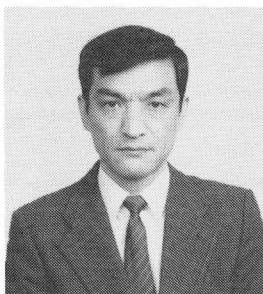
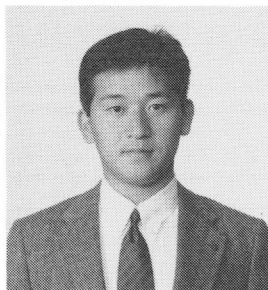


ON THE STRESS RELAXATION OF CONCRETE IN EARLY AGES
AND ITS ESTIMATION METHOD

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SYNOPSIS

Stress relaxation of concrete, especially in early ages, is one of the most important properties for thermal stress analysis in concrete structure. But, available information on the relaxation is very little because of the difficulties in the experiment in maintaining a specimen at a constant deformation. This paper describes the results of compressive and tensile relaxation tests of concrete in early ages. Tests were performed with electro-hydraulic closed-loop testing machine system for compression and with loading frame having high rigidity for tension. Based on the test results, the influences of the age of loading and stress level on the compressive and tensile relaxations are fully discussed and estimating equations for the relaxations at various ages are proposed.

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1. Introduction

Thermal stresses in concrete, caused by the hydration of cement, depend largely on a relaxation or creep property of concrete. In order to make thermal stress analysis more accurate the rational constitutive equation is needed for concrete, where visco-elastic properties are taken into account. Creep analysis has been adopted in the thermal stress analysis, in general, because a plenty of creep data have been accumulated until now. Representative works were made by McHenry(1), Raphael(2), Carlson(3), Arutyunyan(4) and Kimishima(5). In these studies, strains under varying stress field are calculated by using creep functions based on the principle of superposition. Basically, thermal stress is generated when thermal strain is restrained and it decreases in the course of time. For this reason, in the thermal stress analysis, relaxation function is considered to be more useful than creep function because it expresses the stress relaxation directly, and, consequently, the physical meaning of the constitutive equation becomes clearer. In addition, the relaxation function has such an advantage that it can be used easily for the approximate analysis of thermal stress relaxation. Though the relaxation is one of the important characteristics of concrete in thermal stress analysis, the studies on the relaxation property of concrete are few, mainly because of experimental difficulties. This paper deals with the results of the experiments, how about ages at loading and initial stress levels affect the relaxation. Empirical formula for predicting the amount of compressive and tensile relaxations are also presented.

2. Testing machine system

2.1 Compressive relaxation test

Compressive relaxation tests were performed through hydraulic testing machine with electric closed-loop feedback control system. Control accuracy of the testing machine is affected by various factors such as accuracy of displacement meter, stability of amplifier, oil temperature, control mechanism etc. The control accuracy of the testing machine in total was within 1% of the strain induced in the specimen.

2.2 Tensile relaxation test

Tensile relaxation tests were made with a high rigidity loading frame. The induction of an initial strain to a specimen was done by holding bolts with nuts, which are attached to the both ends of the specimen. Elastic deformation of the frame influences the result of the relaxation test. In order to make the loading frame, numerical analysis was carried out to examine the effect of rigidity ratio k_1/k_2 on the test result, where k_1 is a rigidity of the frame and k_2 is a rigidity of the specimen. An analytical model for finite element analysis is shown in Fig. 1. The analysis was made for 6 cases of rigidity ratio, i.e. 0.5, 1.0, 5.0, 10.0, 20.0, and infinity. Results of the analysis is shown in Fig. 2. Rigidity ratio k_1/k_2 affects the apparent relaxation of the specimen largely. Relaxation of the specimen decreases when k_1/k_2 become small. On the other hand, the speed of relaxation is almost the same for all k_1/k_2 values. When the ratio K_1/K_2 takes the values of 0.5, 1.0, 5.0, 10.0 and 20.0, the ratios of final relaxation with the value when k_1/k_2 is infinity become 77%, 63%, 26%, 16% and 9%, respectively. From the results of the numerical simulation, it may be concluded that the rigidity ratio K_1/K_2 of higher than 10 is needed for relaxation tests in obtaining the reliable test results with a sufficient accuracy. In this study, loading frame was made with 250×250 H-shape steel and 250×90 channel-shape steel so that the deformation of the

frame might become 1/10 or less in comparison to that of the specimen. Specimens were supported with rubber belts and the values of strains induced in the specimen were monitored by strain gages attached to the four sides of the specimen. The relaxation of the specimen was measured by strain gages attached to the holding bolt.

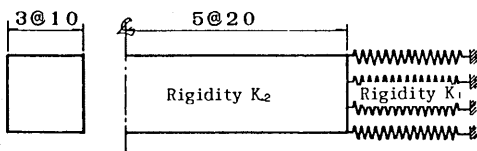


Fig.1 Model for analysis

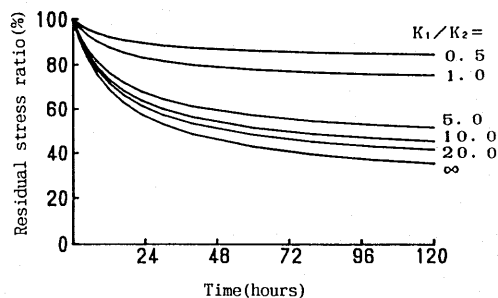


Fig.2 Result of analysis

3. Experiments

3.1 Procedure

In order to evaluate the relaxation property of concrete, ages of loading and induced stresses were changed. The ages of loading were chosen to be 1, 3, 7, 14, and 21 days. The initial stress was determined by stress-strength ratio with 3 levels for compressive and tensile test, i.e. 30%, 50% and 80% for compressive test, and below 30%, 30% to 60%, and 60% to 80% for tensile test. Factors considered in the experimental study are listed in Table 1. The sizes of a specimen for compressive test was 10×10×40 cm, and for tensile test, 10×10×86 cm. After casting, the specimens had been stored in a room with temperature of 20°C and more than 95% RH just before the loading. In order to avoid the effect of drying shrinkage, specimens were covered with paraffin wax at the time of loading so as to keep moisture in the specimens constant during the test. The test was done under the condition of 20°C and of more than 95% RH.

3.2 Materials and mix proportion of concrete

The materials used for the test were ordinary portland cement, river sand (Specific gravity= 2.58, F.M.= 2.53) and gravel (Specific gravity= 2.60, F.M.= 6.92, Max. size= 25mm). The mix proportion of the concrete is shown in Table 2. Table 3 shows compressive and tensile strength, and Young's modulus of concrete at the ages of 1, 3, 7, and 28 days.

4. Relaxation function

The same type equations were adopted for compressive and tensile relaxation function. That is

$$\frac{\sigma_t}{\sigma_i} = \frac{A + C t}{A + t} \quad (1)$$

where, σ_t = stress at time t after loading
 σ_i = induced initial stress
A, C = experimental constant

A, and C have the following meaning. Suppose $t=\infty$, we find $\sigma_t/\sigma_i=C$ from Eq. 1. That is, C corresponds to a final residual stress ratio. Therefore, $(1-C)$ corresponds to a final relaxation ratio. In addition, assume $\sigma_t=(\sigma_i+\sigma_u)/2$ at the time of $t=t_a$, where σ_u =final residual stress, we find $t_a=A$. That is, A shows the time when the relaxation ratio become a half of the final one, and it is named as 1/2 relaxation time.

Table 1 Experiment factors

Specimen	Stress	Age of loading (days)	Stress level (%)	Specimen	Kind of stress	Age of loading (days)	Stress level (%)
C1D30	Compression	1	30	T1D30	Tension	1	0 ~ 30
C1D50	Compression	1	50	T1D60	Tension	1	30 ~ 60
C1D80	Compression	1	80	T3D30	Tension	3	0 ~ 30
C3D30	Compression	3	30	T3D60	Tension	3	30 ~ 60
C3D50	Compression	3	50	T7D60	Tension	7	30 ~ 60
C3D80	Compression	3	80	T7D100	Tension	7	60 ~ 80
C7D30	Compression	7	30	T14D60	Tension	14	30 ~ 60
C7D50	Compression	7	50	T14D100	Tension	14	60 ~ 80
C7D80	Compression	7	80	T21D30	Tension	21	0 ~ 30
C14D50	Compression	14	50	T21D60	Tension	21	30 ~ 60
C21D50	Compression	21	50	T21D100	Tension	21	60 ~ 80

Table 2 Mix propotion of concrete

Slump (cm)	Air (%)	Fine aggregate ratio (%)	Water cement ratio (%)	Unit weight(kg/ m ³)			
				C	W	S	G
8	1.5	44	50	346	173	793	996

Table 3 Properties of concrete

Age (days)	Compressive strength(kgf/ cm ²)	Tensile strength (kgf/ cm ²)	Young's modulus (×10 ⁵ kgf/ cm ²)
1	44	4.3	1.0
3	177	15.7	2.0
7	280	23.3	2.4
28	388	25.0	2.9

5. Test results and discussions

5.1 Compressive relaxation

Fig. 3 through 7 show the results of compressive relaxation tests of 1, 3, 7, 14, and 21 days' loading. The relaxation function decided by the least square method is shown in each Figure. Relaxation occurred rapidly in the beginning and within 1 hour after loading it attained about 25 to 40% of the final one. The amount of the relaxation almost reaches its final value about 100 hours after loading. That is, relaxation ceased rapidly as compared with creep. For each loading age, no distinct difference among relaxation curves is observed at each stress level of 30% to 80%. Therefore, relaxation seems to be proportional to the initial stress as far as the stress level is under 80%. As shown in Table 4, mean values of final relaxation at the loading ages of 1, 3, 7, 14, and 21 days were 95, 63, 45, 35 and 40%, respectively. 1/2 relaxation time falls within 7 to 30 hours. Final relaxation and 1/2 relaxation time tend to become small with the increase of the age of loading. Relaxation functions shown in each Figure are obtained from measured values by applying the least square method. Although a little difference exists between the estimated relaxation function and the corresponding measured value within 6 hours after the loading, both curves are generally in good agreement. Fig. 8 and 9 show the values of A and C obtained in this study. C has an increasing tendency with the increase of the loading ages. On the other hand, the value of A decreases with the age. In this study, both A and C were expressed by two straight lines which meet at the age of 7 days. Eq. 2 and 3 are the estimating formulas for A and C.

$$\begin{aligned}
 A &= -8.25 \log t + 49.74 & (t < 168\text{h}) & (2\cdot a) \\
 &= 7.43 & (t \geq 168\text{h}) & (2\cdot b) \\
 C &= 0.25 \log t - 0.75 & (t < 168\text{h}) & (3\cdot a) \\
 &= 0.07 \log t + 0.18 & (t \geq 168\text{h}) & (3\cdot b)
 \end{aligned}$$

Calculated values from Eq. 2 and 3 are shown in Fig. 8 and 9, respectively, by the solid lines. From the analysis of test results shown in this chapter, compressive relaxation in each loading age can be evaluated from Eq. 1, 2 and 3.

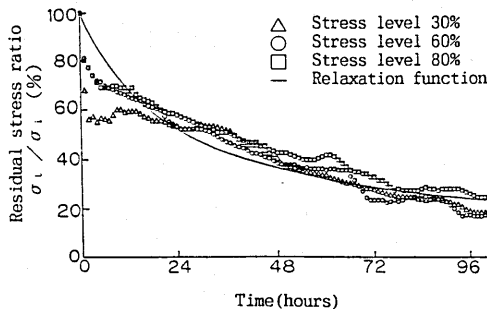


Fig.3 Compressive relaxation curve
(loading age 1day)

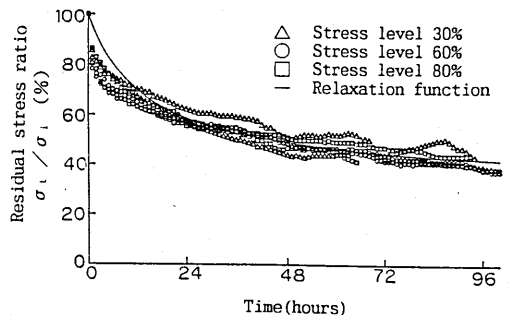


Fig.4 Compressive relaxation curve
(loading age 3days)

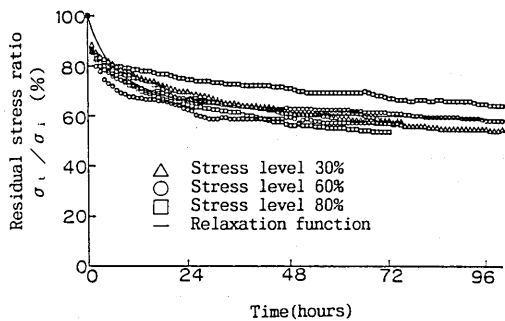


Fig.5 Compressive relaxation curve
(loading age 7days)

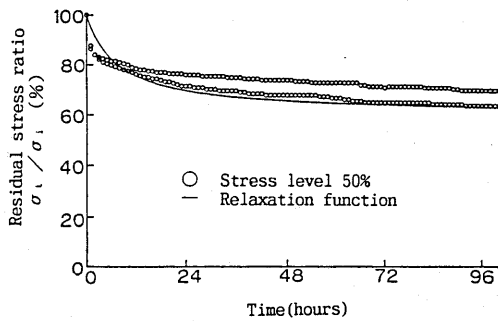


Fig.6 Compressive relaxation curve
(loading age 14days)

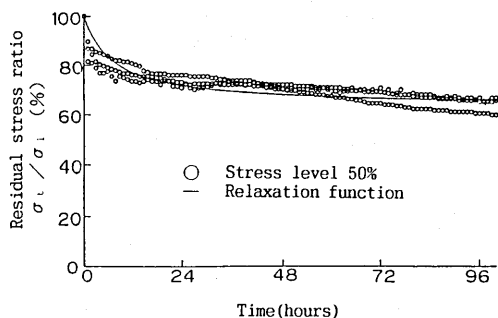


Fig.7 Compressive relaxation curve
(loading age 21days)

Table 4
Compressive final relaxation
and 1/2 relaxation time

Age of loading (days)	1/2 Relaxation time (hours)	Final relaxation (%)
1	25 ~ 30	95 ~ 100
3	10 ~ 14	60 ~ 65
7	5 ~ 15	40 ~ 50
14	7 ~ 8	30 ~ 40
21	7 ~ 15	35 ~ 45

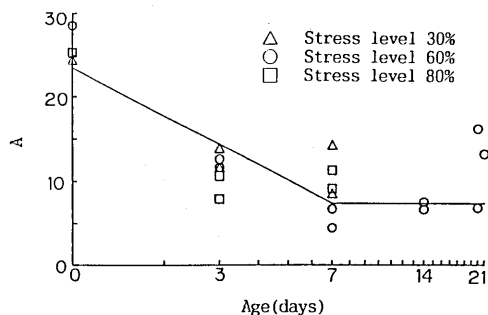


Fig.8 Value of the constant A

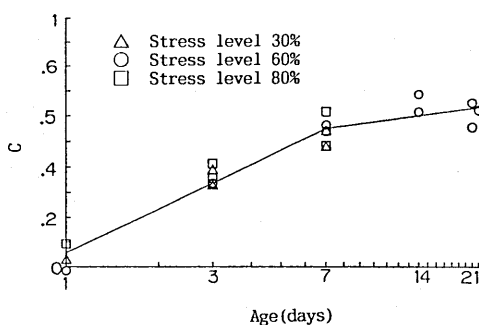


Fig.9 Value of the constant C

5.2 Tensile relaxation

Fig. 10 through 14 show the results of tensile relaxation tests of the loading ages of 1, 3, 7, 14, and 21 days. Tensile relaxation finishes earlier than compressive relaxation. The tensile relaxation reaches its final value within 2 or 3 hours after the loading for every ages. Final relaxation is about 25% at the age of 1 day, and about 15% at other age. The relaxation curves are almost unchanged in spite of the change of stress level. Therefore, tensile relaxation, as well as the compressive one, is proportional to the initial stress. From Fig. 10 through 14 and Table 5, the tensile relaxation is concluded to be constant after 3 days' loading. Final relaxation of the age of 1 day is about 10% greater than that of the other days. Relaxation function obtained by the least square method has a good agreement with the measured value. Fig. 15 and 16 show the values of A and C. The changes of factors A and C according to the loading ages are smaller than those of compression. A may be considered as a fixed value. On the other hand, the amount of C approximately shows a constant value after 3 days' loading. Relations between A, C and the loading age are expressed by Eq. 4 and 5, and they are shown in Fig. 15 and 16.

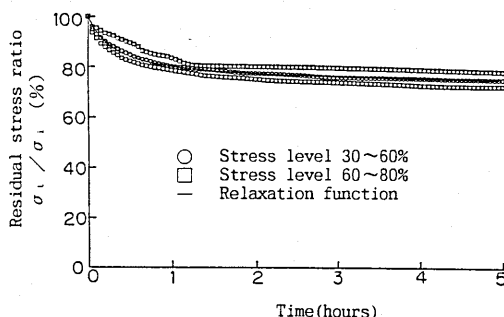


Fig. 10 Tensile relaxation curve
(loading age 1day)

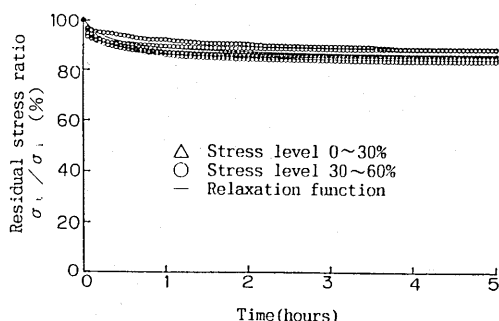


Fig. 11 Tensile relaxation curve
(loading age 3days)

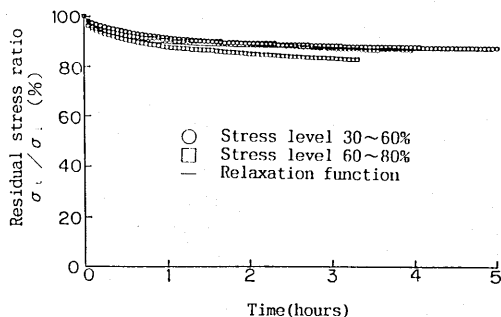


Fig. 12 Tensile relaxation curve
(loading age 7days)

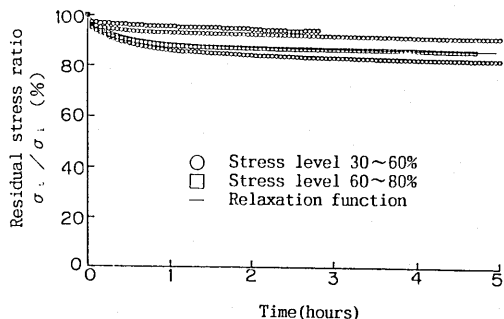


Fig. 13 Tensile relaxation curve
(loading age 14days)

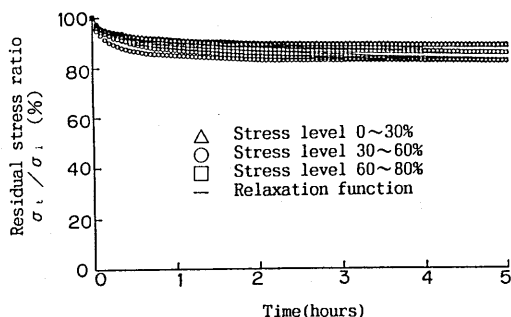


Fig.14 Tensile relaxation curve (loading age 21days)

Table 5
Tensile final relaxation
and 1/2 relaxation time

Age of loading (days)	1/2 Relaxation time (hours)	Final relaxation (%)
1	0.37	26
3	0.22	14
7	0.48	15
14	0.30	13
21	0.32	13

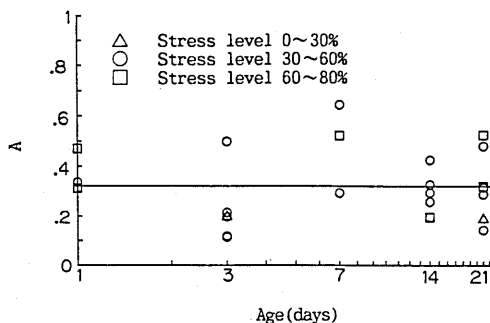


Fig.15 Value of the constant A

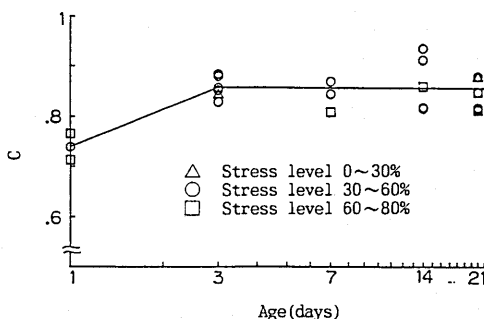


Fig.16 Value of the constant C

$$\begin{aligned}
 A &= 0.32 \\
 C &= 0.10 \log t + 0.39 \\
 &= 0.85
 \end{aligned}$$

$$\begin{aligned}
 &(4) \\
 &(t < 72h) \quad (5 \cdot a) \\
 &(t \geq 72h) \quad (5 \cdot b)
 \end{aligned}$$

By the analysis of test results shown in this chapter, tensile relaxation in each loading age can be evaluated from Eq. 1, 4, and 5.

5.3 Comparison of compressive relaxation with tensile relaxation

Compressive and tensile relaxation curves at the loading ages of 1, 7, and 21 days are shown in Fig. 17 through 19 and in table 6. The initial stress level of each relaxation test was about 50%. The amount of final tensile relaxation for each age test shows the values of 25% to 40% of the amount of the final compressive relaxation. On the other hand, 1/2 relaxation time becomes 1/100 to 4/100 times to that for compression. In other words, tensile relaxation is much smaller and has more shortly ending characteristics than compressive relaxation. Therefore, it is considered that there are some differences in the mechanism of relaxation between compression and tension.

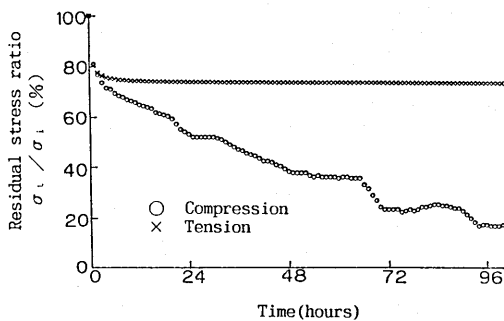


Fig.17 Compressive relaxation and tensile relaxation (loading age 1day)

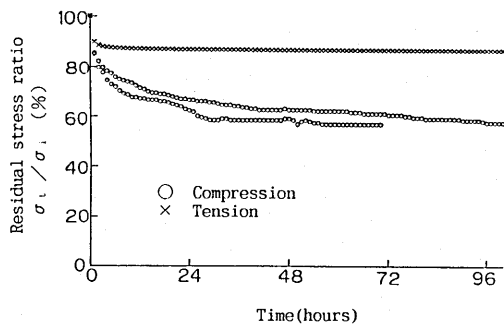


Fig.18 Compressive relaxation and tensile relaxation (loading age 7days)

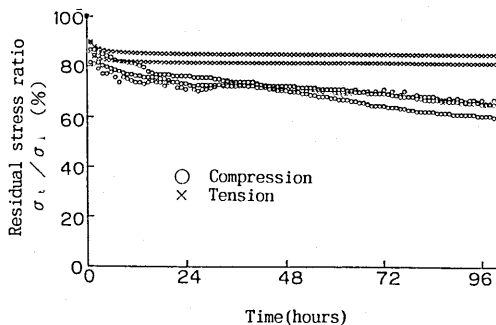


Fig.19 Compressive relaxation and tensile relaxation (loading age 21days)

Table 6 Final relaxation and 1/2 relaxation time of compressive and tensile relaxation

Age of loading(days)	1/2 relaxation time(hours)		Final relaxation(%)	
	Compression	Tension	Compression	Tension
1	7 ~ 28	0.3	95	26
7			45	14
21			40	16

6. Conclusions

The compressive and tensile relaxation tests were performed to study the effects of initial stress level and loading age on relaxation characteristic and the evaluating equations for relaxation were presented. The conclusions obtained from the study are summarized in the followings:

- (1) Not only in compression but also in tension, relaxation is proportional to the initial stress level as far as the stress level is below 80% of its strength.
- (2) Mean values of the final relaxation in compression for loading ages of 1, 3, 7, 14, and 21 days were 95%, 63%, 45%, 35%, and 40%, respectively. 1/2 relaxation time falls within 7 to 30 hours.
- (3) The final tensile relaxation for the loading age of 1 day was about 25% and about 15% for other loading ages. 1/2 relaxation time was about 0.3 hours in spite of the change of loading ages.
- (4) The final relaxation and 1/2 relaxation time became smaller with the increase of loading ages.
- (5) The final tensile relaxation became 25% to 40% of the final compressive relaxation of the corresponding loading ages.
- (6) Compressive and tensile relaxation can be estimated from Eq. 1, 2, 3 and 1, 4, 5, respectively.

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