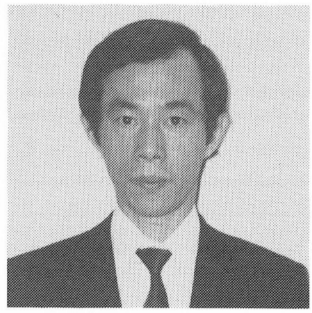
CONCRETE LIBRARY OF JSCE NO. 11. JUNE 1988

EVALUATION OF THERMAL CRACK OCCURRENCE IN MASSIVE CONCRETE STRUCTURES

(Reprint from Transaction of JSCE, Vol.6, No.378, 1987)



Katsuhiko KIMURA

Sadamu ONO

SYNOPSIS

Authors propose a probabilistic approach for the evaluation of thermal crack occurrences, which introduces thermal crack index, random variable defined in terms of the ratio of the concrete tensile strength to the thermally induced tensile stresses due to external restraints.

Furthermore, this report discusses some factors affecting the thermal crack occurrence probability. It becomes clean that the degree of external restraint, tensile strength and calculation methods of thermal crack index are the most contributing factors.

Keywords: mass concrete, thermal crack, external restraint, thermal crack index.

K. Kimura is a chief engineer at Civil Engineering Division of Shimizu Corporation, Tokyo Japan. He received his Master of Engineering Degree from Nagoya Institute of Technology in 1975. He is a member of JSCE and JCI.

S. Ono is a chief engineer at Civil Engineering Division of Shimizu Corporation, Tokyo Japan. He received his Doctor of Engineering Degree from Hokkaido University in 1985. He is a member of JSCE and JCI.

---

1. INTRODUCTION

It is considered that the thermal cracking of concrete due to the heat of hydration has an adverse effect on the functional properties, durability, and the authentic appearance of the concrete structures . In the case of water storage structures, for instance, there were many cases in which the thermal cracking of concrete was the major cause of water leakage, having an adverse effect on the functions of the structures [1]. In view of these past experiences, it is considered that the control of thermal cracking is important in order to guarantee the qualities of the structures, particularly for massive concrete used for the substructures of large bridges i.e, piers, abutments, anchorages, bottom slab and side wall of underground storage tanks, and wall concrete used for culverts, water storage structures, etc. Therefore, a number of studies have been carried out to date on this subject.

The following factors must be considered for controlling the thermal cracking of concrete in the planning, design, and construction stages of concrete structures; the prediction of concrete temperature after placement, thermal stresses and the characteristics and evaluation of thermal cracks. For this reason, many experimental and analytical studies have been conducted with respect to the prediction of concrete temperature and thermal stress. However, few studies have been or are being made concerning the evaluation of thermal crack occurrence in spite of its practical importance.

Generally, the cracking phenomenon occurs as a result of a combination of numerous factors. In the case of massive concrete, in particular, the occurrence of thermal cracking extends over a wide time period from the time it is in the initial plastic state to the time it becomes sufficiently hardened. It is thought that more factors are influential in the occurrence of thermal cracking in massive concrete than in other concretes in general. Because of this, it is presumed that even if an advanced analytical method, such as FEM (Finite Element Method), is used in the deterministic analysis of the concrete temperature and thermal stress when evaluating thermal crack occurrence, the results of the analysis are lacking in precision. The reason for this is various constants used in the analysis such as thermal characteristics, heat transfer coefficient, Young's modulus, and the creep characteristics of concrete inherently have scatters in their values. From this fact it is considered that there is a limit to the appropriateness of a deterministic evaluation method of thermal crack occurrence.

However, a deterministic method of evaluating thermal crack occurrence on the basis of the criteria that the induced tensile stress is greater than tensile strength or vise versa is widely in use [2] as a method for the evaluation of thermal crack occurrence in massive concrete.

One of the authors pointed out that the thermal cracking phenomenon of massive concrete inherently contains a large number of indefinite factors and that a method of evaluating thermal crack occurrence from the probability viewpoint is proper. He further defined an index named thermal crack index and proposed and put into practice this method using this index for evaluating thermal crack occurrence from the probability viewpoint [3]-[5]. This method hereinafter is referred to as Ono's method. However, Ono's method, which describes the thermal crack index in terms of the temperature ratio, has problems in obtaining a solution; for example, in the case where stress due to external forces and thermal stress work simultaneously, its effect cannot be evaluated correctly. Later, a method of evaluation based on such a concept was also proposed by Yoshioka et al. [6] and Osaki et al. [7]. Among past studies, few have dealt with the effects of various factors on the probability of thermal crack occurrence.

In the present study, the authors investigated the relation between the thermal crack index defined by the ratio of tensile strength to the induced tensile stress and the probability of thermal crack occurrence on the basis of experimental data. Furthermore they investigated the effects on the probability of thermal crack occurrence due to the calculation methods for the degree of external restraint, tensile strength, and the thermal crack index.

2. THE PRESENT STUDY INVESTIGATION OF THERMAL CRACKS

The thermal cracks investigated in the present study were thermal cracks caused by external restraint as shown in Fig. 1. They were those produced at the construction stage. In actual structures, thermal stress caused by internal restraint was generated along with that caused by external restraint. However, when a member reaches an age at which the quantity of temperature drop becomes greater and thermal stress caused by external restraint is predominant, the thermal stress caused by internal restraint is small in many cases. In this study, therefore, the effect of thermal stress caused by internal restraint was disregarded to simplify the problem. The data used for this study was short-term data collected about one month after the placement of concrete. One reason for having confined this study to short-term data was that sufficient data was not obtainable from the studies of medium- and long-term thermal cracks. In many cases regarding temperature and thermal cracks, and the evaluation of the degree of external restraint (hereinafter, referred to as the degree of restraint) would become difficult with changes in the geometry of the structure during the progress of construction.

The structures studied were slab-shaped massive concrete structures and wall-type massive concrete structures, restrained mainly on one side. The cement content per unit volume of concrete used for these structures was in the range of 200 - 400 kg/m^3 .

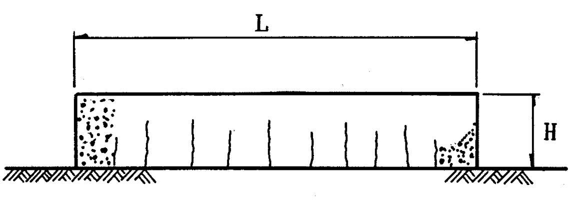


Fig. 1 Thermal cracks due to external restraint

3. THE BASIC CONCEPT OF THE EVALUATION OF THERMAL CRACK OCCURRENCE

(1) Criteria for the Occurrence of Thermal Cracks

a) Assumptions for Mathematical Formulations

① Temperature Distribution within Concrete Members [8]

The temperature distribution in concrete members was assumed as parabola as shown in Fig. 2 and was given by equation (1)

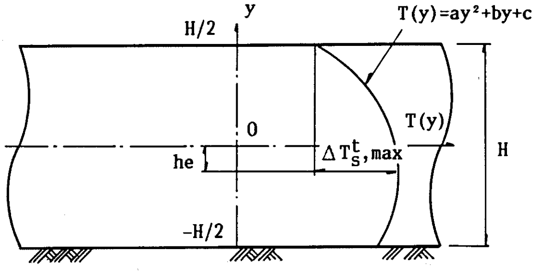


Fig. 2 Temperature distribution in members

$$T(y)=ay^2+by+c \quad \cdots\cdots (1)$$

The locations within members at which the internal temperature marks the maximum value are from past records, he = H/6 in the case of slab-shaped massive concrete structures. Furtheremore it was assumed that he = Unity in the case of wall-type massive concrete structures, since the temperature distribution in them was symmetrical. Consequently, it was assumed that the average temperature, T_m^t , in members could be given by equation (2) in the case of slab-shaped massive concrete structures and by equation (3) in the case of wall-type massive concrete structures:

$$T_m^t=\frac{1}{H}\int_{-\frac{H}{2}}^{\frac{H}{2}} T(y)dy=\frac{1}{4}(3\,T_{max}^t+T_s^t) \quad \cdots\cdots (2)$$

$$T_m^t=\frac{1}{3}(2\,T_{max}^t+T_s^t) \quad \cdots\cdots (3)$$

where

T_m^t: average temperature in member at age t, (°C)
H : thickness of member, (m)
T_{max}^t: maximum value of internal temperature in member at age t, (°C)
T_s^t: surface temperature of member at age t, (°C)
h_e: distance from center of member to location where T_{max}^t is marked, (m)

ⓘ Mechanical Properties of Concrete

The following assumption was made with respect to the mechanical properties of concrete:

$$\sigma_{ct}(t)=0.5\sigma_{cB}(t)^{\frac{2}{3}} \quad \cdots\cdots (4)$$ [9]

$$E_c(t)=33\,000\sigma_{cB}(t)^{0.4} \quad \cdots\cdots (5)$$ [10]

$$B(t)=\frac{1}{1+\varphi(t)} \quad \cdots\cdots (6)$$

where
$\sigma_{ct}(t)$: tensile strength of concrete at age t, (kgf/cm²)
$\sigma_{cB}(t)$: compressive strength of concrete at age t, (kgf/cm²)
$E_c(t)$: Young's modulus of concrete at age t, (kgf/cm²)

$B(t)$: rate of stress relaxation due to creep at age t
$\varphi(t)$: creep coefficient at age t

When there was no available data regarding the creep coefficient, it was obtained by assuming the relative humidity as 70 % (generally considered value) by referring to the CEB/FIP Code (1978 edition) [11] and by taking the virtual thickness, kind of cement and effective age of the concrete into consideration. As for the concrete age upon loading to be used in the calculation of the creep coefficient, the age in which its internal temperature marked the maximum value was adopted. The creep coefficient was approximated with the hyperbola formula as a function of the age of the member after loading. Furtheremore it was assumed that the creep coefficient was the same if the members experienced same loading period with respect to the incremental increase in tensile stress given by equation (10) even if the ages of the loaded members were different.

b) Criteria for the Occurrence of Thermal Cracks

As the criteria for thermal crack occurrence (fracture criteria), the theory of the maximum principal stress was adopted and it was assumed that cracks would occur in the direction of the maximum principal stress and also in the orthogonal direction, when the largest principal stress induced in members exceeded the tensile strength of concrete. The conditions of thermal crack occurrence can be given by formula as follows:

$$\frac{\sigma_{ct}(t)}{\sigma_{te}(t)} \geq 1.0 \quad \cdots (7)$$

where
$\sigma_{te}(t)$: tensile stress induced due to a temperature drop at age t (kgf/cm^2))

c) Stress induced by External Restraint

It was assumed that the temperature stress induced by external restraint could be given by equation (8) (hereinafter referred to as the incremental method) and compressive stress due to temperature rise was not considered.

$$\sigma_{te}(t) = \sum \Delta\sigma_{te,i}(t') \quad \cdots (8)$$

$$\Delta\sigma_{te,i}(t') = B(t') \cdot \Delta\sigma_{t,i} \quad \cdots (9)$$

$$\Delta\sigma_{t,i} = R \cdot \alpha_c \cdot E_c(t'_i) \cdot (T_m^{t_{i-1}} - T_m^{t_i}) \quad \cdots (10)$$

where
$\Delta\sigma_{te,i}(t)$: incremental tensile stress $\Delta\sigma_{t,i}$ at age t (kgf/cm^2)
$\Delta\sigma_{t,i}$: incremental tensile stress occurring from age t_{i-1} to age t_i (kgf/cm^2)
R : degree of external restraint
α_c : coefficient of thermal expansion of concrete ($=10\times10^{-6}$/°C)
$T_m^{t_i}$: average temperature in the member at age t_i (°C)

$$t'_i = \frac{1}{2}(t_{i-1} + t_i), \quad t' = t - t_{i-1}$$

Fig. 3 shows the relation between the average temperature and temperature stress schematically.

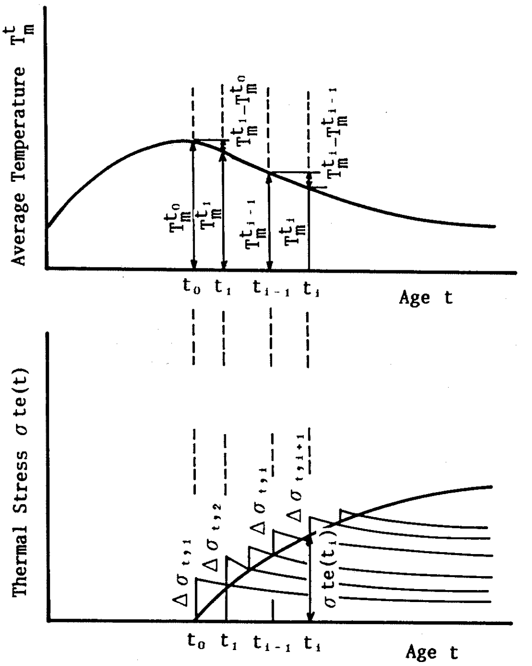


Fig. 3 Schematic calculation diagram of thermal cracks due to external restraint

d) Degree of External Restraint R

For the estimation of the degree of restraint, ACI's method, Nagayama et al.' method and Mori's method are have been proposed. In this study, the method proposed by Ono [4] was used. The degree of restraint was calculated by making reference to Fig. 4 and 5. In the case of direct placement on the ground or footing structures for a pile foundation, the restraint was considered to be small and therefore, R was assumed as 0.1 [12]. In many cases, the thermal cracks were caused by external restraint progress from the neighborhood of construction joints where the degree of restraint is large. Therefore, the value at 0.1 H point near the construction joint, which was considered to have a significant effect on the occurrence of thermal cracks, was adopted as a degree of restraint. The ratio of Young's modulus used in the calculation for the degree of restraint varies according to the age of the member. In this study, the value at the age of 30 days was used, taking simple solution into consideration. However, when the next lift was jointed within 30 days or when thermal cracks occurred within 30 days, the ratio of Young's modulus at the age at jointing time or at the age at which thermal crack occurrence was confirmed was used in the calculation for the degree of restraint.

(2) The Definition of the Thermal Crack Index

The ratio of the concrete tensile strength to the induced stress is defined as the index of thermal crack occurrence (hereinafter, abbreviated to thermal crack index) and is given by equation (11) as follows:

$$ET_s = \frac{\sigma_{ct}(t)}{\sigma_{te}(t)} \quad \cdots\cdots (11)$$

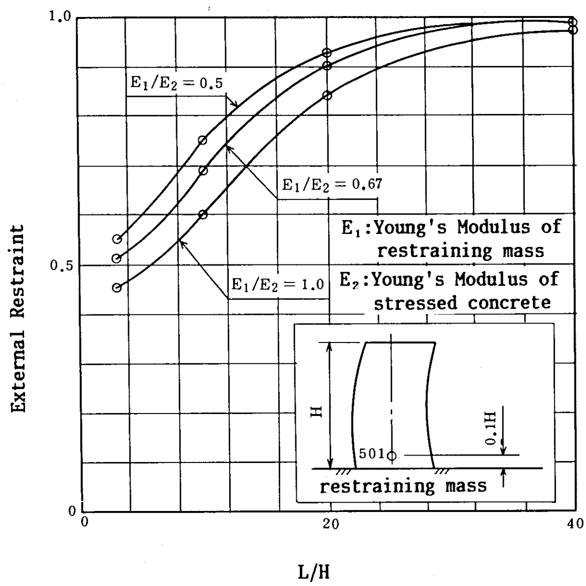


Fig. 4 Calculation diagram of the degree of external restraint (In the case the lengths of the restraining mass and stressed concrete are equal)

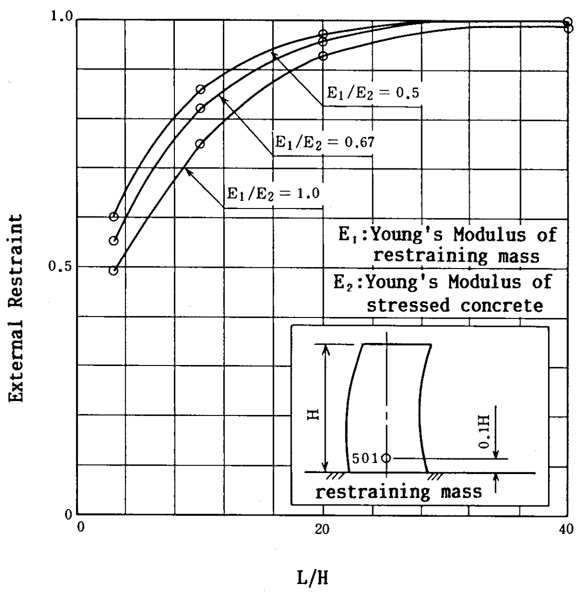


Fig. 5 Calculation diagram of the degree of external restraint (In the case length between restraining mass and stressed concrete is different)

where

ET_s: thermal crack index for thermal cracks caused by external restraint
$\sigma_{ct}(t)$: tensile strength of concrete at age t
$\sigma_{te}(t)$: tensile stress induced at age t due to temperature drop

(3) The Definition of Probability of Thermal Crack Occurrence

The probability of thermal crack occurrence for a given range of thermal crack index (hereinafter, abbreviated to probability of thermal crack occurrence) is defined as the ratio of the number of the observed thermal cracks for the given index range to the total number of observations for the same range and is given by equation (12). For reference, Yoshioka et al. obtained the probability of thermal crack occurrence from the cumulative frequency of crack occurrences and that of non-occurrences.

$$P_i=\frac{n_i}{N_i} \quad \cdots\cdots (12)$$

where

P_i: probability of thermal crack occurrence between $ET_{s,i-1}$ and $ET_{s,i}$
n_i: number of observed thermal cracks between $ET_{s,i-1}$ and $ET_{s,i}$
N_i: total number of observations between $ET_{s,i-1}$ and $ET_{s,i}$

4. ANALYSIS FOR THE CHARACTERISTICS OF THE THERMAL CRACK INDEX BASED FROM THE RESULTS OF INVESTIGATIONS ON ACTUAL STRUCTURES

(1) Data

Investigations were carried out at 47 construction sites. In the analysis, 655 pieces of data were used, of which thermal crack occurred in 233 pieces. Thermal cracks were not present in the remaining 422 pieces.

The structures investigated can be divided into two groups: wall-type massive concrete structures (hereinafter, abbreviated to Mass 2a) and slab-shaped massive concrete structures (hereinafter, referred to as Mass 2c). The numbers investigated were as follows:

Mass 2a ....293 pieces at 19 construction sites
Mass 2c ....362 pieces at 35 construction sites

Part of the results of the investigations is shown in Table 1(a), and Table1(b).

2) Calculation for the Thermal Crack Index

It is necessary that the thermal crack index be obtained by calculating by using equation (8), the stress induced from the time-dependent variation in the average temperature and then calculating the indices by the use of equation (11). However, since there is a relation, as described below, between the thermal crack index ET's obtained by calculating the stress by equation (13) (hereinafter, referred to as simple method) and ETs given by equation (11). In this study ETs was obtained from ET's and its relation to ETs. The difference between ETs and ET's is due to the difference in the method of calculation of the induced stress, that is, ETs are calculated by the incremental method, while ET's by the simple method. In the simple method, the creep coefficient is obtained for age t by the method described in 3.(1) a) and by assuming Young's modulus as $B(t)\cdot E_c(t)$, taking stress relaxation into consideration.

Table 1(a) Investigation data on massive concrete (No.1)

Factor / No.	W/C (%)	types of cement	unit cement weight (kg/m^3)	slump (cm)	pipe-cooling period (days)	pre-cooling application	slab dimensions h (m)	L (m)	H (m)
1	59.2	MBB	260	10.0	primary 4 2ndary 48	Yes	1.50	30.0	15.9
2	〃	〃	〃	〃	〃 3 〃 41	〃	〃	〃	〃
3	〃	〃	〃	〃	〃 3 〃 27	〃	1.40	31.0	15.9
4	〃	〃	〃	〃	〃 3 〃 0	〃	〃	〃	〃
5	56.0	〃	280	10.5	detail not available	〃	1.50	17.2	12.6
6	〃	〃	〃	〃	〃	No	〃	30.0	15.9
7	54.8	NP	290	12.0	0	〃	3.00	18.2	14.0
8	〃	〃	〃	〃	〃	〃	〃	14.3	10.5
9	57.0	〃	309	〃	〃	〃	2.50	47.0	36.0
10	45.5	FC	295	8.0	7～8	〃	4.50	22.0	18.0

remarks W/C: water cement ratio h: slab thickness L: slab longitudinal dimension
H: slab lateral dimension MBB: JIS R5211 Type B Blast-furnace cement specially blended for moderate heat
NP: JIS R5210 Normal portland cement FC: JIS R5213 Type B Fly-ash cement

Table 1(b) Investigation data on massive concrete (No.2)

Factor / No.	Tp (℃)	Placing Season	Curing Method	Curing Period (days)	Tr,max (℃)	Ts,r,max (℃)	DAYmax (days)	ΔT_m^t (℃)	R	Crack Occurrency
1	29.8	summer	sprinkling	over 5	27.2	5.5※	2.0	9.5 (13)	0.94	No
2	28.8	〃	〃	〃	28.2	5.2※	〃	13.5 (21)	0.93	〃
3	28.5	〃	〃	〃	23.5	5.2※	〃	15.2 (22)	0.94	〃
4	25.0	autumn	〃	〃	26.0	5.0※	3.0	21.0 (30)	0.93	〃
5	25.4	summer	〃	〃	〃	5.6※	1.0	18.3 (23)	0.82	Yes
6	26.6	〃	〃	〃	33.4	5.1※	2.0	20.2 (16)	0.94	〃
7	12.0	winter	foaming polystyrol	〃	49.0	17.0	〃	20.0 (30)	0.50	No
8	13.0	spring	〃	〃	47.0	27.0	3.0	24.6 (30)	0.48	〃
9	13.0 ※	winter	foaming polystyrol +sheet	25 ※	37.0※	23.0※	3.5	14.0 (30)	0.82	〃
10	14.0	spring	sheet	7～8	40.5	26.0	4.0	24.0 (30)	0.49	〃

remarks Tp: placing temperature(℃) Tr,max: maximum internal temperature(℃) DAYmax: age showed Tmax(days)
Ts,r,max: maximum surface temperature(℃) value in ( ) is concrete age when temperature become ΔT_m^t
※ estemated values

$$\sigma'_{te}(t)=B(t)\cdot R\cdot \alpha_c\cdot E_c(t)\cdot \Delta T^t_m \cdots\cdots\cdots\cdots(13)$$

$$ET'_s=\frac{\sigma_{ct}(t)}{\sigma'_{te}(t)} \cdots\cdots\cdots\cdots(14)$$

where

$\sigma'_{te}(t)$: tensile stress induced in the member at t due to temperature drop
ΔT^t_m: drop in the average temperature in the member at t
ET'_s: thermal crack index in the case induced stress was given by equation (15)

The relation between the thermal crack index ETs and ET's was obtained as below. Out of the specimens for which the exact detailed data was available, 17 pieces were extracted from mass 2a and 20 pieces from Mass 2c, both at random, and the thermal crack indices were calculated. The relation between ETs and ET's is shown in Figs. 6 and 7, according to the presence or absence of thermal cracks. The relation between ETs and ET's in the case of thermal cracks were present as is shown in equation (15) and in the case cracks were absent is shown in equation (16).

(Cracks were present)

$$ET_s=0.334+0.455\,ET'_s \cdots\cdots\cdots\cdots(15)$$

(Cracks were absent)

$$ET_s=0.219+0.798\,ET'_s \cdots\cdots\cdots\cdots(16)$$

The coefficients of correlation between ETs and ET's in the case that thermal cracks were present was 0.831 and if absent was 0.845 and as the result of test of significance, the coefficients of correlation were recognized as highly significant with r (11,0.01) = 0.684 and r(22,0.01) = 0.517. The results of variance in the regression analysis and test of F are shown in Tables 2 and 3 respectively. As the result of the test of F, regression was recognized to be highly significant.

Table 2 Table of the analysis of variance (In the case thermal cracks were present)

factor	S	ϕ	V	F_0	remarks
R	0.2719	1	0.2719	24.9**	F(1、11、0.01)
e	0.1198	11	0.0109		=9.65
T	0.3917	12			

R : variation between groups e : variation within group
T : total variation S : sum of squares V : variance
ϕ : degree of freedom F_0 : variance ratio

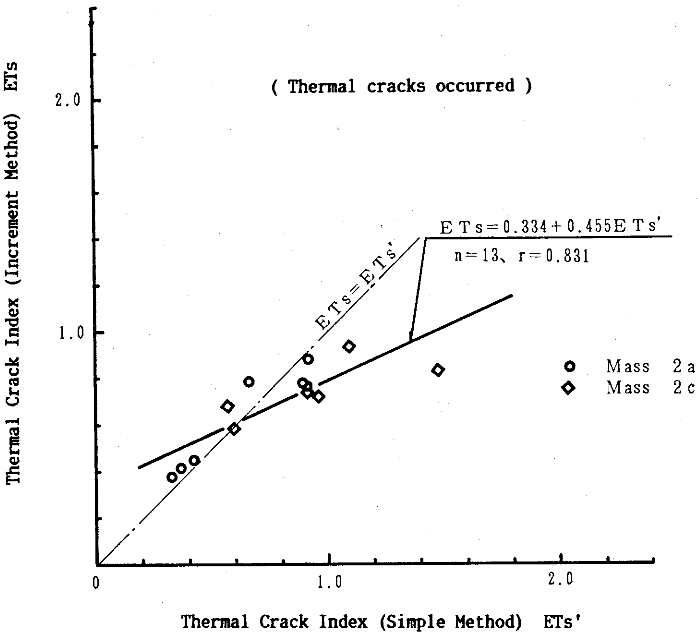


Fig. 6 Relation between the thermal crack indexes obtained by the incremental method and those obtained by the simple method

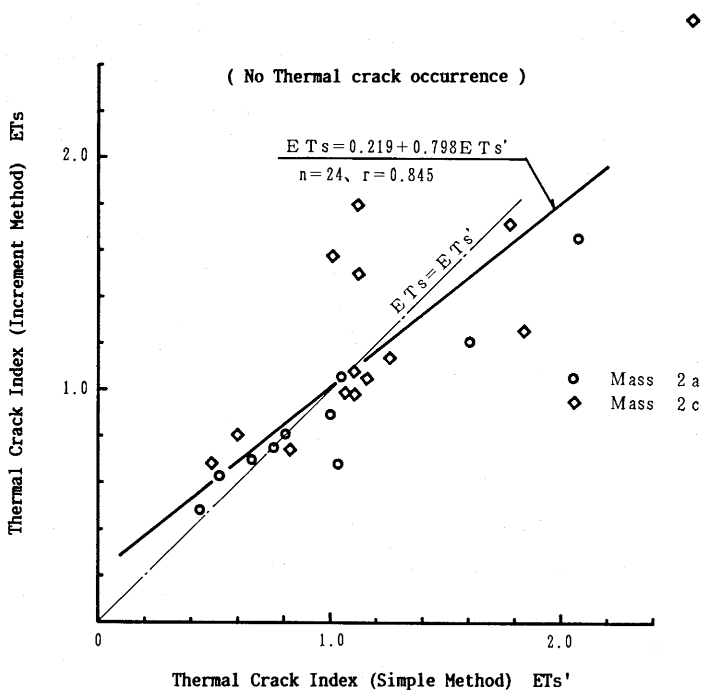


Fig. 7 Relation between the thermal crack indxsed obtained by the incremental method and those obtained by the simple method

Table 3 Table of the analysis of variance (In the case thermal cracks were absent)

factor	S	ϕ	V	F_0	remarks
R	3.9343	1	3.9343	51.9**	F(1, 22, 0.01)
e	1.5777	22	0.0717		=7.94
T	5.512	23			

It was judged from the above mentioned results that it would be reasonable to obtain a thermal crack index, ETs, based on the incremental method from ET's based on the simple method. (Hereinafter, the thermal crack index is referred to as ETs given by equation (11) as a rule.) It is thought that the method of obtaining generated stress by using equation (8) and calculating ETs by the use of equation (11) is more efficient when evaluating of the possibility of thermal crack occurrence than the method of calculating ETs by using equations (13) to (16).

(3) The Statistical Properties of the Thermal Crack Index

Figs. 8 and 9 show histograms of the thermal crack index in the cases where thermal cracks occurred and did not occur respectively. The average value of the thermal crack index was, $\bar{x}$ = 0.824 (0.813) and square root of variance was, $\sqrt{V}$= 0.297 (0.272) in the case thermal cracks occurred, while in the case they did not occur, the average value of the thermal crack index was, $\bar{x}$ = 1.832 (1.128) and square root of variance was $\sqrt{V}$= 1.894 (0.416). Since thermal cracks occurred within the range ET <2.0 and the relation between the probabilities of thermal crack occurrence and the thermal crack indexes were prepared on the basis of the data concerning this range, and the values corresponding to them are shown in parentheses.

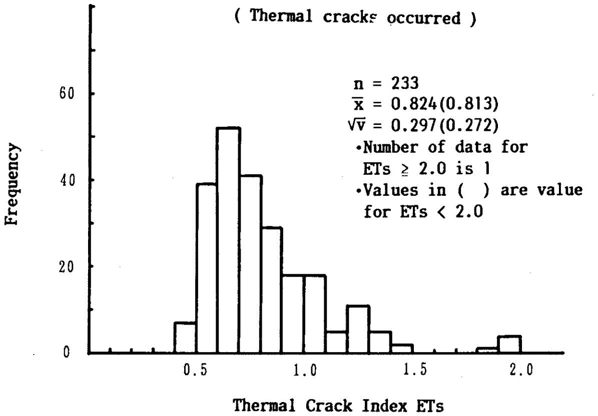


Fig. 8 Histogram of thermal crack indexes

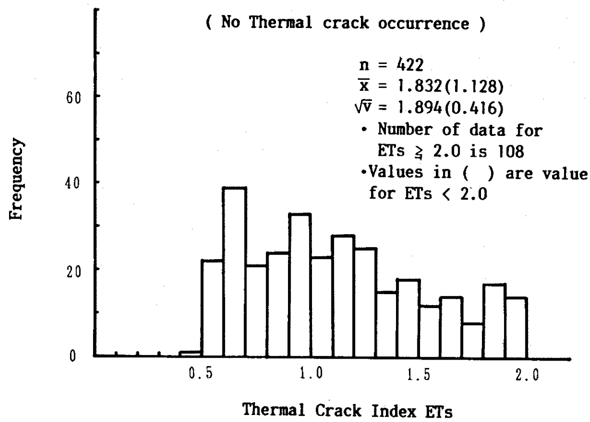


Fig. 9 Histogram of the thermal crack indexes

Distribution of the thermal crack indexes in the case that thermal cracks occurred presents a tendency toward normal distribution. In the case thermal cracks did not occur, on the other hand, the thermal crack indexes were spread over a wide range and the tendency of normal distribution, such as that shown in the case thermal cracks occurred, was not perceived. It is presumed to be because thermal crack occurrence can be grasped momentarily or by point theoretically, but the state in which thermal cracks do not occur is indefinite.

(4) The Probability of Thermal Crack Occurrence

The probability of thermal crack occurrence in each class of the thermal crack indexes was calculated from the histograms shown in Figs. 8 and 9 by using equation (12) and the results were plotted on normal probability paper, as shown in Fig. 10.

In order to investigate the effects of the ranges of data on probability of thermal crack occurrence and the thermal crack index used in this study, the following two cases were studied: the case in which ETs, which exhibited a great change in its tendency of data distribution on the normal probability paper, was smaller than 1.5, i.e., the number of (ETs, P) set, n, was 11 and the case in which ETs<1.8 i.e., n = 14 covered all data on the probability paper.

The coefficient of correlation between the probability of thermal crack occurrence and thermal crack index was obtained as, r =0.919 in the case n = 11 (ETs<1.5) and as r = 0.903 in the case n = 14 (ETs<1.8).

As the result of calibration of the significance of the coefficients of correlation, they were recognized to be highly significant from the results obtained, namely, r(9,0.01) = 0.744 and r(12,0.01) = 0.661. From the result of analysis on the population coefficient of correlation for n = 11, it is estimated to be between 0.71 and 0.98 with the level of significance being 5%.

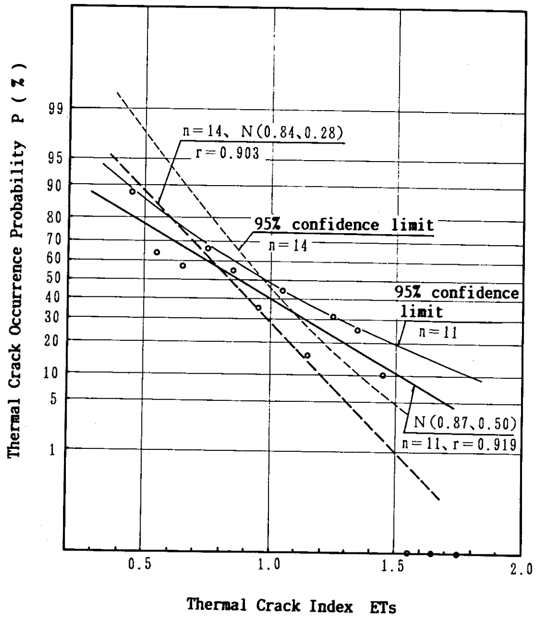


Fig. 10 Relation between the thermal crack index and probability of thermal crack occurrence

Next, the regression characteristics of thermal crack occurrence in Fig. 10 was analyzed as the function of the thermal crack index and the resulting straight lines regression and their 95% confidence limits were shown in the same figure. The results of the dispersion analysis and of the calibration of F in the case n=11 are shown in Table 4. As the result of the calibration of F, it was recognized that regressions in the case n was 11 and 14 were both highly significant. Furthermore, the probability distribution functions of thermal crack occurrence presented normal distribution, N being 0.87 and 0.50 in the case n was 11 and 0.84 and 0.28 in the case n was 14. When the probability of thermal crack occurrence reached 50% against regression straight line, ETs = 0.87 in the case n = 11 and ETs = 0.84 in the case n = 14, and against 95% confidence limit ETs was about 1.0 regardless of n. The data concerning the probability of thermal crack occurrence are all included within the 95% confidence limit in the case n = 11.

Table 4 Table of the analysis of variance (In the case = 11)

factor	S	ϕ	V	F_0	remarks
R	35.00	1	35.00	49.2**	F(1、9、0.01)
e	6.40	9	0.71		=10.6
T	41.40	10			

Then, it was considered which would be more adequate, to use a regression straight line in the case of n = 11 or in the case of n = 14 to express the relation between the probability of thermal crack occurrence and the thermal crack index. Generally, in the study of thermal crack control, it is desired that the probability of thermal crack occurrence in the range below 50% should be estimated with good accuracy. Also it is considered more adequate from the viewpoint of safety in the judgment of thermal crack occurrence, to use a regression straight line in the case n = 11, which gives a high probability of occurrence against the same thermal crack index. From the above mentioned, it is considered adequate to use a regression straight line in the case n = 11 in the study of thermal crack control. A regression straight line in the case n = 11 is used in the following study too.

Fig. 11 shows a probability diagram of thermal crack occurrence in the case n = 11 proposed in this study.

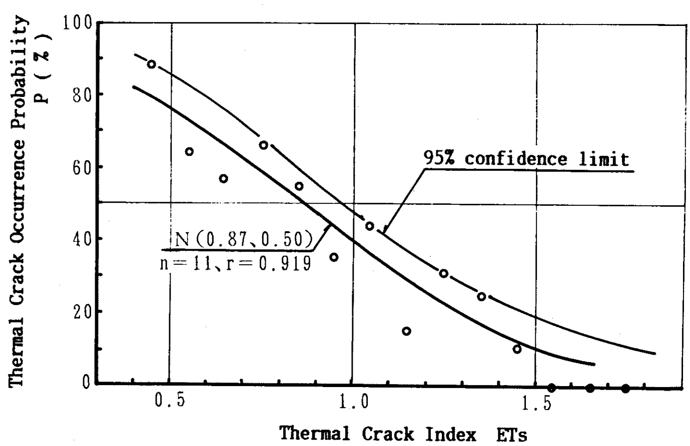


Fig. 11 Probability diagram of thermal crack occurrence

5. EFFECTS OF THE DEGREE OF EXTERNAL RESTRAINT AND TENSILE STRENGTH ON THE PROBABILITY OF THERMAL CRACK OCCURRENCE

(1) Degree of External Restraint R

Here, the effect of the evaluation of the degree of restraint included in the calculation equation (11) of thermal crack index on the probability of thermal crack occurrence is studied.

First, give attention to the fact that the thermal crack index can be obtained from the following equation and then the relation between the thermal crack index and the degree of restraint can be obtained, as expressed by equation (17) :

$$ET_s = \frac{\sigma_{ct}(t)}{\sigma_{te}(t)} = \frac{\sigma_{ct}(t)}{R \cdot \alpha_c \sum B(t') \cdot E_c(t'_i) \cdot \Delta T_m^{t_i}}$$
$$= \frac{1}{R} f(t, \Delta T_m) \cdots\cdots (17)$$

Fig. 12 shows the relation between the thermal crack index and the degree of restraint, assuming $f(t,\Delta T_m)$ in equation (17) as constant. In the case the degree of restraint is evaluated to be larger against the same $f(t, \Delta T_m)$, the thermal crack index increases and the probability of thermal crack occurrence decreases. This means that a conservative value is given from the viewpoint of thermal crack control. In the case the degree of restraint is evaluated smaller, on the contrary, the result is reversed; the probability of thermal crack occurrence decreases. For example, the probabilities of thermal crack occurrence in the case f = 0.5 and R = 0.4, 0.5, and 0.6 as calculated from Fig. 12 become as follows. The probabilities of thermal crack occurrence in the case R = 0.4 and 0.6 against the probability of thermal crack occurrence, P = 40%, in the case R = 0.5, become 23% and 53% respectively. And in the case the degree of restraint is evaluated 0.1 larger, the probability of the thermal crack occurrence increases by 13% and in the case it is evaluated 0.1 smaller, the probability of thermal crack occurrence decreases by 17%. It is understood from Fig. 12 that the effect of the evaluation of the degree of restraint increases with a decrease in the degree of restraint.

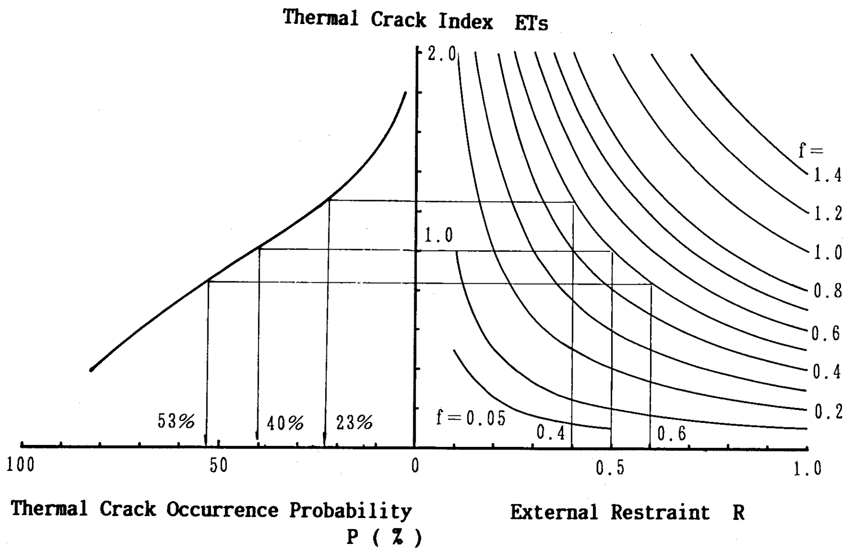


Fig. 12 Relations among probability of thermal crack occurrence, the thermal crack index and the degree of external restraint

In this study, induced stress is calculated from equation (8) as the value of the degree of restraint at the age of the member investigated. However, the degree of restraint inherently varies according to the age of the member, so it can be expected that the probability of thermal crack occurrence varies according to the age of the member which provides the degree of restraint. Based on this assumption, a study was made on the degree of variation in the probability of thermal crack occurrence according to the age of the member providing the degree of restraint, giving concrete examples. As experimental models, slabs, each with a restraining body and a restrained body of equal length, L/H of 8 and 4 weeks' (hereinafter, abbreviated to 4 w) intervals between jointings were used and were investigated in two cases: σ_{28} = 250 kgf/cm^2 (24.5MPa) and 400 kgf/cm^2 (39.2 MPa). As for the age providing the degree of restraint, 1 w and 4 w were adopted. 1 w is an intermediate age at which the average temperature in members attains the peak value where it gets almost stabilized. 4 w is nearly equal to the longest age of the members shown in the data used in this study as mentioned in 2. Here, in the case of σ_{28} = 250 kgf/cm^2 (24.5 MPa) and 400 kgf/cm^2 (39.2 MPa), σ_7/σ_{28} was assumed as 0.53 and 0.64, σ_{35}/σ_{28} as 1.04 and 1.03 and σ_{56}/σ_{28} as 1.11 and 1.09, respectively [13]. The results of the calculation of the ratio of Young's modulus of restrained body to that of restraining body, E_1/E_2 and the degree of R in the case the degree of restraint was provided at 1 w and 4 w become as follows:

In the case σ_{28} = 250 kgf/cm^2 (24.5 MPa): E_1/E_2 is 0.76 and 0.96, R is 0.60 and 0.56, respectively and in the case σ_{28} = 400 kgf/cm^2 (39.2 MPa): E_1/E_2 is 0.83 and 0.97,R is 0.59 and 0.56, respectively.

Young's modulus was obtained from equation (5). In the case of σ_{28} = 250 kgf/cm^2 (24.5 MPa), the probabilities of thermal crack occurrence P_1 and P_4 became 53% and 48% respectively when f = 0.5, their difference being 5%. P_1 and P_4 represent the probabilities of occurrence in the case R was provided at 1 w and 4 w respectively. Assuming that f = 0.7, P_1, and P_4 become 27% and 23% respectively, their difference being 4%. On the other hand, in the case of σ_{28} = 400 kgf/cm^2 (39.2MPa), P_1 and P_4 become 52% and 48% respectively when f = 0.5 and become 26% and 23% respectively when f = 0.7. It is found from these results notwithstanding that the age of the member providing the degree of restraint is 1 w or 4 w. The difference between the probabilities of occurrence evaluated by them is about 5%, indicating that the effect of the age of the member providing the degree of restraint on the probability of occurrence is slight and that the effect of the age of the member providing the degree of restraint on the occurrence of crack decreases with an increase in concrete strength. This is attributed to a difference in the ratio of Young's modulus due to a difference in the ages of the members during the calculation of the degree of restraint.

(2) Tensile Strength

Here, the effect of the difference in the evaluation methods of tensile strength from the compressive strength of concrete on the probability of thermal crack occurrence is examined by expressing the relation between the compressive strength σ_{cB} and tensile strength σ_{ct} by the following three equations and Fig. 13 shows the relationship between σ_{ct} and σ_{cB} .:

$$\sigma_{ct1}=0.5\,\sigma_{cB}^{2/3} \quad\cdots(4)$$

$$\sigma_{ct2}\fallingdotseq 0.1\,\sigma_{cB} \quad\cdots(18)$$

$$\sigma_{ct3}=1.77\sqrt{\sigma_{cB}}\ [14] \quad\cdots(19)$$

Since σ_{te} is not a function of σ_{ct} as shown in equation (8), the following relation holds among the thermal crack indexes obtained in the case tensile strength is given by equation (4), (18), and (19):

$$ET_{si}=ET_{s1}\cdot\frac{\sigma_{cti}}{\sigma_{ct1}} \quad\cdots(20)$$

where

ET_{si}: ET_s corresponding to σ_{cti}

Fig. 14 shows the relation between the compressive strength and the ratio of tensile strength, the relation between the ratio of tensile strength and thermal crack index and the relation between the thermal crack index and the probability of thermal crack occurrence in the case tensile strength is given by equations (4), (18), and (19), respectively. As is clear from Fig. 14, the thermal crack index is greatly affected by the method of evaluating tensile strength even if the compressive strength and generated stress are the same. For example, in the case σ_{cB} = 100 kgf/cm^2 (9.8 MPa), when ET_{s1} = 1.0, ET_{s2} and ET_{s3} become 0.92 and 1.63 respectively and the probabilities of crack occurrence corresponding to them are 40%, 48%, and 6%, respectively, showing great differences. It is also understood that the effect of $\sigma_{ctl}/\sigma_{ct1}$ on the probability of thermal crack occurrence increases with an increasing thermal crack index.

Then, by expressing the relation between tensile strength and compressive strength by equation (4), the effect of the scatter in compressive strength on the probability of thermal crack occurrence is considered by the use of Fig. 14. Assuming that the coefficient of variation of compressive strength v = 10%, the estimated values of the upper and lower limit values of compressive strength σ_{cB} can be given by the following equation:

$$\sigma_{cB}=\overline{\sigma_{cB}}\pm 3\sqrt{V}=1.3\,\overline{\sigma_{cB}},\ 0.7\,\overline{\sigma_{cB}} \cdots\cdots\cdots (21)$$

where

$\overline{\sigma_{cB}}$: average value of compressive strength

$\sqrt{V}$: square root of variance

The ratio of tensile strength of σ_{cB} to that of $\overline{\sigma_{cB}}$, $\sigma_{ct}/\overline{\sigma_{ct}}$, becomes 1.19 and 0.79. Assuming that the thermal crack index and the probability of thermal crack occurrence against $\overline{\sigma_{ct}}$ are 1.0 and 40% respectively; in the case $\sigma_{ct}/\overline{\sigma_{ct}}$ is 1.19, ET_s = 1.19 and P = 27%, and in the case $\sigma_{ct}/\overline{\sigma_{ct}}$ is 0.79, ET_s = 0.79 and P = 57%. As in this case, the probability of thermal crack occurrence greatly varies according to the scatter in the quality of concrete used even if the same concrete is used. Thus, it is considered that the scatter in the quality of concrete used in construction gives a great effect on the probability of thermal crack occurrence, which complicates the phenomenon of thermal crack occurrence and makes dealing with thermal crack occurrence on probability basis an effective method.

As described above, the probability of thermal crack occurrence is greatly affected by the method of evaluation of the degree of restraint and tensile strength, and therefore, it is considered necessary when evaluating thermal crack occurrence, to calculate the degree of restraint and tensile strength by the method used in the preparation of the probability diagram.

6. THE EFFECT OF CALCULATION METHOD ON THE RELATION BETWEEN THERMAL CRACK INDEX AND THE PROBABILITY OF THERMAL CRACK OCCURRENCE

Here, investigation was made as to differentiate between the two kinds of thermal crack index calculation methods, for example, the method proposed in this study and Ono's method, and their respective effects on the relation between the probability of thermal crack occurrence and thermal crack index. In order to set calculation conditions to Ono's conditions, tensile strength is put as $\sigma_{ct}=0.1\,\sigma_{cB}$. Thermal crack index by Ono's method is defined by the following equation [4]:

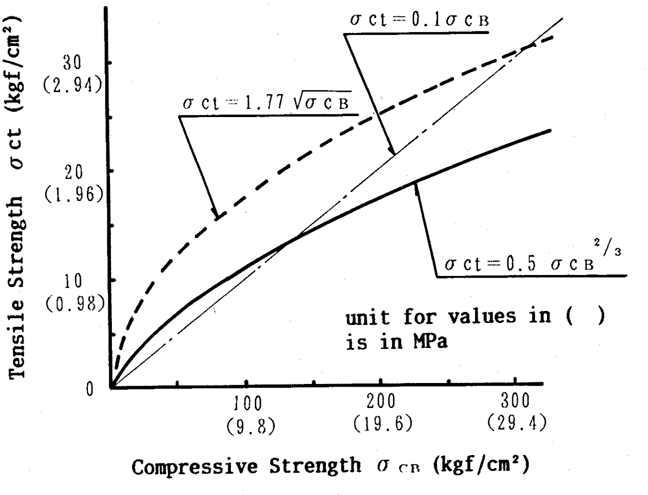


Fig. 13 Relation between the tensile strength and compressive strength

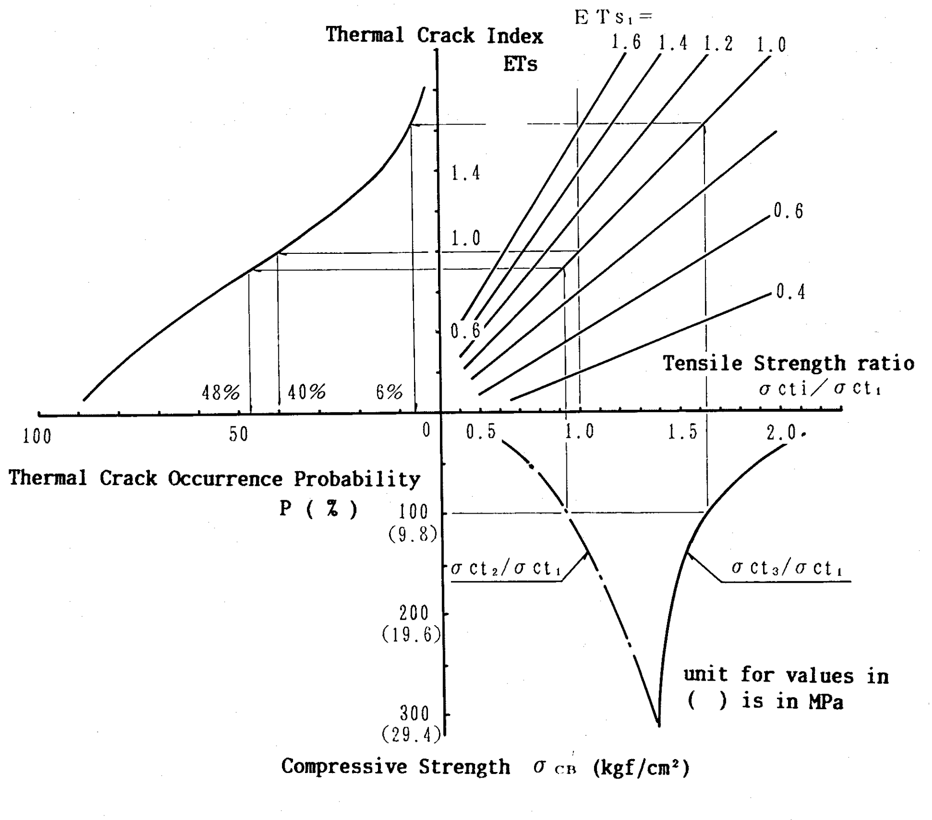


Fig. 14 Relations among the probability of thermal crack occurrence, the thermal crack indexes, the ratio of tensile strength and compressive strength

$$ET_T=\frac{(T^t_{\max})_a}{T_{\max}} \cdots\cdots (22)$$

where

$(T^t_{\max})_a$: limit temperature to thermal crack occurrence due to external restraint at age t (°C)

$$(T^t_{\max})_a=\frac{1}{3}\left[\frac{4}{10\cdot\alpha_c\cdot A(t)\cdot B(t)\cdot R}+4\,T^t_m-(T_p+T^{(t)}_{s,r})\right]$$

$T_{\max}$: maximum value of internal temperature (°C)
$A(t)$: ratio of $E_c(t)$) to $\sigma_{cB}(t)$ at age t
T_p: placing temperature (°C)
$T^{(t)}_{s,r}$: rise in surface temperature at age t when internal temperature showed maximum value (°C)

The data used in this investigation is common to those in literature [4] out of the data introduced in 4.(2). The relations between the thermal crack indexes obtained by the two methods are shown in Figs. 15 and 16 according to presence or absence of thermal cracks. In the case where cracks are present, the correlation between each thermal crack index is high, but in the case where cracks are absent, the correlation is low. It can be presumed from Fig. 15 that the histogram of ET_{s2} in the case where thermal cracks are present assumes a form of the histogram of ET_T but compressed with $ET_{s2} \fallingdotseq 0.7$ as the center, i.e., the variance becomes smaller. On the other hand, it can be presumed that in the case where thermal cracks are absent, there is no consistent tendency between ET_{s2} and ET_T . Thus the form of the histogram of ET_{s2} exhibits a larger variance when compared with that of ET_T .

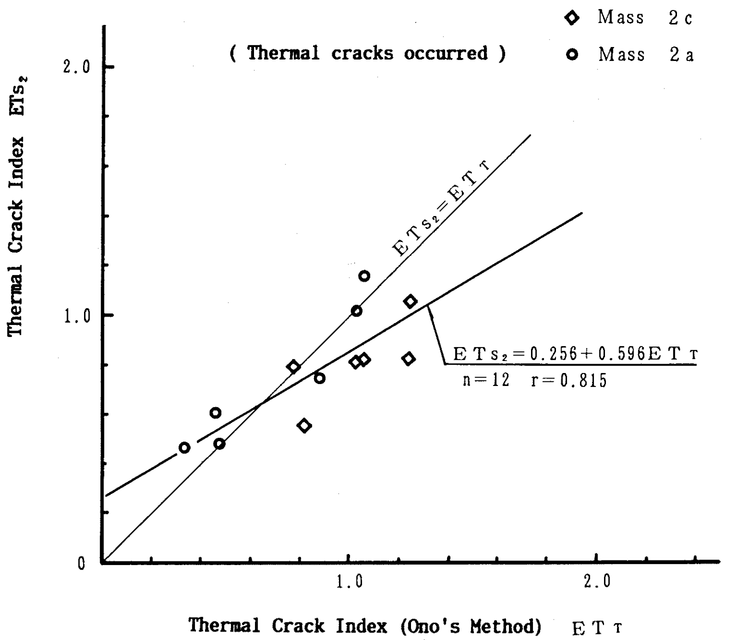


Fig. 15 Relation between thermal crack indexes obtained by different methods.

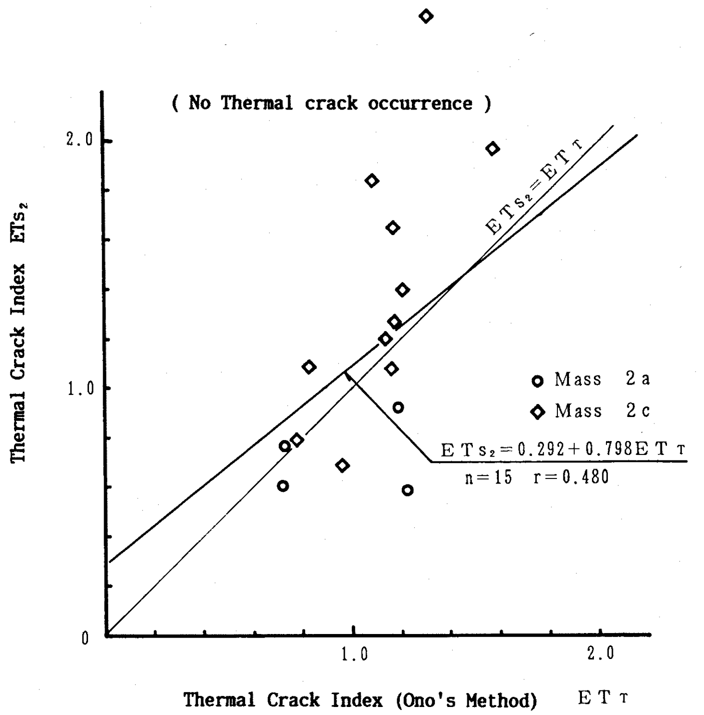


Fig. 16 Relation between thermal crack indexes obtained by different methods.

From the abovementioned, it is recognized that the relation between the probability of thermal crack occurrence and the thermal crack index varies according to the difference in the calculation methods of the thermal crack index.

7. EXAMPLES OF THE APPLICATION OF THE THERMAL CRACK INDEX METHOD

The applicability of the thermal crack occurrence evaluation method using the thermal crack index method proposed in this study was investigated by using the data obtained from Tsukayama's study of actual structures [2]. The data used in the investigation was obtained at four construction sites and are shown in Table 5. Those marked with * in the table contain supplementary explanations by the authors as the data proposed by Tsukayama were found to be deficient. The coefficient of thermal expansion, α_c , was put as $10 \times 10^{-6}/°C$ and Young's modulus was estimated from σ_{cB} by use of equation (5).

The degree of restraint was estimated from Figs. 4 and 5. The creep coefficient was calculated by the CEB/FIP method.

Table 5 shows the calculated values of the thermal crack indexes, and the probabilities of thermal crack occurrence was obtained from Fig. 11. Fig. 17 shows the correspondence of the probabilities of thermal crack occurrence to the presence or absence of thermal crack occurrence. The probabilities of thermal crack occurrence in No. 4, 6 and 7, in which thermal cracks occurred, are higher than 60%, showing a good agreement with the phenomena of thermal crack occurrence. On the other hand, the probabilities of thermal crack occurrence, in No.1, 2, 3, 5, 8 and 9, in which thermal cracks did not occur, where under 9% except for No.2, showing a good agreement with the phenomena of non-occurrence of thermal cracks. Since the results of the judgment of thermal crack occurrence by the method using the thermal crack index proposed in this study can evaluate the presence or absence of thermal crack occurrence

Table 5 Data and probabilities used in the study of applicability of the thermal crack index method

No.	Data Name	αc ($\times 10^{-6}$ 1/℃)	σc (kgf/cm²)	※ $\varphi(t)$	※ R	T^t_{max} (℃)	T^t_s (℃)	T^t_m (℃)	T_p (℃)	$T^{(t)}_{s,r}$ (℃)	T_{max} (℃)	ETs	Occurrence Probability (%)	Crack Occurrency
1	A-site slab,1st lift	10 ※	$\frac{10^2 t}{1.37+0.35t}$	$\frac{t}{2.27+1.14t}$	0.1	46 (8)	37	43.8	24	5 ※	52.5	17.21	under 5	No
2	A-site slab,2nd lift	10 ※	$\frac{10^2 t}{1.26+0.40t}$	$\frac{t}{2.56+1.07t}$	1.0	37 (30)	29	35	19	16	58.3	0.79	56	No
3	B-site slab	11.4	$\frac{10^2 t}{1.27+2.71t}$	$\frac{t}{2.44+1.01t}$	0.1	45.9 (30)	28.3	41.5	28.4	11.6	70.0	8.44	under 5	No
4	C-site retaining wall	10 ※	$\frac{10^2 t}{1.37+0.35}$	$\frac{t}{2.75+1.27t}$	0.77	10 ※ (14)	10 ※	10 ※	14	8.5	35.3	0.72	61	Yes
5	D-site lowerslab,1st lift	11.2	$\frac{10^2 t}{1.30+0.24t}$	$\frac{t}{2.19+1.15t}$	0.1	46 (7)	28	41.5	26.1	5 ※	63.1	7.39	under 5	No
6	D-site lowerslab,2nd lift	11.2	$\frac{10^2 t}{1.30+0.24t}$	$\frac{t}{2.93+1.06t}$	1.0	30 (14)	26	29	29	11	50	0.73	60	Yes
7	D-site side wall	11.2	$\frac{10^2 t}{1.35+0.18t}$	$\frac{t}{1.85+1.14t}$	0.85	33 (14)	21	29	31	16	69	0.54	74	Yes
8	D-site middle slab	11.2	$\frac{10^2 t}{1.35+0.18t}$	$\frac{t}{3.23+0.86t}$	0.4	18 (30)	18	18	31	10	56	1.55	9	No
9	D-site upper slab	11.2	$\frac{10^2 t}{1.30+0.24t}$	$\frac{t}{2.58+1.12t}$	0.3	15 (30)	15	15	19	8	57	1.72	5	No

※ estemated values values in ( ) are ages when become T^t_{max}, T^t_s, T^t_m

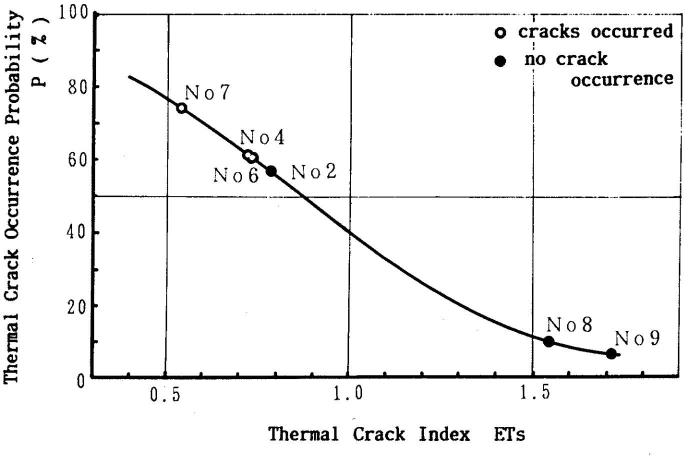


Fig. 17 Correspondence of probability of thermal crack occurrence

to presence or absence of thermal crack occurrence comparatively well, this method is considered to be a practical and effective evaluation method. Furthermore when applying this method to the study of short-time control of thermal cracks, it may be used to judge thermal crack occurrence by the use of the degree of restraint at 30 days. In the case where there is a possibility of thermal crack occurrence before 30 days, it is possible re-study thermal crack occurrence by changing the age of member needed for calculating the degree of restraint.

8. CONCLUSIONS.

This study was aimed at proposing a method of evaluating thermal crack occurrence in massive concrete structures due to external restraint from a probability viewpoint and furthermore, investigating the effects of the degree of external restraint and tensile strength on the probability of thermal crack occurrence. As the result of this study the following information was obtained:

(1) By defining thermal crack index in terms of the ratio of tensile strength to the tensile stress generated from the point of time, at which concrete temperature marked the maximum value, or by defining the probability of thermal crack occurrence in terms of the ratio of the number of data on thermal crack occurrence in a given range of thermal crack index to the total number of data in the same range, the phenomenon of thermal crack occurrence can be evaluated comparatively well by the relation between the probability of thermal crack occurrence obtained for actual structures by use of actual measurements and the thermal crack index. Thermal crack occurrence can be practically and effectively evaluated by using this relation.

(2) The method of evaluation of the degree of external restraint and tensile strength gives a great effect on the probability of thermal crack occurrence. On the other hand, the effect of the age of the member providing the degree of restraint gives little effect on the probability of thermal crack occurrence.

(3) The relation between the probability of thermal crack occurrence and the thermal crack index varies according to the calculation methods of the thermal crack index (thermal crack occurrence probability diagram). Therefore, when evaluating thermal crack occurrence, it is necessary to use the thermal crack indexes corresponding to the thermal crack occurrence probability diagram used.

Evaluation of thermal crack occurrence from the probability viewpoint in the construction stage of massive concrete structures is an effective method for the study of thermal crack control. However, in order to evaluate thermal crack occurrence with a higher accuracy hereafter, it is essential to accumulate accurate data concerning the temparature, thermal cracks and strength of many actual structures and make a higher accuracy evaluation of stresses.

REFERENCES

[1] Japan Concrete Institute: Recommendation for Repair and Reinforcement of Cracking in Concrete, May 1980.

[2] For Example, Ryuichi Tsukayama: Study on the Temperature Rise and the Thermal Cracks of Massive Reinforced Concrete Structures, Thesis for a Degree of Tokyo University, 1974.

[3] Sadamu ONO: A Study on the Evaluation Method of Thermal Crack Occurrence in Massive Concrete, Proceedings of the 36th Annual Conference of JSCE, 5 October 1981.

[4] Sadamu ONO: Study on the Thermal Crack Control of Massive Concrete Structures, Thesis for a Degree of Hokkaido University, September 1984.

[5] Norio YAMAMOTO, Haruki AKIYAMA, Yuji NAKAMOTO: The Construction of the Anchorage of Shimotsui Seto Bridge, Honshi Giho, Vol.8 No. 3, October 1984.

[6] Yasuhiko YOSHIOKA, Masahiro MOROZUMI, Shinji WATANABE: Development and Application of Computer-aided Crack Control System of Massive Concrete Structures, Proceedings of JCI 2nd Colloquium on the Mechanism of Thermal Stress Generation in Massive Concrete Structures, March,1984

[7] Yukio OSAKI, Akira SHONO, Ritsu SUGIYAMA, Akizumi KAWADA: Easy Estimation Method of Thermal Cracking in Massive Concrete Structures, JCI 7th Conference, 1985.

[8] Sadamu ONO: A Consideration of Thermal crack Control of Massive Concrete, CAI of the 33th General Meeting,1979.

[9] Japan Society of Civil Engineers: Recommendation for Limit State Design of Concrete structures, Concrete Library No. 52, November 1983.

[10] Hideo YOKOMICHI: Concrete Bridges, Gihodo Publishing Co.

[11] CEB/FIP Model Code for Concrete Structures, 1978.

[12] Kenzo SEKIJIMA: The Restraint of Foundation Piles in Massive Concrete Structure, CAI of the 36th General Meeting, 1982.

[13] Special Committee on Concrete of CAI: Long-term Strength of Concrete using Various Kinds of Cement, Cement Concrete No. 246, 1967.

[14] ACI Committee 207: Effect of Restraint Volume Change and Reinforcement on Cracking of Massive Concrete, ACI Journal, Vol 70, No. 7, July, 1973