the shear spans although no stirrup was provided in the bending span. The shear span-depth ratio (a/d) was 2.62.

The mechanical properties of the materials used in the both series are shown in Table 2.

One directional monotonic loading was applied to beam specimens by two-point load using hydraulic jacks. The standard loading pattern was monotonic increment until the yield load. Then, displacement control method was used until the flexural compressive failure or cracking of 10 mm wide. The midspan displacement of the specimens was measured continuously using an X-Y recorder.

## 3. EXPERIMENTAL RESULTS AND DISCUSSIONS

### 3.1 FLEXURAL BEHAVIOR OF LOW REINFORCED BEAMS

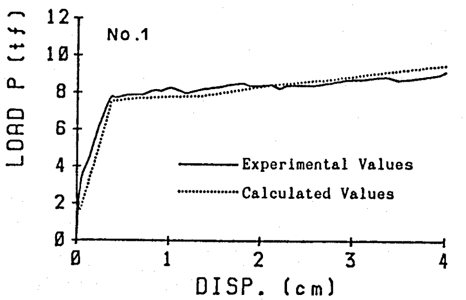



Fig.3 - Load-Displacement Curves (Specimen No. 1)

Fig.3 shows the load-displacement curve of specimen No.1 with tensile reinforcement ratio of 0.8 %. Fig.4 shows the cracking pattern of this specimen under the yield load. These figures show the typical flexural behavior of ordinary RC beams and the experimental displacement curve coincides well with the calculated one (analytical MODEL Ⅱ , refer to 4.1). On the other hand, the load-displacement curve of specimen No.3 with tensile reinforcement ratio of 0.2 % was significantly different as shown in Fig.5. The experimental yield load exceeded the calculated values (analytical MODEL Ⅱ ) shown by a dotted line in the figure and the load-displacement curve fluctuated greatly after the yield of reinforcement. This phenomenon was related to the growth of crack as shown in Fig.6 as the fall of the flexural capacity occurred when existing cracks expanded. It should be noted that the number of cracks which appeared in the specimen at the first stage was not increased until the final loading.

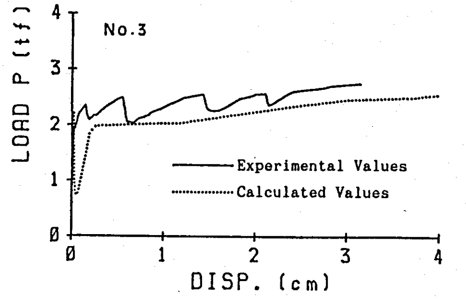



Fig.5 - Load-Displacement Curves (Specimen No. 3)

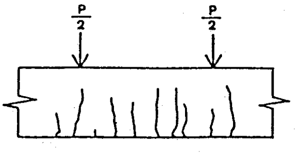



FIg.4 - Cracking Pattern (Specimen No. 1)

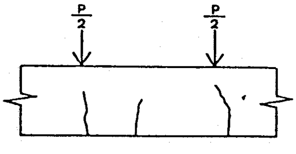



FIg.6 - Cracking pattern (Specimen No. 3)

Fig.7 shows the effect of the compressive strength of concrete on the flexural behavior of low reinforced beams. The compressive strength of concrete of specimen No.5 and No.3 were 200 kgf/cm$^2$ and 630 kgf/cm$^2$ , respectively. The load-displacement curve fluctuated largely when the compressive strength of concrete increased. Fig.8 shows the effect of the thickness of concrete cover on the flexural behavior of low reinforced beams. The thickness of concrete cover of specimen No.2 and No.4 were 10 mm (1$\phi$, here $\phi$:bar diameter) and 50 mm (5$\phi$), respectively. The load-displacement curve fluctuated largely as the thickness of concrete cover increased. The increase of the flexural capacity and its fluctuation seems to be related to an increase of the tension stiffening of concrete and the bond strength between reinforcement and concrete.

Fig.9 shows the effect of the existence of stirrups in bending span. The load-displacement curve of specimen No.8 having the stirrups in the bending span almost coincided with that of specimen No.7. Hence, it can be said that the existence of stirrups in bending span make no influence on the flexural behavior.

Fig.10 shows the load-displacement curve of specimen No.9 which has two tensile reinforcements having the tensile reinforcement ratio of 0.2 %. This result shows the same tendency as the flexural behavior of other specimens with low reinforcement. Hence, it can be said that the number of tensile reinforcement make little influence on the flexural behavior so far as a space between the reinforcements is enough to avoid deterioration of bond.

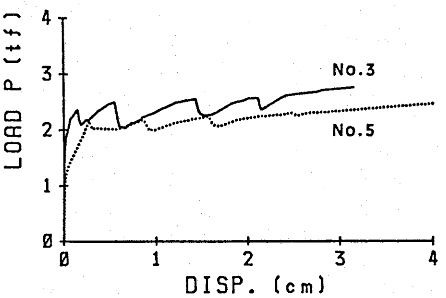



Fig.7 - Load-Displacement Curves (Effect of Compressive Strength of Concrete)

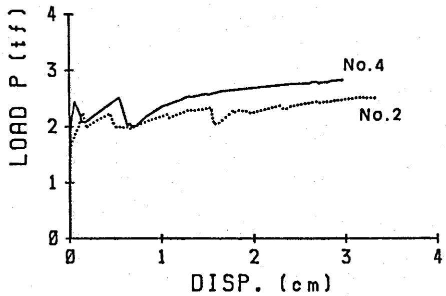



Fig.8 - Load-Displacement Curves (Effect of Thickness of Concrete Cover)

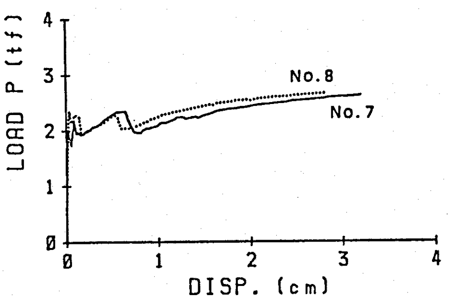



Fig.9 - Load-Displacement Curves (Effect of Stirrups in Bending Span)

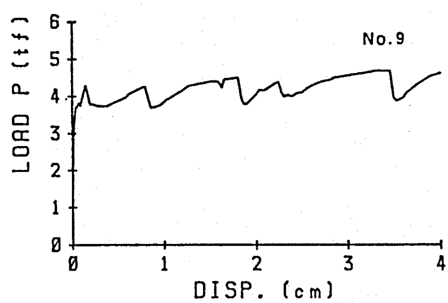
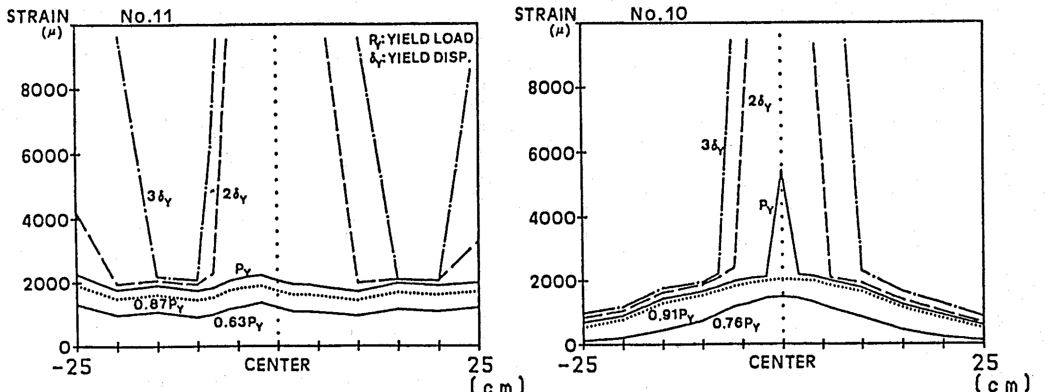



Fig.10 - Load-Displacement Curve (Specimen No. 9)




(a) Specimen No.11 (Tensile Reinforcement Ratio 0.8%)

(b) Specimen No.10 (Tensile Reinforcement Ratio 0.2%)

Fig.11 - Strain Distributions of Tensile Reinforcement in Series B

### 3.2 STRAIN DISTRIBUTION OF TENSILE REINFORCEMENT

Fig.11 shows the strain distributions of tensile reinforcement which were measured in the bending span in the experiment of series B. Specimen No.10 had only one crack in the bending span. Specimen No.11 had about seven cracks with the intervals of 5~7 cm. In the case of specimen No.11, the strain of tensile reinforcement increased uniformly according to the increase of the load as shown in Fig.11(a). And the values of strain reached uniformly 0.002 in bending span under the yield load. While, in the case of specimen No.10 having low reinforcement, the strain of tensile reinforcement were the largest at the crack portion, and the strain at loading points were only 0.0007 when those at crack portion reached the yield point. Then according to the increase of the load, the strain of tensile reinforcement at the crack portion increased immediately to reach the strain hardening region. Thus, in the case of the low reinforced beams, it was found that the strain of tensile reinforcement at the crack portion reached locally the strain hardening region.

### 3.3 MECHANISM OF INCREASE AND FLUCTUATION OF FLEXURAL LOADING CAPACITY

From the experimental results, the mechanism of the crack portion of low reinforced beams can be modeled as shown in Fig.12(a)~(c). First, after the cracks occurred in the specimen, the strain of tensile reinforcement at the crack portion becomes localy greater according to the increase of the load as shown in Fig.12(a). Next, as shown in Fig.12(b), the reinforcement of that portion yields locally and becomes the strain hardening. The flexural loading capacity increases due to the strain hardening of the reinforcement. And, as shown in Fig.12(c), the bond between reinforcement and concrete fails near the crack portion, and the crack extends. At this time the local increment of the strain of tensile reinforcement at the crack portion is released, so that the flexural loading capacity decreases suddenly. Hence, the recurrence of increase and decrease of the flexural loading capacity is explained by this mechanism. If enough tensile reinforcement exists, the above mechanism is hardly seen because of the rather uniform strain distribution in the reinforcement.

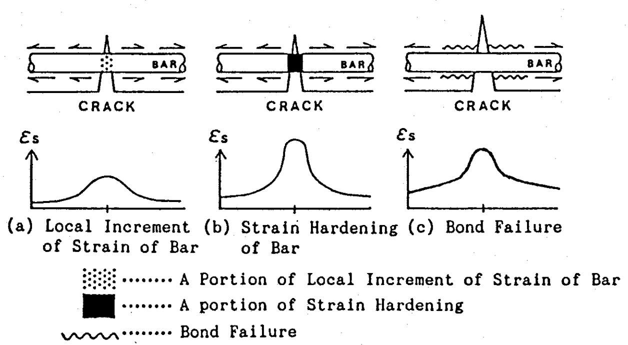



Fig.12 - Mechanism of Increase and Fluctuation of Flexural loading Capacity

## 4. ANALYSIS

### 4.1 METHOD OF ANALYSIS

In order to analyze the peculiar behavior of low reinforced beams, an analytical model considering the local increment of the strain of tensile reinforcement at the crack portion was proposed. This analytical model had an element having the length of $\Delta \ell$ in the longitudinal direction of the beam (hereafter $\Delta \ell$ element) which represents the crack portion at the span center of the beam as shown in Fig.13. It was assumed that the tensile strength of the concrete in the $\Delta \ell$ element was zero and the concrete except the $\Delta \ell$ element had a property of tension stiffening without occurrence of cracks. Hereafter this proposed analytical model was called MODEL I.

Fig.14(a) and (b) show the stress-strain curves of concrete in the $\Delta \ell$ element and the other portion used for MODEL I respectively. The length of $\Delta \ell$ becomes larger as the strain of tensile reinforcement increases after yield strain. In the present calculation, the given initial value of $\Delta \ell$ was equal to 3 % of the effective depth of the RC beam and the degree of extending was determined according to the tensile reinforcement ratio of the specimen and the compressive strength of the concrete.

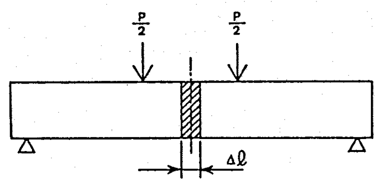
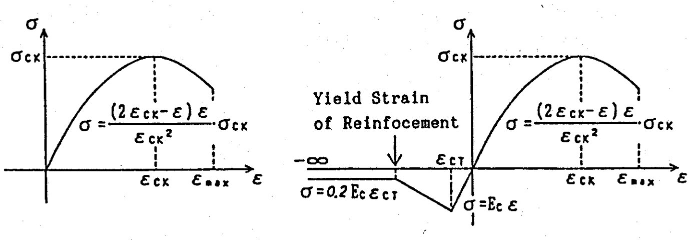


Fig.13 - Analytical Model used for MODEL I



(a) In $\Delta \ell$ Element (b) Except $\Delta \ell$ Element
Fig.14 - MODEL. I for Concrete

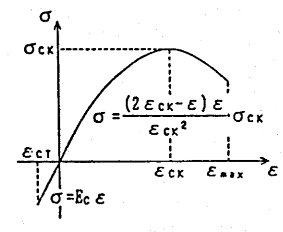


Fig.15 - MODEL II for Concrete

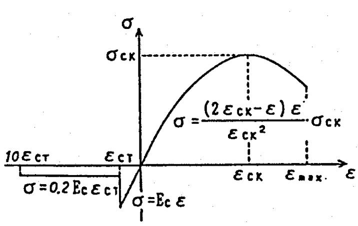


Fig.16 - MODEL III for Concrete

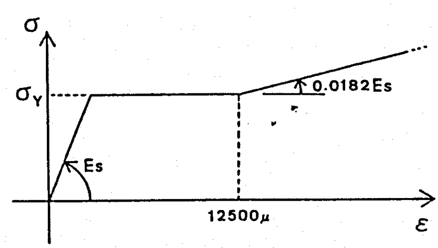


Fig.17 - Stress-Strain Curve of Reinforcement

A fiber model in which the cross section was replaced into many fiber elements was adopted to the method of the calculation. The displacements of the RC beams were obtained by integrating the relationship of moment-curvature of the cross section along the longitudinal direction. Here, the shear displacement was not considered.

In order to compare with MODEL I, an ordinary model not considering the $\Delta \ell$ element was used for the analysis. Hereafter this model was called MODEL II. Fig.15 shows the stress-strain curve of the concrete used for MODEL II. In addition, the analytical model considering the tension strain softening of the concrete as shown in Fig.16 was used. Hereafter this model was called MODEL III.

The mechanical property of the reinforcement used for MODEL I ~ III was assumed as trilinear model as shown in Fig.17.

The analyses were carried out for specimen No.1, No.3 and No.5. It should be noted that the crack load of specimen No.3 became larger than the yield load since specimen No.3 was the low reinforced beam and the compressive strength of the concrete was high.

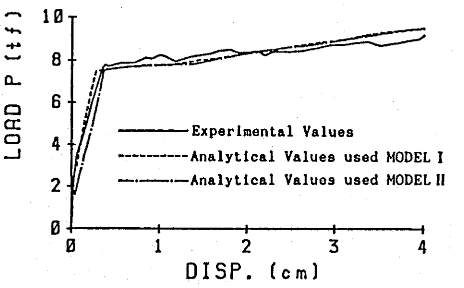


Fig.18 - Specimen No.1

4.2 RESULTS OF ANALYSIS

Fig.18~20 show the comparison of the load-displacement curves derived from the analysis and the experiment. In the case of specimen No.1, both the calculated value using MODEL I and the one using MODEL Ⅱ coincided well with the experimental value as shown in Fig.18.

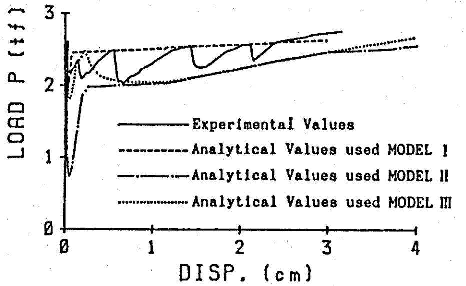


Fig.19 - Specimen No.3

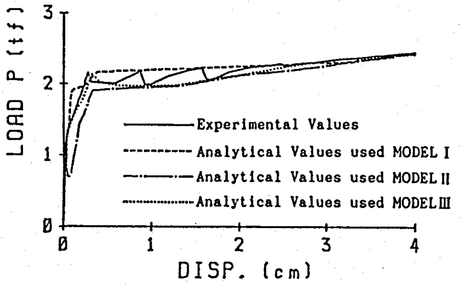


Fig.20 - Specimen No.5

Fig.18 ~ 20
Comparisons of Analytical Value and Experimental Value

Fig.19 and 20 show the load-displacement curves of specimen No.3 and No.5, respectively. The calculated load-displacement curves using MODEL Ⅱ falled suddenly at the calculated crack load. The reason of this phenomenon is that the calculative flexural resistance of the reinforced concrete is smaller than the crack load because of the low reinforcement. And, after the yielding of the reinforcement, the calculated values coincided with the bottom of the fluctuation of experimental values. The calculated load-displacement curves using MODEL Ⅲ considering the tension strain softening of the concrete were coincided well with the experimental results until yielding of the reinforcement, but they could not express the increase of the flexural loading capacity after the yielding of reinforcement similar to the results of MODEL Ⅱ.

On the other hand, MODEL I could express the local strain hardening of the reinforcement, so that the calculated load-displacement curves coincide with the increase of the flexural loading capacity of the low reinforced beams. Here, the bond failure between the reinforcement and the concrete confined the top of the fluctuation of this flexural loading capacity. In the case of this calculation, the first bond failure was assumed to occur when the tensile strain of the reinforcement in the $\Delta \ell$ element of specimen No.3 reached 0.032 and that of specimen No.5 reached 0.022 respectively so as to coincide with the top of the fluctuation of the flexural loading capacity as shown in Fig.19 and 20.

From these results, the top and bottom of the fluctuation of the flexural loading capacity in the low reinforced beams can be expressed by the simple one-dimensional fiber models such as MODEL I and MODEL Ⅱ.

## 5. CONCLUSION

The followings are the conclusions of this study:

(1) In the case of low reinforced beam, it was recognized that the flexural loading capacity under the yield load became significantly larger than the calculated values using ordinary nonlinear method (MODEL II ) and that the flexural loading capacity fluctuated greatly according to the extending of the cracks.

(2) The fluctuation of flexural loading capacity in the post yielding stage is influenced by the compressive strength of concrete and the thickness of concrete cover.

(3) The increase of flexural loading capacity occurs due to the local increment of the strain of tensile reinforcement at the crack portion up to the strain hardening region.

(4) The mechanism of the fluctuation of flexural loading capacity is explained by means of a recurrence of the local strain hardening of the reinforcement at the crack portion and the bond failure near the crack.

(5) The increase of flexural loading capacity in low reinforced beams can be expressed by using the proposed analytical model of MODEL I in this study. The bottom value of the fluctuation of the load-displacement curve coincides with the calculated value by using the ordinary nonlinear analysis (MODEL II).

## REFERENCE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