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Analytical Model for Reinforcement Concrete Panel Element Subjected to Reversed Cyclic In-plane Forces

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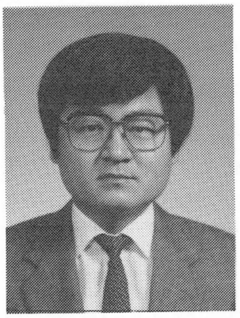
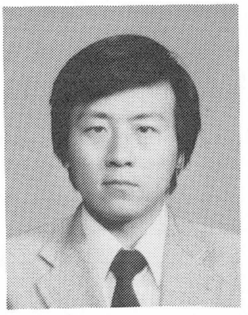


Junichi Izumo　Hyunmock Shin　Kouichi Maekawa　Hajime Okamura

SYNOPSIS

The aim of this study is to develop a constitutive law for a RC panel subjected to reversed cyclic in-plane forces. The authors have formulated the constitutive law for a RC panel by using the constitutive laws for cracked concrete and for steel that have been developed and verified with the reversed uniaxial testing of RC. The proposed constitutive law for the RC panel has been verified with the experiments conducted by Ohmori et al. and Stevens et al.. Further, it has been confirmed that the response of the RC panel under reversed cyclic loading can be predicted by the proposed model.

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1. INTRODUCTION

Finite Element Method (FEM) is considered very effective as an analytical method for reinforced concrete (RC). However, in order to get sufficient accuracy in tracing the responses of RC members and/or structures, FEM requires an analytical model that can describe the behavior of the RC panel element composing the structures.

Aoyagi and Yamada[1] and Collins and Vecchio[2] have done the experimental studies upon the RC panel subjected to membrane stresses, aiming at the development of the constitutive model for FEM. After their studies, many experimental and analytical studies upon the constitutive law for the RC panel have been done until now. The authors have also developed the the constitutive model for the RC panel upon monotonic loading[3]. And it has been reported that the proposed model can describe the responses of the test specimens conducted by Aoyagi and Yamada[1] and Collins and Vecchio[2].

However, to evaluate the earthquake resistance of RC, it is necessary to predict the response of the RC structure under cyclic loading at the range from its cracking to its failure. There are several studies upon the constitutive law for the RC panel under cyclic loading done by Stevens et al.[4] and Inoue[5]. As their proposed models were derived from their own experiments, it is considered that their models should be verified by the other test results.

This study also aims at the development of the constitutive law for the RC panel subjected to reversed cyclic in-plane forces which is applicable to FEM. The study can be characterized in that it has established the constitutive law for the RC panel upon cyclic loading by adopting the several existing constitutive laws for cracked concrete[6][7][8][9] and for steel[3][7]. Moreover, through comparison of the calculated results with the existing test data, the proposed model for the RC panel upon reversed cyclic loading has been evaluated.

2. CONSTITUTIVE LAW FOR THE RC PANEL SUBJECTED TO REVERSED CYCLIC LOADING

2.1 Premise conditions and the range of the application

The RC panel which this study treats is a plane member where many cracks will be generated and reinforcement is distributively arranged. For these types of members, the behavior of RC including many cracks being more important than that of each crack, the smeared crack model has been adopted. Therefore, the stresses and the strains upon the RC panel mean the average stresses and the average strains respectively.

The stresses of the RC panel can be represented by superposition of the stresses of concrete and those of steel (see Fig.1). The constitutive laws for cracked concrete are decided to be given with respect to a local coordinate system which is composed of the x-axis orthogonally crossing the cracked face and the y-axis paralleling it. This is because the constitutive law for cracked concrete is represented with the three stress components such as the stress normal to the cracked face; σ_x, the stress paralleling it; σ_y and the shear stress; τ_{xy}. These stresses are given by the analytical models obtained from the uniaxial testing. Under reversed cyclic loading, the main cracks will be generated in the two directions. However, the concrete stresses are always given with respect to the x-y coordinates which is formulated by the first crack.

The stress of cracked concrete; $\{\sigma_c\}$ is expressed by Eq.(1).

$$\{\sigma_c\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{1}$$

On the other hand, steel bars are assumed to be arranged along two orthogonally crossing directions. The stresses of steel are given with respect to the x'-y' coordinates, the axes of which coincide with the longitudinal direction of the steel bars. It is also assumed that their resistance only acts in the longitudinal direction and that the shear resistance of the steel bars is so small as to be ignored when it is compared with that of concrete. From those assumptions, the stress of steel in RC; $\{\sigma_s\}$ is expressed by Eq.(2).

$$\{\sigma_s\} = \begin{Bmatrix} \sigma_{x'} \\ \sigma_{y'} \\ 0 \end{Bmatrix} = \begin{bmatrix} p_{x'} \cdot E_{sx'} & 0 & 0 \\ 0 & p_{y'} \cdot E_{sy'} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_{x'} \\ \varepsilon_{y'} \\ \gamma_{x'y'} \end{Bmatrix} \tag{2}$$

where the notation of the stresses and the strains are referred to Fig.1, $E_{sx'}$, $E_{sy'}$ is the average stiffness of steel in RC and $p_{x'}$, $p_{y'}$ is the reinforcement ratio. The subscript; x', y' indicates the x'-axis and the y'-axis respectively.

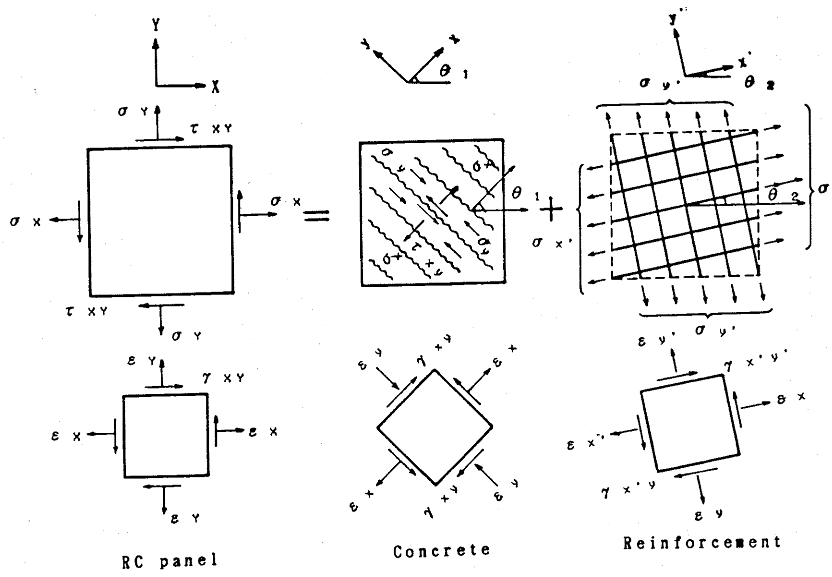


Fig.1 Stresses and strains in a RC panel

The stresses of concrete and steel in RC can be transformed in any direction by using the coordinate transformation matrix. Therefore, the stress of the RC panel; $\{\sigma_{RC}\}$ can be obtained by summation of the stresses of concrete and steel with respect to the global coordinates (the X-Y coordinates) and it is expressed by the following equation.

$$\{\sigma_{RC}\} = [T(-\theta_1)]\{\sigma_c\} + [T(-\theta_2)]\{\sigma_s\} \tag{3}$$

where $[T(\theta)]$ is the coordinate transformation matrix, that is the function of the angle; θ, θ_1 is the angle between the X-axis and x-axis and θ_2 is the angle between the X-axis and x'-axis as referred to Fig.1.

The strains, like the stresses, can also be transformed in any direction as the following equation.

$$\{\varepsilon_{RC}\} = [T(-\theta_1)]\{\varepsilon_c\} = [T(-\theta_2)]\{\varepsilon_s\} \tag{4}$$

where $\{\varepsilon_{RC}\}$, $\{\varepsilon_c\}$ and $\{\varepsilon_s\}$ is the strain vector with respect to the X-Y coordinates, the x-y coordinates and the x'-y' coordinates respectively.

The stress of concrete; $\{\sigma_c\}$ and the stress of steel;$\{\sigma_s\}$ expressed by the strains with respect to the local coordinates can also be rewritten by the strains with respect to the global coordinates with Eq.(4). Therefore, Eq.(3) expresses the constitutive equation for the RC panel.

2.2 Constitutive laws for tension stiffening

The tension stiffening under the cyclic loading is represented by the Okamura and Maekawa model[6] upon monotonic loading and the Shima model[7] upon reloading and unloading (see Fig.2). The Okamura and Maekawa model is numerically expressed by

$$\sigma_x = f_{tb}(\varepsilon_{tu}/\varepsilon_x)^c \tag{5}$$

where f_{tb} (kgf/cm^2) is the tensile strength of concrete under biaxial stress condition[3], ε_{tu} is twice times of the strain at cracking[3] and c is the parameter which expresses bond characteristics of steel.

On the other hand, the tension stiffening model upon unloading and reloading is expressed as the following equation, the Shima and Tamai's research work[7] being adopted.

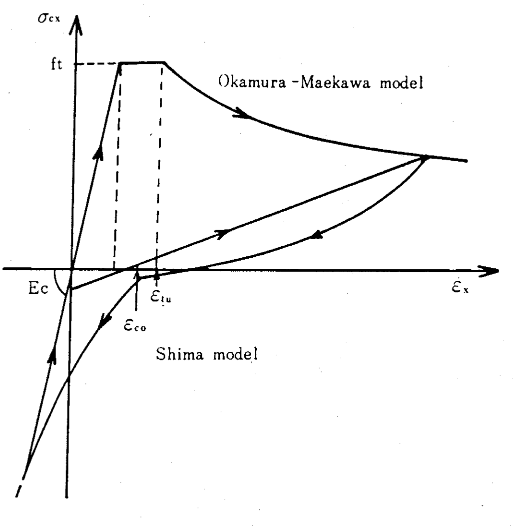


Fig.2 Tension stiffening model for cyclic loading

$$\sigma_x = \sigma_{cc} + \sigma_{cb} \tag{6}$$

where σ_{cc} is the stress generated by the cracked face contacting and σ_{cb} is the stress generated by the bond action which Shima and Tamai defined.

σ_{cb} is expressed by the following equation and it is illustrated in Fig.3(a).

For unloading;

$$\begin{aligned} \sigma_{cc} &= 0 && \text{for } \varepsilon_x \geqq \varepsilon_{co} \\ &= \frac{E_c}{3}(\varepsilon_x - \varepsilon_{co}) && \text{for } \varepsilon_{c1} \leqq \varepsilon_x < \varepsilon_{co} \\ &= E_c \varepsilon_x && \text{for } \varepsilon_x < \varepsilon_{c1} \end{aligned} \tag{7.1}$$

For reloading;

$$\begin{aligned} \sigma_{cc} &= E_c \varepsilon_x && \text{for } \varepsilon_x < 0 \\ &= 0 && \text{for } \varepsilon_x \geqq 0 \end{aligned} \tag{7.2}$$

where E_c is the initial elastic modulus of concrete, ε_{co} is the strain at the start of the crack contacting and ε_{c1} is the strain at the crack completely closing.

σ_{cb} is expressed by Eq.(8) and illustrated in Fig.3(b).

For unloading;

$$\sigma_{cb} = \frac{\sigma_{cbmax} - \sigma_{cbo}}{\varepsilon^2_{xmax}} \varepsilon^2_x + \sigma_{cbo} \tag{8.1}$$

For reloading;

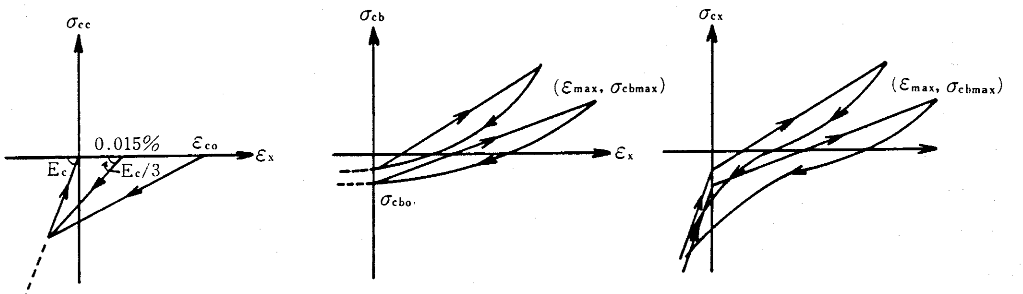
$$\sigma_{cb} = \frac{\sigma_{cbmax} - \sigma_{cbmin}}{\varepsilon_{xmax} - \varepsilon_{xmin}} \varepsilon_x + \sigma_{cbmin} \tag{8.2}$$

where ε_{xmax} is the strain at the start of unloading, ε_{xmin} is the strain at the start of reloading, σ_{cbmin} is the stress generated by the bond action when unloading starts and σ_{cbo} is given by Eq.(9).

$$\sigma_{cbo} = -0.0016E_c \varepsilon_{xmax} \tag{9}$$

Fig.3 illustrates the Shima model. The constitutive equation for tension stiffening upon monotonic loading follows Eq.(5) and the constitutive equation upon unloading follows Eq.(6). Therefore, σ_{cbmax} is given by Eq.(10) so that these tension stiffening models are continuous at the point where unloading starts.

$$\sigma_{cbmax} = f_{tb}(\varepsilon_{tu}/\varepsilon_{xmax})^c \tag{10}$$



(a) Concrte stress produced by contact of the crack

(b) Concrete stress produced by the bond action

(c) Tension stiffening for unloading and reloading.

Fig.3 Tension stiffening model for unloading and reloading

When concrete is again exposed to tension after unloading, the reloading curve for tension stiffening returns the point where unloading has started, on the assumption that the effect of cyclic loading on the tension stiffening is negligible.

Shima and Tamai reported that the value of ε_{co} was roughly 0.015% from their experiment[7]. For a in-plane stress field on which shear deformation will be generated at the cracked face, the residual plastic shear strain along the cracked face is supposed to expedite the crack closing. However, the evaluation of ε_{co} is considered difficult, by taking the effect of residual plastic shear strain into consideration. Moreover, as it is more convenient to use the uniquely expressed reloading curve at the process of the calculation upon reloading, ε_{co} is assumed to be expressed by Eq.(11). In Eq.(11), the effect of residual plastic shear strain is indirectly expressed by the maximum shear strain that has ever been experienced.

$$\varepsilon_{co} = 0.00015 + 0.1 \mid \gamma_{xymax} \mid \tag{11}$$

where γ_{xymax} is the the maximum shear strain that has ever been experienced.

Therefore, σ_{cc} proposed by Shima and Tamai is modified into such that it is represented by the straight line passing the two points; (ε_{co},0) and the intersection of the straight line with the slope of $E_c/3$ and that with the slope of E_c (see Fig.3(a)).

Further, σ_{cb} upon unloading is represented by the quadratic curve passing the point; (ε_{xmax}, σ_{cbmax}) and the vertex; (0, σ_{cbo}) and that upon reloading by the straight line passing both points; the points at the start of reloading and unloading .

2.3 Constitutive law for compressive concrete

The compressive stiffness of cracked concrete in the direction parallelling the cracked face become smaller than that of uncracked concrete[2][10]. Maekawa successfully developed the constitutive law for compressive concrete under any loading path by using the fracture parameter and the compressive plastic strain (hereafter referred as the Maekawa model[8]). The authors considered that only by reducing the fracture parameter with the tensile strain in the direction orthogonally crossing the cracked face, the compressive cracked concrete deformation can be represented by the Maekawa model.

Therefore, the constitutive equation for compressive cracked concrete under cyclic loading is expressed by

$$\sigma'_y = KE_c(\varepsilon'_y - \varepsilon'_{yP}) \tag{12}$$

where σ'_y is the concrete stress, ε'_y is the strain in the y'-direction, ε'_{yP} is the plastic strain expressed by the maximum strain that has ever been experienced[8], K is the fracture parameter of cracked concrete. K is expressed by the following equation;

$$K = K_O\,\omega \tag{13}$$

where K_O is the fracture parameter of uncracked concrete[8] and ω is the reduction factor and is expressed as

$$\omega = 1.0 \qquad \text{for } \varepsilon_x \leqq \varepsilon_1$$

$$\omega = 1.0 - 0.4\frac{(\varepsilon_x - \varepsilon_2)}{(\varepsilon_1 - \varepsilon_2)} \qquad \text{for } \varepsilon_1 < \varepsilon_x \leqq \varepsilon_2$$

$$\omega = 0.6 \qquad \text{for } \varepsilon_x > \varepsilon_2$$

where ε_1 is substituted by 0.12% and ε_2 by 0.44% [3].

The reduction factor upon unloading and reloading is assumed not to recover and it is represented by the maximum tensile strain; ε_{xmax}.

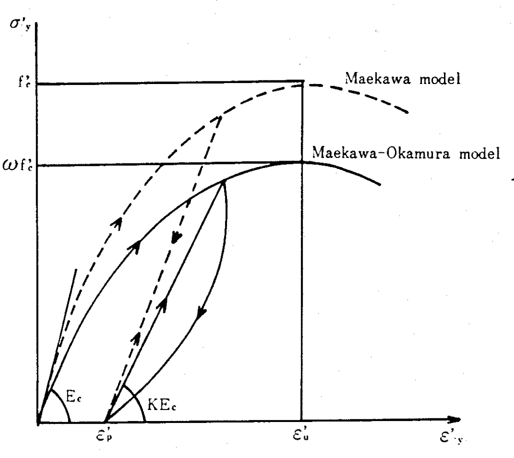


Fig.4 Constitutive model for compression

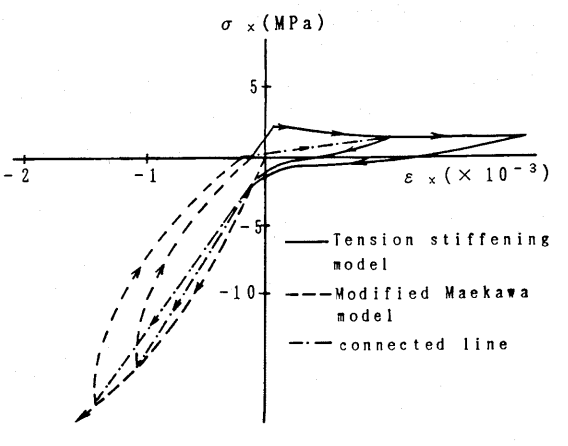


Fig.5 Hysteresis loop for concrete

Fig.4 illustrates the Maekawa model and the constitutive equation for cracked compressive concrete (hereafter referred as the modified Maekawa model), which is adopted for the RC panel model. As the Maekawa model assumes that the fracture parameter and the plastic strain upon unloading and reloading don't change, the behavior upon unloading and reloading becomes linear. From the observation of the experimental results[10][11], it can be seen that the curves has a tendency to get convex toward the bottom upon unloading. The convexity in the unloading curve may be attributed to the delayed elastic of the microscopic damage.

Lack of such convexity in the unloading curve results in also the lack of convexity upon unloading in the behavior of the RC panel. So, in order to give the necessary convexity to the unloading curve as an analytical model, the circular arc that has the infinite tangential stiffness at the start of unloading and passes the residual strain point at completion of unloading has been interpolated for the unloading curve. This is because the unloading curve can be approximately expressed by the circular arc from the known experimental data[10][11]. Such an analytical model with the circular arc can be considered to work well in treating unloading condition, while it is necessary to further study the behavior of concrete under unloading.

2.4 Constitutive law for concrete under tension and compression stress field

In the RC panel subjected to reversed cyclic in-plane stresses, concrete is exposed to tension and compression. In order to develop the constitutive model upon reversed cyclic loading, the above mentioned tension stiffening model and the modified Maekawa model have been adopted.

Fig.5 shows the stress-strain relationship for cracked concrete under reversed cyclic loading. When concrete is exposed to the compressive virgin load and then to the tensile virgin load, the behavior of concrete is described by the modified Maekawa model until concrete is completely unloaded. And after that, it follows the tension stiffening model from the point shifted from the origin by the corresponding compressive plastic strain.

The both the tension stiffening model and the modified Maekawa model should be continuous at their intersections. When concrete is exposed to compression after being exposed to tension, the behavior of concrete is represented by the envelop curve of the modified Maekawa model in the case of the compressive virgin loading. Otherwise, it follows the straight line passing the point at the start of unloading upon the modified Maekawa model.

When concrete is exposed to tension after being exposed to compression, the behavior of concrete follows the modified Maekawa model until concrete is completely unloaded and after that, it follows the straight line passing the points at the start of reloading and unloading.

The constitutive law for cracked concrete under cyclic loading is the continuous function expressed by the average strain of concrete as shown in Fig.5. Therefore, the stress of concrete is given regardless of the open and close cracking condition.

2.5 Constitutive law for the shear behavior along the cracked face

The Li and Maekawa[9] has been adopted to describe the shear behavior in the RC panel. This is not only because their model can accurately trace the shear behavior of the cracked face under the reversed cyclic loading but also because it is the simple one for the analysis. In their model, the shear stress along the cracked face and the compressive stress in the direction orthogonally crossing the cracked face are expressed by the integration equations relating to the direction of the contact face as shown in Eq.(14) and it can predict the shear transfer mechanism for any type of the loading path.

$$\tau_{xy} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 10250 f'_c{}^{1/3} {}_1(\omega_\theta - \omega_{\theta P}) \sin\theta \cos\theta \, d\theta \qquad (\mathrm{kgf/cm^2}) \tag{14.1}$$

$$\sigma_c = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 10250 f'_c{}^{1/3} {}_1(\omega_\theta - \omega_{\theta P}) \sin^2\theta \, d\theta \qquad (\mathrm{kgf/cm^2}) \tag{14.2}$$

where f'_c is the compressive strength ($\mathrm{kgf/cm^2}$), θ is the direction that the contact stress acts on, ω_θ is the plastic displacement in the direction orthogonally crossing the cracked face and $\omega_{\theta P}$ is the plastic displacement of ω_θ. ω_θ is expressed by Eq.(15).

$$\omega_\theta = -\delta \sin\theta + \omega \cos\theta \tag{15}$$

where δ is the shear displacement along the crack and ω is the crack width.

In the case of the monotonic loading, it is proved that the Li and Maekawa model is uniquely represented by $\gamma_{xy}/\varepsilon_x$ [9]. Consequently their model can be used for the smeared crack model, regardless of the length between the cracks.

On the other hand, in the case of the cyclic loading, it requires numerical integration of Eq.(14) and to express the crack width and the shear displacement by the average strains in the RC panel. Assumed that the tensile strain and the shear strain of concrete between cracks are negligibly small compared with the tensile strain and the shear strain due to cracks, the crack width; ω and the shear displacement; δ can be expressed with the average length between the cracks; ℓ as

$$\omega = \ell \varepsilon_x \tag{16.1}$$
$$\delta = \ell \gamma_{xy} \tag{16.2}$$

Therefore, when the Li and Maekawa model is adopted for the smeared crack model, it becomes necessary to settle the average length between the cracks. However, as mentioned in the next chapter, there recognized little influence by the average length between the cracks, even though the average length is varied from 5 cm to 100 cm. It could be said that the Li and Maekawa model which doesn't require the estimation of the average length between the cracks is suitable for the analysis of the RC panel.

Their model can also give the compressive normal stress to the cracked face. This stress reduces the tension stiffening effect. When the compressive normal stress is generated by the shear stress along the cracked face, the tensile stress of concrete is modified so that the compressive stress is subtracted from it.

2.6 Constitutive law for steel in the RC panel

A constitutive law for steel in RC is determined to be expressed by the three types of curves; the curve before yielding, the envelope curve after yielding and the unloading and reloading curve after yielding. The stress of steel; σ_s is expressed by the following equation.

The curve before yielding is expressed as

$$\sigma_s = E_s \varepsilon_s \tag{17.1}$$

The envelop curve after yielding is expressed as

$$\sigma_s = K_1(\varepsilon_s - K_2) \tag{17.2}$$

The unloading and reloading curve after yielding is expressed as

$$\sigma_s = E_s(\varepsilon_s - \varepsilon_{smax}) + \sigma_{smax} \quad \text{for the straight line part} \tag{17.3}$$

$$\sigma_s = \frac{-a(a+1)\sigma_{smax}\varepsilon_{smax}}{\varepsilon_s - \varepsilon_{smax}(a+1)} - a\sigma_{smax} \quad \text{for the curve part} \tag{17.4}$$

where ε_s is the strain of steel, E_s is the stiffness of steel, K_1 and K_2 are the coefficient that are introduced on the assumption that the stress distribution in RC is expressed by the sine function[3], σ_{smax}, ε_{smax} is the maximum stress and the maximum strain that has ever been experienced respectively and a is the parameter represented by the following the equation.

$$a = E_s / \{E_s - (\sigma_{smax} / \varepsilon_b)\} \tag{18}$$

where ε_b is the difference between the plastic strain accumulated by the cyclic loading and ε_{smax} as referred to Fig.6.

As steel can be considered elastic before yielding, the stiffness of the bare bar is used for that of steel in the RC panel. The yielding of steel starts, when the stress of steel at the cracked face reaches the yield point. After yielding, the proposed model has been used for the envelope part[3].

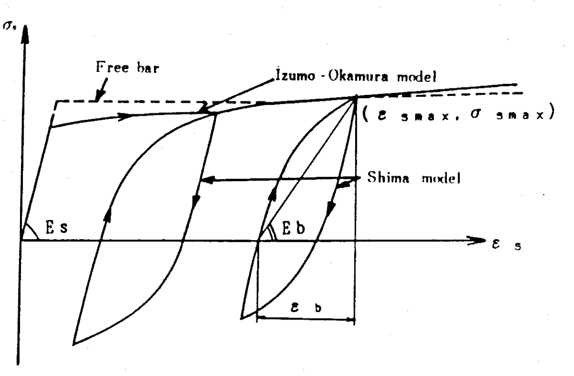


Fig.6 Steel model for cyclic loading

For the unloading and reloading part, the Shima model which proposed to use the Kato model[12] for the constitutive law for steel in RC has been adopted.

3. Analytical results of the RC panel

3.1 Analytical method

The established constitutive law for the RC panel results in the form of the three-element non-linear simultaneous equation for the stresses and the strains. The analysis has been done by solving it to get the values of the strains with the known stresses acting on the RC panel. Consequently, the relationship between the stress and the strain of the RC panel has been obtained through the analysis. The non-linear simultaneous equation has been solved by the Newton-Raphson method.

The tangential stiffness in the hysteresis curve is apt to be discontinuous and it possibly makes the convergence worse. The value of stiffness used in the calculation is such constant stiffness as always greater than the tangential stiffness in order to get the stable solution. Negative stiffness has been substituted for zero value to prevent solutions from divergence that can be occurred when the corresponding stiffness matrix becomes peculiar due to the negative stiffness value in it.

The secant stiffness at the completion of the former load step has been used at the present load step for the envelop part and the reloading part. The maximum tangential stiffness upon the reloading curve has been used for the reloading part. Moreover, to make convergence faster, the stiffness has been substituted for the latest stiffness only once per five cycles of the repeated routine.

3.2 Analytical conditions on the RC panel

The verification of the proposed model can be done by the test data by Stevens et al.[4] and by Ohmori et al.[13]. These specimens were subjected to the reversed cyclic in-plane stresses.

Table 1 The outline of test specimens

Specimen Size (Researcher)	Name	Loading Rate $\sigma_x:\sigma_x:\tau_{xy}$	Reinforcement Ratio Px' (%)	Py' (%)	Angle (degree) θ_2	Yield Point fy (MPa)	Elastic Modulus Ec (MPa)	Compressive Strength f'c (MPa)	Maximum Strength τ_{xymax} (MPa)
(Ohmori et al.) 250×250×14(cm)	KP6	0 : 0 : 1	2.00	2.00	0	469	2.08×10-5	29.0	8.03
(Stevens et al.)	SE8	0 : 0 : 1	2.94	0.98	0	x':492 y':479	1.85×10-5	37.0	5.76
152×152×29(cm)	SE9	0 : 0 : 1	2.94	2.94	0	422	1.85×10-5	44.2	9.55
	SE10	-1 : -1: 3	2.94	0.98	0	x':422 y':479	1.85×10-5	34.0	8.25

Table 1 shows the testing parameters for the specimens. The loading were the proportional loading. In the case of the specimen SE10, the shear stress and the compressive stress which was 1/3 of the shear stress acted on the specimen. The other specimens were subjected to pure shear. In these analyses, the parameter c was set at 0.4 because the deformed bars were arranged in these specimens.

The tensile strength; f_t (kgf/cm^2) has been given with the compressive strength as the following equation;

$$f_t=0.58kf_c'^{2/3} \tag{19}$$

where k is the modification factor of the tensile strength.

The tensile strengths obtained from the test specimens were approximately at the range from 55% to 70% of the tensile strength obtained from the cylinder strength. It could be considered that this was due to the shrinkage and the size effect[14]. From this observation, the tensile strength obtained from the cylinder strength could not be used as the tensile strength in the analysis.

The tensile strength of concrete affects the estimation of the cracking load and the analysis of the RC panel after cracking. However, it is difficult to obtain the tensile strength analytically, which can be used for the analysis of the RC panel. Therefore, the value of k has been given in a way that the tensile strength may coincide with the cracking load of the test result through trials and errors. Table 2 indicates the value of k used in the analysis.

Table 2 Analytical conditons and results

Specimen	k	τ_{peak} 1) (MPa)	τ_{peak}/τ_{XYmax} 2)	τ_{yexp} 3) (MPa)	τ_{ycal} 4) (MPa)	τ_{yexp}/τ_{ycal}
KP6	0.70	6.20	0.77	–	–	–
SE8	0.60	5.76	1.00	5.00	5.49	0.90
SE9	0.55	9.55	1.00	–	–	–
SE10	0.55	8.25	1.00	7.30	7.94	0.92

1)Maximum shear strength in the analytical loops
2)Observed maximum shear strength in the test
3)Observed shear strength at yielding
4)Calculated shear strength at yielding

For easy comparison between the calculated results and the observed ones, three loops from the hysteresis curve are used. Three loops including the observed last hysteresis curve are selected from the specimens done by Ohmori et al.. In this specimens, the ratio of the shear stress at the peak of the last hysteresis curve to the observed maximum shear stress is approximately 0.8.

In the specimens done by Stevens et al., several loops have been obtained in the vicinity of the maximum shear stress. The deterioration caused by the cyclic loading fairly affected the hysteresis curve in the vicinity of the maximum shear stress. As the proposed model doesn't describe the deterioration caused by the cyclic loading, three loops including the first loop selected from the loops in the the vicinity of the maximum strength have been used in this analysis.

3.3 Analytical results and remarks

Before analyzing the test specimens, the effects of the reloading curve for cracked concrete on the hysteresis curve for the RC panel have been examined. As the effects of the envelop curve for cracked concrete on that for the RC panel have already been examined[3][6], these kinds of the sensibility analysis have been omitted in this study.

In order to examine the effects of tension stiffening model, the analysis has been done by varying the ε_{co} and σ_{cbo} with the test data of the specimen KP6. Fig.7 shows the analytical results when the value of ε_{co} is changed by changing the parameter; α in Eq.(20).

$$\varepsilon_{co} = \alpha \, (0.00015 + 0.1 \, | \, \gamma_{xymax} \, | \,) \tag{20}$$

The value of α has been set at 1.0 and 10.0. Fig.7 shows the observed loop and the analytical one. The effect of the varying α is indicated by the oblique lines in Fig.7. As the value of α increases, the contact starts earlier and the unloading curve tends to have more convexity toward the bottom in the course of unloading. However, from the observation of the test result, such convexity which is affected by crack contacting could not be clearly recognized.

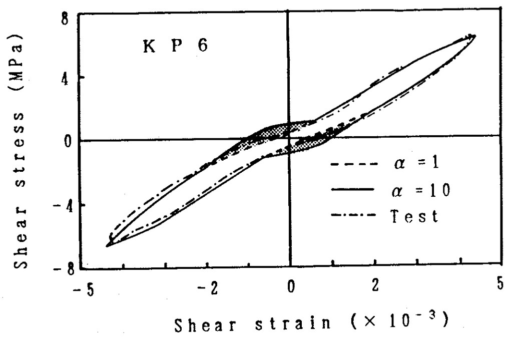


Fig.7 The sensitivity of the contact stress to the analysis

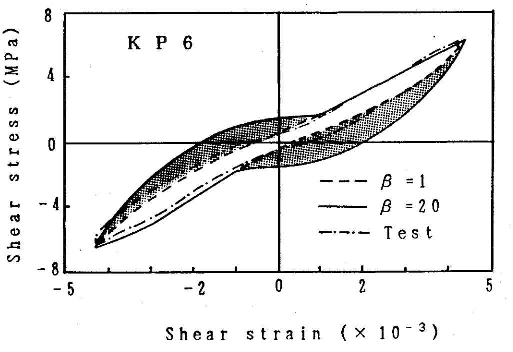


Fig.8 The sensitivity of the stress by the bond action to the analysis

The analysis also has been done when the value of σ_{cbo} is changed by changing the parameter; β in Eq.(21). In this case, α has been set at 1.0.

$$\sigma_{cbo} = -0.0016 \, \beta \, E_c \, \varepsilon_{xmax} \tag{21}$$

Fig.8 shows the observed loop and the analytical one, when β =1.0 and β =20.0. From the analysis, the unloading curve tends to have more convexity through its entire curve, as the parameter; β increases. From comparison between the observed loop and the calculated one, β =1 seems to give the good result.

In order to examine the effect of the constitutive law for compressive concrete on the RC panel, the analysis has been done by varying the value of ε'_{yp}. Gunatilaka[11] studied on the effect of the loading speed on the plastic strain. From her study, the plastic strain at the loading speed of 0.14 MPa/sec and 0.014 MPa/sec is expressed by Eq.(22) and Eq.(23) respectively.

$$\varepsilon'_{yp} = \varepsilon'_{ymax} - 0.00320\ (1 - \exp(-312\,\varepsilon'_{ymax}) \tag{22}$$

$$\varepsilon'_{yp} = \varepsilon'_{ymax} - 0.00198\ (1 - \exp(-505\,\varepsilon'_{ymax}) \tag{23}$$

Fig.9 shows the plastic strain expressed by Eq.(22) and Eq.(23). It could be recognized that the plastic strain is affected by the loading speed. Fig.10 indicates the calculated hysteresis loops when the plastic strain expressed by the Maekawa model, Eq.(21) and Eq.(22). is used. In this analysis, α and β are kept 1.0. Moreover, the third loop of the selected loops is shown in Fig.10. The softening of concrete progresses as the plastic strain increases. Therefore, there is a tendency that the stiffness in the vicinity of the peak upon the hysteresis loops becomes lower and it affects the stiffness after that. The plastic strain expressed by the Maekawa model, which was derived from the test at the loading speed of 0.1 MPa/sec gives almost the same analytical result when the plastic strain loaded at 0.14 MPa/sec. However, when the loading speed is extremely slow or fast, attention should be paid to the usage of the constitutive equation for concrete.

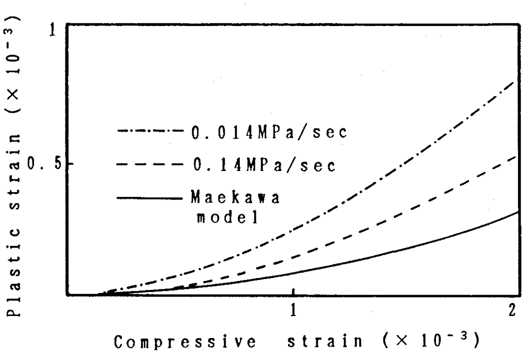


Fig.9 The effect of loading rate on a compressive plastic strain

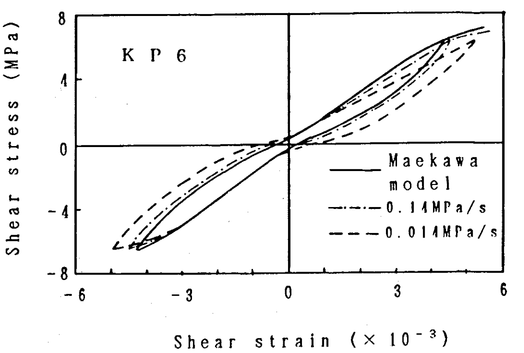


Fig.10 The sensitivity of a plastic strain to the analysis

From these sensitive analyses, in this study, α and β are determined to be 1.0. The difference of the processing speed of the plastic strain affects not only the hysteresis loops but also the maximum strength of the RC panel. From comparison with the analytical result and the tested one in the specimen KP6, it is concluded that the plastic strain given by the Maekawa model is appropriate to describe the behavior of the RC panel.

The relationships between shear stress and shear strain obtained from the test and the analysis in the specimen KP6 are shown in Fig.11 for each hysteresis loop. The reinforcement in the specimen KP6 has not been recognized to be yielded even in the last loop. It is due to the high reinforcement ratio. In the case of the specimen KP6, it was subjected to pure shear and had the reinforcement equally arranged in the both directions. Therefore the tension stiffening model and the modified Maekawa model mostly affect the analysis. Each hysteresis loop obtained from the analysis is good agreement with the test result.

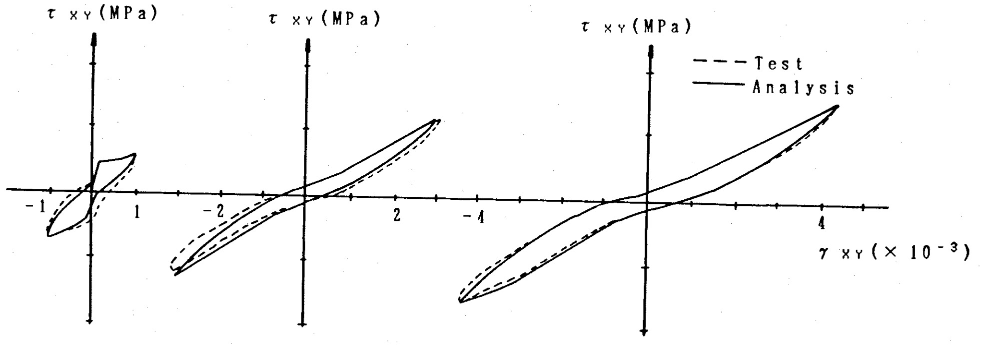


Fig. 11 The test results and the analysis (Ohmori et al.)

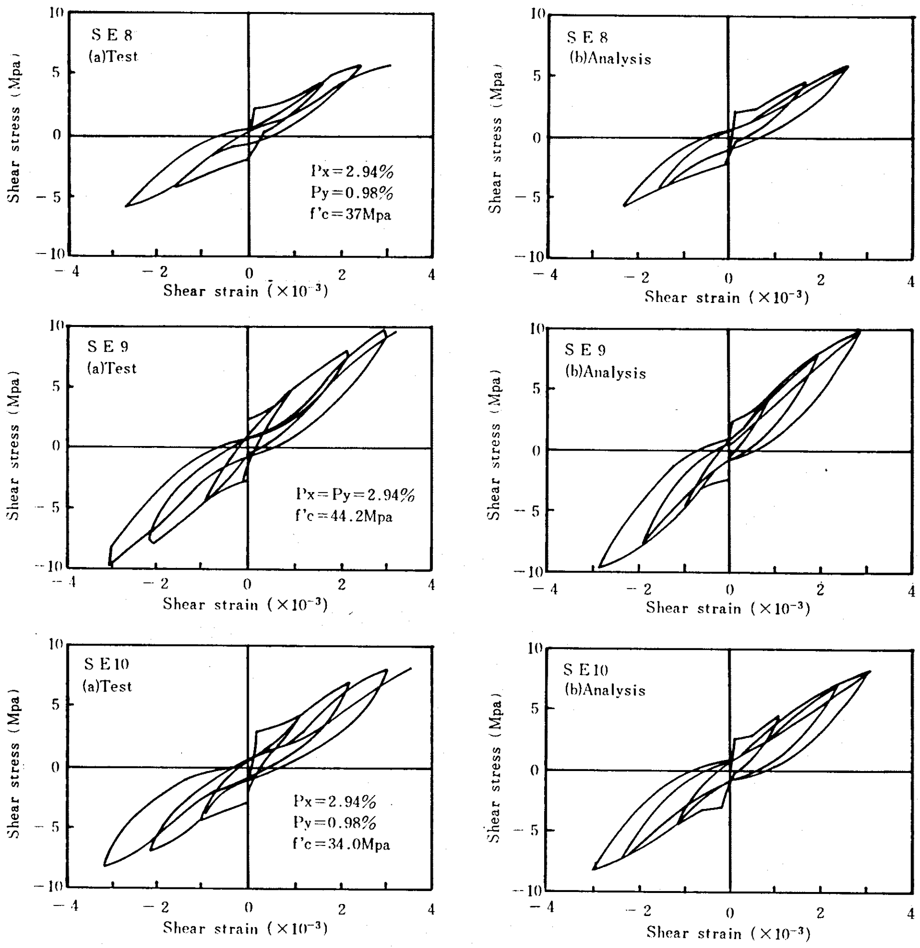


Fig.12 The test results and the analyses (Stevens el al.)

Fig.12 shows the analytical result of the test specimen conducted by Stevens et al.. In the case of the specimen SE9, it was subjected to the reversed pure shear and had the reinforcement equally arranged in the both directions. Therefore, in this case, the tension stiffening model and the modified Maekawa model can have considerable influence on the analysis. This proposed model being incapable of expressing the influence of damage due to cyclic loading, the hysteresis curves obtained by the analysis always return to the starting point of unloading. Nevertheless, it can be said that the analysis results may well describe the loop of the RC panel.

On the other hand, in the case of the specimen SE8 and SE10, the shear stress acted on the cracked face due to the anisotropic arrangement of their reinforcement. Before analyzing these specimens, the authors have performed calculation, giving two extremely different values 5cm and 100cm for ℓ as the average length between the cracks. However, the results have been completely the same in both cases and we have convinced that the Li and Maekawa model works well also in analysing RC panel regardless of the length between the cracks.

In the case of the specimen SE8 and SE10, the lower reinforcement in the y'-direction was yielded. Table 2 indicates the shear stress at the reinforcement yielding obtained from the test and the analysis. The analytical results seem to have good correspondence with the tested ones, even in the presence of shear stress acting on the cracked face. As far as the verification has been done, the proposed model has been proved effective for the analysis of the RC panel subjected to reversed cyclic loading.

4. Conclusion

In this study, the authors have developed the existing constitutive laws for cracked concrete and for steel into the constitutive law for the RC panel subjected to reversed cyclic loading and verified it through comparison with the test data.

As far as verification has been done for the existing test data on the RC panel, the proposed model has proved effective. However, the fact that the number of existing test data on the RC panel now is limited and the number of verification is also limited being taken into consideration, it can be said that the available test data should be accumulated to verify the proposed model in future. Further, the proposed model integrated into FEM should be verified by comparing it with the test data for the RC membrane structures such as shear walls.

Through this study, it seems that the constitutive model for the materials considerably affect the characteristics of the restoring force. However, the authors consider that the effects of the loading speed and the cyclic loading on the materials should be further studied.

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