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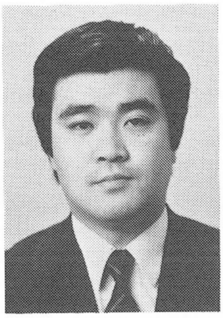
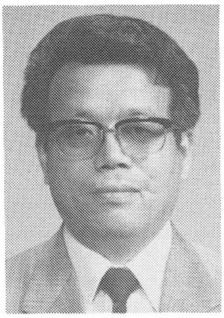
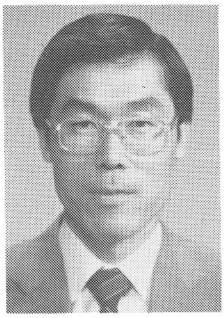
REVALUATION OF THE EQUATION FOR SHEAR STRENGTH OF REINFORCED CONCRETE BEAMS WITHOUT WEB REINFORCEMENT

(Translation from Proceedings of JSCE No.372/V-5 1986-8)


Junichiro NIWA Kazuie YAMADA Kazuo YOKOZAWA Hajime OKAMURA

SYNOPSIS

In the past, the equation for the shear strength of reinforced concrete beams without web reinforcement had been proposed, whereas the result of large-sized beam tests carried out recently revealed that the nominal shear strength was inversely proportional to the fourth root of the effective depth. Taking this fact into consideration, the proposed equation is revaluated and a new equation is derived. The validity of the new equation is verified by the authors using test results of large-sized beams subjected to a concentrated load. Finally, suggestions concerning the application of the new equation for practical design are mentioned.

J.NIWA is an associate professor of civil engineering at Yamanashi University, Kofu, Japan. His research interest is in strength, deformation and design of reinforced concrete members under shear and torsion. He is a member of JSCE, JCI, IABSE and ACI.

K.YAMADA is the director of Technical Research Institute of Maeda Construction Company, Tokyo, Japan. His research interest is in the design and construction of reinforced concrete containments and mass concrete. He is a member of JSCE and JCI.

K.YOKOZAWA is a chief research engineer of the section of concrete engineering at Technical Research Institute of Maeda Construction Company, Tokyo, Japan. His research interest is in the design and construction of reinforced concrete structures. He is a member of JSCE and JCI.

H.OKAMURA is a professor of civil engineering at the University of Tokyo, Tokyo, Japan. His research interest is in the application of finite element method of analysis to reinforced concrete structures. He is a member of JSCE, JCI and IABSE and a fellow of ACI.

1. INTRODUCTION

For the shear strength of a reinforced concrete (RC) beam without web reinforcement and having the ratio of the shear span to the effective depth "a/d" more than 2.5~3.0 which would fail immediately after generation of a diagonal crack, the equation (1) was proposed in 1980 [1]. And the design equation for such beams based on that equation was adopted by JSCE as "Recommendation for limit state design of concrete structures" [2].

$$fv=0.20\ fc'^{1/3}\ (1+\beta p+\beta d)[0.75+1.4/(a/d)] \qquad (1)$$

where, fv: ultimate shear strength (MPa), fc': compressive strength of concrete (MPa), a: shear span, d: effective depth, bw: breadth of web, As: cross-sectional area of tensile reinforcing bars

$$pw=100\ As/(bw\ d),\quad \beta p=\sqrt{pw}-1 \leq 0.732$$
$$\beta d=d^{-1/4}-1,\ d[m]$$

Eq.(1) is derived from the data of the experiments that have been carried out in the past both in Japan and abroad; the outline of which is presented in Table 1 classified by the authors' name. As seen in Table 1, most of these data are for the beams of around d= 0.1~0.5 m and those within around the range of pw $\geq$ 0.5%. Especially, data for the beams of pw < 0.5% and d > 1 m are nearly nil.

However, among the RC structures such as footings and culverts whose cross section is determined solely by the shear strength (fv) of concrete for without using web reinforcement, cases of pw < 0.5% and d > 1 m are rather common.

Accordingly, some equations which would give high accuracy over these ranges are desirable for the purpose of design. While eq.(1) takes into consideration the influence of so-called scale effect in the form of βd, the data supporting the equation are mostly those for the cases of d $\leq$ 1 m. Therefore, whether eq.(1) could be applied for the purpose of estimating the shear strength with satisfactory accuracy or not would be subject to further study.

Table 1 Outline of the experimental data in the past (values in the table indicate the range of parameters, figures in the parentheses are the average value)

Researcher	No. of data	fc' (MPa)	pw (%)	d (m)	a/d
Aster	5	28 ~ 28 (27.9)	0.4 ~ 0.9 (0.62)	0.50 ~ 0.75 (0.55)	3.70
Chang	12	31 ~ 39 (33.0)	1.0 ~ 2.9 (2.38)	0.137	3.72
Diaz de Cossio	22	14 ~ 29 (23.8)	1.9 ~ 2.9 (2.18)	0.08 ~ 0.17 (0.11)	4.00
Higai	7	31 ~ 37 (32.9)	2.39	0.160	3.5 ~ 6.5 (4.36)
Kani	44	18 ~ 35 (27.0)	0.5 ~ 2.9 (2.13)	0.13 ~ 1.10 (0.39)	2.6 ~ 8.0 (4.36)
Krefeld	53	12 ~ 39 (24.6)	1.1 ~ 4.5 (3.06)	0.24 ~ 0.48 (0.26)	3.6 ~ 8.5 (5.28)
Leonhardt	18	30 ~ 39 (35.5)	1.3 ~ 2.1 (1.71)	0.07 ~ 0.60 (0.25)	3.0 ~ 5.9 (3.59)
Malthey	7	24 ~ 31 (26.2)	0.5 ~ 0.9 (0.75)	0.403	2.8 ~ 3.8 (3.37)
Mattock	6	16 ~ 47 (24.3)	1.0 ~ 3.1 (2.24)	0.254	3.0 ~ 5.4 (4.20)
Moody	24	12 ~ 41 (26.0)	0.8 ~ 2.4 (1.90)	0.26 ~ 0.27 (0.27)	2.9 ~ 3.4 (3.26)
Morrow	11	15 ~ 46 (29.9)	1.2 ~ 3.8 (2.26)	0.34 ~ 0.36 (0.35)	3.8 ~ 7.9 (4.70)
Rajagopalan	10	25 ~ 37 (29.8)	0.3 ~ 1.7 (0.75)	0.26 ~ 0.27 (0.27)	3.9 ~ 4.3 (4.14)
Taylor	35	19 ~ 37 (26.6)	0.9 ~ 2.3 (1.61)	0.22 ~ 0.22 (0.22)	3.8 ~ 4.1 (4.00)
Van den Berg	34	15 ~ 66 (34.5)	4.35	0.359	3.5 ~ 4.9 (3.78)
Total	288	12 ~ 66	0.3 ~ 4.5	0.07 ~ 1.10	2.6 ~ 8.5

[Remark] Regarding the individual sources, refer to the literature [1]

2. EVALUATION OF SCALE EFFECT

2.1 Investigation based on the data in the past

In these circumstances, a series of experiments has recently been carried out over this range, and it is pointed out that a concept that fv decreases proportionally to $d^{-1/4}$ is reasonable within this range [3].

Therefore, the authors hit on the idea to replace the portion of the sum of βd and βp with regard to the effective depth and the reinforcement ratio with the form of the product which directly introduced the functional form of $d^{-1/4}$ and finally concluded that the following equation would be most suitable after investigating from various aspects:

$$fv = 0.20\ (pw\ fc')^{1/3}\ d^{-1/4}\ [0.75+1.4/(a/d)] \qquad (2)$$

where, no limitation is set for pw.

Comparison is made between the experimental values and the calculated ones with respect to the 288 data in all in Table 1 from which eq.(1) was derived. As its result, the average value of the ratio of the experimental values to the calculated ones μ and the coefficients of variation C.V. are obtained as the followings:

Eq.(1) $\mu = 1.00$
C.V.= σ/μ =9.2%
Eq.(2) μ = 1.01
C.V.= σ/μ =9.1%

where, σ is the standard deviation.

This means that both equations give nearly same accuracy.

By the way, out of the whole data, those which do not present good conformity commonly with both equations are extracted and closely examined. It turned out that among the data by Kani[4], those of 4 cases in which flexural failure was supposed to have occurred were included. Also, the data given by Diaz de Cossio[5] are the loads at the time of generation of diagonal cracks and do not present the shear failure load in a strict sense. Thus, these data are excluded and the 3 data by Taylor[6] for the beams with large values of "d" are newly added. Examination is made again with regard to 265 data altogether. Its result is as shown below:

Table 2 Distribution of data and degree of conformity of equation for calculation
(Total number of data 265)

pw (%) \ d (m)	0 ~ 0.1	0.1 ~ 0.2	0.2 ~ 0.35	0.35 ~ 0.7	0.7 ~ 1.5
0 ~0.3			2 1.125 (10.8%) 1.144 (10.8%)		
0.3 ~ 0.7			5 0.980 (11.9%) 0.977 (12.0%)	4 0.981 (6.3%) 0.941 (6.8%)	1 1.049 (–) 0.942 (–)
0.7 ~ 1.3			27 1.301 (8,8%) 1.030 (9.1%)	6 0.985 (4.5%) 0.980 (4.9%)	
1.3 ~ 2.2	2 1.121 (0.7%) 1.085 (0.7%)	9 1.068 (6.8%) 1.061 (6.8%)	67 0.978 (9.1%) 0.999 (9.3%)	10 0.974 (4.0%) 1.010 (3.9%)	1 1.025 (–) 1.075 (–)
2.2 ~ 3.4		20 1.026 (9.9%) 1.028 (9.9%)	43 0.999 (5.6%) 1.036 (6.0%)	12 0.972 (3.8%) 1.070 (3.9%)	5 0.899 (6.9%) 1.071 (6.9%)
3.4 ~ 5.0			17 1.048 (5.7%) 0.987 (6.0%)	34 1.031 (5.7%) 0.990 (5.7%)	

[Remark] Guide for reading the table

No. of data
Average of experimental value divided by calculated one by eq. (1)
(ditto. coefficient of variation)
Average of experimental value divided by calculated one by eq. (2)
(ditto. coefficient of variation)

Eq.(1) μ = 1.01, C.V.= 8.2%
Eq.(2) μ = 1.02, C.V.= 8.2%

In this case, it is also confirmed that the accuracy is nearly the same.

When the data are densely distributed within a certain range, there might arise problems with regard to the applicability of the equation even if C.V. for the data as a whole is small enough. Therefore, the authors divided the whole ranges of "pw" and "d" into subdivisions so that $pw^{1/3}$ and $d^{-1/4}$ take nearly equal intervals respectively as shown in Table 2 and by assuming that the average value within a subdivision represents the range, re-examination was conducted. As its result, the mean of the average within each subdivision and C.V. are obtained as the followings:

Eq.(1) μ = 1.02, C.V.= 5.4%
Eq.(2) μ = 1.02, C.V.= 5.2%

Furthermore, if the four subdivisions in which the number of data is two or less are disregarded,

Eq.(1) μ = 1.00, C.V.= 4.2%
Eq.(2) μ = 1.01, C.V.= 3.8%

are gained for the remaining thirteen subdivisions. In either case, conformity of both equations is nearly the same.

With regard to eq.(2), the coefficient of variation in each subdivision falls within the range of 3.9～12.0%. The average value of the coefficients of variation in the subdivision that includes data more than or equal to 4 is 7.0% and the standard deviation is 0.023. The coefficient of variation for the whole data is 8.2%. Thus, differences among them are not so large.

In Table 3, for each subdivision of pw, d and fc', the ratio of the experimental value to the calculated value by eq.(1) and the same to the calculated value by eq.(2) are shown in comparison. From Table 3, it is also confirmed that conformity of eq.(1) and eq.(2) with past data is in the same order.

Table 3 Accuracy of calculation of pw, d and fc' for each subdivision of range

pw (%)	No. of data	Experimental value/Eq. (1) Average (Coefficient of variation)	Experimental value/Eq. (2) Average (Coefficient of variation)
0～0.3	2	1.125 (10.8%)	1.144 (10.8%)
0.3～0.7	10	0.987 (9.5%)	0.959 (9.8%)
0.7～1.3	33	1.023 (8.5%)	1.021 (8.7%)
1.3～2.2	89	0.990 (8.9%)	1.009 (8.7%)
2.2～3.4	80	0.995 (7.5%)	1.041 (7.2%)
3.4～5.0	51	1.037 (5.8%)	0.989 (5.8%)

d (m)	No. of data	Experimental value/Eq. (1) Average (Coefficient of variation)	Experimental value/Eq. (2) Average (Coefficient of variation)
0～0.1	2	1.121 (0.7%)	1.085 (0.6%)
0.1～0.2	29	1.039 (9.2%)	1.038 (9.2%)
0.2～0.35	161	1.002 (8.5%)	1.014 (8.6%)
0.35～0.7	66	1.004 (5.9%)	1.003 (6.2%)
0.7～1.5	7	0.939 (8.7%)	1.053 (7.3%)

fc' (MPa)	No. of data	Experimental value/Eq. (1) Average (Coefficient of variation)	Experimental value/Eq. (2) Average (Coefficient of variation)
10～20	37	0.979 (7.8%)	0.999 (7.7%)
20～30	120	0.993 (8.0%)	1.013 (8.6%)
30～40	92	1.027 (8.3%)	1.013 (8.6%)
40～50	14	1.039 (6.2%)	1.013 (5.7%)
50～60	2	1.016 (1.1%)	0.975 (1.1%)

2.2 Examination with the data of the large-sized beams with low reinforcement ratio

In case a RC beam without web reinforcement is subjected to a concentrated load, conformity of eq.(2) in which the influence of the reinforcement ratio and the effective depth is represented by the form of product $pw^{1/3}\ d^{-1/4}$ and eq.(1) in which the same is represented by the form of sum $(1+\beta p+\beta d)$ with the past experimental data is of nearly the same order. However, it must be noticed that over the range where the past data are abundant, that is, the range of $d \leq 1$ m and $pw \geq 0.5\%$, there is practically no difference between the calculated values by both equations, whichever form the portion that represents the influence of the reinforcement ratio and effective depth may take, product or sum.

The case in which the difference between the calculated values by the two equations becomes very large happens in the range where the thickness of the cross section is determined from the shear strength for a beam having a small reinforcement ratio and a large effective depth that is practically designed as one-way slab.

Therefore, the authors have undertaken to examine the validity of these two equations using the data[3] of experiments of the beams with large effective depth and small ratio of reinforcement which were recently carried out. However, in these experiments, uniformly distributed load is applied by way of hydraulic pressure, which must be taken into consideration. Accordingly, prior to examining the validity of both equations to the large-sized beam with low reinforcement ratio, investigation on how to deal with the uniformly distributed load is going to be made in advance.

3. DEALING WITH UNIFORMLY DISTRIBUTED LOAD

3.1 Method by Iguro et al.[3]

In dealing with the case in which a beam is subjected to the uniformly distributed load, Iguro et al. consider one quarter of the span "ℓ" to be equivalent to the shear span "a" when the beam is subjected to a concentrated load, referring to Kani's idea[4].

Assuming that the location where the diagonal crack that leads to failure of the beam generates (the distance between the center of support and the location where the diagonal crack generates at the lower edge of the beam is defined to be "x") is the middle point of "a", they compare the calculated value of the shear force applying at the position of $x = 12\ d/(4\times2) = 1.5\ d$ from the center of support because $\ell/d = 12$ (calculated assuming $a/d = 3.0$) and that of the shear strength. And to support this assumption, they point out that the values of "x" observed in the experiments are nearly 1.5 d.

3.2 Method of dividing the uniformly distributed load into a number of concentrated loads

Although the method employed by Iguro et al. is very simple, it is difficult to reasonably explain the assumption that 1/4 of the span of a beam to which a uniformly distributed load is applied is equivalent to the shear span of a beam with a concentrated load and that the position at which the diagonal crack that leads to failure of the beam generates is located at 1/2 of the said shear span. And the authors presume, if 1/4 of the span should be assumed to be equivalent to the shear span, the applying shear force have to be calculated based on the same assumption. Anyway, generality of evaluating "x" to be $\ell/8$ can not be confirmed, partly because of the condition that the ratio of the span to the effective depth ℓ/d is kept constant throughout the series of experiments.

Therefore, as one of the analytical approaches, the authors tried a new method to divide the uniformly distributed load into a number of concentrated loads and replace the former with the latter.

First, the condition of a beam to which a unit uniformly distributed load, namely $w = 1$ is applying is considered. Then, by assuming this unit uniformly distributed load to be a group of a number of concentrated loads, the span is divided into the sections of a finite number and let one each concentrated load equal to the sum of the uniformly distributed load on each section apply at the middle point of the said section.

For a virtual concentrated load thus obtained, the applying shear force diagram is drawn. Then, the increase in the shear strength around the points of support and the loading points is to be taken into consideration by reducing the shear force virtually, and the reduction of the applying shear force is made based on the applying shear force diagram drawn.

In choosing the function of the reduction used for this purpose, possible change in the failure mode, that is, from the mode of diagonal tension failure to the failure such as deep beams in accordance with the distance between the center of support and the loading point is taken into consideration. Thus, a function that could be applicable to either mode of failure is chosen beforehand.

Within the shear span between the loading point and the center of support, the authors assumed that influences from both the load and the reaction at the support are combined and averaged influence from the both are taken into consideration, by referring to the method by Ishibashi et al.[7] for investigating the shear strength of the footing supported by a few piles.

In this case, it may be assumed that failure happens in the shear span on whichever side, right or left. If the shear span in question is the one on the left side, for instance, out of the shear forces due to the virtual concentrated loads in the vicinity of the support on the left, the shear force between the center of support on the right side and the virtual concentrated load (negative shear force) is supposed to affect favorably for this pattern of failure and tends to apply to increase the shear strength. Therefore, in this method, only the shear force between the concentrated load and the center of support on the left (positive shear force) is to be reduced and the shear force between the concentrated load and the center of support on the right (negative shear force) is to be used as it is.

The reduced applying shear force diagrams obtained for each virtual concentrated load in the manner as mentioned above are summed up for all the virtual concentrated loads. The largest value of the shear force (positive shear force) in the shear force diagram as a whole thus summed up is denoted as Vmax.

Finally, the fundamental shear strength Vco is calculated by assuming the portion of the function of a/d, F(a/d) in eq.(1) or eq.(2) to be unity and Vco/Vmax is obtained. This value is considered to be the calculated value w_{cal} of the uniformly distributed load at the time of shear failure.

In performing numerical calculation, change in the calculated values due to increase in the number of division of the span is investigated. Since it is known that the calculated value gets close asymptotically to a certain value, if the number of division is taken at around 50, the authors decided to carry out the calculation in this method by dividing the span into 50 equal sections. The flow of calculations in the above is illustrated in Fig. 1.

As the value of the reduction function βx, the average value of $\beta x1$ to be determined by taking the influence from the support into consideration and $\beta x2$ to be determined likewise from the load is employed. By multiplying this to the aforesaid positive shear forces only, the reduced applying shear forces are obtained. And each βxi is to be so determined that it can adapt to the change of the failure mode.

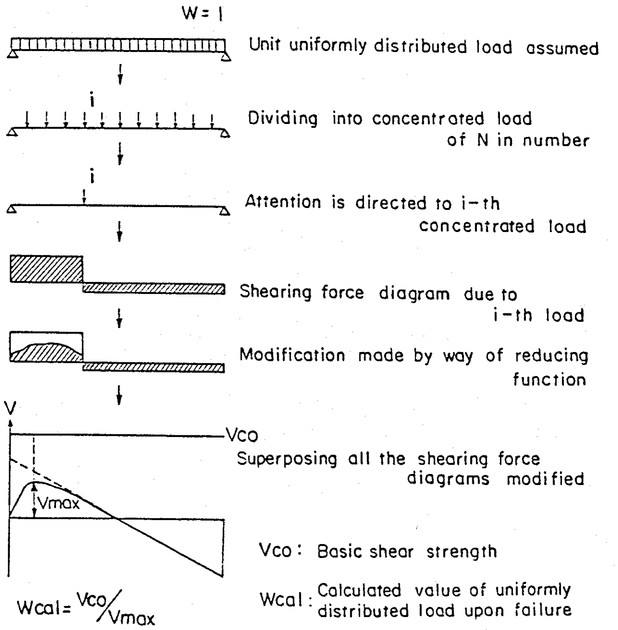


Fig.1 Sequence of calculation of the method for dividing the uniformly distributed load into concentrated load

In so doing, it is assumed that the influence from the support and that from the load can be represented in the same way and each βxi is to be determined solely by the position within the shear span in question and the distance "x" (absolute value) from the center of support or the loading point.

In practice, referring to F(a/d) = 0.75+1.4/(a/d), the functional form of a/d in eq.(1) or eq.(2) and aiming at the case of a/d=5.6 which makes the value of these equations unity, the value of each βxi is assumed to be 1.0, when the distance "x" from the center of support or the loading point exceeds 2.8d, that is one half of 5.6d. When "x" becomes less than 2.8d, the functional form is changed from that in eq.(1) or eq.(2) to the functional form corresponding to the failure like deep beams in accordance with the mode of failure. Anyway, by adopting this method, it becomes also possible to duly evaluate the shear strength when the concentrated loads are applied to a beam in general.

From the above, the reduction function βx by which respective diagram of the applying shear force is to be multiplied is gained as the following (Fig. 2):

$$\beta x = (\beta x1 + \beta x2)/2 \tag{3}$$

where,

$$2.8d \leq x \qquad \beta xi = 1 \tag{4}$$

$$0 \leq x < 2.8d \qquad \beta xi = 1/[0.75+1.4/(2x/d)] \tag{5}$$

or

$$\beta x i = 0.21\ fc'^{-1/6}\ [1+(2x/d)^2] \quad (6)$$

whichever smaller one among eq.(5) or eq.(6).

In this case the shear strength for deep beams is assumed to be calculated by the following equation:

$$fv = 0.94\ fc'^{1/2}\ pw^{1/3}\ d^{-1/4}\ /[1+(a/d)^2] \quad (7)$$

Eq.(6) is what is gained by dividing the fundamental shear strength $fvo = 0.20\ fc'^{1/3}\ pw^{1/3}\ d^{-1/4}$ by eq.(7).

3.3 Result of calculation of the shear strength by the both methods

Among the experiments in which an idealistic uniformly distributed load is applied, the one carried out by Leonhardt is well known. In order to verify the validity of each method of calculation, the data by Leonhardt[8] are referred to.

Table 4 presents the results of calculation by both methods, namely the method by Iguro et al. and the one by dividing the uniformly distributed load into a number of concentrated loads in comparison with the experimental values. For the purpose of comparison, the calculation is made by both eq.(1) and eq.(2).

According to Table 4, the dispersion of the ratios of the experimental values to the calculated ones is small and their averages are well within the permissible range, whichever method may be employed, the method by Iguro et al. or the one by dividing the uniformly distributed load into a number of concentrated

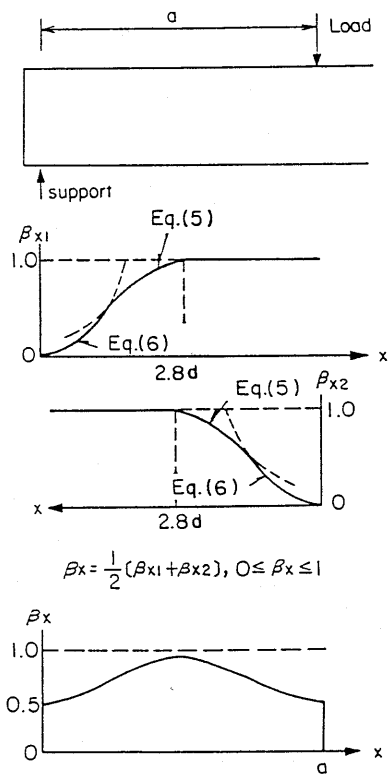


Fig.2 Reduction coefficient βx

Table 4 Calculation of shear strength of small-sized beams to which uniformly distributed load is applied

Specimen	d (m)	Pw (%)	ℓ/d	fc' (MPa)	Experimental value/ Calculated value by the method of Igro etal.		Experimental value/ Calculated value by the method of dividing into concentrated loads	
					Eq. (1)	Eq. (2)	Eq. (1)	Eq. (2)
14/1	0;273	2.04	11.0	33.0	1.04	1.08	1.10	1.13
14/2	0.273	2.04	11.0	33.0	1.04	1.08	1.10	1.14
15/1	0.272	2.05	14.7	35.0	1.01	1.05	1.05	1.08
15/2	0.273	2.05	14.7	35.0	1.08	1.11	1.11	1.15
16/1	0.273	2.04	18.3	34.5	1.09	1.13	1.12	1.16
16/2	0.274	2.04	18.2	34.5	1.09	1.13	1.12	1.15
17.2	0.274	2.04	21.9	32.4	1.07	1.11	1.08	1.11

[Remark] Data from Leonhardt [8], compressive strength is converted by $fc' = 0.85\ \beta z$

ones. And, if the results of calculation by eq.(1) and eq.(2) are mutually compared, the results by eq.(2) tends to give slightly smaller values. Anyway, these data are based on the following ranges of variables:

fc': 32.4 ~35.0 MPa, pw: about 2%, d: about 27 cm, ℓ/d: 11~22

4. VERIFICATION OF THE EQUATION FOR SHEAR STRENGTH BASED ON THE DATA OF LARGE-SIZED BEAM WITH LOW REINFORCEMENT RATIOS

4.1 Case of distributed load

Since the shear strength could be estimated with reasonable accuracy by a method in which the distributed load is divided into a number of concentrated loads for small specimens to which uniformly distributed load is applied, the authors went further to apply this method to the large-sized beams with low reinforcement ratio to which uniformly distributed load is applied and compare the results with the experimental values of the shear strength. The results are presented in Table 5.

According to Table 5, with regard to the large-sized beams with low reinforcement ratio, considerable differences are observed between the calculated values and the experimental ones.

In case eq.(1) is applied, the ratio of the experimental values to the calculated ones varies over the range of 0.81 ~1.18. Among them, the specimen KS-4 whose ratio of the experimental value to the calculated one is 0.81 has "d" of 1 m and the maximum size (MS) of coarse aggregate of 10 mm, and its strength is lower by about 10% than that of the specimen KS-5 which has the same details as those of KS-4 except that its MS is 25 mm. Assuming that KS-4 be excluded, because such difference is supposed to be caused by lower interlocking action of aggregate, the ratio of the experimental data to the calculated ones becomes 0.93 ~1.18 which may be said that estimation is made more or less reasonable accuracy.

However, if the results of calculation are closely examined, the trend that the ratio of the experimental value to the calculated one is relatively increasing as the effective depth increases is clearly observed. In other words, the influence of the scale effect that is taken into consideration in eq.(1) is a little too large as compared with the actual case and it means that its effect is being enlarged as the effective depth increases.

On the other hand, in the case of eq.(2), while there is no such change in the ratio of the experimental value to the calculated one as seen in the case of

Table 5 Calculation of shear strength of large-sized beams with low reinforcement ratio to which uniformly distributed load is applied

Specimen	d (m)	pw (%)	ℓ/d	fc' (MPa)	Experimental value/Calculated value by the method of Iguro etal.		Experimental value/Calculated value by the method of dividing into concentrated loads	
					Eq. (1)	Eq.(2)	Eq. (1)	Eq. (2)
KS – 3	0.60	0.42	12.0	21.1	0.88	0.81	0.93	0.86
KS – 4	1.00	0.40	12.0	27.2	(0.77)	(0.66)	(0.81)	(0.69)
KS – 5	1.00	0.40	12.0	21.9	0.92	0.79	0.97	0.83
KS – 6	2.00	0.40	12.0	28.5	0.99	0.75	1.04	0.79
KS – 7	3.00	0.41	12.0	24.3	1.11	0.79	1.18	0.84

eq.(1), the value of the ratio itself is as small as 0.79~0.86 which shows the trend of overestimating the actual value to a certain extent. After all, neither equation can be appreciated in estimating the experimental value with satisfactory accuracy.

In case concentrated loads are applied to the beam, the conformity of eq.(2) is not so bad even for the beam with comparatively large effective depth, say d = 1 m or so. However, for the data of the large-sized beam with low reinforcement ratio which is subjected to the uniformly distributed load, there remains some problems in the estimation.

As mentioned previously, between eq.(1) and eq.(2), no significant difference is observed with respect to the great majority of the results of experiments in the past. However, in the range for which these equations are going to be applied, for instance in the case of d = 3 m and pw = 0.4%, very wide difference would arise (Eq.(2) gives a value about 1.4 times as large as the one given by eq.(1)).

As the possible cause of the large difference between the experimental value and the calculated one as seen in this example, the problem of accuracy of eq.(1) or eq.(2) may be pointed out first on the side of calculated values and then problem of adequacy of the method of dealing with the uniformly distributed load. However, because the specimens are the large-sized beams with low reinforcement ratio, the experimental values themselves might possibly include errors to a certain extent either.

In the experiment of this large-sized beam, for instance, the time of loading is as long as about 30 minutes per one step (10 min. for loading and 20 min. for measurement) and failure happened during measurement while the load was kept constant. Since the number of steps of loading until failure took place was around 12 with equal loading intervals, if the increase in the load could be made smoothly the strength might possibly be raised further by 1 more step or so (about a little less than 10%)[9].

In any case, the influence of the method of dealing with the uniformly distributed load is supposed to be very large. Therefore, the authors determined to newly carry out the experiments of shear failure of the large-sized beams with low reinforcement ratio to which concentrated loads are applied, partly for the purpose of eliminating the vague portion of knowledge due to the uniformly distributed load.

4.2 Loading tests of concentrated loads

The shapes, dimensions and arrangement of the reinforcing bars of the specimens are shown in Table 6 and Fig.3. The number of specimens is four in all. Specimen No.1 and No.2 have the effective depth d = 2 m and their reinforcement ratios pw are 0.28% and 0.14%, respectively. Both specimen No.3 and No.3-CR have d = 1 m and pw = 0.14% and No.3-CR is provided with flexural cracks in order to investigate the influence of existence of flexural cracks on the shear strength.

Besides, in order that the flexural failure would not take place prior to the shear failure, deformed prestressing bars (Gebinde Stab) which have higher yield point are used for the tensile reinforcement. As a matter of course, no web reinforcement is made within the span. At the fixing ends of the reinforcing bars, anchor plates are fixed with nuts, and furthermore, stirrups are arranged to reinforce the anchoring parts. The Gebinde Stab are being offered

Table 6 Variables of specimens of large-sized beams with low reinforcement ratio

Specimen	Overall length L (m)	Span ℓ (m)	Depth h (m)	Effective Depth d (m)	Breadth bw (m)	Cross-sectional area of tensile reinforcement As (cm²)	Ratio of tensile reinforcement pw (%)	Remark
No. 1	13.5	12.0	2.1	2.0	0.6	33.24 ϕ23-8	0.28	
No. 2	13.5	12.0	2.1	2.0	0.6	16.62 ϕ23-4	0.14	
No. 3	7.0	6.0	1.1	1.0	0.3	4.15 ϕ23-1	0.14	
No. 3 – CR	7.0	6.0	1.1	1.0	0.3	4.15 ϕ23-1	0.14	Flexural cracks introduced

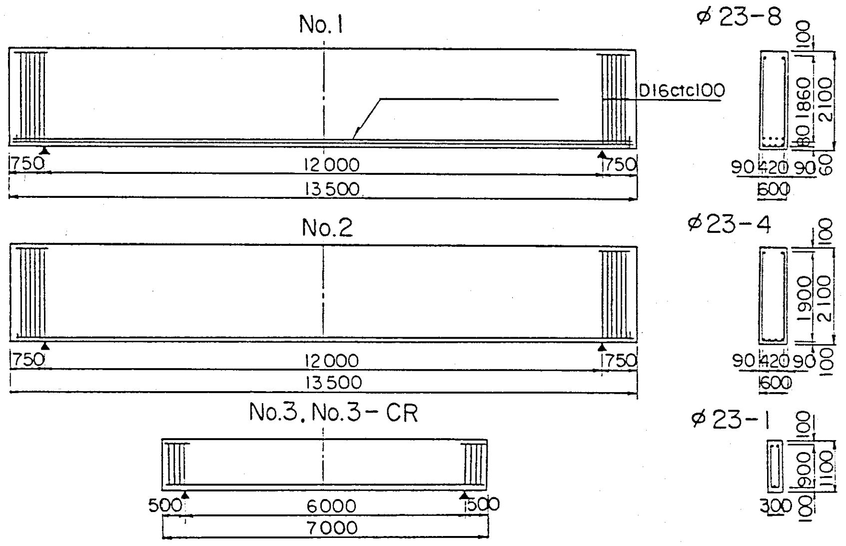


Fig.3 Shape and dimensions of each specimen

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The mix proportion of concrete is common throughout all the specimens. The maximum size of coarse aggregate is 25 mm. The mix proportion is shown in Table 7.

As the tensile reinforcement, the aforementioned "Gebinde Stab" ϕ23 is used. Its yield point, tensile strength, modulus of elasticity and bearing area coefficient[10] are as shown in Table 8. The "Gebinde Stab" is a prestressing bar with screw lugs, having the bearing area coefficient of 0.068. In view of the fact that the bearing area coefficient of the conventional deformed reinforcing bars is around 0.06, the "Gebinde Stab" is considered to have the bond characteristics of similar order to that of the conventional deformed reinforcing bars.

Concrete is cast, while the specimens are placed in a horizontal position with the side surfaces of the beam at the top and bottom. The specimens No.1 and No.2 are made by two-layers casting with 30 cm thickness of one layer, while the specimens No.3 and No.3-CR are made by single casting. Concrete is fully compacted with high frequency vibrators. After casting is finished, they are covered with sheets and cured under temperature control and water spray.

Table 7 Mix proportion of concrete used

Maximum size of aggregate (mm)	Air (%)	Ratio of fine aggregate (%)	Slump (cm)	C (kg)	W (kg)	S (kg)	G (kg)	Admixture (kg)
25	4	43.4	12 ± 1	322	177	880	1023	Pozzolith No. 70 3.45

Table 8 Characteristics of deformed prestressing bar (Gebinde Stab)

Name	Nominal diameter mm	Nominal cross -section area cm	Yield point MPa	Tensile strength MPa	Modulus of elasticity MPa	Bearing area coefficient
ϕ23	23	4.155	999	1130	2.01×10^5	0.068

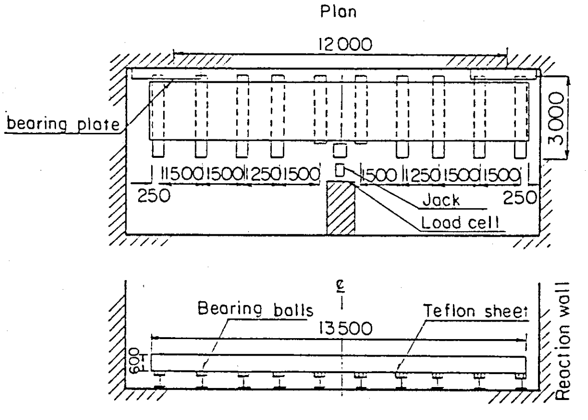

Fig.4 Method of loading for specimens No.1 and No.2

As the method of loading, one point loading at the middle of the span is adopted. The shear span ratio a/d is 3.0. The specimens No.1 and No.2 are placed in a horizontal position with one of its side surfaces down and loaded against the reaction wall. The specimen is placed on the holders. Between the specimen and the holders supporting it, numerous steel bearing balls are placed in order to eliminate frictional constraint between the holders and the specimen and steel plates are inserted on the balls. Between the steel plates and the specimen, Teflon sheets are laid. At the loading point and the points of support, bearing plates with the length of 13 cm and 10 cm respectively are used for loading over the full breadth of the beam.

In order to examine whether frictional constraint could be eliminated or not, the specimen is actually moved by way of a pantograph jack. It is known that the specimen with its own weight of about 390 kN can be moved sliding on the holders being pushed with the horizontal force less than 5 kN, thus, it is considered that frictional constraint is almost fully eliminated. This method of loading is illustrated in Fig. 4.

The specimens No.3 and No.3-CR are held in a beam testing machine in the vertical position and loaded by way of the bearing plates having the length of 6.5 cm at the point of loading and the points of support as well over the full breadth of the beam.

The cycle of loading follows the standards mentioned below:

1st cycle 0 kN → Generation of flexural cracks → 0 kN

2nd cycle 0 kN → Until τ = V/(bw d) = 3.92 MPa (4 kgf/cm^2) is reached → 0 kN

3rd cycle 0 kN → Failure

During the loading cycle, occasionally the load is held constant to measure the strain in the tensile reinforcing bars, the strain of concrete in flexural compressive zone and the lateral displacement of the beam and at the same time the development of cracks are observed. The locations of various instruments used in this experiment are shown in Fig. 5.

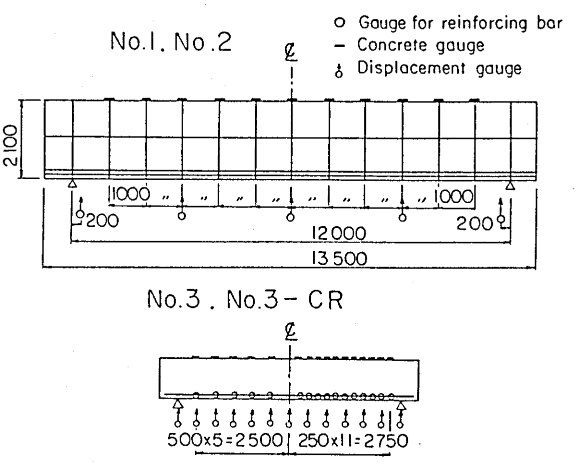

Fig.5 Arrangement of measuring instruments

Specimen No.3-CR is the one that is provided with flexural cracks prior to loading. The flexural cracks are introduced into the specimen by applying a constant flexural moment using the rigid frame constructed with the beam and H-shaped steels as shown in Fig. 6 in which the flexural moment is generated by applying tension between the ends of the H-shaped steels. During this operation, the beam is kept horizontal being placed on the holders. And along the flexural tension side edge of the beam, notches of 1 cm depth are provided at the interval of 0.25d so that flexural cracks are easily generated. The flexural cracks thus introduced are shown in Fig. 7.

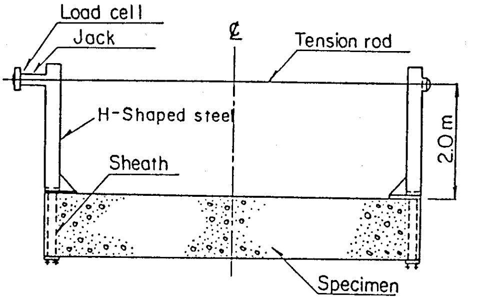

Fig.6 Method to introduce flexural cracks

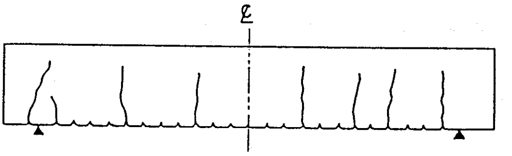

Fig.7 Conditions of introduced flexural cracks (No.3-CR)

4.3 Outline of the test results

As the result of the experiments, the compressive strength of concrete, modulus of elasticity, the load at which flexural cracks generate, the maximum load and modes of failure are shown in Table 9 and the conditions of the specimens after failure are presented in Fig. 8.

(a) Specimen No.1 (ℓ = 12 m, d = 2 m, pw = 0.28%)

At the load P = 706 kN (shear stress τ = 0.294 MPa), flexural cracks almost reaching the neutral axis generated at once around the middle point of the beam with a loud bursting sound.

In the 2nd cycle of the test, at P = 588 kN (τ = 0.245 MPa) flexural cracks

Table 9 Outline of test results

Specimen	Concrete		Load at which flexural crack generates kN	Maximum load kN	Mode of failure
	Compressive strength MPa	Modulus of elasticity MPa			
No. 1	28.0	2.60×10^4	706	804	Diagonal tension failure
No. 2	27.1	2.38×10^4	627	764	Diagonal tension failure
No. 3	25.4	2.50×10^4	98*	204*	Diagonal tension failure
No. 3 – CR	25.4	2.50×10^4	—	311*	Flexural tension failure

* Values excluding their own weight for No.3 and No.3-CR

began to generate, gradually increased their number. A diagonal crack started at the lower edge about 1.5d apart from one of the points of support at P = 784 kN (τ= 0.327 MPa) and developed. Finally, while loading was suspended and measurement was carried out at P = 804 kN (τ = 0.335 MPa), failure occurred suddenly.

(b) Specimen No.2 (ℓ = 12 m, d = 2 m, pw = 0.14%)

At the load P = 627 kN (τ= 0.261 MPa), flexural cracks generated almost reaching the neutral axis at the middle part of the beam. The conditions of generation of the first flexural cracks were nearly same as those of the specimen No.1.

In the second cycle of the test, flexural cracks started to generate at P = 529 kN (τ= 0.221 MPa). However, if compared with the specimen No.1, the range over which flexural cracks generated is narrower and they tends to concentrate in the vicinity of the flexural cracks generated at the middle part. The number of cracks is less.

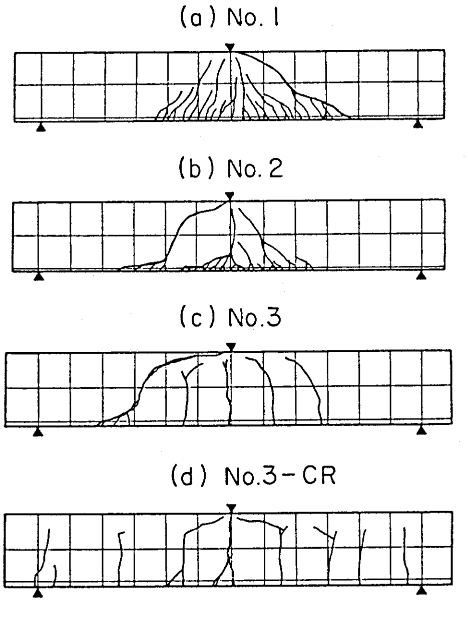

Fig.8 Conditions of each specimen after failure

Failure occurred when the diagonal crack width generated at the point of the lower edge about 2d apart from one of the points of support at the load of P = 764 kN (τ= 0.319 MPa) quickly developed. Anyway, both specimens No.1 and No.2 failed before the load originally aimed at in the 2nd cycle of the test, namely τ = 0.392 MPa had been reached.

(c) Specimen No.3 (ℓ = 6 m, d = 1 m, pw = 0.14%)

At the load of P = 98 kN (τ = 0.163 MPa)(excluding the beam's own weight, ditto hereinafter), flexural cracks almost reaching the neutral axis are generated at the middle of the span. While in the test of the 2nd cycle, the number of the flexural cracks gradually increased, the intervals of the cracks are more regular in comparison with the case of No.2 and they generated within the range of 1.5d on both sides of the point of loading at about 70 cm pitch (0.7d).

Failure occurred when the diagonal crack that was generated on the lower edge of the beam at around 1.5d apart from one of the points of support at P = 204 kN (τ = 0.340 MPa) quickly developed. This specimen also failed in the same manner as specimens No.1 and No.2 before the load of τ = 0.392 MPa is reached.

(d) Specimen No.3-CR (ℓ = 6 m, d = 1 m, pw = 1.4%, with flexural cracks)

This specimen which is provided with flexural cracks that reach the height of about 2/3 of the beam depth in advance presented a behavior remarkably different from those of other specimens.

At P = 108 kN (τ = 0.180 MPa)(excluding the beam's own weight, ditto hereinafter), flexural cracks started at the middle of the beam where there had been no flexural cracks given in advance. At around P = 147 MPa (τ = 0.245 MPa), diagonal cracks began to develop at the tops of the flexural cracks introduced prior to the test and expanded little by little, as the load was increased, but did not develop quickly. The flexural cracks generated at the middle part developed gradually as the load is increased and finally, at P = 311 kN (τ = 0.518 MPa) following abrupt expansion of widths of flexural cracks at the middle part, concrete around the loading point crushed and the beam failed.

Although flexural cracks had existed on the lower edge of the beam at around 1.5d apart from the points of support, they did not induce shear failure.

4.4 Discussions on the test results

(a) On the load at which flexural cracks generate

In view of the large effective depth of the beams, the authors decided to calculate the load at which flexural cracks generate by the theory of elasticity, taking the influence of the tensile reinforcing bars into consideration.

As shown in Table 10, the loads at the time of generation of flexural cracks calculated by the theory of elasticity for specimens No.1 and No.2 taking the influence of the tensile reinforcing bars into consideration give higher values by about 7% and 4% respectively than those calculated neglecting the reinforcing bars, which shows that these values estimate the experimental values with reasonably good accuracy.

With regard to specimen No.3, while the load at which flexural cracks generate was calculated taking the influence of the specimen's own weight into account assuming the unit weight of the specimen to be 24.5 kN/m^3, the result of calculation gave the value on fairly higher side compared with the

Table 10 Calculation of load at which flexural crack generates

Specimen	Experimental value Pcr	Calculated value considering tensile reinforcing bars P_{cr1} kN	Calculated value neglecting tensile reinforcing bars P_{cr2} kN	$\frac{P_{cr}}{P_{cr1}}$	$\frac{P_{cr}}{P_{cr2}}$
No. 1	706	668	625	1.09	1.13
No. 2	627	635	613	0.99	1.03
No. 3	122*	165	161	0.74	0.76

* Value compensated against the influence of its own weight

experimental one either for the case in which the influence of the reinforcing bar is considered or neglected. As one of the reasons for this, the fact that specimen No.3 has been under the influence of drying shrinkage is considered since longer time has passed than specimens No.1 or No.2 until it is loaded in the test after curing under temperature control and water spray finished. Actually, in the case of specimen No.3-CR which was introduced flexural cracks soon after curing was finished, the flexural crack is first generated at the flexural moment of 250 kNm. If this value is converted to the concentrated load at the middle, about 155 kN is obtained. This is very close to the calculated value at which the flexural crack is generated.

(b) On the maximum load

The experimental values of the maximum load and the calculated values by eq.(1) and eq.(2) are presented in Table 11. The mode of failure was diagonal tension failure for specimens No.1, No.2 and No.3 and flexural failure for specimen No.3-CR to which flexural cracks had been introduced prior to test.

If comparison between the experimental values and the calculated ones is made among the specimens No.1 ~ No.3, it is known that eq.(1) gives a smaller calculated value than the experimental one for every specimens. All these calculated values are on the safe side, but the ratios of the experimental value to the calculated one disperse widely over the range 1.2 ~ 2.0 and accuracy of calculation is not satisfactory enough. Over the range of variables of the present experiments which lacks sufficient number of data that would constitute the basis of eq.(1), namely the range of $d \geq 1.0$ m, and $pw \leq 0.5\%$, the result that might suggest drop in conformity of eq.(1) was gained as previously feared.

In contrast to this, eq.(2) that contains the influence of "d" and that of "pw" in the form of their product referring to the information that the shear strength falls down proportionally to $d^{-1/4}$ which was obtained from the experiments of large-sized RC beams subjected to uniformly distributed loads, gives the ratio of the experimental values to those calculated ranging over 0.83 ~1.03 for specimens No.1 to No.3 throughout, from which it is made clear that the experimental values can be estimated with deviation of tolerable extent. With regard to specimen No.1, failure occurred during measurement was being done by keeping the load constant. There was possibility of getting somewhat higher experimental value, if loading could have been stepped up.

Specimen No.3-CR that was provided with flexural cracks in advance presented flexural failure against expectation. The calculated flexural strength is as shown in Table 11.

While the yield point of the "Gebinde Stab" is defined as the stress that leaves the residual strain of 0.2%, the difference between this value and the tensile strength is not so large. Taking the above fact and the conditions that spe-

Table 11 Calculation of shear failure load by Eq.(1) and Eq.(2)

Specimen	Failure load P kN	Calculated value by Eq. (1) P_{cal1} kN	Calculated value by Eq.(2) P_{cal2} kN	Flexural c capacity Pf kN	P/P_{cal1}	P/P_{cal2}
No. 1	804	651	972	2350	1.23	0.83
No. 2	764	374	762	1210	2.04	1.00
No. 3	227*	160	221	303	1.42	1.03
No. 3 – CR	334*	160	221	303	(2.09) above	(1.51) above

* Value compensated against the influence of its own weight

cimen No.3-CR is a beam of low reinforcement ratio and that the width of the cracks at the time of failure was considerably spread into consideration, the flexural strength was calculated by using the tensile strength.

(c) On the modes of failure

The specimens No.1～No.3 failed with the mode of diagonal tension failure as intended by design. In the case of specimen No.3-CR, however, the mode was of the flexural failure against expectation.

The reasons why specimen No.3-CR was added to this series of experiments are the followings: As the result of close investigation on the conditions of generation of cracks and failure of the specimens No.1 and No.2, it is recognized that because they have low reinforcement ratio, generation of flexural cracks concentrates at the middle of span. Consequently generation of flexural cracks in the middle part of the shear span that would induce diagonal cracks and furthermore cause diagonal tension failure may be prevented. Since reasoning that the above fact might have caused higher value of the shear strength than that which is given by eq.(1), it is determined to conduct the test of specimen No.3-CR, anticipating that the value of lower limit of the shear strength would be obtained by generating flexural cracks prior to the test.

In providing flexural cracks, investigation was made on the problem how deep these flexural cracks should be made. In consideration of the prerequisite that the purpose of the test requires that flexural crack should have been generated for sure, and in anticipation that if the flexural cracks provided are deep enough, diagonal cracks would generate on the way along them, the flexural cracks are so made as to cover more than half of the depth of the beam. However, since the reinforcement ratio of the beam is very small, fine control of the depth of cracks is so difficult that the depth of flexural cracks introduced have reached the level of about 2/3 of the depth of beam from its lower edge.

In the actual test, however, diagonal cracks never generate on the way along the flexural cracks, instead slight ones are observed at their tops only, nor they develop quickly. The beam finally failed in flexural failure mode. As the factors caused such failure, the following are considered: Deformation is concentrated to the cracked part due to drop of the flexural rigidity of the beam influenced by the introduced flexural cracks and in the vicinity of flexural cracks the tensile stress normal to them is freed and tensile forces do not apply any more in concrete, thus preventing generation of diagonal cracks that branch from the flexural cracks.

Accordingly, it is understood that diagonal cracks that would have generated when the conditions of deformation of concrete within the beam exceed the critical limit for generation of diagonal cracks if there were no flexural cracks, have not generated in this case and the beam ended up with flexural failure. The load-displacement curves for the specimens No.3-CR and No.3 are

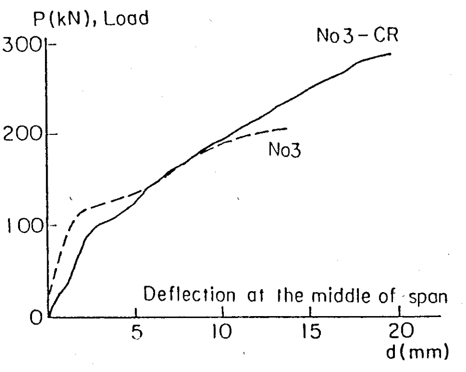

Fig.9 Measured value of load and deflection at the middle of span (No.3, No.3-CR)

shown in Fig. 9. As clearly seen in the diagram, the flexural rigidity in the initial stage falls heavily.

5. REVALUATION OF EQUATIONS FOR SHEAR STRENGTH AND THE APPLICATION TO DESIGN

According to Table 11 in which comparison between the result of experiments newly conducted for the large-sized beams with small reinforcement ratio and eqs.(1) and (2), it is known that the experimental values are fairly larger than the calculated ones by eq.(1) and rather closer to the values gained by eq.(2), though the number of experiments is not so many.

Without any doubt, eq.(1) shows excellent conformity with the great majority of the experimental data in the past. However, though eq.(1) can represent the qualitative trend of the experimental data, it can not confirm its conformity for the range within which the data of the past experiments are not sufficient because it is an empirical equation. According to the result of experiments conducted this time for the range where the past experimental data have been scarce and where the RC structures are often used in practice, it has been made clear that eq.(1) estimates the shear strength fairly on the safe side.

Although eq.(2) itself is an empirical equation derived based on eq.(1), its conformity with the great majority of the past experimental data is as good as that of eq.(1) and furthermore its conformity over the range of $d \geq 1.0$ m and $pw \leq 0.5\%$ is judged to be better than that of eq.(1).

However, since the number of experiments is small, problems are left unsolved whether eq.(2) could be applied to design as it is or not. It is considered to be possible to cope with such problems for the time being by taking a larger partial safety factor for calculating the strength of member in the design practice. From now on, as the experimental data over this range are accumulated, clearer knowledge on the value itself of the partial safety factor for calculating the strength of member will be established.

In addition, when eq.(2) is applied to design, there is a problem in calculating pw to what extent the reinforcing bars in the axial direction could be taken into account. In other words, in case the designed cross section is not a rectangular but box-shaped one, the problem is whether all of the cross-sectional area of the reinforcing bars along the axial direction arranged in its bottom slab could be taken into account or not. When the effect of pw on shear strength should mainly arise from restriction of expansion of the width of cracks and dowel action, the influence of the latter on the reinforcing bars along the axial direction arranged far away from the webs can not be expected. Therefore, it would be recommended to establish a certain limit for the value of pw used in design (say $pw \leq 3.0\%$ and so forth).

Among the large-sized beams of low reinforcement ratio, the tensile reinforcement ratio of specimen No.3-CR to which flexural cracks are introduced is 0.14%. This value almost equals to the minimum reinforcement ratio which is determined from the structural details. While it is considered that the minimum reinforcement is arranged for the level of the design load at which flexural cracks would not generate, so far as known from the result of this experiments, even if cracks have generated by the influences of such factors as thermal stress, drying shrinkage etc., its shear strength may be evaluated by eq.(2) without risk.

6. CONCLUSIONS

This study was undertaken aiming at the following two problems: The first one is whether the equation for the shear strength of the beam without web reinforcement that was previously proposed (eq.(1)) could be reasonably applied or not over the range of $d > 1.0$ m and $p_w < 0.5\%$ in which past experimental data are rarely found; and the second one; how closely the new equation for the shear strength (eq.(2)) to which the influence of the scale effect that the strength decreases proportionally to $d^{-1/4}$ which was pointed out from the experiments of the large-sized beams of low reinforcement ratio is introduced could conform with the experimental value ?

Thus, specimens of the large-sized beams with low reinforcement ratio are manufactured and the experiments are carried out by applying the concentrated load. As the result of a series of investigation, the following conclusions are obtained:

(a) For the range containing past abundant data, both eqs.(1) and (2) are applicable for estimating the experimental values with reasonable accuracy and their conformity differs little.

(b) For the large-sized beams with low reinforcement ratio, eq.(1) estimates the values considerably on the safe side, while eq.(2) gives those very close to the experimental ones.

(c) Although both of eqs.(1) and (2) are empirical ones, it would be desirable to adopt eq.(2) for design, judging from both aspects of accuracy of the values calculated and the range of application. However, considering that the number of experimental data for the large-sized beams with low reinforcement ratio is scarce, it would be suitable for the time being to take larger value of the member factor (for instance, around $\gamma_b = 1.3$).

ACKNOWLEDGMENTS

The authors wish to express their gratitude to Prof. Jun Yamazaki, Faculty of Engineering, Tokyo Metropolitan University, Prof. Takeshi Higai, Faculty of Engineering, Yamanashi University and Mr. Toshiyuki Shioya, The Institute of Technology of Shimizu Construction Co., Ltd. who gave them valuable advice in the course of this study as well as the members in The Technical Research Institute of Maeda Construction Co., Ltd. who cooperated them in the performance of the experiments.

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