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RELATION BETWEEN MIX PROPORTION AND DRYING SHRINKAGE OF HARDENED CEMENT PASTE, MORTAR AND CONCRETE

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Tadashi FUJIWARA

SYNOPSIS

It has been suggested that drying shrinkage of hardened cement paste, mortar and concrete is strongly related to mix proportion. The purpose of this experiment is to examine past theories about the relation between mix proportion and drying shrinkage, using many specimens made with various mix proportions. The relations of factors of mix proportion, such as water content in the original mix, to drying shrinkage already pointed out by many investigations were also observed in this experiment over a comparatively wide range of mix proportion, except for lean mixture of extremely stiff consistency. These relations can be systematically explained by the theoretical law of drying shrinkage derived from multiphase approach. It is expected that the law can properly predict the effect of mix proportion on drying shrinkage of concrete.

T. Fujiwara is an associate professor of civil engineering at Iwate University, Iwate, Japan. He received his Doctor of Engineering Degree in 1981 from Tohoku University. His research interests include mechanism of drying shrinkage of concrete, properties of concrete made with low quality aggregate and durability of concrete to freezing and thawing actions. He is a member of JSCE and JCI.

1. INTRODUCTION

It has become very important in design of concrete structure to estimate properly the value of drying shrinkage of concrete, in accordance to the shift from the allowable stress design method to the limit state design method. The survey of many past investigations about drying shrinkage of concrete suggests that the purpose of investigations at the initial stage was to find out factors relating to drying shrinkage, and that the concern of the latest investigations is mainly to establish the estimate method of drying shrinkage, considering these factors. Some estimate methods[1],[2] are actually adapted in the specifications of design, and they are considered to contribute to the development of the limit state design method. On the other hand, some investigations point out that there are still problems in the adaptability of these estimate methods. In order to improve the reliability of the estimate methods, it is necessary to go back to the starting point which tries to find out factors relating to drying shrinkage of concrete.

There is no doubt that mix proportion of concrete is also one of the main factors relating to drying shrinkage. However, there are many opinions about the relationship between factors of mix proportion and drying shrinkage of concrete, such as water content in the original mix is a determining factor, so it seems that the relationship is not yet systematically understood. Furthermore, it is necessary to investigate the relationship over a wide range of mix proportion, because concretes with unusual mix proportion, such as superplasticized concrete with low unit water content and RCD concrete with lean mix proportion, are put into practical use.

The purposes of this experiment are to examine past theories about the relation between mix proportion and drying shrinkage, using many specimens made with various mix proportions, and to explain systematically these theories from the standpoint of multiphase approach.

2. OUTLINE OF EXPERIMENT

The objects of this experiment are cement paste, mortar and concrete whose mix proportion is shown in Fig.1. Sand-aggregate ratio of concrete is 45% in every mix proportion.

The number of kinds of mix proportion whose flow value could be determined were four among six kinds in the case of cement paste, and 35 among 85 kinds in the case of mortar. According to flow test of concrete in the same manner as mortar, the flow value of 50 kinds of mix proportion among 81 kinds could be determined. Mix proportions whose flow value could not be obtained had extremely stiff consistency or wet consistency. These unusual mix proportions were also used for shrinkage test in order to clarify the relation between mix proportion and drying shrinkage over a wide range. Total number of kinds of mix proportion used in this experiment reached 172.

Normal portland cement, river sand (specific gravity: 2.54, absorption: 2.9%) and river gravel (specific gravity: 2.52, absorption: 3.2%, maximum size: 15mm) were used. The size of prism specimens for the shrinkage test was 4x4x16cm. After curing for 28 days in water, specimens were dried in an atmosphere at 20°C and relative humidity of 60%. Length change of specimens was measured by a comparator and the value of drying shrinkage at drying period of 105 days was used for analysis.



Fig.1 Mix proportion

The object of the analysis was mainly the experimental results about mortar involving cement paste, because the experimental conditions about concrete, such as the maximum size of coarse aggregate, the size of specimen and the measuring method of consistency, were not necessarily standard. Therefore, the main purpose of this investigation is not to have a quantitative understanding of the relation between mix proportion and drying shrinkage in conformity to the purpose of application, but to understand the relation over a wide range of mix proportion and to analyze the relation from various standpoints.

3. RELATION BETWEEN FACTORS OF MIX PROPORTION AND DRYING SHRINKAGE

Figure 2 shows the relation between volume of cement paste per unit volume of concrete (paste content) and drying shrinkage of concrete. This result was obtained by another experiment on poor mix concrete. It is generally pointed out that drying shrinkage of concrete is considerably effected by water content per unit volume of concrete, and that drying shrinkage is in proportion to paste content. This figure shows, however, that there is a comparatively large difference in drying shrinkage of each concrete in spite of constant water content, and that the shrinkage is in inverse proportion to paste content. This result suggests the possibility that general opinions about the relation between mix proportion and drying shrinkage do not agree to the actual facts in the case of unusual mix proportion.

The relation between water content and drying shrinkage is shown in Fig.3, which includes all data obtained by this experiment. It is obvious that larger the



Fig.2 Relation between paste content and drying shrinkage of concrete



Fig.3 Relation between water content and drying shrinkage



Fig.4 Relation between paste content and drying shrinkage

water content, the larger drying shrinkage in the case of mix proportion of medium and wet consistency. In order to prevent large drying shrinkage and the occurence of cracks, it is very important to decrease water content, as much as possible, within the range of these mix proportions. On the other hand, mix proportions with very stiff consistency show shrinkage larger than expected, in spite of their small water content. Specimens using these mix proportions are very porous, so they seem to be susceptible to deformation.

The relation between paste content and drying shrinkage is shown in Fig.4. Drying shrinkage increases as paste content increases, except in the cases of very small paste content whose drying shrinkage is larger than expected. The result shown in Fig.2 seems to be within the range of small paste content. Many previous investigations[3] have given a linear correlation in the case of usual mix proportions, but with a greater increase in shrinkage with increasing paste content in the case of wide range of mix proportions. Generally speaking, drying shrinkage increases exponentially with paste content.

As is evident from Fig.3 and 4, there is no significant difference between the tendency in mortar and concrete, so the results of just mortar including cement paste will be presented after this.

The relation between water cement ratio and drying shrinkage per cement content ("unit shrinkage") shows essentially straight line within the range of medium consistency, as shown in Fig.5. The experimental equation is as follows. $\epsilon_m/C = 6.06(W/C)-0.62$ (1)

(2)

 $\epsilon m/C = 6.06(W/C) - 0.62$ $\epsilon m = 6.06W - 0.62C$

Em: shrinkage strain of mortar(x10⁻⁶) W: water content(kg/m³)

C: cement content(kg/m^3)



Fig.5 Relation between water cement ratio and unit shrinkage

The coefficients of equation (2) also mean that drying shrinkage is strongly affected by water content.

This relationship has already been pointed out by some investigations, but it does hold only in the limited range, the range of medium consistency, in the case of this experiment. Especially, mix proportions with extremely stiff consistency considerably depart from this straight line and have excessive unit shrinkage, as shown in Fig.6.



Fig.6 Relation between water cement ratio and unit shrinkage

Figure 7 shows the relation between cement content and drying shrinkage. The larger the cement content, the larger the drying shrinkage when water cement ratio is constant, except for very lean mixture, which show comparatively large shrinkage. This suggests that cement content has a certain effect on drying shrinkage. However, if lines through the points representing equal water content on this figure are drawn, these lines are nearly horizontal, indicating that as long as water content is not varied, shrinkage will be nearly constant for a wide range of cement contents. This also means that water content is more important than cement content.





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The above results obtained by this experiment show that the relations of factors of mix proportion to drying shrinkage already pointed out by many investigations can hold over a comparatively wide range of mix proportion, except for lean mixture of extremely stiff consistency. The effect of water content upon drying shrinkage is the largest among several factors of mix proportion relating to drying shrinkage, within usual range of mix proportion. In order to decrease shrinkage, proper selection of water content is desirable. Some factors other than water content, such as water cement ratio and paste content, are also related to drying shrinkage to a certain extent. This means that it is difficult to relate the value of drying shrinkage to just the one factor of mix proportion.

4. EXAMINATION OF ESTIMATE METHOD OF DRYING SHRINKAGE

According to the Standard Specification for Design and Construction of Concrete Structures by Japan Society of Civil Engineers (JSCE), the determination of drying shrinkage is defined as follows. "It is a rule that drying shrinkage of concrete shall be determined, considering humidity around structures, shape and dimension of members, mix proportion of concrete and others." The standard values of drying shrinkage strain is also given in this specification for ordinary normal concrete and lightweight concrete, considering weight environmental conditions and age at start of drying concrete. Furthermore, when magnitude and rate of drying shrinkage strain are to be obtained, with consideration for large influences of environmental humidity and member size, following equation may be applied generally, as is described in the comments of this specification.

 $\varepsilon'_{cs}(t,t_o) = \varepsilon'_{so}[\beta_s(t)-\beta_s(t_o)]$

(3)

 ε 'so: basic shrinkage stain

 $\beta_{S}\left(t\right)$: coefficient relating to concrete age and apparent thickness of a member

Effect of mix proportion upon drying shrinkage is not considered in the standard value of drying shrinkage strain and equation (3), although the specification insists the necessity of considering mix proportion. It is only commented that the standard value of drying shrinkage are preferably increased in the case of mix proportion with relatively wet consistency. This comment is based on the Model Code for Design and Construction of Concrete Structure by CEB-FIP which accepted the proposal by Rüsch[1]. The former Model Code had considered the effect of water cement ratio and cement content. On the other hand, the proposal paid attention only to consistency in order to simplify the effect of mix proportion, considering that water cement ratio of usual concrete is limited to a comparatively narrow range, and that cement content is related to consistency. Specifically speaking, the basic shrinkage strain is increased and decreased by 25% for wet consistency and stiff consistency respectively.

Figure 8 shows the relation between cement content and flow value. There exists an obvious quantitative relation between cement content and flow value when water cement ratio is constant. This means that the intention of Rüsch to pay attention to consistency in place of cement content is basically proper. Flow value, however, is comparatively susceptible to the difference in water cement ratio.

The relation between flow value and drying shrinkage is shown in Fig.9. Shrinkage strain is, on the whole, in proportion to flow value. This result also shows the propriety of the method of Rüsch which represents the effect of mix









proportion upon drying shrinkage by using consistency. In this figure, the range of flow value is divided into three equal parts and each part is assumed as stiff, medium and wet consistency. On the assumption that the average shrinkage strain within the range of medium consistency is the basic strain, increased and decreased strain by 25% of this basic strain are also shown in this figure. In the case of stiff consistency, experimental values of shrinkage strain approximately agree with decreased strain. On the other hand, not a few experimental values considerably depart from increased shrinkage strain. This result is caused by scattering on the relation between flow value and shrinkage strain, especially within the range of wet consistency, and suggests that there is still some room for further improvement on the method of Rüsch.

Flow value per cement content ("unit flow value") has an adequate proportional correlation with water cement ratio, as shown in Fig.10, regardless of paste content. The experimental equation is as follows. F/C = 0.96(W/C) - 0.17 (4)

F/C = 0.96(W/C) - 0.17F:flow value (cm)

The relation between this unit flow value and the above-mentioned unit shrinkage is shown in Fig.11. There exists a better proportional correlation, compared with the result shown in Fig.9 which was carried out according to the proposal of Rüsch, and the experimental equation is as follows.

 $\varepsilon_{\rm m}/{\rm C} = 6.34({\rm F}/{\rm C}) + 0.44$

 $\varepsilon m = 6.34F + 0.44C$

(5) (6)

(7)

This equation means that drying shrinkage is basically determined by flow value. However, drying shrinkage can be predicted, with a higher accuracy, by considering cement content in addition to flow value. By substituting equation (4) into equation (5), the following equation can be obtained.

 $\varepsilon_m/C = 6.34\{0.96(W/C)-0.17\}$

= 6.08(W/C) - 0.64

This equation is nearly equal to the above equation (1), and results in the expression that drying shrinkage is effected mainly by water content and secondly by cement content. Therefore, it seems to be more convenient and rational to represent the effect of mix proportion in the form of equation (1),



Fig.10 Relation between water cement ratio and drying shrinkage



Fig.11 Relation between unit flow value and unit shrinkage

(8)

(9)

which can predict drying shrinkage by factors of mix proportion only, compared with paying attention to consistency.

In the Standard Specification by JSCE, there are the following comments. "There are equations to predict drying shrinkage other than equation (3), which are the equations given by Bažant, by ACI-209 Committee and others. These equations have also wide applicable ranges, so that they may be used, considering the purpose of the use." The parts relating to the effect of mix proportion upon drying shrinkage in Bažant's equation[2] are as follows.

cs	=	121	0-88	Uy
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 $y = (390z^{-4}+1)^{-1}$

 $z = \{1.25(A/C)^{1/2} + 0.5(G/S)^2\} \{(1+S/C)/(W/C)\}^{1/3} \cdot f_{C}^{1/2} - 12$ (10)

Es: final shrinkage

A/C: aggregate-cement ratio by weight

- G/S: gravel-sand ratio by weight
- S/C: sand-cement ratio by weight
- W/C: water-cement ratio by weight
- f'c: 28 days strength(ksi)

These equations were derived by using optimization techniques and include most of the factors of mix proportion which seem to be related to drying shrinkage. After calculating y from equation (9) applying experimental conditions and values of mortar and cement paste used in this investigation, the relation between y and drying shrinkage under absolute drying state was obtained as is shown in Fig.12. There is no obvious correlation between y and drying shrinkage, in the case of mix proportion with large paste content. The values of drying shrinkage are almost the same regardless of the value of y in the case of mix proportion with 50% and 30% of paste content, and an inverse proportional relation between y and drying shrinkage, which is expected from equation (8), does not hold. Similar tendency were pointed out by other investigations, so it seems to be necessary to reinvestigate the part on the effect of mix proportion in Bažant's equation.



Fig.12 Examination of Bažant's equation

5. EXAMINATION BY MULTIPHASE APPROACH

Various equations have been proposed to relate shrinkage of concrete to properties of concrete components. Among these equations, the following equation derived theoretically by Pickett[4] seems to be useful when the relation between mix proportion and drying shrinkage of mortar is considered. (11)

 $\varepsilon_m = \varepsilon_p (1 - V_s)^{\alpha} = \varepsilon_p V_p^{\alpha}$

 $\alpha = \frac{3(1-\nu_m)}{1+\nu_m+2(1-2\nu_s)E_m/E_s}$

 ε_p : drying shrinkage of paste(x10⁻⁶) Vs: fine aggregate volume concentration Vm: Poisson's ratio of mortar vs: Poisson's ratio of fine aggregate Em: Young's modulus of mortar(kgf/cm²) Es: Young's modulus of fine aggregate(kgf/cm²)

equation suggests that drying shrinkage of mortar is basically determined This by shrinkage of paste and paste content. If water cement ratio is constant, drying shrinkage of mortar will increase exponentially with paste content because shrinkage of paste is considered to be constant. This tendency is obvious in Fig.4. It is expected from this example that several past theories the relation between mix proportion and drying shrinkage can be about systematically explained by using the composite theory. Furthermore, there seems to be a possibility that a proper estimate method of drying shrinkage, including the effect of mix proportion, can be established by investigation of composite theory on the background of mix proportion.

Equation (11) includes the properties of composite materials, such as Young's modulus and Poisson's ratio of mortar, so that this is not real composite theory which must represent drying shrinkage of composite materials only in properties of constituent elements, such as aggregate and cement paste. This equation also involves the problem that drying shrinkage of aggregate is overlooked. Hansen et al.[5] derived the following equations, improving these defects in Pickett's equation.

 $\frac{\varepsilon_{\rm m}}{\varepsilon_{\rm p}} = (1-m) \frac{n+1+(n-1)V_{\rm s}^2-2nV_{\rm s}}{n+1} + m$ (n≧1) (12) $\frac{\varepsilon_{m}}{\varepsilon_{p}} = (1-m) \frac{n+1-(n+1)V_{s}}{n+1+(n-1)V_{s}} + m$ (n≦1) n = Ea/Ep, $\varepsilon m = \varepsilon s/\varepsilon p$

Ep: Young's modulus of paste (kgf/cm^2) Es: shrinkage strain of fine aggregate $(x10^{-6})$

several factors included in the above equation, drying shrinkage of paste Among is in approximately proportion to water cement ratio as shown in Fig.13, and the experimental equation as follows. (14)

(13)

 $\varepsilon_p = 67(W/C)$



Fig.13 Relation between water cement ratio and drying shrinkage

Because drying shrinkage of aggregate has no relation to mix proportion, it seems to be beyond the scope of this investigation to consider the drying shrinkage of fine aggregate as equation (12) and (13). It is necessary, however, understand the properties of used materials before determining mix to proportion, so that it will be proper, if possible, to include the properties of used materials into the estimate method of drying shrinkage which represents the effect of mix proportion. Previously, it has been generally thought that drying shrinkage of concrete is caused mainly by shrinkage of paste, and that drying shrinkage of aggregate can be disregarded. Although the Standard Specification by JSCE recognizes the necessity of considering the influence of properties of aggregate upon drying shrinkage of concrete, it also neglects drying shrinkage of aggregate. Drying shrinkage of aggregate also is not included in the estimate methods of drying shrinkage proposed by Rüsch and Bažant. According to our previous investigations, however, drying shrinkage of aggregate can never be disregarded and its effect on drying shrinkage of concrete is comparatively large. The quality of aggregate has become lower and recently it is difficult to obtain proper aggregate for concrete. Therefore, there is a strong suspicion that drying shrinkage of aggregate increases drying shrinkage of concrete, so that it seems to be more useful to predict drying shrinkage of concrete based on the composite theory which considers the effect of drying shrinkage of aggregate.

Because it is difficult to measure shrinkage strain of fine aggregate, it was assumed that the strain, ε_P , is equal to 300×10^{-6} . This value corresponds to the shrinkage strain of mortar at W=O in Fig.3. Young's modulus of fine aggregate, the measurement of which is also difficult, was derived from the measured value of Young's modulus of mortar and paste, using the following composite theory given by Hansen et al.[6]

$$\frac{E_{m}}{E_{p}} = \frac{n+1+(n-1)V_{s}}{n+1-(n-1)V_{s}}$$
(15)

Considering the relations and the assumptions described above, equation (12) and (13) are developed as follows.

$$\varepsilon_{\rm m} = 67(W/C) \{ 1 - \frac{300}{67(W/C)} \} \frac{n+1+(n-1)V_{\rm s}^2 - 2nV_{\rm s}}{n+1} + 300 \quad (n \ge 1) \quad (16)$$

$$\varepsilon_{\rm m} = 67(W/C) \ 1 - \frac{300}{67(W/C)} \frac{n+1-(n+1)V_{\rm s}}{n+1+(n-1)V_{\rm s}} + 300 \quad (n \le 1) \quad (17)$$

Shrinkage strains of mortar calculated from the above equations approximately agree with the measured strain as shown in Fig.14, except for mix proportion with extremely stiff consistency.



Fig.14 Examination of Hansen's equation

Figure 15 to 19 show the relation between factors of mix proportion and drying shrinkage calculated from above equations. They correspond to Fig.3, 4, 5, 7 and 13 respectively. Although there are a few points, which still have to be considered in detail, it is clear that the tendency in calculated shrinkage approximately agree with that in measured shrinkage. This means that several past theories about the relation between mix proportion and drying shrinkage can be systematically explained by the composite theory of drying shrinkage.

In other words, the composite theory seems to be very useful to predict drying shrinkage because it gives a reasonable explanation of the effect of mix proportion and it includes properties of used materials. However, some factors, the measurement of which is difficult in general, are included in the composite theory, so that there is a practical difficulty with this theory. Among these factors, Young's modulus of paste has a certain relation with cement water ratio, as shown in Fig.20, and the relation can be expressed in the form as follows.

 $E_p = f(C/W)$

(18)



Fig.15 Relation between water content and calculated drying shrinkage



Fig.17 Relation between water cement ratio and calculated unit shrinkage



Fig.16 Relation between paste content and calculated shrinkage



Fig.18 Relation between cement content and calculated shrinkage





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According to another experiment on crushed stone, Young's modulus of aggregate has an obvious correlation with its absorption as shown in Fig.21, and therefore,

Ea = f(wa)
Ea: elastic modulus of aggregate
wa: absorption of aggregate

(19)

(20)

Drying shrinkage of aggregate is in proportion to its internal surface area, but it is generally difficult to measure this quantity. Adsorption seems to be related to internal surface area, so there is a possibility that drying shrinkage of aggregate is expressed by adsorption instead of internal surface area. Figure 22 obtained from another experiment shows the relation between adsorption of gravel and drying shrinkage of concrete made with thatgravel. Adsorption of gravel is expressed by the percentage of weight of water adsorbed in gravel, which was left in air with relative humidity of 100% for 24 hours after an oven-dry state, to weight of gravel in an oven-dry state. It is comparatively easy to measure adsorption of aggregate. There is an obvious proportional relation between these quantities in this figure, and there is no doubt that adsorption of aggregate is strongly related to drying shrinkage of concrete. This result suggests that drying shrinkage of aggregate is also related to adsorption of aggregate because the difference in each concrete specimen used in this experiment was only properties of gravel. Therefore, drying shrinkage of aggregate can be expressed as follows.

 $\varepsilon_a = f(w'_a)$

Ea: drying shrinkage of aggregate w'a: adsorption of aggregate

Substituting equation (18), (19) and (20) into equation (12) and (13), drying shrinkage of composite materials will be expressed by factors of mix proportion and basic properties of used materials. Because further experiments will be required in order to obtain the established form of equation (18), (19) and (20), and because aggregate used in this experiment is only of one kind, the adaptability of the final expression can not be investigated at this stage. However, the procedure shown here is expected to be useful in including the effect of mix proportion and properties of used materials into the estimate method of drying shrinkage.





6. CONCLUSIONS

The followings were learned within the scope of this investigation.

- (1) The relations of factors of mix proportion to drying shrinkage already pointed out by many investigations were also observed in this experiment over a comparatively wide range of mix proportion, except for lean mixture of extremely stiff consistency.
- (2)These relations can be systematically explained by the theoretical law of drying shrinkage derived from multiphase approach.
- (3) The method of Rüsch for predicting drying shrinkage, which represents the effect of mix proportion by using consistency, is basically proper. However, it seems to be more convenient and rational to represent the effect in combination of water content and cement content, compared with consistency. The method of Bažant method which includes most of the factors of mix proportion has no obvious correlation with this experimental result.
- (4) The composite theory about drying shrinkage seems to be very useful to predict drying shrinkage because it gives a reasonable explanation of the effect of mix proportion and it includes properties of used materials.

REFERENCES

- [1]Rüsch, H and Jungwirth, D. : Berücksichtigung der Einflüsse von Kriechen und Schwinden auf das Verhalten der Tragwerke, Werner-Verlag GmbH, Dusseldorf, 1976
- [2]Bažant, Z.P. and Panula, L. : Practical Prediction of Time-Dependent Deformations of Concrete, Materiaux ET Constructions, Vol.11, No.75, pp.307-316, 1979.
- [3]Lyse, I. : Shrinkage and Creep of Concrete, Jour. of ACI, Vol.56, pp.775-786, 1960.
- [4]Pickett, G. : Effect of Aggregate on Shrinkage of Concrete and Hypothesis Concerning Shrinkage, Jour. of ACI, Vol.52, No.5, pp.581-590, 1956.
- [5]Hansen, T.C. and Nielsen, K.E.C. : Influence of Aggregate Properties on Concrete Shrinkage, Jour. of ACI, Vol.62, No.7, pp.783-794, 1965.
- [6]Hansen,T.C. : Influence of Aggregate and Voids on the Modulus of Elasticity of Concrete, Cement Mortar and Cement Paste, Jour. of ACI, Vol.62, pp.193-216, 1965.