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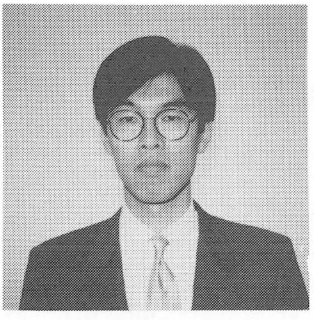
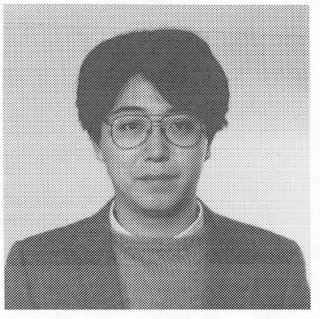
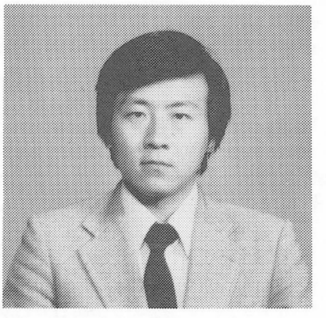
NONLINEAR BEHAVIOR OF CRACKED REINFORCED CONCRETE PLATE ELEMENT UNDER UNIAXIAL COMPRESSION

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SYNOPSIS

In recent studies on predicting behaviors of reinforced concrete, reported is that compressive strength of cracked concrete parallel to cracks is lower than uncracked concrete strength. However, macroscopic deformational behaviors and mechanisms of the strength reduction due to cracks has never been clarified.

The authors conducted the experimental research with hollow-cylinder type of reinforced concrete specimens for examining the deformational behaviors and strength of cracked concrete. The results show us that the strength reduction and stiffness can be expressed by the crack opening and spacing. Within the test results, however, we found that the average tensile strain normal to cracks, which is a function of the crack opening and spacing, uniquely governs nonlinear deformational behaviors of cracked concrete.

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1. INTRODUCTION

The application of nonlinear finite element analysis (FEM) to the design of reinforced concrete has become a great trend in modern concrete engineering. Compared with the macroscopic design formulae such as truss models in beams, FEM has a great advantage in dealing with multi-dimensional shape of structures, any boundary conditions and loads. The active research on FEM for reinforced concrete has solved some analytical problems [1),2)]. Especially, behaviors of reinforced concrete composed of plate (in-plane) elements such as tanks, containers and shear walls are expected to be analytically simulated in the near future. As this type of structures are concerned, there exist no computational problems. The advance of material models directly contributes to the enhanced analysis reliability.

Reinforced concrete consisting of planar members is idealized as a set of finite elements including dispersed cracks [3)]. The structural response is numerically simulated on the basis of the predicted section in-plane forces corresponding to in-plane deformation. In-plane constitutive equation for reinforced concrete generally derives from 1) properties of reinforcement, 2) model of concrete between cracks, 3) stress transfer modeling across cracks and 4) constitutive equation as to bond. Active research works [4)-6)] in each mechanics have been investigated and some are now going on. Among them, the compressive model of concrete between cracks is the authors' discussing point.

Reported was that the compressive constitutive law of concrete involving cracks differs from that of plane concrete without cracks originated by principal tension [7),8)]. The reductions of the compressive stiffness and strength parallel to cracks has been quantitatively discussed, but the origin of non-linearity under compression is one of the discussing points unfinished and not investigated. The main subject of this paper is to make clear the compressive nonlinear mechanics of concrete parallel to cracks. As one of new findings, we discuss correlation between the non-linearity and cracking conditions represented by the crack opening and spacing.

2. Influence Factors on the Stiffness and Strength Reduction

Collins and his co-workers [3)] first formulated the compressive strength reduction of reinforced concrete elements including several cracks as a function of the maximum mean principal strain of elements. This research work influenced the following macroscopic models for torsion [10)] in beams and for the evaluation of shear capacity of wall type structures. Furthermore, it was pointed out that the modeling of concrete parallel to cracks becomes a governing factor for the ultimate capacity of reinforced concrete elements [3)]. However, the applicability of proposed models for the strength of cracked concrete is not clear. The authors believe that the "computational form" of constitutive models must match the mechanics of non-linearity to be dealt with. Then, let us review some possibilities to reduce the compressive stiffness and strength.

(A) Uniformity of Stress Distribution in Cracked Concrete

Cracks are mainly produced in concrete normal to the maximum principal tensile direction, but we have the complex and meandering crack lines because of coarse aggregates' existence as shown in Fig.1. Even though a cracked concrete be uniaxially compressed, the stress distribution between cracks in compression would not become uniform due to the eccentricity of resultant force in each column section surrounded by cracks [11]. Generally speaking, the capacity of the eccentric loading is smaller than the pure compression.

(B) Effect of Multi-Axial Stress

The concrete between cracks is laterally confined by the hoop reinforcement (x-direction in Fig.1) when uniaxial compression in the longitudinal direction is applied to concrete. This confinement produced by the reinforcement and Poisson's effect of concrete improves the strength and ductility. On the other hand, let us consider the general stress states of concrete in reinforced concrete elements where the tensile and shear stresses are transferred by bond and across cracks shown in Fig.1. We should take into account biaxial compression-tension stress states which may reduce the compressive capacity of concrete [12]. From a view point of transferred tension due to bond, Noguchi and et,al. [13] pointed out the stiffness of bond as one of influence factors on the compressive capacity of reinforced concrete walls.

(C) Size Effect

The greater size of coarse aggregates to the specimen's dimension makes the lower compressive strength. Similar effect would be expected provided that distributed cracks in concrete possess smaller crack spacing. The size effect may appear with regard to the thickness of concrete plates compared with the size of crack spacing. Generally, the size of reinforcing bars cannot be ignored against the thickness of members so that the cracks are often produced along reinforcing bars due to stress concentration around them. There exists a possibility that the damaged concrete located around reinforcing bars would affect the compressive strength, especially after yielding of steel. This is one of size effects for composites.

As mentioned above, there are many factors which can be considered, but the individual discussion of each influence factor has never been employed. The authors focused their efforts on the factor stress uniformity in (A) and conducted uniaxial compression test of reinforced concrete in the direction parallel to distributed cracks so that the multi-axial effects in (B) and the size effect by reinforcement and the bond deterioration after yielding of steel bars shall be avoided as much as possible.

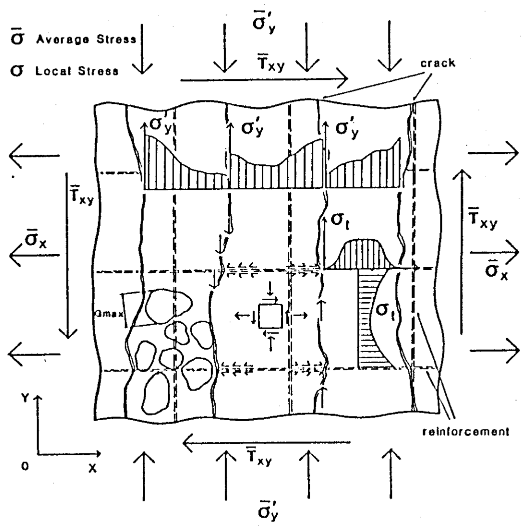


Fig.1 Stress State in Concrete between Cracks.

3. EXPERIMENTS

(1) Test Specimens

We utilized cylindrical shells of concrete (outer diameter of 332mm, thickness of 37mm) in which reinforcement was arranged only in the hoop or transverse "t" direction shown in Fig.2. For introducing cracks in the "l" direction (See Fig.2), the authors applied hydrostatic pressure inside of the shells. Hoop reinforcing bars were connected by lap splices which were randomly placed to avoid stress concentration. Edging away from the confinement effects discussed in Chapter 2, we adopted lightly reinforced specimens having the reinforcement ratio of 0.9% as a maximum. We used small deformed bars of 3mm in diameter to sidestep on the stress concentration around bars. Fig.3 shows the arrangement of reinforcement.

The finishing precision of specimen shapes and arrangement of bars was limited within 0.5mm and 1mm by which the uniaxial compressive strength are not affected [14]. Taking the cross-sectional size and spacing of reinforcement into account, we adopted the maximum size of 10mm for coarse aggregates. The mixture proportion of concrete used is shown in Table 1. After the curing period of about 4 weeks, the loading tests were performed.

(2) Loading Method

The pressure to generate cracks in the hoop direction of specimens was supplied by the pressure vessel made of steel pipes surrounded by fiber reinforced lubber shown in Fig.4. After generating prescribed crack conditions, the pressure vessel was removed, then uniaxial compression was applied to specimens in the axial direction. No external force in the hoop direction was applied. The teflon sheet with silicon grease was placed between specimens and loading apparatus to cut off the friction. Furthermore, we injected super high early portland cement paste between the sheet and a specimen so as to make the contact condition as smooth as possible. After hardening of cement paste, cyclic loads were applied with the constant stress speed of approximately 20 micro-strain per second.

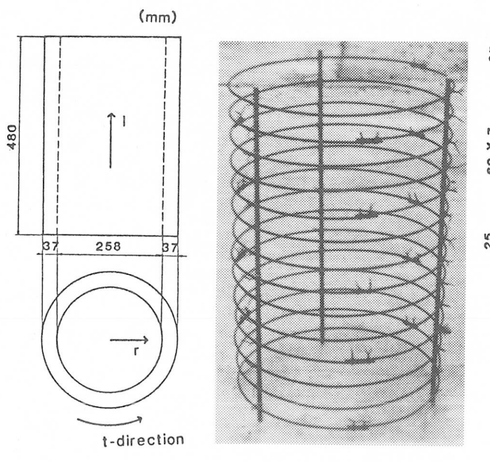


Fig.2 Hollow Cylindrical Specimens and Reinforcement Arrangement

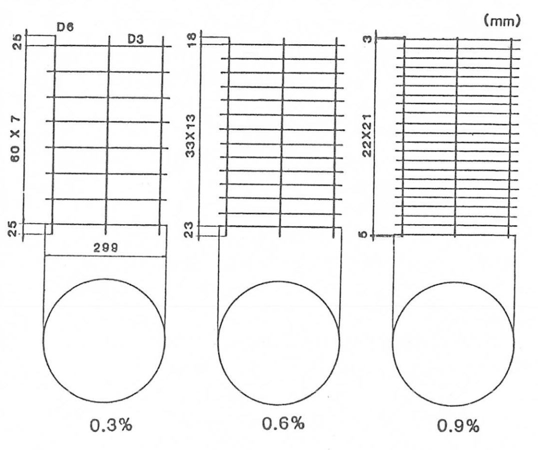


Fig.3 Details of Reinforcement in Each Reinforcement Ratio

(3) Measurements

The mean strain ε_t in the hoop direction in Fig.2 can be assumed as the mean strain ε_r in the radial direction if the thickness of shells is relatively small enough compared with the radius. Then, we computed the mean radial strain in place of the hoop one by measuring mean displacement at three points devided by the specimen radius as shown in Fig.4. Let ε_{t1} denote the initial transverse strain just before the compressive loading.

The strain and the stress in the axial direction parallel to cracks are defined as ε_1' measured at three points by displacement transducers, and σ_1', respectively. The positive stress and strain are defined as tension, but the superscript of ['] indicates the turn of the definition.

Table 1 Mixture of Concrete

type	W/C	W	C	S	G
A	63	184	292	951	878
B	54	157	302	982	907

note: unit of W/C ; "%"
unit of W,C,S,G ; kgf/cm3
high early portland cement
& river sand (S) used.
maximum size of G ; 10 mm

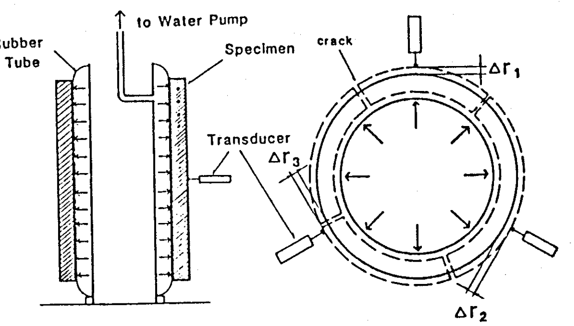


Fig.4 Crack Formation and Measurement

4. Compressive Deformation of Cracked Concrete

(1) Stress-Strain of Cracked Concrete

The compressive stress-strain diagram (σ_1'-ε_1') of cracked concrete for each initial transverse tensile strain ε_{t1} (0 - 7000μ) normal to cracks is shown in Fig.5(a)-(c) where the uniaxial compressive strength by standard test is defined as fc'. Test results and specifications of specimens are arranged in Table 2. With varying reinforcement ratio, we could obtain several combinations of mean crack spacing of lc with mean crack opening of ω to the similar mean hoop strain of ε_{t1}. Fig.5 and Fig.6 include the sketch of crack patterns with solid lines and at the failure by dotted ones. After the ultimate capacity of specimens, the microscopic cracks in concrete between pre-introduced cracks came into naked eyes and the authors observed the out-plane failure with the slide lines similar to that of the cylinder test specimens.

The stress versus strain relationship of Specimen C1 having the light reinforcement ratio of 0.6% without pre-cracking is almost the same as that of specimen C0 made of plane concrete without any reinforcement. Furthermore, we cannot find any difference of the compressive strength. This means no confinement and size effects due to reinforcement mentioned in Chapter 2. Accordingly, the authors concluded that the compressive strength of shells coincides with the cylinder strength of fc', which can be used as a standard value for uncracked condition.

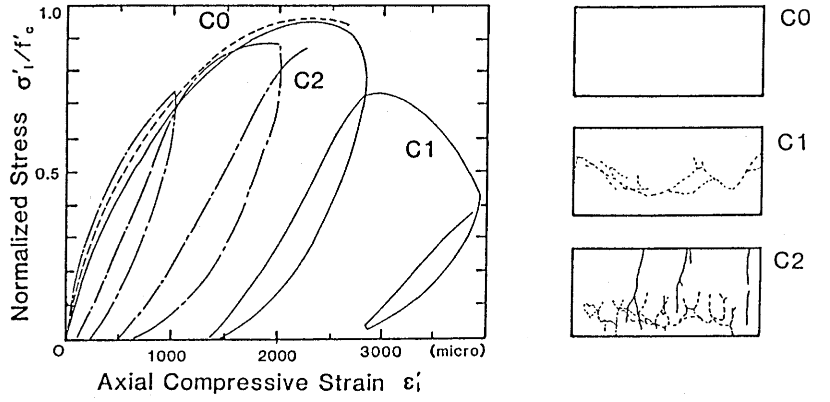


Fig.5(a) Stress-Strain Diagrams of Slightly Cracked Concrete (Specimens C0, C1 and C2, ε_{t1} = 0 - 760μ)

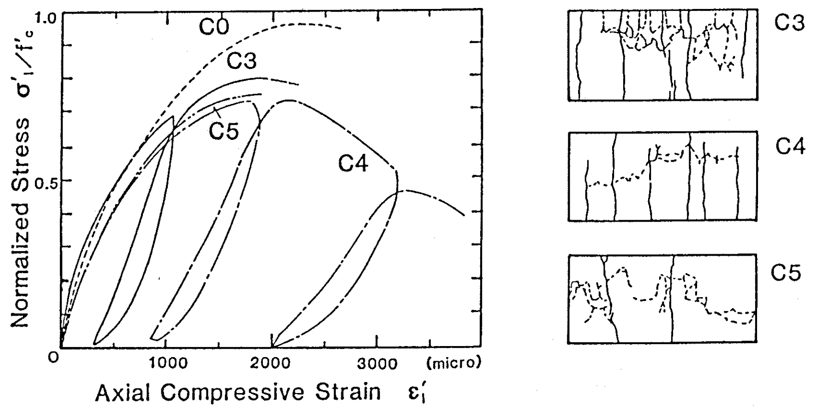


Fig.5(b) Stress-Strain Diagrams of Representative Cracked Concrete (Specimens C3, C4 and C5, ε_{t1} = 1900 - 2170μ)

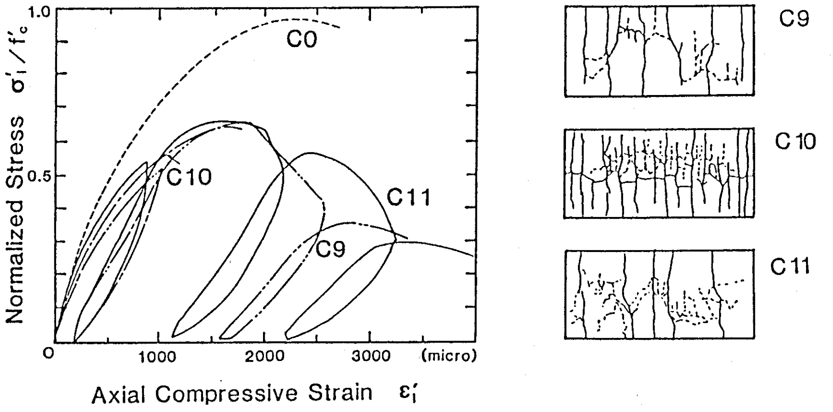


Fig.5(c) Stress-Strain Diagrams of Heavily Deteriorated Cracked Concrete (Specimens C9, C10 and C11, ε_{t1} = 7080 - 8070μ)

Let λ denote the ratio of the strength of cracked concrete σ_{1u}' to the cylinder strength ($=\sigma_{1u}'/fc'$). The introduction of cracks exactly reduces both the compressive strength of specimens and the maximum compressive strain defined as ε_{1u}'. The greater strain of ε_{t1} gives us the smaller compressive strength of σ_{1u}'. However, we can see from Fig.5 that the stress-strain relation hardly depends on the crack spacing and width provided that the mean strain in the transverse direction is kept constant.

Compared with the reduction of the strength and the maximum strain, the initial compressive stiffness is slightly reduced by cracking. In other words, the deformational behaviors under the low stress level is not affected so much by the cracking, but the non-linearity parallel to cracks appears at the early stage of loading according to the transverse strain level normal to cracks. The stress-strain relation normalized by the capacity values of σ_{1u}' and ε_{1u}' is shown in Fig.7. The envelope curve of each cyclic response appears to be similar to each other, but we cannot judge whether the inner curves at unloading and reloading are similar to others or not only with Fig.7. The cyclic response will be discussed later.

Table 2 Test Results of Cracked Concrete Specimens.

No.	$\varepsilon_{t1}(\mu)$	p(%)	Fc'(Mpa)	λ	cracks	lc(mm)	ω(mm)	Mixture
C0	0	0	30.4	0.95	0	-	-	A
C1	0	0.6	21.4	0.96	0	-	-	A
C2	760	0.9	28.6	0.90	3	348	0.26	A
C3	1900	0.9	36.7	0.80	5	209	0.40	A
C4	1910	0.6	21.4	0.74	6	174	0.33	A
C5	2170	0.3	28.6	0.74	2	522	1.13	A
C6	3750	0.3	27.3	0.67	4	149	1.05	A
C7	3865	0.6	44.2	0.71	6	174	0.67	B
C8	3950	0.6	36.7	0.73	3	348	1.37	A
C9	7080	0.6	44.2	0.67	7	149	1.05	B
C10	7600	0.9	28.6	0.65	15	70	0.53	A
C11	8070	0.3	27.3	0.67	5	209	1.68	A

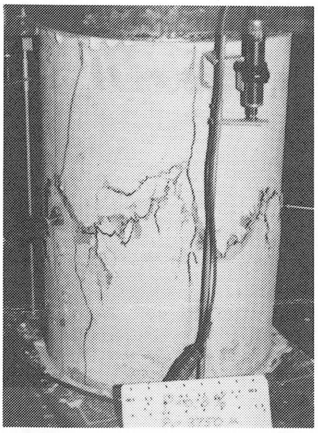


Fig.6 Failure Mode of C6.

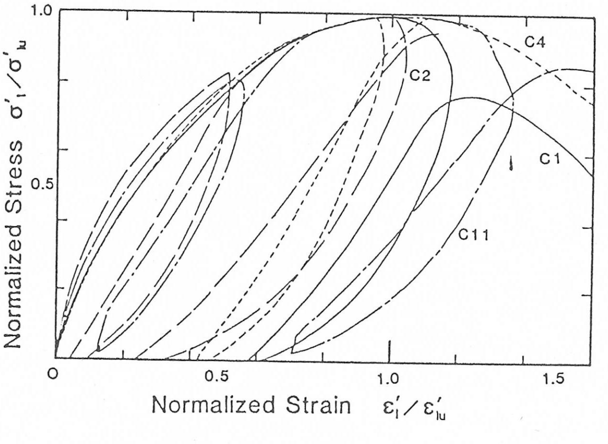


Fig.7 Normalized Stress-Strain Relation

(2) Deformation normal to Cracks -- Crack Strain --

Generally, the mean strain of ε_t normal to cracks is measured as the strain of continuous concrete denoted by ε_{ct} between cracks plus the mean strain originated from the localized crack opening as ε_{cr} (See Fig.8). Using the sum of crack openings $\Sigma\omega$ in a reference length L, we have [11],

$$\varepsilon_t = \varepsilon_{ct} + \varepsilon_{cr} \quad \cdots\cdots\cdots\cdots\cdots (1)$$

$$\varepsilon_{cr} = \Sigma\omega/L$$

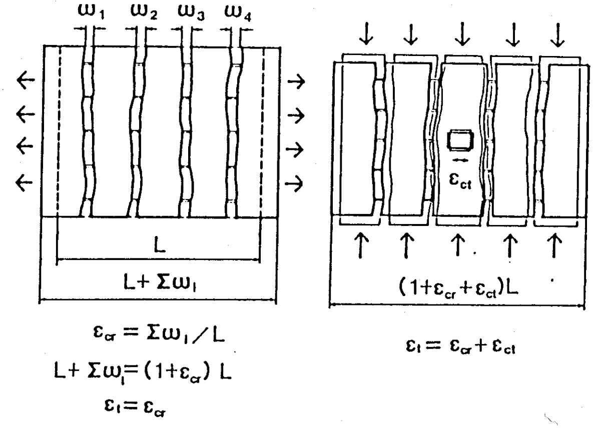
where L was the outer peripheral length of shells in this experiment. Just before the compressive loading, we can expect ε_{ct} as zero and the initial transverse strain of cracked concrete comes mainly from the crack strain of ε_{cr} as,

$$\varepsilon_{t1} = \varepsilon_{cr} \quad \cdots\cdots\cdots\cdots\cdots (2)$$

The variation of ε_t during compression is shown in Fig.9 where the origin coincides with ε_{t1} in each specimen. According to the increase in compression, the strain ε_t normal to cracks is gradually growing, but under low stress level approximately 50% of the uniaxial strength, the authors' recognition is that the difference of Poisson's effect of cracked concrete from that of the uncracked one is negligible. Accordingly, we can consider the constant crack strain not affected by the crack formation in concrete.

(3) Reduction of Compressive Strength due to Cracks

The relation between the strength reduction rate λ and the crack strain ε_{t1} is shown in Fig.10(a), including data under different crack conditions represented by the crack spacing lc and the opening ω. We should note the unique correlation between them. This means that the strength reduction is apparently governed only by the crack strain regardless of the crack numbers introduced in reinforced concrete specimens. This independency of crack conditions on the strength is the great advantage in simplifying FE material models based on the "Smeared Crack" idealization, but not clarified from a view point of mechanics.



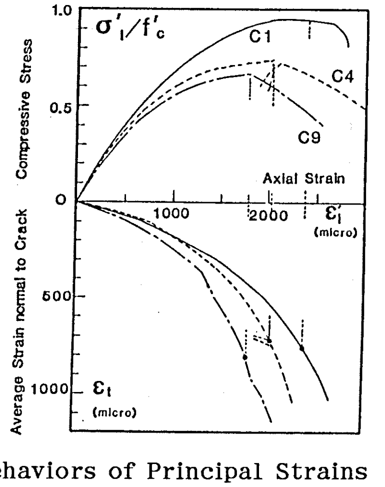


Fig.8 Crack Conditions and Strain Fig.9 Behaviors of Principal Strains

The authors show the relation of the mean crack opening ω versus the strength reduction λ. We see the distinct correlation between them in each crack spacing range (lc>350mm, 150-210mm, 70mm>lc) as shown in Fig.10(b). In any crack spacing, the greater opening of cracks makes the lower compressive strength, and furthermore, the reduction rate with reference to ω is highly affected by the smaller crack spacing of lc. Regarding the strength reduction, the effects of the crack spacing and the opening clash. Let us consider the crack condition where the crack strain denoted by (ω /lc) is kept constant. The test results shown in Fig.10(a) indicates that the effect of the greater crack opening cancels the effect of the greater crack spacing in appearance if (ω /lc) is constant.

As shown in Fig.10(b), reality is that the convergence of the strength reduction where the crack opening is large enough to drop the interaction between concrete exists. Similar trend on the behavior as to the maximum compressive strain of ε_{1u}' corresponding to the initial crack strain of ε_{t1}' are observed as shown in Fig.11. The mechanism from which the nonlinear deformational behaviors mentioned earlier derive will be again discussed in a later chapter.

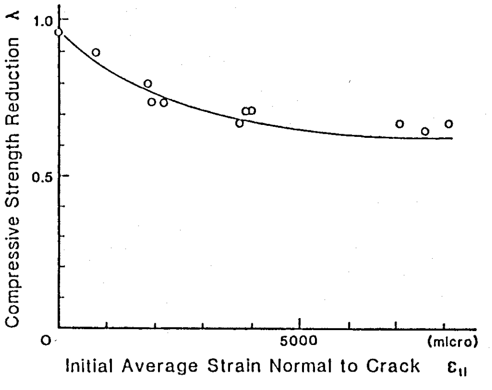
Let us compare previously reported models for reduced capacity of compression with authors' experimental results. The proposals by Collins [8] and Hsu [10] are rewritten on the basis of notations used in this paper as,

$$\lambda = \frac{1}{0.85 + 0.27(\varepsilon_{tu}/\varepsilon_1)} \quad \cdots\cdots\cdots\cdots (3.1)$$

where ε_{tu} is defined as the transverse strain normal to cracks at the ultimate capacity in compression. Furthermore, Vecchio and Collins [9] proposed the following model by modifying Eq.(3a) as,

$$\lambda = \frac{1}{0.80 + 0.34(\varepsilon_{tu}/0.002)} \quad \cdots\cdots\cdots\cdots (3.2)$$

where the value of λ is not greater than unity. Cervenka [15] analyzed reinforced concrete panels in using,



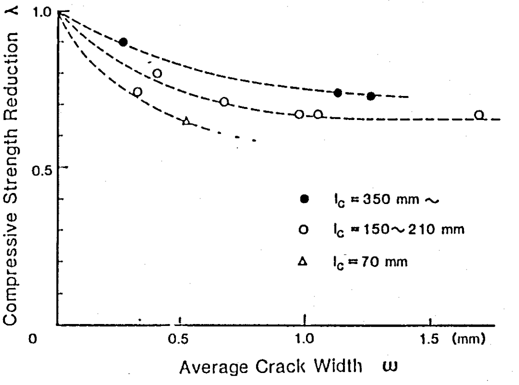


Fig.10 Strength Reduction versus Crack Strain (a) and Crack Opening (b)

$$\lambda = 1.0 - 0.45(\varepsilon_{tu}/0.005) \quad \cdots\cdots\cdots\cdots (3.3)$$

Maekawa et.al took into account the strength reduction based on the experiment using plane concrete with single crack when they joined the international competition [11] in 1983. Fig.12 involves these models described by the crack strain at the capacity of failure and experimental data. The models proposed by Collins and Cervenka assume the monotonically descending branch as shown in Fig.12. On the other hand, experimental results tell us the existence of lower bound in the strength. The difference of proposed strengths from empirical facts may arise from the difference of stress states by Collins' tests from the authors' experimental condition in which purely uniaxial compression parallel to cracks was intended to reproduce for neglecting other factors than the effect of cracking.

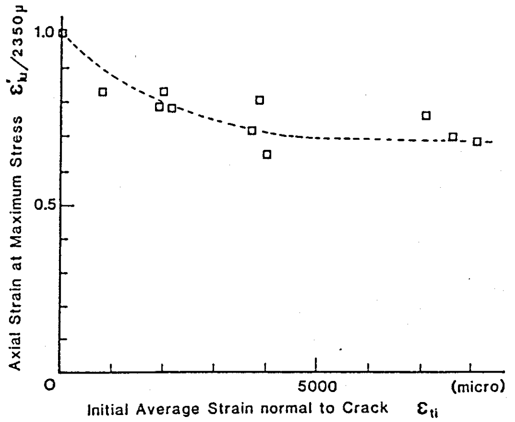


Fig.11 Maximum Compressive Strain versus Initial Crack Strain

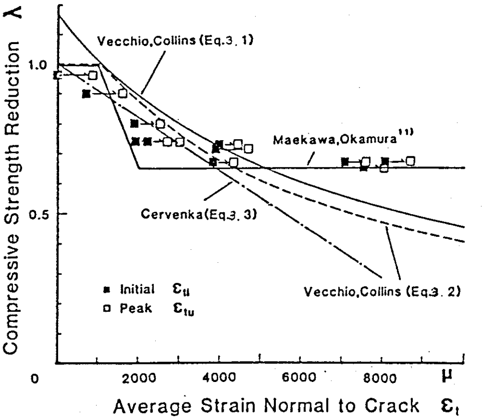


Fig.12 Prediction of Strength Reduction by Proposed Model

5. Plasticity and Fracture of Cracked Concrete - Mechanism of Strength -

The deformational behaviors of cracked concrete appear to be complicated, but the authors believe in some basic governing rules simple to be understood. Then, let us divide the deformation of cracked concrete into elastic and plastic deformations as ε_{1e}' and ε_{1p}'. The latter is defined as the residual total strain when external load is removed [11].

(1) Plasticity

Involving unloading paths on the loading program, we can directly measure the plastic compressive strain of ε_{1p}'. The relationship between ε_{1p}' and maximum compressive strain ε_{1max}' on the past loading history is shown in Fig.13, where we cannot see any effect of crack conditions described by the crack spacing and opening on the plasticity. Even though the initial crack strain has a great influence on the stiffness and strength in compression parallel to cracks, the progress of the compressive plasticity is not affected by the crack strain exactly, then we have,

$$\varepsilon_{1p}' = \varepsilon_{1p}'(\varepsilon_{1max}') \quad \cdots\cdots\cdots\cdots (4)$$

The similar test results were reported as the plasticity of concrete subjected to three dimensional confinement is concerned. The plasticity in the maximum principal direction in compression has no correlation with the confinement pressure. Although this fact will be a general rule for concrete, we should not forget the effect of loading speed on the plasticity. The plastic strain used in Fig.13 may include large amount of time-dependent plasticity. In the future research, we must formulate the path dependency including time.

(2) Elasticity and Fracture

The linear relation between stress and elastic strain can be roughly assumed, but their stiffness is not constant but generally varies according to loading paths [11] as follows.

$$\sigma_1' = E_o K \varepsilon_{1e}' \qquad \text{or}$$

$$\sigma_1' = E_o K (\varepsilon_1' - \varepsilon_{1p}') \quad \cdots\cdots\cdots\cdots \quad (5)$$

where, Eo is the initial stiffness, and K is the stiffness reduction factor defined as fracture parameter, which conceptually represents the effective unfractured area having the load carrying capacity in concrete [12].

From experimental results, the right hand side of Eq.(5) must include the crack strain as a governing parameter, but the term of the plasticity was empirically confirmed to be independent on the crack strain. This gives us an important suggestion for the mechanism of the strength reduction that only the fracture parameter is affected by the cracking, on the other hand, the plasticity not by the cracks. Similar to the plasticity, the relationship between the fracture parameter and the maximum compressive strain as one of the path-dependent parameters is shown in Fig.14. We note the smaller value of K under the higher crack strain. It may be easy to understand that the greater crack strain (larger crack opening and/or smaller crack spacing) causes the progress of the microscopic fracturing. From a view point of the nonlinear behaviors of cracked concrete (strength and stiffness reduction), the authors understand the following equation as an appropriate form of constitutive laws.

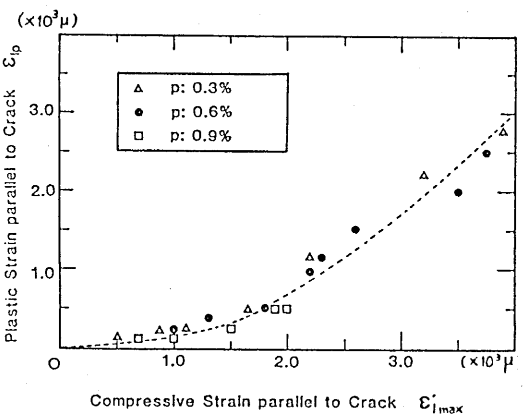


Fig.13 Plasticity of Cracked Concrete

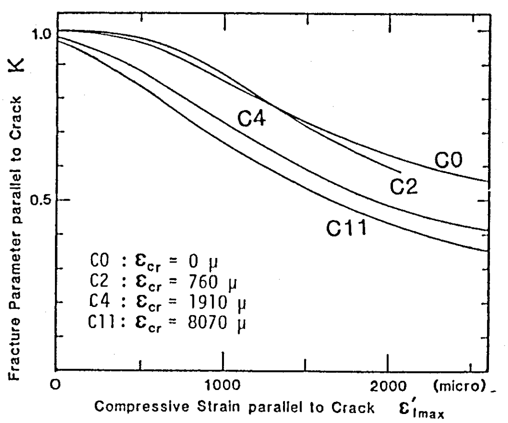


Fig.14 Fracturing Process

$$\sigma_1' = Eo\ K\,(\varepsilon_1' - \varepsilon_{1p}')$$

$$= Eo\ K(\varepsilon_{1max}', \varepsilon_{cr})\{\varepsilon_1' - \varepsilon_{1p}'(\varepsilon_{1max}')\} \quad \cdots\cdots\cdots (6)$$

We can simultaneously explain the reduction of both the compressive stiffness and the strength of cracked concrete by the variation of the fracture parameter only. However, we should take into account the plate thickness, size of aggregates, dimension of steel bars, loading speed, multi-axial stress and others for deciding the practical function of K in Eq.(6).

(3) Mechanism of the Decrease in Fracture Parameter

The reduction of the fracture parameter according to the crack strain was found to be a rational method to deal with the non-linearity caused by cracks. Then, let us consider the mechanism of the reduced fracture parameter due to cracks. The authors focused our view point on the stress distribution mentioned in Chapter 2.

After the crack formation, approximately 30 strain gauges were attached to cracked concrete and the strain distribution between cracks were directly measured as shown in Fig.15. In this test, the larger crack opening (0.6mm) and the small cracker spacing (70mm) were intended to form to get as large drop of the strength as possible. We can recognize the steep strain gradient in concrete between cracks (See Fig.15) where the load level was just before the ultimate capacity. The strain varies linearly in concrete similar to the eccentric loads to concrete columns. Cracks do not propagate straight because of the aggregates. If concrete columns resist the external compression without any interaction through cracks, the center of resultant force generally deviates from the gravity center of each concrete column between cracks due to its angulated crack lines. This eccentricity of compression causes the apparent reduction of the capacity.

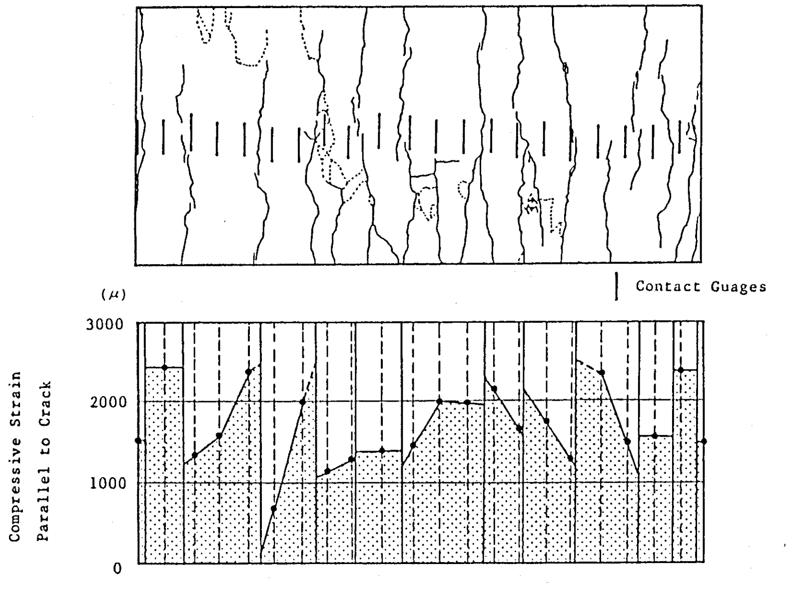


Fig.15 Local Strain Distribution between Cracks

Fig.16 shows the response of concrete strains to the external mean stress. It was observed that the cracked concrete fails when the maximum value of the compressive strain reaches the ultimate strain corresponding to the uniaxial strength of uncracked concrete. After that, concrete may loose the capability of redistributing stress. Therefore, the mean stress and strain at the ultimate of cracked concrete must be smaller than the stress and strain for plane concrete without pre-cracking. The eccentricity of loading represents the deficient usage of material section, in other words, the loss of effective cross sectional area which corresponds to the variation of the fracture parameter.

The authors now understand that the crack opening governs the uniformity of strain distribution and the interaction through cracks is considered to exist and to play an important role. Here, the crack spacing has much to do with the convergence of the strength to the stable limit value when the cracks are open too much.

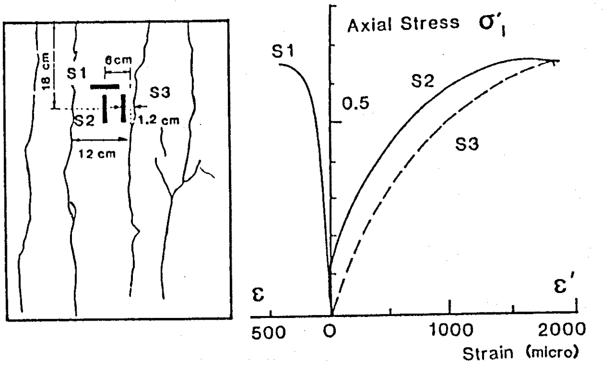


Fig.16 Relation of the Mean Stress and Local Strain

6. CONCLUSIONS

(1) The uniaxial compressive strength of cracked concrete parallel to cracks surely decreases. The strength reduction rate is dependent on the crack spacing and opening, but it was clarified that the mean crack strain normal to cracks not greater than 4000μ govern the reduction of strength in appearance. Applying this behavior to the evaluation of the capacity of reinforced concrete, we should further take into account the effect of reinforcement and size, tension stiffness transferred by bond, stress rotation whose effects were neglected in the authors' experiments.

(2) Within the experiment, the strength reduction has a lower bound and is kept constant if the crack strain becomes greater than 4000μ.

(3) The plastic deformation parallel to cracks is governed by the maximum strain in past loading history, not dependent on the mean crack strain normal to cracks. The stress-strain relation including cyclic stress path can be rationally described by the elasto-plastic and fracture model in which the fracture parameter, on the contrary, was investigated to be affected by the crack strain.

(4) One of the reasons why the compressive strength of cracked concrete decreases derives from the non-uniform stress distribution similar to the eccentric loading in concrete surrounded by cracks. When we have the smaller crack opening and/or the larger crack spacing, we can expect more uniform stress distribution enough to guarantee the high strength.

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