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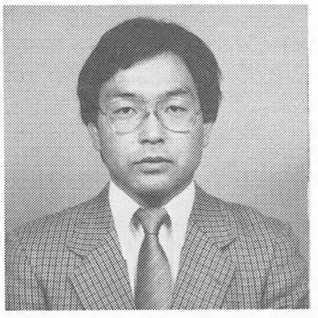
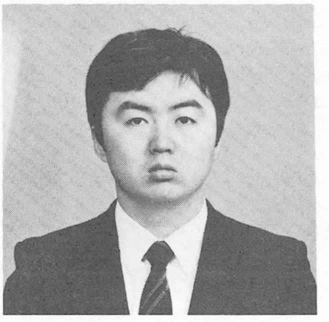
# EVALUATION OF ULTIMATE DEFLECTION OF REINFORCED CONCRETE MEMBERS





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SYNOPSIS

Cyclic loading tests were carried out using cantilever type reinforced concrete beams, in order to obtain a reliable equation to estimate the ductility, which is one of the most important properties in earthquake resistant design. The variables adopted were main reinforcement ratio, web reinforcement ratio, shear span ratio, axial compressive stress, compressive strength of concrete, number of repetitions of loading, maximum size of coarse aggregate and so on. The effects of these variables on the ductility were investigated one by one. The results were summarized to a series of equations to estimate the ductility as a term of ductility factor. It was proved that the derived equations can essentially evaluate the effects of the valuables on the ductility and can estimate the ductility factor with satisfactory accuracy, even a little modification may be needed on the effect of the maximum size of coarse aggregate and the effect of longitudinal reinforcements arranged along the side faces.


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## 1. INTRODUCTION

In seismic resistant design of reinforced concrete structures, it has been recognized that such the design concept that, for the possible maximum earthquakes, some plastic deformation is allowed although the collapse must be avoided is acceptable. In fact, some plastic deformation is being taken into account in almost of all design specifications in some forms. The most straight form of this design concept may be such that the response deformations of the members in a structure are compared with the ultimate deformation capacities of the corresponding members. At the present stage, however, any design specification has not adopted this straight form of the seismic design procedure. This is because there has not been established any reliable method to evaluate the ductility of a member subjected to alternating cyclic loading.

As a matter of fact, several researchers have tried to establish the method to evaluate the ductility of a member subjected to repeated cyclic loading. That is, T. Arakawa and et.el [1] and T. Shimazu [2] have proposed the methods to evaluate the ductility of columns in buildings, and also in the field of civil engineering structures, T. Higai and et. el [3], T. Ishibashi and et. el, [4] and the authors [5], [6] have tried to establish a method to evaluate the ductility. Since the variables which have a dominant effect on the ductility, that is, the reinforcement ratio, shear span ratio, axial compressive stress and so on, in the members in buildings are considerably different from those in civil engineering structures, however, the results given by T. Arakawa and et.el or T. Shimazu are considered not applicable to civil engineering structures, and also, none of the results by T.Higai and et. el, T.Ishibashi and et. el, or authors, though they are intended to apply to civil engineering structures, has a sufficient accuracy and reaches such a level that could be applied to the seismic resistant design of reinforced concrete structures. The reason for this can be attributed to the fact that the portion which controls the displacement of a member is different from the portion where the displacement is measured and the fact that the complicated effects of various factors on the ductility is treated inclusively as the ratio of shear strength to the flexural strength ( hereinafter this ratio is referred to the strength ratio). These problematic points were tried to be solved of course, but complete solution has not been found out yet.

The investigation reported herein was conducted to meet the situation described above and to obtain a reliable equation, with such an accuracy that can be applied to the seismic resistant design, to evaluate the ductilities of the reinforced concrete members in ordinary civil engineering structures. That is, at first, the reversed cyclic loading tests were carried out on the specimens in which the influencing factors were varied individually, and the cracking patterns, deformation characteristics, failure characteristics were investigated in detail. Then, based on the results, the problematic points in the previous studies were investigated, and the relations between each influencing factor and the ductility were formulated to numerical equations. Finally, a series of equations to evaluate the ductility, which contains all of the factors adopted in the loading tests was, formulated combining the above equations, and the reliability of the equations was evaluated using test results previously reported.

## 2. METHOD OF EXPERIMENTS

### 2.1 Specimens

The specimens used in the reversed cyclic loading tests were the ones of a cantilever type, as shown in Fig.1. The variables of the tests were tensile reinforcement ratio ($p_t$), web reinforcement ratio ($p_w$), compressive strength of concrete ($f_c'$), shear span ratio (a/d), axial compressive stress( $\sigma_0$ ), maximum size of coarse aggregate $G_{max}$ and the number of repetitions of cyclic loading (n). All specimens were designed so that the range of variations of the variables covers that of ordinary civil engineering structures. That is, $p_t$=0.59-1.66% ; $p_w$=0-0.24% ; a/d=2.5-6 ; $\sigma_0$ =0-30kg/cm2 ; $f_c'$=128-565kg/cm2 ; Gmax=5-25mm ; n=1-30. The dimensions of specimens used were as shown in Table 1, and the mechanical properties of reinforcements used were as shown in Table 2.

## 2.2 Method of Loading and Measurement

The loading apparatus and its set-up were shown in Fig.2. As can be seen from this figure, a specimen of a cantilever type was set horizontally by fixing the footing portion. The reversed cyclic load was applied vertically at the top portion of the specimen using the actuator of the maximum capacity of ±50 tf. When the axial force was to be applied, a load cell and a hydraulic jack were arranged at the top and the footing portion, respectively, and they were tightened to the specimen with four prestressing bars. In order to keep the axial force in exactly horizontal direction ( in a real structure this corresponds to the exactly vertical direction), another hydraulic jack was arranged vertically at the footing, and using this jack, the jack to apply the axial force was moved

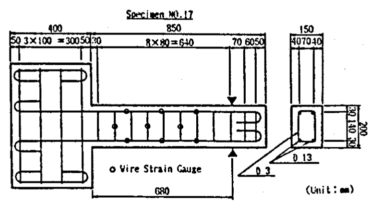


Fig.1 Dimensions of Specimens

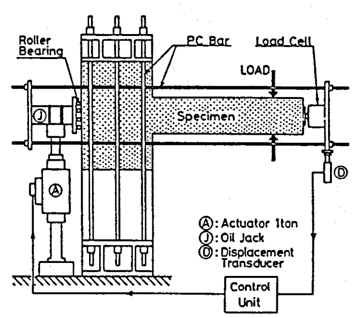


Fig.2 Testing Apparatus

Table 1 Specimens and Cracking Patterns

| No. | Pt (%) | Pw (%) | a/d | σo (kg/cm²) | fc' (kg/cm²) | Gmax (mm) | n (cycles) | μut | V·a/M | Failure Pattern |
|---|---|---|---|---|---|---|---|---|---|---|
| 5 | 1.06 | 0.12 | 4.00 | 0 | 203 | 12.5 | 10 | 5.5 | 1.38 | 2 |
| 13 | | | | | 279 | | | 4.7 | 1.43 | 2 |
| 4 | 0.59 | 0.12 | 4.00 | 0 | 408 | 12.5 | 10 | ≧7.1 | 2.19 | — |
| 3 | 0.89 | | | | 357 | | | 7.1 | 1.71 | 2 |
| 9 | 1.66 | | | | 338 | | | 3.9 | 1.22 | 2 |
| 8 | 1.06 | 0.00 | 4.00 | 0 | 413 | 12.5 | 10 | 3.6 | 1.18 | 3 |
| 7 | | 0.08 | | | 400 | | | 4.9 | 1.37 | 2 |
| 6 | | 0.23 | | | 318 | | | 7.5 | 1.76 | 1 |
| 2 | 1.06 | 0.12 | 3.00 | 0 | 309 | 12.5 | 10 | 3.8 | 1.17 | 2 |
| 11 | | | 5.00 | | 389 | | | 6.5 | 1.75 | 1 |
| 12 | | | 6.00 | | 363 | | | ≧4.3 | 2.01 | — |
| 14 | 1.06 | 0.12 | 4.00 | 10 | 294 | 12.5 | 10 | 4.0 | 1.31 | 2 |
| 15 | | | | 20 | 301 | | | 3.4 | 1.21 | 2 |
| 1 | 0.89 | 0.12 | 3.00 | 0 | 330 | 12.5 | 10 | 6.5 | 1.39 | 2 |
| 10 | 1.66 | | 5.00 | 0 | 376 | | | 4.2 | 1.47 | 2 |
| 16 | 1.06 | | 3.00 | 10 | 307 | | | 3.3 | 1.07 | 3 |
| 3 | 0.90 | 0.12 | 4.00 | 10 | 330 | 12.5 | 10 | 4.5 | 1.41 | 1 |
| 1 | 0.99 | 0.12 | 4.00 | 0 | 308 | 12.5 | 10 | 5.6 | 1.55 | 1 |
| 2 | | | | 5 | 298 | | | 5.8 | 1.46 | 1 |
| 4 | | | | 20 | 321 | | | 4.3 | 1.28 | 1 |
| 5 | | | | 30 | 326 | | | 4.2 | 1.19 | 3 |
| 6 | 0.99 | 0.12 | 4.00 | 10 | 335 | 5.0 | 10 | — | 1.42 | 2 |
| 7 | | | | | 254 | 25.0 | | 4.0 | 1.36 | 2 |
| 8 | 0.99 | 0.12 | 4.00 | 10 | 565 | 12.5 | 10 | 4.3 | 1.53 | 1 |
| 12 | | | | | 140 | | | 5.1 | 1.22 | 2 |
| 10 | 0.90 | 0.12 | 4.00 | 10 | 327 | 12.5 | 1 | 6.0 | 1.41 | 1 |
| 11 | | | | | 323 | | 3 | 4.9 | 1.41 | 1 |
| 9 | | | | | 319 | | 30 | 4.3 | 1.41 | 1 |
| 13 | 0.99 | 0.12 | 2.50 | 10 | 337 | 12.5 | 10 | 4.3 | 1.02 | 3 |
| 14 | | | 5.50 | | 348 | | | 4.6 | 1.82 | 1 |
| 15 | 0.99 | 0.24 | 4.00 | 10 | 128 | 12.5 | 10 | 5.4 | 1.53 | 1 |
| 16 | | 0.12 | | | 128 | | | 4.4 | 1.21 | 2 |
| 17 | | 0.06 | | | 128 | | | 3.5 | 1.05 | 2 |

Pt:Tensile reinforcement ratio, Pw:Web reinforcement ratio, a/d:Shear span ratio
$\sigma_0$:Axial compressive stress, fc':Compressive strength of concrete,
μut:Ductility factor ratio, V·a/M:Ratio of shear strength to flexural strength,
Gmax:Maximum size of coarse aggregate, n:Number of repetitions
Failure patterns.See Fig.3

Table 2 Mechanical Properties of Reinforcing bars

| Type | | Elastic Modulus (ton/cm²) | Yielding Stress (kg/cm²) | Yielding Strain (μ) | Ultimate Stress (kg/cm²) | Area of Reinforcement Provided (cm²) |
|---|---|---|---|---|---|---|
| SD30,D10 | | 1500 | 3650 | 2440 | 5380 | 0.7133 |
| SD30,D13 (*) | A | 1490 | 3960 | 2480 | 5470 | 1.267 |
| | B | 1630 | 3800 | 2330 | | |
| | C | 1830 | 3840 | 2100 | | |
| SD30,D16 | | 1710 | 3580 | 2090 | 5910 | 1.986 |
| SD30,D3 (*) | A | 1350 | 2740 | 2030 | 5690 | 0.06905 |
| | B | 1450 | 2400 | 1650 | 4160 | 0.07280 |
| | C | 1450 | 2540 | 1750 | | |

Note (*) : Type A is used for Specimens of Series 1
Type B is used for Specimen No.1～14 of Series 2
Type C is used for Specimen No.15～17 of Series 2

avertically by the amount equal to the displacement of the specimen.

The load was increased monotonously, controlling the actuator by the magnitude of the load until the yield load, and the measured displacement at the yield load was defined as the yield deflection $\delta_y$. The yield load is a calculated one by using the elastic theory and assuming the ratio of Young's modulus n=15, at which the stress in the main reinforcements reaches the actual yield point. When the measured strain in the main reinforcements at the fixed end had reached the yield strain before the load reached the yield load, however, the displacement when the measured strain reached the yield strain was defined as the yield deflection. After the load reached the yield load, the deflectio-ns of the integral multiples of the yield displacement, that is $\pm\delta y, \pm 2\delta y$, $\pm 3\delta y$, were applied cyclically by controlling the actuator by the magnit-ude of displacement. The number of repetitions at a certain deflection was the predetermined one.

In the experiments, the displacement due to pulling out of the reinforcemen-ts from the footing was measured using four displacement transducers, in addi-tion to the load and the deflection at the loading points. The deformation characteristics in the stem of a specimen was also investigated in detail measuring displacements of at most thirty points by spring type displacement transducers.

## 3. CRACKING AND DISPLACEMENT CHARACTERISTICS OF REINFORCED CONCRETE MEMBERS SUBJECTED TO REVERSED CYCLIC LOADING AND APPLICABILITY OF RESOLUTION OF DISPLACEMENT TO EVALUATION OF DUCTILITY

### 3.1 Cracking Patterns

Almost of all specimens lost the load carrying capacities showing features of diagonal tension failure or shear-compression failure after flexural yielding occurred. The cracking patterns changed as follows during the loading stage between the yield displacement ( hereinafter, the yield displacement will be denoted as dy) and the ultimate stage. That is, at dy, only flexural cracks occurred, and the ones near the footing ( the boundary between the footing and the stem will be called as the fixed end, hereinafter) developed through the whole sections; when the displacement became larger than 2dy, diagonal cracks occurred gradually from the tips of the flexural cracks already occurred, and two of them, which developed from the top and bottom surfaces of the stem became to show x-shaped pattern due to the reversed loading. The cracking patterns after this stage could be divided into three categories, as shown in Fig.3. That is, the pattern in which a x-shaped diagonal crack occurs and the deterioration is concentrated to this cracked region (pattern 1); the pattern in which several x-shaped diagonal cracks occur and cover concrete is spalled off in a wide region (pattern 2); the pattern in which one of the diagonal cracks is widened without spalling off of cover concrete and the features of diagonal tension failure is shown (pattern 3). The cracking patterns of each specimen were shown in Table 1. As shown in this table, the failure patte-rns varied depending on the relation between the magnitude of a working shear force and that of a shear streng-th, and show the tendency changing from the pattern 1 to the pattern 3, as the acting shear force becomes relatively larger, that is, the strength ratio becomes smaller.

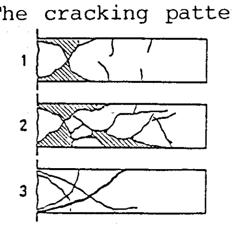


Fig.3 Cracking Patterns

### 3.2 Index of ductility

In this investigation, ductility factor, that is, the ratio of ultimate displacement to yield displacement, was adopted as a qualitative index of ductility of reinforced concrete members. This is because the ductility factor is the most direct index of ductility in specimens of a cantilever type. In determining the ductility factor, a problematic point, that is, how to determine the ultimate displacement, arises though the yield displacement can be determined relatively easily, as described in 2.2. The authors have already reported [6] that it is reasonable under the static loading to define the ultimate displacement as the limit displacement where, on the restoring force-displacement envelope curve, the restoring force does not become smaller than the one at the yield displacement, from such a point of view that ultimate displacement of reinforced concrete members subjected to reversed cyclic loadings should be defined as the displacement where a severe damage including the diagonal cracks is observed from the external appearance and, at the same time, restoring force is greatly reduced. The studies were proceeded following this definition at first. In some specimens, however, it was observed that the restoring forces, which had been lowered to a value slightly smaller than the one at the yield displacement just after the displacement had exceeded the yield displacement, was maintained till a greatly larger displacement without showing any severe damages. It is not reasonable to apply the above definition to such specimens because the defined ultimate displacement is greatly smaller than the displacement at which it is regarded from the external appearance that the specimen reached the ultimate state. The study was carried out to establish a reasonable definition which satisfies the regulations described above and, at the same time, includes these special cases. It was concluded from the result that the reasonable definition is " the limit displacement at which the restoring force does not exceed 80% of the maximum value". The ultimate displacements were determined and the ductility factors were calculated using this definition afterwards. The ultimate displacements based on this definition gave almost the same values as the ones based on the definition described above except the cases of the restoring force - displacement curves of the special shape described above.

### 3.3 Resolution of displacement

The displacement at the tip of specimens was resolved into several components shown below, to investigate the deformation characteristics and to make clear whether it is proper or not to express the ductility in terms of the displacement at the tip. That is, firstly, the total displacement dy was resolved into displacement due to pulling-out of reinforcements ($\delta_4$) and the displacement of the stem ($\delta_0$); secondly, $\delta_0$ into the displacement of the portion severely damaged with diagonal cracks ($\delta_2$), the displacement of the portion slightly damaged with flexural cracks only ($\delta_1$) and the displacement due to slip at the fixed end ($\delta_3$); finally, $\delta_0$ into the flexural displacement $\delta f$ and the shear displacement $\delta s$, where the flexural displacement means the displacement which causes displacement angle, and the shear displacement means the displacement which does not cause the displacement angle and includes displacement due to slip between the cracks. The measured pulling-out of the reinforcements and relative displacements of each part of specimens were used in the resolution.

### 3.4 Results of resolution and possibility of its application to the evaluation of ductility

An example of relations between total displacement and various displacement components is shown in Fig.4. As shown in this figure, $\delta_3$ shared a little in the whole displacement in all specimens, and did not increase rapidly at ultimate stage. It may be concluded from this figure, though $\delta_3$ has not been noticed in previous studies, that it dose not affect the ultimate stage greatly. Also, the displacement $\delta_4$ is considered to have no effect on the ultimate stage essentially. Therefore, the relations between d1 and $\delta_2$ or between $\delta f$ and $\delta s$ were investigated afterwards.

It has been tried in several studies [4] - [6] to evaluate the ductility by noticing the behavior of $\delta_3$, which has a dominant effect on the ultimate stage. These attempts were based on the fact that there is a linear relationship between the strength ratio, which is the ratio of shear strength to flexural strength, and the ductility expressed in terms of $\delta_3$. They have not been necessarily succeeded though the concept is considered to be reasonable as a principle. As for the reason for this, the authors have pointed out several facts [6]. That is, though the length $L_2$, where $\delta_2$ is caused, had been assumed to be constant value in the previous studies, but it is not correct; there are many factors which affect $L_2$, and there is roughly linear relationship between $L_2$ and the strength ratio; it can be expected that the preciseness of evaluation by the attempt of this kind will be improved if $L_2$ is determined reasonably. Based on these, the ductilities were calculated using the linear equation between $L_2$ and the strength ratio, which was determined, based on the test data obtained in this investigation. It was indicated from the results that the preciseness was not improved unexpectedly. The reason for this is considered to be the assumption, the linear relationship between $L_2$ and the strength ratio, is an over-simplified one. That is, as can be predicted from Table 1 and 3, $L_2$ is affected by very many factors and it may be impossible to treat their effects inclusively as the strength ratio. Therefore, it can be concluded that the attempt to estimate ductility based on d2 and $L_2$ could not easily be come out well.

New attempt to evaluate the ductility was studied at the next step, in which the displacement characteristics, which governs the ultimate state, were made clear by investigating $\delta f$ and $\delta s$. The trend of $\delta f$ and $\delta s$ of the specimens, in which only $\sigma_0$ was varied, was shown in Fig.5, as an example of the results. It can be recognized from this figure that, regardless values of $\sigma_0$, $\delta s$ increased accompanied by the increase of the displacement but the behavior of $\delta f$ was greatly influenced by the value of $\sigma_0$, That is, it increased in the specimens of smaller $\sigma_0$, and decreased in the specimens larger $\sigma_0$. Since the increase of $\delta s$ accompanied by the increase of displacement was recognized in all specimens, it was expected that the equation to calculate the ductility could be established if $\delta s$ was evaluated properly. However, it was also indicated that the ultimate state of members was not always governed by $\delta s$ but

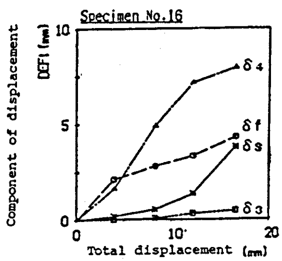
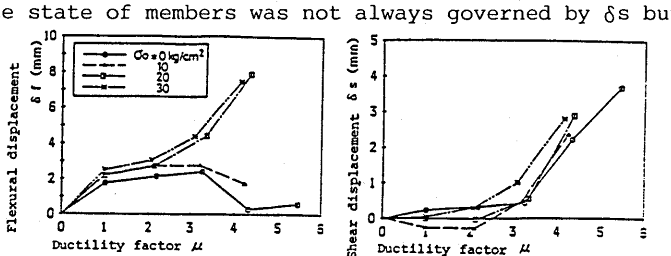



Fig.4 An Example of Relations between Total Displacement and Displacement Components




(1) Flexural Displacement (2) Shear Displacement

Fig.5 An example of Relations between df and ds sometimes governed δy df. In order to evaluate the ductility using δf or δs, therefore, it is needed to determine which of δf and δs governs the ultimate state, considering the cracking patterns or combinations of governing factors. Within the limit of the experiment, however, any special relations could not be found out. Therefore, the authors could not help interpreting as it is very difficult to evaluate the ductility by this procedure.

The procedure, in which the ductility is evaluated by the displacement components chosen as governing the ultimate state considering the displacement characteristics, was considered essentially reasonable because it is intended to model the actual phenomena faithfully. As described above, however, it could not be expected fruitful results from this procedure to obtain a precise equation to calculate the ductility because the displacement component, which governs the ultimate state, is affected by various complicated factors. Therefore, it is interpreted that the displacement must be treated inclusively as the displacement at the tip of members of a cantilever type though there are many questionable points remained essentially.

## 4. EFFECT OF VARIOUS FACTORS ON DUCTILITY OF REINFORCED CONCRETE MEMBERS AND ITS FORMULATION

In order to make clear whether it is proper or not to express the effects of various factors on the ductility inclusively by the strength ratio and if necessary, to propose a new method, the relations between each adopted factor and the ductility were investigated one by one, based on the test results described above. It was indicated from the results, if the conclusion is shown beforehand, that the inclusive expression of the effect of various factors on the ductility is immoderate and the formulations of each factor are necessary. Afterwards, therefore, the effects of each factor on the ductility factor will be discussed qualitatively, and the results of formulation will be described. In the formulation procedure, the results by M. Ohta and T.Higai and et.el were used as well as the ones by authors.

### 4.1 Effect of tensile reinforcement ratio pt

It was indicated from the results in which only $p_t$ was varied keeping the other factors constant that the ductility factor is smaller, the larger $p_t$. When $p_t$ becomes large, the increase of flexural strength is much larger than that of shear strength though both of them increase, and the shear strength

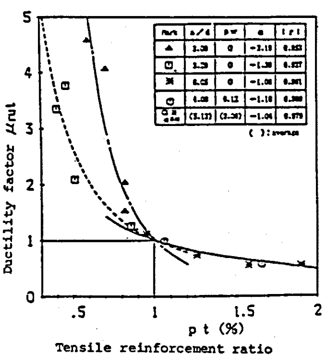


Fig.6 Relation between pt and mrut

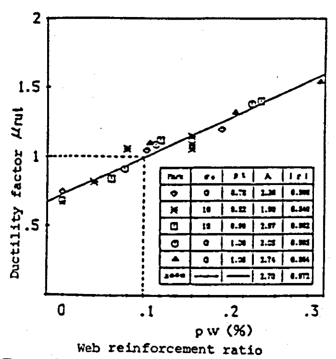


Fig.7 Relation between pw and mrut

to the flexural strength. Therefore, the ductility factor becomes smaller for larger $p_t$.

Figure 6 was drawn to formulate the relation between the ductility factor and $p_t$, and indicates the relations between them in terms of the ratio of the ductility factor for an arbitrary $p_t$ to that for $p_t$=1%. According to this figure, it is recognized clearly that μrut increased rapidly in the range of $p_t \leq 1\%$. Although it coincides qualitatively with the relations between the strength ratio and the ductility factor previously established that the ductility factor increases when $p_t$ decreases, it is questionable whether such a rapid increase as shown in Fig.6 is taken into account in the previous equations. From Fig.6,

$$\beta_t = \mu_{rut} - 1 = (p_t)^{\alpha} - 1 \quad \cdots\cdots (1)$$

where, βt is a coefficient which expresses the effect of $p_t$ on the ductility factor, and equals to 0 when pt = 1%; a is a constant depending on $p_w$ and a/d. The variations of a accompanied by the variations of $p_w$ and a/d were investigated using the least mean square method with the result shown in Fig.6, and it was indicated that the effect of $p_w$ is negligible and a decreases hyperbolically when a/d decreases in the range less than 4.0 though it is almost constant in a/d≧4. Based on this result,

$$\alpha = -0.146/(a/d-2.93) - 0.978 \quad \cdots\cdots (2)$$

The reasonable applicable range of this equation is a/d ≧ 3.0 because the tests were carried out in the range of a/d ≧ 3.0. Since a/d of members in general civil engineering structures is almost within this range, this limit of application may not cause any serious problem.

## 4.2 Effect of pw

The ductility factor increases when $p_w$ increases keeping the other factors constant. In order to investigate this relation qualitatively, the ratio of μurt to that for $p_w$ = 0.1% was calculated. The result was as shown in Fig.7, and it was recognized that the relations between them was almost linear. That is,

$$\beta_w = \mu_{rut} - 1 = 2.70\,(p_w - 0.1) \quad \cdots\cdots (3)$$

where $\beta_W$ is a coefficient to express the effect of $p_w$. The relations between $\beta_W$ and $p_w$ are affected by $\sigma_0$, $p_t$, a/d and so on, but the the effects of $\sigma_0$ and $p_t$ were neglected because their effects were small as shown in Fig.7. As for a/d, its effect on μurt-$p_w$ or βw-pw relations could not be made clear because only the data of a/d = 4.0 were available.

## 4.3 Effect of a/d

It was indicated from the envelop curve of restoring force displacement hysteresis curves (hereinafter, it will be referred to the envelop) when only a/d was varied within 3 - 5, keeping the other factors constant that the ductility factor becomes smaller for smaller the a/d.

Generally speaking, the shear strength is larger, the smaller a/d. The reason why the above result was obtained in spite of this fact is considered to be that the working shear force becomes larger, the smaller a/d and the increase of the shear force is larger than that of the shear strength, when the working bending moment at the fixed end of members such as the tested specimens is not changed. This coincides with the fact on the shear strength previously recognized, that is, in the range of a/d ≧ 3, the strength ratio is larger, the larger a/d is.

In order to formulate the effect of a/d on the ductility described above, the relation between a/d and $\mu rut$, which was normalized by the one for a/d = 4, was derived from the results of $\sigma_0$ = 0 kg/cm2 and 10 kg/cm2 as well as the ones described above. The result was as shown in Fig.8, and it was indicated that $\mu urt$ increases accompanied by the increase of a/d and keeping the linear relationship, but the degree of the increase is smaller, the larger 0. The reason for this was revealed as follows from the investigation on the relation between $\delta f$ and $\delta s$, cracking patterns and so on. That is, if 0 does not work in members of small a/d, failure is initiated by the decrease of the capacity of shear transfer at the region between the shear cracks and if $\sigma_0$ works, the ductility is increase, the capacity being increased; whereas, if $\sigma_0$ does not work in members of large a/d, the rotation of the plastic hinge at the x-shaped cracks near the fixed end shares large part of the displacement and if $\sigma_0$ works, the ductility could not be increased because of tendency of buckling of compression reinforcements and spalling-off of cover concrete due to compressive stress; as a result, the change of the ductility factor accompanied by the the change of a/d becomes small when $\sigma_0$ works. The fact described above could not be expressed by using the strength ratio, and this is a typical example such that it is not proper to express the ductility factor as a function of the strength ratio.

It can be concluded from the above experimental evidence that $\sigma_0$ does affect the slope of the linear relation between murt and a/d, but the effect could not be made clear quantitatively. Therefore, it has no choice to assume the linear relationship to formulate the effect of $\sigma_0$ on the ductility factor. The formulated equation is

$$\beta_a = \mu_{rut} - 1 = A(a/d - 4) \quad \cdots\cdots (4)$$

where

$$A = -0.0153\sigma_0 + 0.175 \qquad (\sigma_0 \leqq 11.4) \quad \cdots\cdots (5)$$

$$A = 0 \qquad (\sigma_0 > 11.4) \quad \cdots\cdots (6)$$

when $\sigma_0$ > 11.4 in eq.(5), A becomes minus because the effect of $\sigma_0$ was assumed linear. Since this does not coincide with the real phenomena, A was determined to be 0 for this case. In formulating the above equations, only the data for $P_W$ = 0.12% used. Therefore, the interaction between $P_W$ and a/d was not taken into account in the above equations. This point as well as the effect of $\sigma_0$ is the theme to be investigated further.

## 4.4 Effect of concrete strength f'c

The relation between murt and $f_c'$ was normalized by the one for $f_c'$ = 300kg/cm2 and was illustrated, based on the envelops in the specimens in which

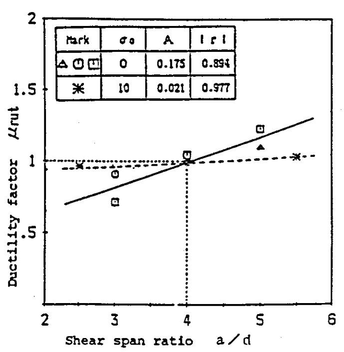



Fig.8 Relation between a/d and mrut

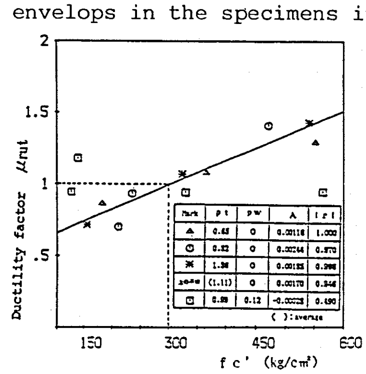



Fig.9 Relation between f'c and mrut

concrete strengths were varied within 128-565kg/cm2 keeping the other factors constant. The result was as shown in Fig.9, and it was indicated that f'c has less effect on the ductility factor if web reinforcements were arranged. The reason for this may be as follows. That is, $f_c'$ affects the range and the loading level of occurrence of diagonal cracks, and the range becomes wider and the level becomes lower for lower $f_c'$, but if the web reiforcements are arranged, the effect of $f_c'$ as described above becomes small and does not affect the ductility factor because fairly large part of working shear force is carried by the web reinforcement. Whereas, if no web reinforcements is arranged, the occurrence of diagonal cracks results in immediate decrease of load carrying capacity, and so, $f_c'$ affects the ductility factor, as shown in Fig.9. As for $f_c'$, therefore, the equation to formulate the ductility factor should be alternated, considering the existence of web reinforcement or not. This is an another example of the limit of the equation based on the strength ratio. From Fig.9,

$$\beta_c = \mu_{rut} - 1 = A\,(f_c' - 300) \quad \cdots\cdots (7)$$

where,

$$A = 0.00170 \qquad (p_w = 0\,\%) \quad \cdots\cdots (8)$$

$$A = 0 \qquad (p_w \neq 0\,\%) \quad \cdots\cdots (9)$$

## 4.5 Effect of axial stress 0

It was indicated from the results of specimens in which $\sigma_0$ was varied within 0-30kg/cm2 that the ductility factor is smaller, the larger $\sigma_0$, if the other factors were kept constant. Based on this fact, the relation between $\mu_{rut}$ and $\sigma_0$, which was normalized by data of $\sigma_0$ =10 kg/cm2, was illustrated (see Fig.10). From this figure,

$$\beta_N = \mu_{rut} - 1 = 2.15\,(\sigma_0 + 10)^{-0.260} - 1 \quad \cdots\cdots (10)$$

The preciseness of the eq.(10) was questionable because the data used to formulate the eq.(10) were the ones of a/d=4.5, $P_t$=1.0% and $P_w$ =0.1% only, but there could not be found great difference from the results by T. Shimazu [2]. Therefore, no serious problem may be remained in the preciseness of eq.(10).

## 4.6 Effect of number of repetitions (n)

It was indicated from the envelops where n was varied within 1-30 keeping the other factors constant that the ultimate displacement was smaller, the larger n and this tendency became more significant in a range of small n. The reason for this is supposed to be that the shear force carried by the cracked surfaces decrease due to abrasion caused by the increase of number of repetit-

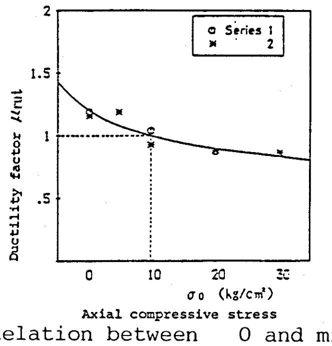


Fig.10 Relation between 0 and mrut

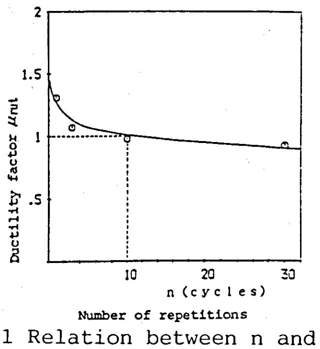


Fig.11 Relation between n and mrut

ions of shear displacement because the displacement at which $\delta s$ begins to increase becomes smaller for larger n. To formulate this, the relation between $\mu rut$ and n, which was normalized by the result of n = 10, was illustrated with the result shown in Fig.11. Since the relation between them was supposed to be able to express by hyperbolic function, the relation $\mu rut = a(n)^{\alpha}$ was assumed, and a and a was determined by the least mean square method. From the result,

$$\beta_n = 1.26 (n)^{-0.0990} - 1 \cdots\cdots (11)$$

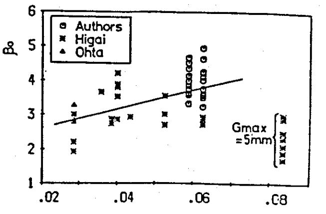


Fig.12 Relation between b0 and 1/d

In an actual earthquake, such a conditions as the loading tests, that is, integral multiples of the yield displacement is loaded repeatedly, could not occur. Therefore, it is less meaningful to include the effect of n in the equation to calculate the ductility. Since it is unquestionable that the ductility is lowered due to increase of n, however, it is feasible to design assuming n = 10. This is reasonable because it was indicated in the experiment that $\beta n$ was little varied within the range of n>10.

The effect of maximum size of coarse aggregate was also investigated. However, it may be practical to treat this effect as will be described in the next chapter because the data used was ones on the relatively small specimens. Therefore, the formulation for this effect was not carried out.

## 5 PROPOSAL OF EQUATIONS TO EVALUATE THE DUCTILITY

The following equation to evaluate the ductility is proposed in the form of total of coefficients which express the effect of each factor, taking into account of effects of all factors described above. That is,

$$\mu_u = \beta_0 \ (1 + \beta_t + \beta_w + \beta_c + \beta_N + \beta_a + \beta_n) \cdots\cdots (12)$$

This equation is formally such that only the main effect of each factor is taken up and the interactions are neglected. As described above, however, each coefficient $\beta$ includes the interactions, and so, the interactions are not neglected though the form shown above is adopted.

The coefficient $\beta o$ in the equation was introduced to express the effects of the other factors which were not taken into account. The effects which should be considered through $\beta o$ are the ones which are not included in parentheses, that is, the maximum size of coarse aggregate Gmax, the effective depth of members d, which is closely related to the maximum size, and so on. In order to investigate the effects of these factors, $\beta o$ were calculated by substituting experimentally obtained ductility factors and the coefficients $\beta t$-$\beta n$, and the relations between the calculated $\beta o$ and d or Gmax were illustrated. The result on d is as shown in Fig.12. It is recognized from this figure that the relation between $\beta o$ and 1/d was almost linear except in the case of Gmax = 5mm. Based on this fact, the relation between them was formulated by the method of least squares. The result is,

b0 = 28.4x1/d+2.03

No equation for Gmax = 5mm was formulated but the constant value of $\beta o$ = 2.33 was adopted because of limitations of numbers of specimens, which were the ones of d = 12cm only. This may cause no great problem because Gmax of 5mm is not used in an actual cases. As for the relations between $\beta o$ and Gmax, the relation between $\beta o$/(28.4/d+2.03) and Gmax was investigated, considering the interaction between d and Gmax. From the results, no close correlations were found between them, the coefficient of correlation being 0.277. This means the effect of Gmax has been taken into account through d. That is, it is not necessary to contain the effect of Gmax in the equation. If many data on d or Gmax will be accumulated, it becomes possible to treat $\beta_0$ in the same manner as the other factors.

The following equations to estimate the ductility factor of reinforce concrete members are derived by summing up all results described above.

$$\mu_u = \beta_o \ (1+\beta_t+\beta_w+\beta_c+\beta_N+\beta_a+\beta_n) \quad \cdots\cdots (12)$$

$$\beta_o = 28.4/d + 2.03 \qquad (G_{max} > 5\text{mm}) \quad \cdots\cdots (13)$$

$$\beta_o = 2.33 \qquad (G_{max} = 5\text{mm}) \quad \cdots\cdots (14)$$

$$\beta_t = p_t^{\alpha} - 1 \quad \cdots\cdots (15)$$

$$\alpha = (-0.146/(a/d-2.93) - 0.978) \qquad (a/d \geqq 3.0) \quad \cdots\cdots (16)$$

$$\beta_w = 2.70\ (p_w - 1) \quad \cdots\cdots (17)$$

$$\beta_a = (0.0153\sigma_o + 0.175)\ (a/d - 4.0) \qquad (\sigma_o \leqq 11.4\text{kg/cm}^2) \quad \cdots\cdots (18)$$

$$\beta_a = 0 \qquad (\sigma_o > 11.4\text{kg/cm}^2) \quad \cdots\cdots (19)$$

$$\beta_N = 2.15\ (\sigma_o + 10)^{-0.260} - 1 \quad \cdots\cdots (20)$$

$$\beta_c = 0.00170\ (f_c' - 300) \qquad (p_w = 0\ \%) \quad \cdots\cdots (21)$$

$$\beta_c = 0 \qquad (p_w \neq 0\ \%) \quad \cdots\cdots (22)$$

$$\beta_n = 1.26n^{-0.0990} - 1 \quad \cdots\cdots (23)$$

where, the units of d, $f_c'$, $P_t$ and $P_w$ are cm, kg/cm2, % and %, respectively.

## 6. EVALUATION OF PROPOSED EQUATIONS

The preciseness of the proposed equations were calibrated as follows. At first, the relations between every influencing factor and the ratio of the experimental ductility factors to the calculated ones were investigated. As an example of the results, a result on $P_t$ is shown in Fig.13. It can be recognized form this figure that the ratio of the experimental values to the calculated ones are in the range of 0.7-1.3 and there are no special correlations between $P_t$ and the ratio. This means the equations is properly evaluating the effect of $P_t$ on the ductility factor. The same investigations were carried out on the factors other than $P_t$. It can be recognized from the results that the equations are properly evaluating the effects of various

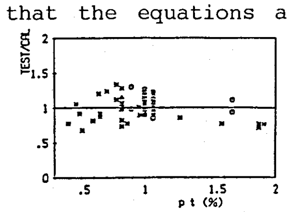



Fig.13 Relation between Ductility and pt

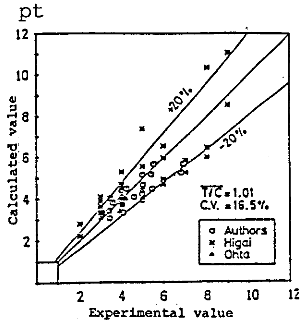



Fig.14 Evaluation of Proposed Equations

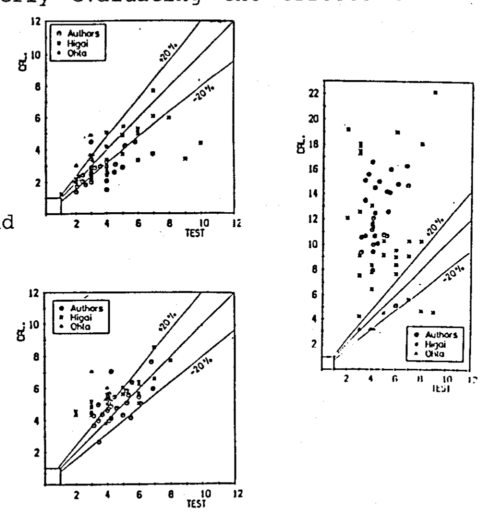



Fig.15 Evaluation of Previous Equations

influencing factors on the ductility except the case of $p_w$ = 0%. In the case of $p_w$ = 0%, it could be hardly drawn the same conclusions as the other factors because the scatterings of the ratio of the experimental values to the calculated ones were relatively wide. This may be caused by the fact that no interaction between a/d and $p_w$ were taken into account, as described in the previous chapter.

As a next step, the ductility factors estimated by the equations were compared with the ones obtained by the experiment using all of the data. The result was as shown in Fig.14, and it was recognized that the errors of the estimation were in the range of ±20% for almost all specimens and the preciseness was almost constant regardless of the magnitude of the ductility factor. In addition, the average of the ratios of the experimental values to the estimated ones was 1.01 and the coefficient of variation was 16.5%. The results of the same estimations by the previously proposed equations were as shown in Fig.15, and it can easily recognized from this figure that the result by the proposed equations is much more precise than the ones by the previous equations.

From the evaluations described above, it was obvious that, as for the deviations of data used for the formulation of the equations, there is no serious problem. Therefore, the equations were evaluated using the data which were not used in the formulation. The data were ones by Institute of Public Works, Ministry of Construction, Japanese Government [9] and the ones by Y. Osaka and et.el [10]. The reasons why these data were used were such that these data included the ones on relatively large specimens, that is, the effective depths of 80-40cm and on specimens in which reinforcements were arranged along the side faces (these reinforcements will be called side reinforcements hereinafter). The data also include the ones on specimens subjected to dynamic loads, of which velocity was up to 70cm/s, as well as the static loads.

The ductility factors obtained by the data described above were compared with the ones calculated by the proposed equations. The result was as shown in Fig.16. According to this figure, it is recognized that the calculated values agreed well with the experimental ones though the formers were a little larger than the latters. The average of the ratios of experimental values to the calculated ones for all specimens was 0.91, and the coefficient of variation was 15.7%. These values also indicate the equations gave proper results. However, it was found from further investigations on the results on three statically loaded specimens with no side reinforcements that the average of the ratio of the ductility factors by the experiments to the ones by the estimation was considerably small, that is, 0.83. These specimens were almost same as the ones used in formulation of the equations except the dimensions. Therefore, this result indicates the equations give unsafe

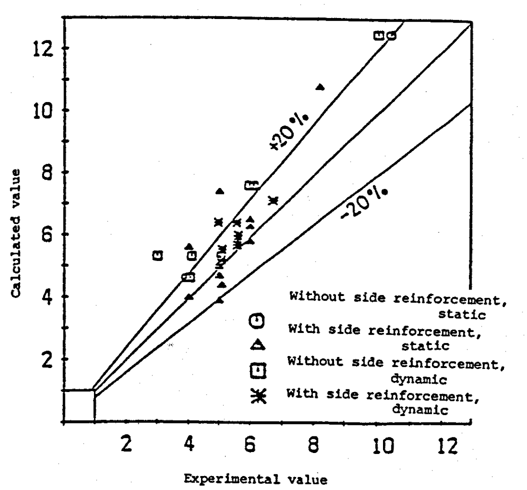


Fig.16 Evaluation of Proposed Equations by Previous Data on Larger Specimens

side estimates for members of larger effective depth. In addition, it was found from the calculations of the ratios of ductility factors of the specimens with and without side reinforcements that the ratios of the specimens with and without side reinforcements were 0.95 and 0.81, respectively, and though the proposed equations did not take into account the effect of side reinforcements, they gave estimates nearer to the experimental values for the specimens with side reinforcements than the ones without. It may be appropriate to conclude that a unsafe side estimations given by the proposed equations for the members of larger sections were canceled by the increase of ductility due to the effect of side reinforcements.

As described above, in the proposed equations, some problems on the applicability to members of larger sections are remained, and the effect of side reinforcements could not be taken into account. However, the fact that the proposed equations gave the ductility factors with the accuracy of 15.8% of coefficient of variation for the specimens greatly different from the ones used in the formulation indicates that the proposed equations could evaluate the effects of various factors on the ductility essentially correctly, and they could be served for practical uses if some modification is made, if necessary.

## 7. CONCLUSIONS

The investigations were carried out to make clear the ductility of reinforced concrete members, which is very important to evaluate the earthquake resisting capacity of the structures, and the equations to evaluate the ductility were proposed, based on the cracking patterns and the displacement characteristics of specimens subjected to the reversed cyclic loadings and adopting the ductility factor as an index of ductility. Within the limits of the investigations, the followings can be concluded.

(1) As a method to evaluate the ductility of reinforced concrete members, there is a method in which equations to estimate the ductility factor are formulated by decomposing the displacements of members and extracting the components of the severely damaged part with diagonal cracks, which govern the ultimate state of the member. This method is superior in its concept per se. To adopt this method to evaluate the ductility, however, it becomes necessary to estimate the length of the severely damaged portion as well as the displacement component in this portion. Besides this procedure is very complicated, it is very difficult to estimate the both of them accurately because they are affected by very many factors. Therefore, a fruitful result can not be expected from this method.

Instead of this method, there can be considered another method in which the total displacement is resolved into flexural and shear displacements, and using one component which gives more dominant effect on the ultimate state, the equation is formulated. The result of the investigation on this method showed that the resolution of the displacement is not effective to evaluate the ultimate displacement though it is effective to make clear the flexural behaviors.. This is because which and how each component arouse the ultimate state is governed by a little change of various influencing factors. Therefore, It can be concluded that a fruitful result could not be expected in the attempt to evaluate the ductility by resolving the total displacement into such components that one of them has a dominant effect on the ultimate state.

(2) There have been many examples of a method to estimate the ultimate displacement using the relation between the ductility factor and the ratio of the

shear strength to the flexural strength, in which the effects of many factors are expressed inclusively by the ratio. This method is based on the fact that the ductility factor is larger, the larger the ratio, and the relation between them has been recognized from experience to be almost linear. To obtain a precise equation to estimate the ductility factor, the degrees of effects of various factors on the ratio should be nearly equal to the effects on the ductility factor. From the result of detailed investigation on the relation between the displacement characteristics of members and the various influencing factors, however, it was found that the above requirement were not necessarily satisfied. For example, the ductility factor became larger for smaller tensile reinforcement ratio, but the degree is not so significant as could be expected from the strength ratio; the effect of a/d is varied depending on the magnitude of axial stress. As for the compressive strength of concrete, it was indicated that the shear strength became larger for higher compressive strength regardless of existence of hoops or not, but the ductility factor was little affected by the concrete strength if the hoops were arranged. All of these facts indicate it is essentially improper to express the effects of various factors inclusively by the strength ratio, though it could be allowed if used as an approximation. In order to obtain a precise equation to estimate the ductility factor based on actual phenomena, therefore, it is concluded to be proper to investigate the effects of each factor one by one and to formulate the equation based on the results.

(3) The displacement at the tip of specimens of a cantilever type was adopted as the target function, following the facts described in (1), and the relations between the ductility factor and each influencing factor were formulated one by one, following the facts described in (2) and using data obtained from in this investigation and previous ones. The results were summarized to a series of equations to estimate the ductility factor. These equations include, as influencing factors, tensile reinforcement ratio, web reinforcement ratio, shear span ratio, axial stress, concrete strength, maximum size of coarse aggregate and number of repetitions of loading. The preciseness of the equations were investigated using the data which were used to formulate the equations. From the result, it was recognized that the ductility factors given by the equations were precise enough because the average of the ratios of the experimental value to the estimated ones was 1.01 and the coefficient of variation was 16.5%, and no wide variation of the estimated values was recognized regardless of any changes of factors. The evaluation was also carried out using data on relatively large specimens which were not used to formulate the equations. From the result, it was recognized that the effects of various factors were essentially successfully taken into account in the equations though a little modification is needed in a few points, which were out of range of data used to formulate the equations.

**ACKNOWLEDGMENT**

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**REFERENCES**

[1] T. Arakawa and et. el, "Cyclic Behavior and Evaluation of Inelastic Capacity of Reinforced Concrete Columns," Proceedings of Annual J.C.I. Meeting, 1983 (in Japanese)

[2] T. Shimazu, "On the Ultimate Values of Deformation Angle for Reinforced Concrete Columns," Transactions of A.I.J., No. 23, 1982

[3] T. Higai and et. el, "Shear Failure of Reinforced Concrete Members Subjected to Large Deflection Reversals," Proceedings of Sixth Annual J.C.I. Meeting, 1984 (in Japanese)
[4] T. Ishibashi and et. el, " Ductility Factor of Reinforced Concrete Members," Data of Structural Design, No.79, 1984 (in Japanese)
[5] K. Toyoda and et. el, "Study on Evaluation of Ultimate Deflection of Reinforced Concrete Members," The 39th Annual Meeting of J.S.C.E, 1985 (in Japanese)
[6] K. Toyoda and et. el, "Experimental Study on Evaluation of Ultimate Deflection of Reinforced Concrete Members," Transactions of J.C.I. , Vol.7, 1985
[7] T. Hiraishi and et. el , "Method to Calculate Shear and Flexural Deformations of Shear Walls", Annual Meeting of A.I.J. 1982 (in Japanese)
[8] M. Ohta, " Study on Seismic Resistant Design of Reinforced Concrete Piers of Single Column Type", Report of Public Works Research Institute, Ministry of Construction, Japanese Government, No.153, 1980 (in Japanese)
[9] Earthquake Disaster Prevention Branch, Public Works Research Institute, "Experimental Study on Dynamic Load Carrying Capacity of Reinforced Concrete Piers (1)", Research Material at Public Works Research Institute, Ministry of Construction, Japanese Government, No.2232, 1985
[10] Y. Osaka, "Study on Characteristics and Their Application to Earthquake Resistant Design of Reinforced Concrete Structures Subjected to Dynamic Random Load", Report of Grant-in-Aid for Scientific Research in 1983, 1984